Polarimetric speckle noise effects in quantitative physical parameters retrieval

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Abstract: Quantitative estimation of physical parameters of the Earth’s surface is currently an active area of research. Multidimensional synthetic aperture radar (SAR) systems appear in this framework as a tool capable of performing this task on a global scale and independently of the weather conditions. In particular, polarimetric SAR data have been successfully employed, in combination with different electromagnetic inversion algorithms, to extract physical information from measured data. Nevertheless, SAR data are affected by speckle noise. The influence of speckle noise and its filtering in the quantitative estimation of physical parameters is analysed. To perform this study, a novel model for speckle noise in multidimensional SAR data is considered. Finally, the effect on the quantitative estimation of surface parameters is investigated.

1 Introduction

Until now, an abundant number of studies and publications have corroborated the validity of multidimensional synthetic aperture radar (SAR) imagery on the retrieval of physical information from the Earth’s surface. The sensitivity of microwaves to the geometric and material properties of natural objects makes the quantitative information extraction possible. The foundations of quantitative remote sensing have to be built to exploit the correlation structure that characterises the multidimensional recorded data. In this field, SAR polarimetry has demonstrated its presence and is playing an important role.

Polarimetric SAR (PolSAR) systems work on the basis of transmitting and receiving electromagnetic waves in a pair of orthogonal polarisation states, which allow exploiting the advantages of SAR technology in a polarimetric framework. SAR systems allow sensing of the Earth’s surface independent of the weather conditions and the day/night cycle. These systems have the capacity to retrieve such information with a high spatial resolution. In the dimension perpendicular to the sensor’s movement, that is, range, a high spatial resolution is achieved by means of pulse-compression techniques. Nevertheless, in the azimuth dimension, which is perpendicular to range, the high spatial resolution is achieved by means of a coherent recording and processing of the scattered electromagnetic waves. Despite this inherently coherent nature, SAR systems are also the origin of speckle noise, which can be considered as one of the most important problems for SAR imagery. Speckle noise is a true electromagnetic measurement, but the spatial resolution at which SAR systems operate means that it has to be considered as a signal-dependent noise term. Consequently, speckle must be eliminated in order to retrieve the useful information contained within the data [1–3].

In one-dimensional SAR systems, speckle noise has been largely studied and its effects on the data have been clearly determined. For this type of configuration, speckle is characterised by a multiplicative noise nature [2–4]. However, the problem of speckle noise in multidimensional SAR systems cannot be considered completely solved, as the one-dimensional multiplicative speckle noise model cannot be extended to multidimensional systems, except in those situations in which the data channels are uncorrelated. Nevertheless, it has been recently demonstrated that for multidimensional SAR systems, speckle is because of the combination of multiplicative and additive noise sources [5]. This combination depends, precisely, on the correlation structure characterising the multidimensional data. Knowledge of this correlation structure makes it possible to optimise speckle noise reduction, which cannot be performed arbitrarily. It addition, multidimensional speckle noise filtering is constrained by additional necessities as, for instance, the maintenance of spatial resolution and spatial details or the estimation of data without a bias [6]. Finally, it is important to determine which are the effects of multidimensional speckle noise on the data estimation process, especially when quantitative physical-parameter inversion is addressed.

The capacity of PolSAR data to retrieve quantitative physical parameters has been already demonstrated in the literature. Different studies have illustrated the capacity of PolSAR systems to perform, for instance, unsupervised terrain classification [7–9] or natural hazards management [10, 11]. Another important application of SAR polarimetry is the estimation of surface parameters, as it allows the retrieval of surface roughness (rms height) and volumetric soil moisture ($m$, %) [12–14]. Basically, all these applications are based on the eigendecomposition of the
scattering matrices that contain second order statistics and completely characterise PolSAR data, that is, the coherency and covariance matrices, represented by \( T \) and \( C \), respectively [15—17]. As a result, it is also important to analyse which are the effects of speckle noise over this eigendecomposition, as the final goal is to determine the impact of speckle noise over the retrieved physical parameters.

The aim of this article is to establish a bridge between the theory of polarimetric speckle noise and the quantitative estimation of physical parameters from PolSAR data. In particular, this study focuses on the quantitative retrieval of surface parameters. The following section starts with a brief theoretical introduction concerning PolSAR data description and characterisation.

2 SAR polarimetry

A PolSAR system measures the \( 2 \times 2 \) complex scattering matrix, denoted by \( S \), for every data sample. This matrix relates the scattered electromagnetic field with the incident field transmitted by the SAR system. For deterministic or point scatterers, \( S \) contains all the necessary information to characterise the target under study [16]. Nevertheless, the main focus of attention for PolSAR systems is description of the status and the changes in natural targets. For example, vegetation cover (forest or agricultural fields), ice/snow and bare surfaces are bio-/geo-physical parameters derived from distributed objects. For these type of scatterers, the \( S \) matrix turns into a multidimensional random variable, which cannot be employed to characterise those scatterers [18]. In order to overcome this limitation, and with the aim of exploiting all the information that PolSAR data can offer, \( S \) is vectorised in order to form the so-called target vectors [16—19]. If the scattering matrix is vectorised by means of the lexicographic basis for \( 2 \times 2 \) matrices, the following target vector is derived

\[
k = \left[ S_{hh}, \sqrt{2} S_{hv}, S_{vv} \right]^T
\]

where \( T \) indicates transpose. In (1), the first and the second sub-indices indicate the polarisation for the received and the transmitted waves, respectively, where \( h \) stands for the horizontal and \( v \) for vertical polarisations. The vector (1) has been derived in the so-called backscattering alignment convention [1], which justifies the term \( \sqrt{2} \) in its second element in order to keep the total power constant.

Those scatterers for which the scattering matrix \( S \) has a random nature are called distributed or random targets. Under the Gaussian scattering assumption for the scattered fields from homogeneous targets, (1) is characterised by a zero-mean, complex and multidimensional Gaussian distribution [20, 21]

\[
p_k(k) = \frac{1}{\pi^n|C|} \exp(-k^H C^{-1} k)
\]

where \( H \) is the complex conjugate transpose. As can be deduced from (2), the average of \( k \) has no information, as it equals zero. Consequently, second-order moments must be considered to extract the useful information. As can also be observed in (2), this distribution is completely determined by the Hermitian, positive, semi-definite covariance matrix \( C \), which presents the following definition

\[
C = E(kk^H) = \begin{bmatrix} E(|S_{hh}|^2) & \sqrt{2} E(|S_{hh}S_{hv}|) & E(|S_{hh}S_{vv}|) \\ \sqrt{2} E(|S_{hv}S_{hh}|) & 2E(|S_{hv}|^2) & \sqrt{2} E(|S_{hv}S_{vv}|) \\ E(|S_{vv}S_{hh}|) & \sqrt{2} E(|S_{vv}S_{hv}|) & E(|S_{vv}|^2) \end{bmatrix}
\] (3)

The covariance matrix \( C \) is able to completely characterise the data associated with distributed scatterers. The matrix \( C \) is the most convenient way to analyse data from a statistical point of view, because it is inherent to the data distribution and, additionally, all its entries consist of the Hermitian product of the elements of the vector \( k \). The SAR data statistics can also be described by two additional formulations that are equivalent to the one based on \( C \). When the interest is on the physics associated with the scattering mechanism, the equivalent coherency matrix \( T \) is the better choice [16]. The \( T \) matrix is generated from a vectorisation of \( S \) in the so-called Pauli basis, resulting in the vector \( k_p \), which entries can be clearly associated to physical-scattering mechanisms. The \( T \) matrix is defined as

\[
T = E(k_p k_p^H) = \begin{bmatrix} E(|S_{hh} + S_{vv}|^2) & E(|S_{hh} + S_{vv}|S_{hv}) & E(|S_{hh} + S_{vv}|S_{vv}) \\ E(|S_{hh} - S_{vv}|S_{hv}) & 2E(|S_{hh} - S_{vv}|) & E(|S_{hh} - S_{vv}|S_{vv}) \\ 2E(|S_{hv}|^2) & 2E(|S_{hv}S_{hv}|) & 4E(|S_{hv}|^2) \end{bmatrix}
\]

The second equivalent matrix representation for the data statistics is called the Mueller matrix, denoted by \( M \). The advantage of this representation is that all the entries of the matrix \( M \) are real.

In order to extract the information contained in \( C \), this matrix needs to be estimated from the data \( k \). By assuming statistical ergodicity and homogeneity, the expectation operator in (3) can be substituted by a spatial averaging that allows the definition of the so-called sample covariance matrix

\[
Z_n = \frac{1}{n} \sum_{i=1}^n k_i k_i^H
\]

where \( n \) denotes the number of samples that have been employed to estimate \( C \). Indeed, the sample covariance matrix consists of the maximum likelihood estimator of \( C \) [22]. Considering the Jacobian associated with the change of variables given by (5) into (2), one can demonstrate that the sample covariance matrix is characterised by the Wishart distribution [20—23]

\[
p_{Z_n}(Z_n) = \frac{m^n|Z_n|^{n-m}}{|C|^m \Gamma(n)} \text{etr}(-nC^{-1}Z_n)
\]

where \( \text{etr}(X) \) is the exponential of the trace of the matrix \( X \), and the multivariate gamma function is defined as follows

\[
\Gamma_m(n) = \pi^{mn/2} \prod_{i=1}^m \Gamma(n - 1 + i)
\] (7)

The value of every data sample \( i \) consists of the target vector \( k_i \), see (1). Thus, for the same sample, the one-look sample
covariance matrix can be defined as
\[ Z = k_k^H \] (8)
that is, \( n = 1 \) in (5). Consequently, the covariance matrix \( C \) has to be estimated from (8). This estimation process is also, in fact, the speckle noise-filtering process. As has been shown, a first possibility to reduce speckle, that is, to estimate \( C \), is to employ the multilook approach presented in (5). The main drawback of this approach is that speckle-noise reduction is obtained at the expense of spatial resolution and the loss of spatial details. Thus, new alternatives to filter speckle noise need to be investigated.

2.1 Multidimensional speckle-noise model

A second option to estimate \( C \) from the data is to exploit the Wishart data PDF given in (6). One way to perform this analysis is to embed the data distribution into a noise model, that is, a function that allows identifying of the useful information to be retrieved, the noise sources, and how these terms interact. This idea can be expressed, for the particular case of (6), as follows
\[ Z = f(C, n_1, n_2, \ldots, n_p) \] (9)
where \( n_i \) for \( i = 1, 2, \ldots, p \) represent the different noise sources and \( C \) is the parameter to recover.

An interesting point in order to derive the noise model for the sample covariance matrix \( Z \) is to notice that all its elements consist of the Hermitian product of the different entries of the target vector \( k \). Consequently, the first step consists of deriving the speckle-noise model for the Hermitian product of two correlated SAR images
\[ S_jS^*_j, \quad i, j = hh, hv, vv \] (10)
The main parameter that characterises (10) is the complex correlation coefficient, defined as
\[ \rho_j = \frac{E[S_jS^*_j]}{\sqrt{E[|S_j|^2]E[|S|^2]}}. \] (11)
in which amplitude \( |\rho_j| \) is called coherence and \( \phi_j \) refers to the average phase difference. The mean power in the two channels is denoted by \( \psi = \sqrt{E[|S_j|^2]E[|S|^2]} \) for \( i, j = hh, hv, vv \). Finally, the complete noise model for \( Z \) shall be obtained by extending the speckle-noise model of (10) to all the matrix entries.

The Hermitian product in (10) can be decomposed as follows
\[ S_jS^*_j = |S_j||S^*_j| \exp(j(\theta_j - \phi_j)) = z_j \exp(j\phi_j), \] (12)
where \( z \) is its amplitude and \( \phi \) denotes the measured phase difference. In order to derive the speckle-noise model for (12), the complex measured phase difference, \( \exp(j\phi) \), is first considered. Then, the amplitude information given by \( z \) is included to derive the final speckle-noise model.

On the basis of the Wishart distribution, the measured phase difference \( \phi \) can be described by the following additive-noise model [24]
\[ \phi = \phi_x + v \] (13)
where \( \phi_x \) denotes the true phase difference and \( v \) is a zero-mean noise term depending on \( |\rho| \) and independent from \( \phi_x \).

If the additive-noise model presented in (13) is now introduced into the complex phase term, \( \exp(j\phi) \), this term can be written as follows [25]
\[ \exp(j\phi) = N_x \exp(j\phi_x) + (v_x + jv_y) \] (14)
where the parameter \( N_x \) has the expression
\[ N_x = \frac{\pi}{4}|\rho|^2 F\left(\frac{1}{2}, \frac{1}{2}; 2; |\rho|^2\right) \] (15)
This parameter, as observed in Fig. 1, contains the same information as the coherence value \( |\rho| \). The components of the complex additive-noise term \( v_x + jv_y \) in (14) are characterised by the following statistical moments
\[ E[v_x] = E[v_y] = 0 \quad \text{var}[v_x] = \text{var}[v_y] \approx \frac{1}{2}(1 - |\rho|^2)^{0.685} \]
\[ E[v_xv_y] = 0 \] (16)
where the exponential of the variance is obtained from an approximation process allowing the derivation of such a simple expression [25]. As one can deduce from (14), the complex phase term \( \exp(j\phi_x) \) can be divided into two different terms. The first one, given by \( N_x \exp(j\phi_x) \), contains basically the information referred to as the complex correlation coefficient (11). This information is corrupted by the complex noise term given by \( v_x + jv_y \), in which, as one can conclude from (16), the lower the coherence value, the more significant its effects.

The speckle-noise model for the Hermitian product of two correlated SAR images is derived by introducing the noise model for the complex measured phase difference (14) into (12), giving, as a result, the next expressions for its real and imaginary parts
\[ \Re[z \exp(j\phi)] = N_x z \cos(\phi_x) + z v_1' \cos(\phi_x) - z v_2' \sin(\phi_x) \] (17)
\[ \Im[z \exp(j\phi)] = N_x z \sin(\phi_x) + z v_1' \sin(\phi_x) + z v_2' \cos(\phi_x) \] (18)
The terms \( v_1' \) and \( v_2' \) represent two random signal terms that can be considered as noise sources. These are defined as
\[ v_1 = \cos(v) = N_x + v_1' \] (19)
\[ v_2 = \cos(v) = N_x + v_2' \] (20)
where \( N_x \) is equal to zero by construction. As observed in (17) and (18), the real and imaginary parts of the Hermitian product of two correlated SAR images can be separated into three additive terms. The statistical analysis of these terms is not a straightforward process because of the existing correlation between the amplitude \( z \) and the phase \( \phi \), which introduces a correlation among the additive terms. The interested reader is directed to the reference [5] for a complete in-depth analysis of (17) and (18). After this

![Fig. 1 Parameter \( N_x \) as a function of the coherence value \( |\rho| \)]
analysis, it has been demonstrated that the complex Hermitian product of a couple of SAR images is described by the following speckle-noise model

$$S_iS_{j'} = \psi z_n n_n C_i \exp(j\phi_s)$$

Multiplicative term
$$+ \psi(|\rho| - N)z_o \exp(j\phi_s) + \psi(n_{ar} + jn_{ai})$$

(21)

where $E[z] = \psi z_o$ has been considered

$$z_n = \frac{\pi}{4} F_1 \left( -\frac{1}{2}, -\frac{1}{2}; |\rho|^2 \right)$$

(22)

and $i, j = hh, hv, vv$. The first noise source in (21) is $n_m$, which consists of a multiplicative noise source characterised by the moments

$$E[n_m] = 1 \quad \text{var}[n_m] = 1$$

(23)

which only corrupts the amplitude of the Hermitian product $S_iS_{j'}$, for $i, j = hh, hv, vv$. On the other hand, the second noise source is because of the complex additive term $n_{ar} + jn_{ai}$, which is characterised by the statistical moments

$$E[n_{ar}] = E[n_{ai}] = 0$$

$$\text{var}[n_{ar}] = \text{var}[n_{ai}] = \frac{1}{2}(1 - |\rho|^2)^{1,32}$$

(24)

Again, the exponential of the variance value results from an approximation process that results in such a simple expression. Because of the complex nature of this speckle-noise term, it introduces noise, both in the amplitude and in the phase of $S_iS_{j'}$ for $i, j = hh, hv, vv$.

As deduced from (21), the final speckle noise in the Hermitian product of two SAR images is the result of the combination of two noise sources: the homogeneous multiplicative speckle-noise source $n_m$ and the non-homogeneous, complex, additive noise source $n_{ar} + jn_{ai}$. The combination of these noise sources is determined by the complex correlation coefficient (11), which characterises the Hermitian product itself.

Finally, let the coherence $|\rho_{ij}|$ be equal to one in (21). First, from (24), one can observe that the additive speckle-noise source $n_{ar} + jn_{ai}$ disappears as the mean and variance values of its real and imaginary parts are equal to zero. Thus, (21) reduces to

$$S_iS_{j'} = \psi n_m \exp(j\phi_s), \quad i, j = hh, hv, vv$$

(25)

In addition, if one considers $S_i = S_j$, which implies that $\phi_s = 0$, (25) simplifies to

$$S_iS_{i'} = |S|^2 = \psi n_m, \quad i = hh, hv, vv$$

(26)

The simplification in (26) corresponds to the multiplicative speckle-noise model for the SAR images intensity. Consequently, the speckle-noise model presented in (21) can be understood as a generalisation of the multiplicative speckle-noise model for the SAR images intensities.

Consider now, on the contrary, that the correlation between the couple of SAR images in (21) is null, that is, $|\rho| = 0$. In this situation, the first and second additive terms of (21) disappear and, as a consequence, the Hermitian product reduces to

$$S_iS_{j'} = \psi (n_{ar} + jn_{ai})$$

(27)

which means that the Hermitian product contains only the additive speckle-noise term. From the extreme cases presented in (25) and (27), it is possible to deduce that for the Hermitian product $S_iS_{j'}$, for $i, j = hh, hv, vv$, the speckle noise is characterised by a multiplicative noise nature for high coherences, whereas it is characterised by an additive noise nature for lower ones. For any other intermediate value of coherence, the speckle noise results from the combination of both sources, as detailed by (21). Finally, it must be also considered that the multiplicative speckle-noise component in (21) is modulated by the phase term $\exp(j\phi_s)$. This dependence implies that the multiplicative speckle-noise term affects differently the real and imaginary parts of the Hermitian product $S_iS_{j'}$, for $i, j = hh, hv, vv$.

### 2.2 Speckle noise reduction

As has been already introduced at the beginning of Section 2, the first alternative to retrieve the covariance matrix $C$ from measured data is to use the multitlook approach presented in (5), but at the expense of spatial resolution. As the high spatial resolution with which data is retrieved is one of the main properties of SAR systems, it is necessary to minimise the loss of spatial resolution in the speckle-filtering process. As a consequence, alternatives to the indiscriminate filtering of the multitlook approach have to be investigated.

In the last 25 years, different alternatives for speckle-noise filtering have been presented in the literature. These alternatives can be divided into two main groups, depending on the final purpose. The first group encompasses all those techniques that consider PolSAR data as a sort of diversity, combining all the data channels to derive a final speckle-free image. As a result, these techniques do not introduce losses in spatial resolution, but the polarimetric information is completely lost as a consequence of the combination of the different data channels. The second group contains those techniques that maintain polarimetric information. All these techniques are based on a spatial processing of the different SAR images affecting, hence, their spatial properties. It can be concluded that polarimetric speckle filter presents a compromise between the maintenance of the polarimetric information and the maintenance of the spatial properties. In order to perform quantitative remote sensing, the preservation of the polarimetric information is of crucial importance. Therefore only those filtering techniques that keep the polarimetric information and, at the same time, do not reduce the spatial resolution, can be considered.

Lee et al. [6] proposed the principles over which the polarimetric speckle filters must be developed:

- Every term of the sample covariance matrix $Z$ must be filtered independently in the spatial domain to avoid the introduction of crosstalk among them. Filtering algorithms, exploiting the degree of statistical independence between the elements of the covariance matrix, will introduce crosstalk.
- All the terms of $Z$ must be filtered by the same amount in a manner similar to multitook filtering.
- The filtering technique must adapt to the spatial properties of the data in order to avoid the loss of spatial resolution and spatial details.

On the basis of these three principles, Lee et al. [6] proposed a polarimetric speckle-filtering technique. This approach is based on the extension of the local statistics filter, applied within the edge-aligned windows in order to maintain spatial resolution. The foundations of the local statistics filter tie on the assumption that the multiplicative speckle-noise model is able to describe speckle noise in all
the entries of $Z$. This assumption is valid only for the diagonal elements of $Z$, as the latter consist of the intensities of the different SAR images. The assumption is not valid for the off-diagonal elements of $Z$ as, for these elements, speckle noise results from the combination of multiplicative and additive noise sources, as demonstrated in Section 2.1.

The availability of the complete speckle-noise model for the Hermitian product of two SAR images, which is able to characterise all the elements of the matrix $Z$, suggests a variation of the polarimetric speckle-filtering principles presented in the previous paragraph. The first of these principles states that polarimetric filtering must be done independently of the statistical dependence of the polarimetric channels. But, as demonstrated previously, for the off-diagonal entries of $Z$, speckle noise depends on it. Hence, the coherence must be estimated to optimise speckle-noise reduction for these elements. In relation with this variation, the second principle should be also revised, as speckle noise for the off-diagonal elements of $Z$ depends on the value of the complex correlation coefficient, and these elements must be filtered according to it. Thus, the condition that they should be filtered by the same amount could be relaxed. However, special care must be exercised to preserve polarimetric information.

On the basis of the novel, multidimensional speckle-noise model presented in Section 2.1 and considering what has been presented previously, a novel, polarimetric speckle-filtering approach is presented here. According to (21), the strategy of this filtering approach is to filter speckle noise in a two-step process. The first step will reduce the additive speckle noise term for each entry of $Z$, whereas the second step will reduce the multiplicative speckle noise component. A PolSAR processing scheme of this filtering approach is sketched in Fig. 2.

The first stage of this filtering approach consists of the estimation of the correlation structure which characterises the data, that is, $\rho_{ij}$ for $i,j=hh, hv, vv$. The complex correlation coefficients are estimated in the frame of the discrete wavelet transform (DWT) with the approach presented by López-Martínez [26]. This approach employs the phase wavelet transform (DWT) with the approach presented by et al. [5]. Finally, the multiplicative terms obtained from the splitting step are filtered by the approach proposed by Lee et al. [6], because this approach is optimal for a multiplicative noise component. The last step of the algorithm consists of a correction for a small bias originated by the fact that when the additive term of (21) is eliminated a portion of the useful signal is also eliminated. This bias does not represent a problem as it is equal to

$$B_{ij} = \frac{\rho_{ij}}{N_{c} \pi/4 |\rho_{ij}|^2 F_i \left(-\frac{1}{2}, -\frac{1}{2}; 1; |\rho_{ij}|^2\right)},$$

that is, it depends only on the coefficient $\rho_{ij}$ for $i,j=hh, hv, vv$, which has been previously estimated.

### 2.3 Eigendecomposition

The eigendecomposition of the covariance or coherency matrices, $C$ and $T$, respectively, has been demonstrated as a useful tool to perform quantitative physical parameter retrieval. Originally, the eigendecomposition is applied to the $T$ matrix because of its closeness with the properties of the physical scattering process [16]. Nevertheless, the description of the speckle noise for this matrix is a difficult task because these elements do not present the same mathematical nature. This drawback can be overcome by considering the eigendecomposition of $C$ instead of $T$, as observed in Section 2.1. Finally, as the covariance matrix $C$ and the coherency matrix $T$ are related by a unitary similarity transformation, they present the same eigenvalues, but not the same eigenvectors [16]. From a physical point of view, the eigenvectors can be considered as a set of orthogonal scattering mechanisms, whereas the corresponding eigenvalue is interpreted as the scatterer power associated with the eigenvector.

Given $Z_n$, the sample covariance matrix, its eigendecomposition is written as follows

$$\Xi = Q^\dagger Z_n Q$$

where

$$\Xi = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

In (29), the $3\times3$, complex, unitary matrix $Q$ contains the eigenvectors of $Z_n$ as its columns and in (30), $\lambda_i$ for $i = 1, 2, \ldots, 3$ consists of the corresponding eigenvalues.

In addition to the primary parameters of the eigendecomposition (29), secondary statistical polarimetric parameters have been derived [16]. The objective of these parameters is to allow an interpretation of and, thus, a

Fig. 2  Scheme of the proposed PolSAR speckle-noise filter
better understanding of the physical scattering process. From the eigenvalues, the entropy $H$ and the anisotropy $A$ are considered, which are defined as

$$H = -\sum_{i=1}^{3} p_i \log(p_i), \quad p_i = \frac{\lambda_i}{\sum_{i=1}^{3} \lambda_i}$$

$$A = \frac{\lambda_2 - \lambda_3}{\lambda_1 + \lambda_3}$$

The entropy provides information about the depolarising properties of the scatterer. For $H$ close to zero, the scatterer is highly polarised, and $Z_0$ can be considered of rank 1, whereas values close to one indicate a depolarising mechanism.

The last parameter of analysis is the average alpha angle, denoted here as $\tilde{\alpha}$. This parameter is obtained from the eigenvectors of the matrix $T$. These eigenvectors take the form

$$e_i = \begin{bmatrix} \cos \alpha_i e^{i\phi_i} & \sin \alpha_i \cos \beta_i e^{i\phi_i} & \sin \alpha_i \cos \beta_i e^{i\phi_i} \end{bmatrix}^T, \quad i = 1, 2, 3$$

This arbitrary three-dimensional vector has five degrees of freedom, and can therefore be written in terms of five angles, whereas only the alpha angle will be considered in (34). The $\alpha_i$ parameter represents an internal degree of freedom of the scatterer. It is continuous within a range of $0^\circ < \alpha < 90^\circ$ and is associated with the ‘type’ of scattering mechanism.

From (33), considering also (31), $\tilde{\alpha}$ is defined as

$$\tilde{\alpha} = \frac{1}{3} \sum_{i=1}^{3} p_i \alpha_i$$

The eigenvector decomposition of rank 3 of the coherency matrix leads to three eigenvalues and three eigenvectors. Hence, 15 angles could be extracted that build up the general unitary matrix. However, the coherency matrix contains only eight independent parameters. Consequently, the 15 angles are not independent from each other. The interpretation of each individual $\alpha$ and $\beta$ angle is problematic. To overcome this, the interpretation is performed not in terms of a dominant scattering mechanism, but in terms of a mean value.

### 3 Surface parameters retrieval

The main parameters employed to characterise surface properties are its geometrical representation as surface roughness and its material representation as a dielectric constant. These two parameters are coupled in the measured backscattering intensities; so a scattering model has to be introduced in order to decouple their contributions. Furthermore, an extension of the observation vector is required for an estimation of both roughness and dielectric properties. The most promising (and mostly used) method for this is the introduction of polarisation diversity. Hence, the observation vector formed by the backscattering intensities is extended by considering the correlation structure of these intensities. In the case of PolSAR data, this extended vector is directly the $C$ or the $T$ matrices.

As mentioned in the Introduction, a signal-formulation based on the coherency matrix $T$ is more convenient to relate the multidimensional SAR data with the physical properties of the illuminated object. The recently developed model, the extended-Bragg model (X-Bragg) [16], is used in the following to provide some qualitative insights on the influence of speckle-noise effects on surface roughness and dielectric constant estimation. The model is an extension of the small perturbation model and assumes reflection symmetry surfaces, where the mean normal to the surface vector defines the axis of symmetry. This model accounts for cross-polarised backscattering as well as depolarisation effects and is modelled with the $T$ matrix as follows

$$T = \begin{bmatrix} C_1 & C_2 \sin(2\beta_1) & 0 \\ C_2 \sin(2\beta_1) & C_3(1 + \sin(4\beta_1)) & 0 \\ 0 & 0 & C_3(1 - \sin(4\beta_1)) \end{bmatrix}$$

where the coefficients $C_1$, $C_2$ and $C_3$ describe the Bragg component of the surface, and are given by

$$C_1 = |R_S + R_P|^2, \quad C_2 = (R_S + R_P)(R_S^* - R_P^*), \quad C_3 = \frac{1}{2}|R_S - R_P|^2$$

In (36), the following definitions apply

$$R_S = \frac{\cos \theta - \sqrt{e - \sin^2 \theta}}{\cos \theta + \sqrt{e - \sin^2 \theta}}$$

$$R_P = \frac{(e - 1)(\sin^2 \theta - \alpha(1 + \sin^2 \theta))}{(e \cos \theta + \sqrt{e - \sin^2 \theta})^2}$$

where $R_S$ and $R_P$ are the Bragg-scattering coefficients perpendicular and parallel to the incidence angle, respectively.

It has been demonstrated by Hajnsek et al. [13] that the surface roughness and the soil moisture can be estimated via the eigendecomposition of $T$, by considering the model presented by (35). In the first step, the surface roughness $ks$ is obtained as

$$ks = 1 - A$$

In the second step, the computed $H$ and $A$ values are used to estimate the dielectric constant and the soil moisture. This estimation is obtained by comparing the measured $H$ and $A$ values by those given by the model in (35). The matrices $C$ and $T$ present the same eigenvalues, as they are related by a unitary similarity transformation. This relation makes it possible to establish a connection between the speckle noise sources identified in the multidimensional speckle-noise model given in (21) and the surface-scattering model presented in (35).

### 4 Experimental results

This section focuses primarily on the determination and the study of the speckle-noise effects on the estimation of the secondary statistical polarimetric parameters and, especially, the information derived from the eigen-decomposition of the matrix $C$, as this matrix allows a better characterisation of speckle noise as detailed in Section 2. On the one hand, the following analysis will concentrate on determining the consequences of number of looks $n$ of the multilook approach, see (5), into the signal-estimation process. This point is of special importance, as it allows us to determine the effects of the filtering strength. On the other hand, the consequences of the multiplicative and additive noise sources presented in Section 2.2 are studied and compared with the effects of $n$. Finally, the
effects of speckle filtering on quantitative surface parameters estimation is considered, concentrating in the surface parameters retrieval model presented in Section 3.

4.1 Experimental polarimetric SAR data

To study the different points which have been presented in Section 2, three different PolSAR datasets have been employed.

The first dataset was acquired by the AIRSAR system, operated by the Jet Propulsion Laboratory (JPL), at L-band over San Francisco Bay. These data was originally 4-look processed by averaging the Stokes matrix. The second dataset was taken by the E-SAR airborne SAR system, operated by the German Aerospace Center (DLR) in L-band, over a test site close to the Elbe River in the northeastern Germany. The third dataset, acquired also by the DLR’s E-SAR system over the Alling test site is close to the DLR site. All data have been acquired at L-band.

4.2 Effects of the number of looks

As shown in (5), the main parameter in the multilook-filtering approach, without considering at this point the spatial resolution losses, is the number of averaged samples or looks denoted by \( n \). The fact that multilook techniques are linear, as they consist basically of the sum of data samples, permits us to derive the statistics of the filtered data. This would not be possible for other more complex and nonlinear techniques. Consequently, in this context, the study of the effects associated with the number of looks is important, as one can determine the effects of the filtering strength. This study is illustrated over the San Francisco PolSAR dataset presented in the previous section.

The sample covariance matrices have been derived in two cases in which the multilook approach (5) has been applied by means of 5 × 5 and 9 × 9 pixel averaging windows. The eigendecomposition presented in Section 2.3 has been applied to both cases. The results are given in Figs. 3 and 4 for the entropy (H), in Fig. 5 for the anisotropy (\( A \)) and in Fig. 6 for the average alpha angle.

The first point that one can notice in the results of entropy, anisotropy and the average alpha angle is the expected blurring because of the loss of resolution from the multilook filtering. A more careful analysis of these results reveals additional differences between the processed images. The entropy values in Fig. 4 present different values between the images. This effect can be clearly observed for entropies greater that 0.95, which are shown in white. As noticed, in average, the entropy values are greater for the 9 × 9 image than those corresponding to the 5 × 5 result. A similar behaviour can be observed for the anisotropy images presented in Fig. 5. In this case, comparing both results, one can notice that the anisotropy values corresponding to the 9 × 9 averaging window are clearly lower than the results corresponding to the 5 × 5 pixel window. Finally, these characteristics are not observed for the average alpha angle presented in Fig. 6. The clearest difference between these three parameters is that whereas the entropy and the anisotropy are derived from the eigenvalues of the eigendecomposition of \( Z \), the average alpha angle is obtained from the eigenvectors of the decomposition. However, the eigenvalues seem to be more affected by the speckle noise than the eigenvectors of the same decomposition.

4.3 Inversion results

This section considers multidimensional speckle-noise reduction from a different point of view, focussing on the effects of the novel additive speckle-noise source.
introduced in Section 2.2. Finally, the effects of this particular noise source are considered in the quantitative estimation of surface parameters. In order to determine the effects of the additive speckle-noise source, two alternatives to filter multidimensional speckle noise are considered: the approach presented by Lee et al. [6] and labelled mult_speckle and the modification of this filter, presented in Section 2.2, labelled mult_add_speckle. These two filtering techniques have been applied over the PolSAR datasets of Elbe and Alling.

After filtering both PolSAR datasets, the entropy ($H$) and average alpha angle ($\bar{\alpha}$) values have been obtained. In the two datasets, four areas of interest have been selected, which average $H$, and $\bar{\alpha}$ values are presented in Fig. 7. As one can observe, the entropy values derived from the data filtered with the mult_add_speckle approach are clearly higher than those obtained with the mult_speckle filter. These differences can be also noticed for the $\bar{\alpha}$ values, although to a minor degree. This type of behaviour has been also observed in Section 4.2 but, in that case, in terms of the number of looks. The increase in the number of looks to filter PolSAR data leads to a larger reduction of noise and, as observed in Section 4.2, also to an increase of the entropy value. Nevertheless, as concluded from Fig. 7, for a fixed number of looks, a reduction of the additive speckle noise component has the same effect as an increase in the number of looks, that is, a larger reduction of speckle noise and an increase of the entropy values. The reduction of the additive speckle noise can lead to a larger reduction of speckle noise, without affecting the spatial resolution and details of the image.

In Fig. 7, the differences between the entropy values derived with the two filtering approaches are larger in the Alling dataset than in the Elbe dataset. These results show that the increase of entropy because of a larger reduction of speckle noise depends on the entropy value itself, in such a way that the higher the entropy, the larger the differences. This observation suggests that low entropy data could be less affected by speckle noise and the filtering process than data characterised by a higher entropy. The additive speckle-noise component introduced in Section 4.2 can explain, at least in a qualitative way, this behaviour. PolSAR data characterised by low entropy normally present a high degree of correlation among the different data channels. As given by (21), these data should be mainly affected by the multiplicative speckle-noise component. Consequently, the reduction or not of this speckle-noise component does not affect the final results because of its low importance. This observation is corroborated in the following quantitative surface-parameter estimation.

In a pre-processing step, areas with entropy values larger than 0.45 and average alpha angles larger that 45° are not considered as surface scattering areas. This masking ensures that only surface scattering areas are selected for the inversion, where it must be also considered that SPM covers an even more restricted region. Consequently, the Alling dataset is not valid for estimating surface parameters, especially for the data filtered with the mult_add_speckle approach. Nevertheless, Elbe data can be employed to derive the physical parameters that characterise the surfaces. In this case, ground measurements of the soil moisture ($m_v$) and the surface roughness ($k_s$) are available, which are presented in Table 1. The ground measurements are compared with the quantitative estimation obtained by the X-Bragg model presented in Section 3. This comparison is presented in Fig. 8, where Fig. 8a corresponds to the soil moisture and Fig. 8b to the surface roughness. As can be deduced from these results, the type of speckle-noise filtering does not affect the inversion results. This can be explained: as data present low entropy values, the additive speckle-noise component can be considered, in this case as negligible. Thus, it can be concluded that the quantitative retrieval of physical parameters in low-entropy environments does not depend on the type of speckle filter employed to estimate the data. Nevertheless, this conclusion cannot be extended to situations in which data present medium or high entropy. In these cases, as observed in the results derived from the Alling dataset, the speckle noise and the process to filter it have clear effects on the final values of the different parameters from which the physical information is extracted. In this sense, more research work is necessary to establish final conclusions.

### Table 1: Ground measurements of the Elbe test site

<table>
<thead>
<tr>
<th>Field No.</th>
<th>Cultivation</th>
<th>$m_v$ (vol. %)</th>
<th>$k_s$ ($\lambda$ $\approx$ 23 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>harrowed</td>
<td>18.5</td>
<td>0.46</td>
</tr>
<tr>
<td>12</td>
<td>seed bed</td>
<td>19.5</td>
<td>0.57</td>
</tr>
<tr>
<td>13</td>
<td>coarse harrowed</td>
<td>21</td>
<td>0.75</td>
</tr>
<tr>
<td>16</td>
<td>ploughed</td>
<td>23</td>
<td>0.95</td>
</tr>
</tbody>
</table>
The main aim of this article is to study the effects of speckle noise and its filtering process in the quantitative estimation of physical parameters from PolSAR data. A complete speckle-noise model for the covariance matrix has been presented. As shown, for the diagonal elements of this matrix, speckle is completely described by a multiplicative noise model. However, in the case of the off-diagonal elements, speckle noise results from the combination of multiplicative and additive noise sources. The combination of these two noise sources is determined by the complex correlation coefficients that characterise every one of the elements. On the basis of this multidimensional speckle-noise model, a novel approach to filter speckle noise is proposed.

The study of the speckle-noise effects is divided into two parts. First, the effect of the averaged sample employed to filter the covariance matrix is considered. It is shown that the number of looks has a clear effect on the entropy, the anisotropy and the average alpha angle values. But, whereas the effects are clearly noticeable over the entropy and the anisotropy, the average alpha angle seems to be less sensitive to speckle noise or to its filtering process. This allows us to conclude that the eigenvectors of the covariance or coherency matrix are more robust parameters to the speckle noise than the eigenvalues of the same transformation. Second, the additive noise component because of speckle in the off-diagonal matrix entries of the covariance matrix is studied. As a result of its filtering, it can be concluded that the reduction of this speckle-noise component leads to a higher noise reduction, but without the loss of spatial details, as the number of averaged samples remains unchanged.

Finally, quantitative estimation of surface parameters is studied. As observed, the effect of speckle noise over the data depends on the entropy of the data itself, in such a way that the higher the entropy, the more severe the effects of speckle noise. Consequently, since surface-scattering data are characterised by low entropies, one can conclude that quantitative estimation of surface parameters is robust in the sense that it almost does not depend on the speckle noise or its filtering process.

5 Conclusions

6 Acknowledgment

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7 References

Fig. 8 Estimated against measured parameters

a Volumetric soil moisture $m_v$

b Surface roughness $k_s$, for the different filtering processes (mult_add_speckle refers to the wavelet approach)
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