On Parameter Tuning for FAST TCP
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Abstract—This paper studies the stability of FAST TCP using a continuous time model of a single-link single-source network. A sufficient condition on asymptotical stability of FAST TCP congestion window is obtained, which relates all the relevant parameters in FAST TCP and decouples the key parameter $\alpha$ from others. A guideline on FAST TCP parameter setting is thus provided. The ns2 simulations validate the theoretical results.

Index Terms—FAST TCP, congestion control, stability, parameter setting.

I. INTRODUCTION

FAST TCP [4], [5] outperforms its predecessors in networks with large bandwidth-delay product due to using queueing delay as congestion indication. However, parameter setting of FAST TCP still remains a challenging task [2], [8] as there may be no single set of parameters that works well under different network scenarios and usually a dozen of parameters are coupled together.

Recently, stability of TCP dynamic has attracted much attention, see, for example [6], [7], [9]. Particularly, local and global stability on FAST TCP have been studied in [1], [10], [12], [13]. Stability of protocol dynamic is seen to play a key role in parameter setting to obtain satisfactory performance. The basic requirements for a stability condition to serve as a guideline of parameter setting are (1) it should relate all the relevant parameters in a comprehensive manner; and (2) the key parameter should be decoupled from other parameters in the condition. From this point of view, the existing stability results on FAST TCP are not straightforward for application of parameter setting. It is still not available a guideline on FAST TCP parameter setting with the aim of obtaining satisfactory performance under various network environments.

In FAST TCP, the parameter $\alpha$ is the number of the packets each source attempts to maintain in the network buffers at equilibrium [4]. This key parameter together with other parameters including the link capacity $c$, the propagation delay $d$ and the source control gain $\gamma$ are responsible for the network performance under FAST TCP. In [2], on a simulation basis, it has been suggested that the value of $\alpha$ should be at least 0.0075 times the link capacity, in the network with a single bottleneck link. However, no theoretical basis is given to support their claim in there. Nevertheless, how this parameter setting is related to other parameter, like the propagation delay, is still unknown.

This paper still studies the stability of FAST TCP. A sufficient condition on asymptotical stability of FAST TCP congestion window is obtained, which relates all the relevant parameters in FAST TCP and decouples the key parameter $\alpha$ from others. Based on this, a guideline on choosing this parameter to obtain satisfactory performance is provided. We provide ns2 simulation results to validate the theoretical results.

II. ANALYSES TO FAST TCP MODEL

Consider a single-link single-source network topology in which the source and destination node is connected through a single bottleneck link with capacity $c$. Let $p(t)$ be the queueing delay and let $x(t)$ be the source sending rate. Let $\tau_f$ denote the forward delay between source and link, and $\tau_b$ the backward delay in the feedback path from link to source. Then, the source rate observed by the link at time $t$ is $x(t-\tau_f)$, and $q(t) = p(t-\tau_b)$ is the queueing delay observed by the source at time $t$. Let $w(t)$ be the source congestion window size at time $t$. The source sending rate $x(t)$ is then defined as

$$x(t) = \frac{w(t)}{d + q(t)},$$

where $d$ denotes the constant propagation delay of the source, and the round trip-time (RTT) at time $t$ is given by $R(t) = d + q(t)$. Here, we interpret $\tau_f + \tau_b = R_0$ as the equilibrium value of $R(t)$.

The original FAST TCP adapts the congestion window periodically according to the discrete time model [4]

$$w[k+1] = (1-\gamma)w[k] + \gamma \left( \alpha + \frac{d}{d+q[k]}w[k] \right),$$

where $\alpha$ and $\gamma$ are protocol parameters. This update is performed once every RTT. We follow [10] to obtain a continuous time approximation of the window control. Rewrite (2) as

$$\frac{w[k+1] - w[k]}{R[k]} = -\gamma \frac{w[k]}{R[k]} + \gamma \left( \frac{\alpha}{R[k]} + \frac{d}{(R[k])^2}w[k] \right).$$

Using a first order Euler approximation of the derivative and applying the identity $R[k] = d+q[k]$, we derive the continuous time window update equation

$$\dot{w}(t) = \gamma \left( \frac{\alpha}{d+q(t)} - \frac{q(t)}{(d+q(t))^2}w(t) \right).$$

Queueing delay has been traditionally modelled by

$$p(t) = \frac{1}{c} \left( x(t-\tau_f) - c \right) = \frac{1}{c} \left( \frac{w(t-\tau_f)}{d+p(t-\tau_b)} - c \right).$$

Noting that $q(t) = p(t-\tau_b)$ and in equilibrium

$$\frac{\alpha}{p_0} = x_0 = \frac{w_0}{d+p_0}, x_0 = c, d+p_0 = R_0,$$
we linearize (3) and (4) around \((w_0,p_0)\) to obtain

\[
\begin{align*}
\delta w(t) &= -\frac{\gamma p_0}{R_0^2} \delta w(t) - \frac{\gamma \alpha d}{p_0 R_0^2} \delta p(t - \tau^p) \\
\delta p(t) &= \frac{1}{R_0 c} \delta w(t - \tau^f) - \frac{1}{R_0} \delta p(t - R_0),
\end{align*}
\]

where \(\delta w(t) = w(t) - w_0, \delta p(t) = p(t) - p_0\). \(\delta w(t)\) and \(\delta p(t)\) are both perturbed variables.

### III. Stability Analyses

By taking the Laplace transformation of the equations (5), we have

\[
\begin{align*}
s \delta W(s) - \delta w(0) &= -\frac{\gamma p_0}{R_0^2} s \delta W(s) - \frac{\gamma \alpha d}{p_0 R_0^2} \delta P(s) e^{-\tau^p s} \\
\delta P(s) - \delta p(0) &= \frac{1}{R_0 c} s \delta W(s) e^{-\tau^f s} - \frac{1}{R_0} \delta P(s) e^{-R_0 s},
\end{align*}
\]

where \(\delta W(s)\) and \(\delta P(s)\) denote the Laplace transform of \(\delta w(t)\), and \(\delta p(t)\), respectively. By direct computing, one has

\[
\left( s + \frac{1}{R_0} e^{-R_0 s} \right) \delta w(s) + \frac{\gamma p_0}{R_0^2} \delta W(s) = \frac{1}{R_0} e^{-R_0 s} \delta w(0) - \frac{\gamma \alpha d}{p_0 R_0^2} e^{-\tau^p s} \delta w(0).
\]

Then, we obtain the so-called characteristic polynomial \(\Delta(s)\)

\[
\Delta(s) = s^2 + \left( \frac{\gamma p_0}{R_0^2} + \frac{1}{R_0} e^{-R_0 s} \right) s + \frac{\gamma p_0}{R_0^2} e^{-R_0 s}.
\]

The characteristic polynomial (8) determines the stability in terms of the congestion window size of the linearized closed-loop time-delayed system (5). We use Routh-Hurwitz stability criteria [3] to study its stability. To analyze the system stability, it is adequate to have the Padé approximation [3]

\[
e^{-R_0 s} = \frac{2 - R_0 s}{2 + R_0 s}.
\]

The approximated characteristic equation of the linearized system (5) is then reached at

\[
\Delta(s) = R_0 s^3 + \left( \frac{\gamma p_0}{R_0^2} + 1 \right) s^2 + \left( \frac{2}{R_0} + \frac{\gamma (2p_0 - R_0)}{R_0^2} \right) s + \frac{2\gamma}{R_0} = 0.
\]

By denoting \(a_3 = R_0, a_2 = 1 + \gamma p_0/R_0, a_1 = 2/R_0 + \gamma (2p_0 - R_0)/R_0^2, a_0 = 2\gamma/R_0\), the characteristic equation (9) becomes

\[
\Delta(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0.
\]

According to the Routh-Hurwitz criteria [3], all of the coefficients of the equation (10) on \(s\) have to be positive, which gives

\[
\frac{2}{R_0} + \frac{\gamma (2p_0 - R_0)}{R_0^2} > 0.
\]

Substituting \(R_0 = d + p_0\) and \(p_0 = \alpha/c\) into the above inequality, one has

(1) \(\gamma\) can take any positive value, if \(\alpha > cd\).

### Table I

<table>
<thead>
<tr>
<th>Routh Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_3)</td>
</tr>
<tr>
<td>(a_2)</td>
</tr>
<tr>
<td>(a_1 - a_0 a_0/a_2)</td>
</tr>
<tr>
<td>(a_0)</td>
</tr>
</tbody>
</table>

\(2\) \(\gamma < (cd + \alpha)/(cd - \alpha) = 2\alpha/(cd - \alpha)\), if \(\alpha < cd\). Combining the above two conditions, we focus on the case of \(\gamma < 2\) in this paper. We now compute and construct the Routh Table by using these coefficients of characteristic equation.

According to Routh-Hurwitz stability test, the linearized system (5) is stable if and only if all the values of the first column of the above Routh Table are positive, i.e.,

\[
\begin{align*}
R_0 &> 0, \quad \frac{\gamma p_0}{R_0} + 1 > 0 \\
2 &+ \frac{\gamma (2p_0 - R_0)}{R_0^2} - \frac{R_0}{R_0^2} + \frac{2\gamma}{R_0} > 0
\end{align*}
\]

Considering \(R_0 = d + p_0\) and \(p_0 = \alpha/c\), the inequalities (11) are equivalent to the following inequality on \(\alpha\)

\[
\gamma^2 + \frac{2}{c^2} - \frac{\gamma^2 + 2\gamma - 4}{d} \cdot \alpha + (2 - 3\gamma)d^2 > 0.
\]

Solving (12), one obtains

\[
\alpha < \frac{(\gamma^2 + 2\gamma - 4) - \gamma \sqrt{16\gamma^2 + \gamma^4}}{2(\gamma^2 + \gamma^2) + 2}\cdot d\cdot c,
\]

or

\[
\alpha > \frac{(\gamma^2 + 2\gamma - 4) + \gamma \sqrt{16\gamma^2 + \gamma^4}}{2(\gamma^2 + \gamma^2) + 2}\cdot d\cdot c.
\]

Let

\[
f(\gamma) = \frac{(\gamma^2 + 2\gamma - 4) + \gamma \sqrt{16\gamma^2 + \gamma^4}}{2(\gamma^2 + \gamma^2) + 2}.
\]

Solving \(f(\gamma) > 0\), one then has \(\gamma > 2/3\).

![Fig. 1. Relationship between \(f(\gamma)\) and \(\gamma\)](image)

Figure 1 illustrates that \(f(\gamma) < 1\) if \(\gamma < 2\). Considering \(\alpha > 0\), we thus obtain the following theorem.

**Theorem 1**: Given the networks parameters \(c, d,\) and \(\gamma,\) FAST TCP described by (5) is asymptotically stable in terms of the variable \(\delta w(t)\), if

\[
\alpha > f(\gamma) \cdot d\cdot c,
\]

when \(2/3 < \gamma < 2\). Here \(f(\gamma)\) is given by (15).

**Remark 1**: In ns2 implementation of FAST TCP [2], it is intuitively suggested that on a path with capacity \(c, \alpha/c\) must be 5 times than the TCL variable \(\text{mi\_threshold},\) which defaults
Remark 2: Unlike the existing stability conditions of FAST TCP which has suggested the scalability of the result, related all the relevant parameters and decoupled the key TCP \cite{10,12,13}, the result in Theorem 1 has actually the source control gain $\gamma$

Remark 3: Note that in the usual FAST TCP setting \cite{4}, the parameter setting for FAST TCP. Therefore, this result is in a particular form that facilitates parameter setting for FAST TCP.

IV. SIMULATION VERIFICATION

We provide ns2 \cite{11} simulations to show how the obtained stability condition can be served as guideline on parameter setting in various network environments. Consider a single-link single-source network, with the link capacity being 1 Gbps and each packet size being 1000 bytes. The propagation delay is 50 ms. Firstly, we assume the source control gain $\gamma = 0.8$, which is used. Theorem 1 also explains the fact that FAST TCP has been stable for all experimental cases studied so far when $\gamma = 0.5$. Besides, the guideline on setting parameter given by Theorem 1 allows a larger source control gain.

Fig. 2. Congestion window when $\gamma = 0.8$, $\alpha = 1250$

Fig. 3. Queue size when $\gamma = 0.8$, $\alpha = 1250$

to 1.5 ms. Theorem 1 gives the condition $\frac{\alpha}{c} > f(\gamma) \cdot d$, which provides a theoretic support for this intuition, and further it can be applied for different settings of source control gain $\gamma$, which has suggested the scalability of the result.

**Remark 2:** Unlike the existing stability conditions of FAST TCP, which has suggested the scalability of the result, related all the relevant parameters and decoupled the key parameters from all the other parameters. This result is in a particular form that facilitates parameter setting for FAST TCP.

**Remark 3:** Note that in the usual FAST TCP setting \cite{4}, the source control gain $\gamma$ is set to be within $\left[0, 1\right]$ and a default value of 0.5 is used. Theorem 1 also explains the fact that FAST TCP has been stable for all experimental cases studied so far when $\gamma = 0.5$. Besides, the guideline on setting parameter given by Theorem 1 allows a larger source control gain range.

IV. SIMULATION VERIFICATION

We provide ns2 \cite{11} simulations to show how the obtained stability condition can be served as guideline on parameter setting in various network environments. Consider a single-link single-source network, with the link capacity being 1 Gbps and each packet size being 1000 bytes. The propagation delay is 50 ms. Firstly, we assume the source control gain $\gamma = 0.8$. From Theorem 1, the stability range of $\alpha$ is calculated to be within $\left(1006, \infty\right)$. Setting $\alpha = 1250$, the simulation results are given in Figures 2 and 3, which have shown that the congestion window and queue size are stable, and they converge to their equilibrium points very fast. Next we change the value of $\gamma$ to 1.2. Then the stability range of $\alpha$ is $\left(3564, \infty\right)$, obtained from the condition (16). Setting $\alpha = 3750$, Figures 4 and 5 have plotted the simulation results. From Figures 4 and 5, one observes that the stability of FAST TCP is achieved once again, by setting the appropriate value of $\alpha$, even though the value of $\gamma$ is greater than 1.

V. CONCLUSIONS

Based on a continuous time model of FAST TCP and by using control theory, we have obtained the sufficient stability condition for FAST TCP. The condition has given a guideline to select FAST TCP’s parameter $\alpha$. The stability condition can also be applied to the environments with different source control gain settings of FAST TCP. We have validated the theoretical result by simulations.

REFERENCES