Discussion on: “Stability Analysis of Finite-Level Quantized Discrete-Time Linear Control Systems”

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1. Contribution

For better understanding, we briefly review the authors’ contribution.

Consider the feedback system in Fig. 1, composed of the SISO discrete-time linear system \( G \) and the static quantizer \( Q \). The system \( G \) corresponds to the pair of a plant and controller, and \( Q \) is a finite logarithmic type, that is, a logarithmic map from a continuous set to a finite set (see equation (4) in the original paper). The main topic is the stability of this feedback system.

In considering this, two challenging issues are involved. First, since the quantizer output takes a value on a finite set, the saturation could be caused by \( Q \). This prevents us to consider the global stability. Second, \( Q \) contains a deadzone around the origin, i.e., \( Q(y) = 0 \) for every \( y \in [-\epsilon, +\epsilon] \) \((\epsilon > 0)\). Thus, in general, the asymptotic stability is unachievable. These are the difference from the case in [1].

As a stability notion, the paper has considered the existence of ellipsoidal sets \( D \), \( A \) in the state space such that

- \( A \subset D \),
- the state starting from \( D \) reaches \( A \) in finite time,
- once the state enters \( A \), it stays there forever,

as shown in Fig. 2. The sets \( D \) and \( A \) are the so-called stability region and attractor. We can understand that these are respectively introduced to handle the saturation and deadzone. A similar idea is found in [2].

Finding the sets is not only for checking whether the system is stable or not but also for evaluating a kind of degree of the stability. In fact, their geometric properties, such as volume and shape, are useful information on how much control is needed. Moreover, by comparing two quantized systems using the sets, one can know which of the two systems is better in the sense of the stability.

In this paper, a sufficient condition for the stability has been obtained as a parameter-dependent linear matrix inequality (although the authors have just called it the “linear matrix inequality”). Furthermore, it has been shown that some design problems of controllers and quantizers can be solved with the inequality. In my view, this is an interesting and novel contribution in the quantized control area.

2. Discussion

I would like to propound two problems.

Q1. Which is better in the sense of the stability, the input quantization or the output quantization?

Three types of quantized systems have been considered in the paper: (a) state feedback systems, (b) output feedback systems with input quantization, and (c) output feedback systems with output quantization. A unified solution has been provided based on the \( G \) and \( Q \) representation in Fig. 1, and also the resulting systems have been demonstrated by numerical simulations. However, I could not find any general answer to the above question, though the geometric properties of \( D \) and \( A \) are closely related to
the question and a case study may be performed by Fig. 3 and 4 in the original paper.

This topic is fundamental in clarifying the most important signal in quantized control systems (e.g., which has to have higher resolution, the actuator signal or the sensor signal?). This interests many researchers of this area, and it is left as a future work.

Q2. Is the use of ellipsoids the best for the stability analysis?

It is reasonable to discuss the existence of the sets $\mathcal{D}$ and $\mathcal{A}$ for the stability analysis. However, do they have to be ellipsoids? The use of ellipsoidal sets allows us to reduce the problem into a linear matrix inequality problem, which is definitely a big advantage. On the other hand, it has never been proven that ellipsoids are the best approximations of $\mathcal{D}$ and $\mathcal{A}$. It could be possible that parallelepipeds are better approximations of them.

The question has a connection to the norm selection for evaluating the performance of quantized systems (what is the best norm?). The ellipsoid based method is concerned with the 2-norm performance, while the $\infty$-norm has been employed in other literatures, e.g., [3].

To my best knowledge, the question, including the norm selection, is still open and quite interesting. I hope that the authors extend the results to a non-ellipsoidal approximation case.

References

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Final Comments by the Authors
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We thank Dr. S. Azuma for his valuable comments on the paper. Firstly, we emphasize that, as pointed out in Remarks 4.2 and 4.4, the main results of the paper are cast in terms of one-parameter-dependent LMIs. Secondly, in his contribution Dr. Azuma has raised two important and challenging issues regarding the impact of the input and the output quantizations on the stability region and attractor, and the conservativeness of using quadratic stability in the context of finite-level quantized control systems, for which we make the following comments:

1. Input or output quantization

It is shown in [1] that in the case of logarithmic input quantization, an observer based output feedback controller with a deadbeat observer achieves the same result as a quantized state feedback controller, in the sense of quadratic stability. Notice that the output feedback controllers considered in our paper are not necessarily observer based and thus, in general, input and output quantizations can achieve different results. The numerical experiments in the paper have revealed that in the case of output
quantization, a relatively larger number of quantization levels is needed to achieve a similar estimate for the stability region and attractor than for the input quantization configuration.

2. Best Lyapunov function candidate

We believe that this question is quite general and applies to many other nonlinear systems. It is very hard to provide a conclusive reply regarding the “best” class of Lyapunov function candidates to deal with quantized linear systems.

In our opinion, quadratic stability is still a very powerful tool for stability analysis of quantized linear systems with good compromise between conservativeness and the required computational effort, as indicated by the numerical examples. Nevertheless, we agree with Dr. Azuma’s claim that there is no clear evidence that ellipsoids are better than (multiple) linear Lyapunov functions (see, for instance, [2]).

References