1 ABSTRACT

Speckle noise reduction in PolSAR data is still an open problem. The authors have recently proposed a PolSAR data speckle noise model able to determine the noise nature for all the elements of the covariance matrix, which describes the polarimetric properties of a distributed scatterer. The main consequence of this model is that speckle presents different nature for the different covariance matrix elements. Consequently, in order to derive an optimum filtering, they have to be processed according to the noise nature. This paper analyzes this approach by demonstrating that to process the covariance matrix elements in a different way does not damages polarimetric information.

2 INTRODUCTION

Speckle noise is one of the most important problems associated with SAR technology as it makes difficult to derive relevant information from measured data. Speckle noise is produced by the coherent nature of SAR systems since each one of the recorded echoes is the result of the complex addition of elementary echoes from each point scatterer within the resolution cell. As a result, speckle reduction techniques are valuable. The goal of these techniques is to reduce speckle noise effects without damaging the properties and features of the signal which wants to be recovered.

The speckle noise problem has been already solved for one-dimensional complex SAR imagery. In this situation, information can only be obtained from amplitude, the complex SAR images phase does not contain useful information [1]. For the amplitude (or intensity), speckle noise can be modelled as a unit mean, unit variance multiplicative random term, whereas the phase is modelled as an additive term uniformly distributed in the interval $[-\pi, \pi)$. This sort of distribution prevents to derive information from phase.

The availability of SAR systems able to provide multi-dimensional SAR data makes possible to increase the amount of information of a particular area under study. Since single SAR images are affected by speckle noise, multi-dimensional imagery will be also affected by it. The one-dimensional speckle noise model can not be considered for the multi-dimensional case as a result of the correlation between the different SAR data channels. SAR polarimetry (PolSAR) is the ideal vehicle to analyze the problem of multi-dimensional speckle.

In order to characterize non-deterministic scatterers from a polarimetric point of view, only second order statistical descriptors can be employed. Assuming a Gaussian scattering process, these are enough to completely characterize this type of scatterers [2]. Recently, the authors have proposed a new speckle noise model for PolSAR imagery under a covariance matrix formulation [3]. As demonstrated, the new noise model is able to identify the speckle noise nature for all the elements of this matrix. The nature of speckle noise depends on the data’s correlation structure, in such a way that additive speckle noise is dominant for low coherences, whereas a multiplicative speckle term is dominant for high coherences. The most important consequence of this new approach is that speckle noise presents a different nature for each covariance matrix element. From a point of view of speckle noise reduction, this fact means that the covariance matrix elements have to be processed according to the particular noise nature.

The purpose of this paper is to present new approaches for PolSAR speckle noise reduction based on the new PolSAR speckle noise model. These alternatives are based on processing the covariance matrix elements considering the particularities of speckle noise in each case. As it will be demonstrated, these filtering approaches does not damages polarimetric information.
3 PolSAR SPECKLE NOISE MODEL

As it has been already mentioned in the introduction, non-deterministic scatterers can be only characterized polarimetrically by means of second-order statistical descriptors. This statistical information can be expressed in different ways as for instance the Mueller matrix, the coherency matrix or the covariance matrix. From an statistical point of view, the later presents the advantage that its entries are the Hermitian product of the different polarimetric channels or SAR images.

The new speckle noise model is based on modelling speckle noise for PolSAR data under a covariance matrix formulation. Most of PolSAR systems measure polarimetric data in the linear polarization basis, denoted \{h, v\}. Polarimetric data channels are defined as \(S_{pq}\), where \(p\) and \(q\) represent the polarizations employed for reception and for transmission respectively. Considering this formulation, for a monostatic system, the covariance matrix is defined as

\[
[C] = \begin{bmatrix}
E\{S_{hh}S_{hh}^*\} & \sqrt{2}E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\
\sqrt{2}E\{S_{hv}S_{hh}^*\} & 2E\{S_{hv}S_{hv}^*\} & \sqrt{2}E\{S_{hv}S_{vv}^*\} \\
E\{S_{vv}S_{hh}^*\} & \sqrt{2}E\{S_{vv}S_{hv}^*\} & E\{S_{vv}S_{vv}^*\}
\end{bmatrix}. \tag{1}
\]

As one observes, this matrix is defined as an average information due to the expectation operator \(E\{\cdot\}\). The covariance matrix \([C]\) has to be estimated from measured data. In this case, the one-look sample covariance matrix, from which \([C]\) has to be estimated, is defined as

\[
[Z] = \begin{bmatrix}
S_{hh}S_{hh}^* & \sqrt{2}S_{hh}S_{hv}^* & S_{hh}S_{vv}^* \\
\sqrt{2}S_{hv}S_{hh}^* & 2S_{hv}S_{hv}^* & \sqrt{2}S_{hv}S_{vv}^* \\
S_{vv}S_{hh}^* & \sqrt{2}S_{vv}S_{hv}^* & S_{vv}S_{vv}^*
\end{bmatrix}. \tag{2}
\]

If one considers a Gaussian scattering process, \([Z]\) is described by a Wishart distribution [4]-[5], which depends on the covariance matrix \([C]\). Each one of the sample covariance matrix entries have the expression

\[
S_{pq}S_{rs}^* = ze^{j\phi_z} = \Re\{ze^{j\phi_z}\} + j\Im\{ze^{j\phi_z}\} \tag{3}
\]

The extension of the multiplicative speckle noise model for each polarimetric channel \(S_{pq}\), \(p, q \in \{h, v\}\) cannot be employed to model the Hermitian product given by Eq. 3, as it prevents to employ the information contained within the phase difference \(\phi_z\). This extension would be only possible when the \([C]\)-matrix is a diagonal matrix, that is, when the polarimetric data channels \(S_{pq}\), \(p, q \in h, v\) are uncorrelated.

Based on the additive noise model for the Hermitian product phase difference \(\phi_z\) [6], it has been recently demonstrated that the Hermitian product of complex SAR images is described by the speckle noise model [7]-[3]

\[
S_{pq}S_{rs} = \psi N c \tau_n n_m e^{j\phi_s} + \psi(|\rho| - N c \tau_n) e^{j\phi_r} + \psi(n_{ar} + jn_{ai}) \tag{4}
\]

where \(n_m, n_{ar}\) and \(n_{ai}\) represent the speckle noise terms and \(|\rho|\) denotes the degree coherence between the pair of complex SAR images. The first additive term of Eq. 4 contains the multiplicative speckle noise term, denoted by \(n_m\), which multiplies the term \(\psi N c \tau_n\). The parameter \(N c\), whose value is

\[
N c = \frac{\pi}{4}|\rho|_2 F_1(1/2, 1/2; 1; |\rho|^2) \tag{5}
\]

has a central role in the noise model given by Eq. 4 as it makes possible the definition of the multiplicative speckle noise \(n_m\). \(\psi\) refers to the average power of the two SAR images, defined as \(\psi = (|S_{pq}|^2|S_{rs}|^2)\) for \(p, q, r\) and \(s \in \{h, v\}\), and \(\tau_n\) refers to the average Hermitian product normalized intensity value

\[
\tau_n = \frac{\pi}{4} 2 F_1(-1/2, -1/2; 1; |\rho|^2). \tag{6}
\]

\(2 F_1(a, b; c; x)\) represents the Gauss hypergeometric function. The second additive term of Eq. 4 is a deterministic term since the multiplicative speckle term \(n_m\) does not affect the complete signal to recover, but only the quantity \(\psi N c \tau_n\). Finally, the term \(\psi(n_{ar} + jn_{ai})\) consist on two zero-mean processes, \(n_{ar}\) and \(n_{ai}\), whose variances equal

\[
\text{var}\{n_{ar}\} = \text{var}\{n_{ai}\} = \frac{1}{2}(1 - |\rho|^2)^{1.32} \tag{7}
\]

These noise terms affect both, the amplitude and the phase of the Hermitian product. In this case, these processes represent additive speckle noise terms in the real and imaginary parts of the Hermitian product of a pair of complex SAR images.
As it can be deduced from the model given by Eq. 4, the speckle noise presents two different mechanisms: a multiplicative given by $n_m$, and an additive given by $n_{ar}$ and $n_{ai}$. The multiplicative speckle term is dominant for high coherences. But, considering Eq. 7, it can be demonstrated that the speckle noise is dominated by an additive nature for low coherences. Despite additive noise terms depend only directly on the coherence $|\rho|$, the actual phase difference $\phi_x$ has also an indirect effect over the final nature of speckle noise for the Hermitian product real and imaginary parts. The phase $\phi_x$ produces the real and imaginary parts of the Hermitian product to present noises of different natures. For instance, for $\sin \phi_x$ close or equal to zero and a high coherence value, the imaginary part will be dominated by an additive speckle noise, whereas the real part will be dominated by a multiplicative speckle noise.

Consequently, if speckle has to be reduced two points have to be considered. First, the nature of speckle noise depends on the coherence $|\rho|$ being additive for low coherences whereas it is multiplicative for high coherence. Second, speckle noise nature is different for the real and imaginary parts of the Hermitian product as a consequence of the indirect dependence on $\phi_x$.

4 PolSAR SPECKLE NOISE REDUCTION

As it has been shown, speckle noise presents two mechanisms in the case of the Hermitian product of complex SAR images. Consequently, the final speckle noise nature is the result of the combination, depending on the coherence value $|\rho|$, of these two mechanisms.

Under the Gaussian scattering assumption, the covariance matrix characterizes completely a non-deterministic scatterer as higher order statistical moments can be derived from the elements of $[C]$ [2]. Thus, if the elements of the covariance matrix are correctly processed, even independently, the polarimetric information is maintained. The main consequence of the proposed speckle noise model, Eq. 4, is that it opens the possibility to define new PolSAR speckle noise reduction algorithms. These new approaches are mainly based on processing the covariance matrix elements as a function of the coherence degree. It can be observed from Eq. 4, that most of the useful signal to recover, given by the term $\psi N_c \overline{z}_n \exp(\phi_x)$ is affected by the multiplicative speckle term $n_m$. The actual complex value to recover is $\psi |\rho| \exp(\phi_x)$ [5]. Therefore, as is can be deduced, the term $N_c \overline{z}_n$ has the same role as the coherence $|\rho|$. Fig. 1 shows the closeness between these two terms. The difference between these two values corresponds to the second additive value in Eq. 4. The value $N_c \overline{z}_n$, whose expression can be obtained from the product of Eqs. 5-6, depends only on the coherence value $|\rho|$. Therefore, if the complex coherence $|\rho|$ is available, the first additive term of Eq. 4 can be obtained by multiplying the Hermitian product amplitude by $N_c$ and the complex term $\exp(\phi_x)$, which corresponds to the complex coherence phase. Since speckle noise has only a multiplicative nature in the first additive term of Eq. 4, algorithms designed to process only multiplicative speckle can be employed, for instance, the algorithm shown in [6]. For comparison, the multilook approach will be also considered to process the multiplicative speckle term.

Since the signal recovered from the first additive term does not correspond to the complete signal to recover, it has to be post-processed to recover it. Consequently, in the recovered value $\psi N_c \overline{z}_n \exp(\phi_x)$, the term $N_c \overline{z}_n$ can be transformed to $|\rho|$ by considering Eqs. 5-6.

The speckle noise reduction technique explained in the previous paragraphs is based basically on eliminating the additive speckle noise term from the real and imaginary parts of each element of the one-look covariance matrix $[Z]$. As the additive component depends on the coherence value $|\rho|$, the process to eliminate it will be different in each covariance matrix entry. Once the additive speckle term, as the multiplicative speckle term affects all the covariance matrix terms in the same way, they are processed in the same form. Fig. 2 presents an scheme of the proposed filtering approach.
5 RESULTS

In this section the approach to reduce PolSAR speckle noise, based on the technique introduced in the previous section, is tested. The main purpose of this section is to show that to process the one-look covariance matrix elements, depending on the speckle model presented on Eq. 4, does not damages polarimetric information. This will be show by considering simulated and real data.

First, a set of three uncorrelated gaussian channels have been obtained and correlated with the covariance structure \[[8]\]

\[
[C] = \frac{1}{4} \begin{bmatrix}
2\beta + \delta & 0 & 2\beta - \delta \\
0 & 2\delta & 0 \\
2\beta - \delta & 0 & 2\beta + \delta
\end{bmatrix}.
\]

These data correspond to the scattering from a cloud of identical particles with random orientations. If \(\beta = 2\) and \(\delta = 1\), the corresponding entropy \(H\) equals 0.7897 and the angle \(\alpha\) equals 0.5236.

In order to compare the recovering of polarimetric information four alternatives to process PolSAR data are compared. The first set of techniques are based on processing all the covariance matrix elements in the same form. These techniques are a multilook filter considering a 7 by 7 pixel window, whereas the second technique is the PolSAR data filter presented in the reference [6]. The second set of techniques consist on applying the previous two techniques only on the multiplicative speckle term of the different covariance matrix elements. In this situation, the complex coherence information is obtained in a previous step by the algorithms presented in [9]-[10]. Fig. 3 presents the \(H - \alpha\) diagrams obtained with the four approaches. The mean values of \(H\) and \(\alpha\) are presented in Table 1.

![H-alpha diagrams](image)

(a) (b) (c) (d)

Figure 3: H-\(\alpha\) diagrams. (a) Multilook with 7 by 7 pixel window. (b) Lee filter. (c) Multilook with a 7 by 7 pixel window applied to the multiplicative speckle term. (d) Lee filter applied to the multiplicative part. The results in the second column have been obtained processing the covariance matrix elements in a different way.

<table>
<thead>
<tr>
<th>Mlook</th>
<th>Mlook over mult. part</th>
<th>Lee filter</th>
<th>Lee filter over mult. part</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H)</td>
<td>0.7742</td>
<td>0.7889</td>
<td>0.7639</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.5703</td>
<td>0.5771</td>
<td>0.5971</td>
</tr>
</tbody>
</table>

Table 1: Mean values corresponding to entropy \(H\) and \(\alpha\) for simulated data.

As it can be deduced from the results presented in Fig. 3 and Table 1, to process the covariance matrix elements
in a different way, according to the multidimensional speckle noise model shown by Eq. 4, does not represent to damage polarimetric information. In addition, the polarimetric signature corresponding to data simulated as given by Eq. 8 is compared with the corresponding obtained in the case the multiplicative speckle noise term is processed with the Lee filter.

Figure 4: Polarization signatures. Original (a) Copolar (CV=0.5), (b) Crosspolar (CV=0.25). Multiplicative speckle term processed with the Lee filter (a) Copolar (CV=0.49), (b) Crosspolar (CV=0.28). CV stands for coefficient of variation.

The results presented in Fig. 4 show that the polarimetric signature is also maintained despite the elements of the covariance matrix are processed differently.

The different one-look covariance matrix elements, whose correlation structure corresponds to the covariance matrix shown in Eq. 8 with $\beta=2$ and $\delta=1$, present elements with very different speckle behavior. The diagonal elements present a fully multiplicative noise behavior. The off-diagonal elements $S_{hh}S_{hh}^*$ and $S_{hv}S_{hv}^*$ are characterized by a coherence equal to zero, presenting therefore, a speckle noise whose nature is completely additive. On the contrary, the off-diagonal element $S_{hh}S_{hv}^*$ presents, for $\beta=2$ and $\delta=1$, a coherence $|\rho|$ equal to 0.6 which produces the speckle to be dominated by a multiplicative nature. These differences produce the covariance matrix elements to be processed in a very different way in order to eliminate the additive part in each case. Despite the large differences in the way each element is processed, as shown, polarimetric information is maintained.

The approach to filter PolSAR data which has been presented in the last section, based on processing the elements of the covariance matrix according to the corresponding coherence, has been also tested with real PolSAR images. In this case PolSAR data acquired with the EMISAR sensor at C-band are employed. PolSAR data are processed with the Lee filter, the Lee filter applied to the multiplicative component of the model given by Eq. 4 and the multilook filter with a 7 by 7 pixel window. Fig 5 shows the histograms of H and $\alpha$ corresponding to each of the filtering approaches.

Figure 5: Histograms for the EMISAR PolSAR data. (a) H. (b) $\alpha$.

As one can deduce from Fig 5, in the three cases the same polarimetric information is recovered. The differences
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References


