A Dynamic Model of Job Networks and Persistent Inequality *

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Abstract

This paper investigates the properties of a local economy in which personal connections are important in finding jobs. The complementarities in the model generate an interesting nonlinear relationship between the distribution of human capital in the economy, the characteristics of the social network, and equilibrium labor market dynamics. The model is shown to be consistent with a number of stylized facts about the increased neighborhood concentration of poverty since 1970. I argue that this type of model is more consistent with the empirical facts about neighborhood poverty than previous models which focus on human capital accumulation.

1 Introduction

Since 1970, cities in the United States have seen a substantial increase in the geographic concentration of unemployment, poverty, and associated social problems. Between 1970 and 1990, the number of high-poverty neighborhoods (census tracts in which over 40% of families have incomes below the poverty line) in U.S. metropolitan areas has more than doubled, and the number of Americans living in high-poverty neighborhoods has nearly doubled [14, p. 30]. Furthermore, as sociologist William Julius Wilson points out, the fraction of adults in such neighborhoods who are employed has fallen precipitously. Wilson's work, in both his classic *The Truly Disadvantaged* [26] and his more recent *When Work Disappears* [27] emphasizes the role of feedbacks from

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the social isolation of extremely poor communities to the perpetuation and expansion of poverty for their residents. These neighborhood feedbacks affect both developmental outcomes such as human capital accumulation and adult problems such as unemployment. Wilson's arguments have inspired a growing body of empirical and theoretical work on such "neighborhood effects".

A substantial body of empirical research [3, 5, 7, 8] has found some evidence that characteristics of one's neighborhood and ethnic group affect individual decisions and outcomes. The dominant strand in the related theoretical literature, exemplified by Bénabou [2], Durlauf [10], and Streufert [23], focuses on the combined role of these local complementarities and income-stratified neighborhoods on human capital accumulation. If complementarities are strong enough, these models predict the emergence of poverty traps or thresholds in which small differences in neighborhood composition or initial conditions lead to large differences in outcomes over time.

This paper addresses a less well-understood type of neighborhood effect emphasized by Wilson, the role of informal job networks in finding employment. In the model, managers' social ties to workers provide information about unobserved differences in productivity between these workers. These information flows induce complementarities in employment. I find that, for a given set of social ties, there is a critical level of human capital in a community below which the community's long-run employment rate goes to zero from any initial state. Above this critical value, substantially higher levels are sustainable. In addition, small increases in community human capital above the critical value produce large increases in long-run employment. This approach is similar in spirit to critical mass or epidemic models of social behavior. It is also complementary to the earlier work which finds critical behavior in human capital accumulation. I argue in this paper that the model here can explain some facts about neighborhood poverty that can not explained by human capital accumulation.

The existing theoretical literature on job networking is limited. In Montgomery [19], the tendency of friends to share characteristics implies that qualified workers are likely to refer other workers who are qualified. He shows that the wage dispersion between workers of high and low ability is increasing in both network density (the probability a worker has a social tie) and inbreeding bias (the probability that a referred worker has similar skills). Montgomery's model is static, and as such can only tell a story about how one-shot dynamics are affected by the job network. The job network is defined by a single parameter, the density of contacts. In contrast, the dynamic model in this paper facilitates the investigation of a richer variety of network characteristics, and their relationship to both short and long run outcomes.

A number of studies using survey data find that networks of friends and relatives are commonly used as a resource in job search. Granovetter's study of job search in the 1970's [11] indicates that about 50% of workers obtained their jobs through friends and relatives. Rees and Shultz [22], in a detailed study of the Chicago labor market, find that in twelve occupations studied, between 23.5% and 73.8% of workers used social contacts. Campbell and Marsden's more recent study [4] finds that over 51% of jobs are filled through referral. On the employer's side, Holzer [13] finds that 36% of firms filled their last position with a referred applicant.

However, the prevalence of job networking is not enough on its own to imply its economic importance. Networking through friends and relatives is not costly, so its substitutes (formal application, employment agencies, etc.) need only be slightly less effective or slightly more costly to be much less prevalent. If this is the case, then variations in the ability of workers to network will have minimal effect on their employment opportunities.

A few micro studies have attempted to estimate the economic importance of job networking. Holzer [13] estimates that approximately 41% of the difference in monthly probabilities of employment for unemployed black youth versus unemployed white youth are due to lower probabilities of obtaining a job offer through friends and relatives and estimates that contacting friends and relatives is much more likely to generate a job offer than a number of other common methods. Bartlett and Miller [1], using data on female executives, find that controlling for a wide variety of other factors, membership in private clubs and service on corporate boards has a significant positive effect on earnings. Datcher [9] finds that obtaining a job through informal channels has a negative effect on the likelihood of quitting.

In addition to these micro studies, further evidence for the importance of networks can be found in the ethnic concentration of many small industries documented by Rauch [21]. Such concentration is particularly strong among recent immigrant groups. In an extreme example, Rauch found that over 75% of employed Korean-born residents of Los Angeles were either self-employed or worked for Korean-owned firms. Additional examples cited by Rauch include Korean and Dominican grocers in New York City, the 80% of doughnut shops in California that are operated by Cambodian immigrants, or the 1/3 of U.S. motels owned by immigrants from the Indian state of Gujarat.

2 Description of the Model

It is helpful to distinguish between a number of different senses in which the term "job networks" is used in the literature. Immigrant networks such as those discussed by Rauch [21] are based on preferential treatment. Employers simply prefer to hire people they know, whether out of nepotism or because they feel that they can work better with friends. Networks also facilitate social learning, in which habits which are valuable or harmful in one's career are transmitted through a social process. Wilson [27] describes how young people raised in high-unemployment social environments have difficulty knowing the informal norms of the workplace. This paper focuses on the use of social ties to gather private information on worker or job characteristics.

In this economy, workers live two periods. Generations overlap, and in each time period there are I members of each generation. There are also I firms. I may be infinite. An worker is denoted by the pair i, t, where t is the time period in which the worker is young. The social network is exogenous and represented by a directed graph called Γ whose node set is $\{1, 2, 3, \ldots, I\} \times Z$. Node i, t represents worker i, t. Workers have social ties to members of the previous generation, and these ties are represented by directed edges in Γ. An example of such a network is depicted graphically in Figure 1. An edge from one worker to another means that they know each other, and is denoted by $(a) \searrow (b)$. If a and b are connected by an edge in the network Γ, this will be denoted by $(a) \searrow (b) \in \Gamma$. For technical reasons, I assume the number of connections to or from any node is bounded above by some finite number. The network can be random, in which case Γ will be a random variable. In the model, information flows through these social connections will be important in matching firms to potential employees.

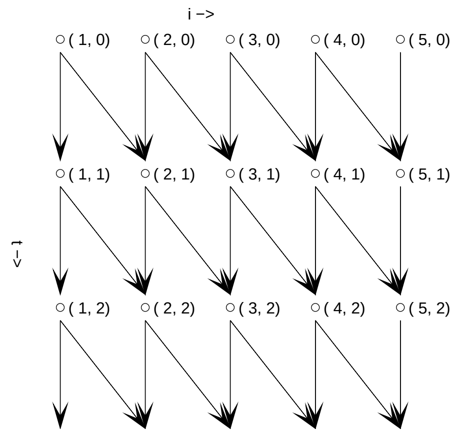

Figure 1: An example of a social network in this model. Each agent (node) is represented by a circle, and each social tie (edge) between agents is represented by an arrow.

Each period, the workers and firms in the model play a simple game. Each firm observes some information on worker productivity, with better information available to those firms which have social contacts with a given worker. Firms simultaneously make individual wage offers to each worker. The worker then chooses to accept one of the offers or to engage in home production, and allocates the resulting income to consumption.

2.1 Workers

In this model, labor supply and consumption decisions are very straightforward. Young workers have one unit of labor which can be allocated to home production or employment at a firm. The resulting income is used to purchase both the home-produced good and the commercially-produced good. The home-produced good is numeraire and the commercially-produced good is traded at an equilibrium price p_t. Old workers neither work nor consume. This assumption is strictly for simplicity and is not important for the results in this paper.

Each young worker receives a separate wage offer from each firm j, denoted by $w_{i,t}^j$. Let the vector of such wage offers be $\mathbf{w}_{i,t}$. Home production exhibits constant returns to scale, and the home-produced good can be traded on the market for the commercially produced good. I make the home-produced good numeraire, and assume that one unit of the good can be produced with one unit of labor. Upon receiving the wage offers, the worker chooses the amount of labor to supply to each firm j, denoted by $n^j_{i,t} \in [0,1]$. Let $n_{i,t}$ be the total labor supplied to the market by worker i,t, i.e., $n_{i,t} \equiv \sum_{j=1}^{I} n^j_{i,t}$. In addition to wage income, each worker in generation t also receives profit π_t from the firms, which he takes as given.

The worker allocates this income between the home-produced good and the commercially-produced good. Let $x_{i,t}$ denote worker i,t's consumption of the home-produced good, and $y_{i,t}$ denote consumption of the commercial good. The worker has CES preferences over these two goods, giving the payoff function:

$$(x_{i,t})^{\sigma} + p(y_{i,t})^{\sigma} \tag{1}$$

subject to the budget constraint:

$$x_{i,t} + p_t y_{i,t} \leq \pi_t + \left(1 - \sum_{j=1}^{I} n^j_{i,t}\right) + \sum_{j=1}^{I} w^j_{i,t} * n^j_{i,t} \tag{2}$$

and the feasibility constraints:

$$\sum_{j=1}^{I} n^j_{i,t} \quad \leq \quad 1 \tag{3}$$

$$n^j_{i,t}, x_{i,t}, y_{i,t} \quad \geq \quad 0. \tag{4}$$

A strategy for the young worker, then, is a function mapping $(\mathbf{w}_{i,t}, p_t, \pi_t)$ into $(n^j_{i,t}, x_{i,t}, y_{i,t})$ which satisfies these constraints.

2.2 Firms

Firms face a substantially more complex decision-making environment than workers. All firms produce identical goods and are competitive in the goods market, but have differential information about workers. As a result firms behave strategically in the labor market. Each firm makes a separate wage offer to each worker, and has only limited information on both the worker's productivity and the wage offers of other firms.

A worker i,t employed at firm j can produce $a^j_{i,t} \in \{0,1\}$ units of output. Productivity $a^j_{i,t}$ is independently distributed across all firm-worker pairs with worker-specific mean $\bar{a}_{i,t}$. I interpret $\bar{a}_{i,t}$ as an individual's human capital, and assume it is observable by all. In contrast $a^j_{i,t}$ is unknown until after employment and can be interpreted as the quality of a match between firm and worker.

Social ties provide additional information on the quality of a particular match in the following manner. Each young worker i,t becomes a "manager" of firm i the next period. A manager in this model has no productive capability. The only purpose he serves is to provide the firm with referrals on younger workers. In order for a manager to provide a referral on a given worker, the manager must have a social tie to that worker and must have experience in the job from the previous period. In the notation of this paper, the conditions are:

$$(j, t-1) \searrow (i, t) \in \Gamma \tag{5}$$

$$n_{j,t-1} = 1. \tag{6}$$

Each firm j receives a signal $s_{i,t}^j$ about the ability level of agent i, t if these two conditions are met.

For convenience, I make a few technical assumptions on the signal. First, assume that the signal has continuous, bounded support with a differentiable conditional probability density function $f(s|a)$. I normalize (the previous assumptions guarantee this can be done without loss of generality) so that the signal's support is the unit interval $[0, 1]$. Furthermore, assume that the strict monotone likelikood ratio property (MLRP) holds:

$$\frac{f(s|a=1)}{f(s|a=0)} > \frac{f(s'|a=1)}{f(s'|a=0)} \quad \text{if } s > s'. \tag{7}$$

Equation (7) implies that the signal is informative, i.e., that a manager's estimate of a worker's productivity is strictly increasing in the value of the signal.

All period t wage offers $w_{i,t}^j$ are made simultaneously. When firm j makes a wage offer to worker i, t, it knows prices p_t, overall worker ability $\bar{a}_{i,t}$, and the value of its own signal $s_{i,t}^j$. If worker i, t accepts its wage offer, firm j will make profits of $p_t a_{i,t}^j - w_{i,t}^j$. However, it will only receive that payoff if the worker accepts, leading to the payoff function:

$$\left(p_t a_{i,t}^j - w_{i,t}^j\right) n_{i,t}^j(\mathbf{w}_{i,t}, p_t) \tag{8}$$

where $\mathbf{w}_{i,t} = (\ldots, w_{i,t}^{j-1}, w_{i,t}^j, w_{i,t}^{j-1}, \ldots)$. I assume that the social network Γ is known by all. As a result, each agent knows how many firms have information on a given worker, which I will denote by $k_{i,t}$. In terms of previously defined variables:

$$k_{i,t} \equiv \sum_{(j,t-1) \searrow (i,t) \in \Gamma} n_{j,t-1}. \tag{9}$$

A strategy for firm j is a function mapping $(\bar{a}_{i,t}, s_{i,t}^j, k_{i,t}, p_t)$ to $w_{i,t}^j$.

2.3 Equilibrium

An equilibrium in this economy is a set of prices $\{p_t\}$, firm profit levels $\{\pi_t\}$, and strategies and beliefs for each agent such that:

1. At price level p_t and profit level π_t, the strategies and beliefs of the agents form a perfect Bayesian equilibrium.
2. At price level p_t the goods market clears, i.e.,

$$\begin{aligned} \sum_{i=1}^{I} x_{i,t} &= \sum_{i=1}^{I} \left(1 - \sum_{j=1}^{I} n_{i,t}^j\right) \\ \sum_{i=1}^{I} y_{i,t} &= \sum_{i=1}^{I} \sum_{j=1}^{I} a_{i,t}^j n_{i,t}^j. \end{aligned}$$

3. The profits received by the young workers are equal to the average profit of the firms, i.e.,

$$\pi_t = \frac{\sum_{j=1}^{I}\left(p_t a_{i,t}^j - w_{i,t}^j\right) n_{i,t}^j}{I}.$$

3 Equilibrium properties of the model

This section characterizes the predictions of this model for both the short-run (one-period) and long-run aggregate behavior of employment, wages, and output. Let $\mathbf{n}_t$ denote the vector of employment status for generation t:

$$\mathbf{n}_t \equiv (\ldots, n_{i-1,t}, n_{i,t}, n_{i+1,t}, \ldots). \tag{10}$$

In this model, $\mathbf{n}_t$ provides the dynamic link across time.

3.1 Short-run dynamics

Because of the CES preferences, the market clearing price for the commercial good is:

$$p_t = p\left(\frac{x_t}{y_t}\right)^{1-\sigma} \tag{11}$$

where x_t and y_t are aggregate output of the home-produced and commmercially-produced goods.

Let $w_{i,t}$ be the equilibrium wage earned by agent i,t. In equilibrium, the worker chooses the highest wage offered, so $w_{i,t} = \max\{w_{i,t}^j, 1\}$ From the firm's perspective, labor is sold to the market in a first price auction with private valuations and a floor equal to the value of one unit of the home-produced good. Each firm j has a valuation of worker i,t equal to $p_t E(a_{i,t}^j | \bar{a}_{i,t}, s_{i,t}^j)$. In terms of the variables described so far:

$$p_t E(a_{i,t}^j | \bar{a}_{i,t}, s_{i,t}^j) = \frac{p_t}{1 + \frac{1-\bar{a}_{i,t}}{\bar{a}_{i,t}}\left(\frac{f\left(s_{i,t}^j | a_{i,t}^j = 0\right)}{f\left(s_{i,t}^j | a_{i,t}^j = 1\right)}\right)}. \tag{12}$$

Any firm that receives a signal that the worker is worth more than the reservation wage will bid more than the reservation wage. Let $s_{i,t}^{\min}$ denote the value of $s_{i,t}^j$ such that employer j is indifferent about hiring agent i,t at the reservation wage:

$$s_{i,t}^{\min} \equiv S \qquad \text{such that} \qquad E\left(p_t a_{i,t}^j \,\middle|\, s_{i,t}^j = S, \bar{a}_{i,t}\right) = 1. \tag{13}$$

In equilibrium, if $\bar{a}_{i,t} \geq \frac{1}{p_t}$, then $n_{i,t} = 1$. Otherwise $n_{i,t}$ is governed by the following conditional distribution:

$$\Pr(n_{i,t} = 1 | \Gamma, \mathbf{n}_{t-1}, \mathbf{n}_{t-2}, \ldots) = \Pr(n_{i,t} = 1 | k_{i,t}) \tag{14}$$

$$= 1 - (1 - q_{i,t})^{k_{i,t}} \tag{15}$$

$$\text{where: } q_{i,t} \equiv \int_{s_{i,t}^{\min}}^{1} \bar{a}_{i,t} f(s | a_{i,t}^j = 1) + (1 - \bar{a}_{i,t}) f(s | a_{i,t}^j = 0) ds. \tag{16}$$

In addition, $n_{i,t}$ and $n_{i',t}$ are independent for $i \neq i'$ conditional on $\mathbf{n}_{t-1}$ and Γ.

Wages are increasing in the number of contacts a worker possesses. In other words, the wage of a worker with k contacts is first order stochastically dominated by that of an identical worker with more than k contacts:

$$\Pr(w_{i,t} < w | \bar{a}_{i,t} = \bar{a}, k_{i,t} = k) \geq \Pr(w_{i,t} < w | \bar{a}_{i,t} = \bar{a}, k_{i,t} = k+1). \tag{17}$$

3.2 Social networks and neighborhood poverty

As stated earlier, my motivation for modeling job networks in this way is to explain some patterns in neighborhood poverty. As Wilson [27] emphasizes, most poor neighborhoods before 1970 had reasonably high employment, even if wages were low. Since then many poor neighborhoods have become what he calls "jobless ghettos" – neighborhoods in which unemployment is the rule rather than the exception. Wilson's argument is that interdependency in employment via job networks creates a feedback effect that leads to this pattern. Social multipliers induced by this neighborhood effect lead to a nonlinearity in the relationship between the economic fundamentals (education levels, for example) of a community and the employment outcomes experienced by its members. A primary purpose of this paper is to establish conditions under which such critical behavior will appear. In the context of the model presented here, I am interested in the relationship between the distribution of human capital $\bar{a}_{i,t}$ in a community, the properties of the social network Γ, and the long-run employment rate. Under certain conditions this relationship exhibits multiple long-run equilibria and a threshold nonlinearity at some critical level of human capital.

In order to analyze long-run employment at the neighborhood level, I make a few assumptions. In this model a neighborhood is a collection of workers which are connected to one another socially in Γ but not connected to any other workers. Each neighborhood contains many workers at a given point in time but is small relative to the economy as a whole. Such a neighborhood can be modeled as an economy of its own in which Γ is a component of a larger network and the home-produced and commercial goods are perfect substitutes. For the remainder of the paper, I assume $\sigma = 1$, which implies

$$p_t = p \qquad \forall t. \tag{18}$$

The assumption that the goods are perfect substitutes is needed for most of the results in the remainder of this paper, particularly those that relate to the critical behavior of the system. This assumption is only sensible for a community or social network which is large enough that aggregate community outcomes are fairly predictable, but small enough to have very little effect on relative prices. Most urban communities have several thousand working-age residents, and the United States economy has 150-200 million. Since I am interested in communities within such a large economy, the assumption of fixed price can be justified.

In addition, I assume workers are homogeneous with respect to ability:

$$\bar{a}_{i,t} = \bar{a} \qquad \forall i, t. \tag{19}$$

This assumption allows me to isolate the social network's role in generating the distribution of employment and wages. I also assume:

$$\bar{a} < \frac{1}{p} < 1. \tag{20}$$

It is then straightforward to establish that:

$$q_{i,t} = q < 1. \tag{21}$$

The following sections will discuss the relationship between q and long-run employment.

3.3 Long run dynamics - General case

As in many macroeconomic models, the aggregate behavior of the model in the long run is of primary interest. The long run analysis in the next few sections focuses primarily on the evolution of $\mathbf{n}_t$, and its relationship with neighborhood human capital. As noted previously, the distribution of employment provides the dynamic link in this model. The results of Section 3.1 can be combined with results on long-run employment to develop implications about the long-run dynamics of wages and output.

In order to look at long-run dynamics it will be helpful to note that the network Γ, the offer rate q, and the initial condition $\mathbf{n}_0$ can be used to define a *percolation* process. This characterization provides a number of mathematical tools (see Grimmett [12]) which have been developed for the analysis of such processes. Take the social network Γ and designate each edge as "open" with probability q and "closed" otherwise. This is called a percolation graph. Node $i, 0$ is called "wet" if $n_{i,0} = 1$. If there is a path of open bonds from a wet node to a node i, t, then that node is wet as well. The terminology arises from the use of percolation to model flows of fluids through an irregular medium, e.g., water or petroleum through rocks. It is straightforward to show that this percolation process is equivalent to the dynamics of the model in this paper. For a given social network Γ, offer rate q and initial condition $\mathbf{n}_0$, the probability of a given worker i, t being employed is equal to the probability of node i, t being wet in the associated percolation process. An example of a percolation graph appears in Figure 2. This model is also equivalent to the contact process often used to model interacting particle systems (see Liggett [16]). A variant of the contact process similar to that seen here has been estimated by Topa [24] for Chicago census tracts.

Intuitively, a better initial condition, a higher offer rate, or a denser network should improve long-run employment rates. The following proposition formalizes that result.

Proposition 3.1 (Monotonicity of employment rate) *For all i and $t \geq 0$,* $\Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0)$ *is increasing in all three arguments, i.e.,*

1. $q \geq q'$ *implies* $\Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0) \geq \Pr(n_{i,t} = 1 | q', \Gamma, \mathbf{n}_0)$.
2. $\Gamma \supset \Gamma'$ *implies* $\Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0) \geq \Pr(n_{i,t} = 1 | q, \Gamma', \mathbf{n}_0)$.
3. $\mathbf{n}_0 \geq \mathbf{n}_0'$ *implies* $\Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0) \geq \Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0')$.

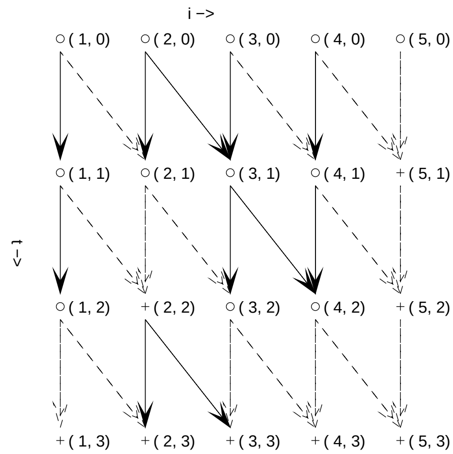

Figure 2: An example of a percolation graph. Open edges are indicated by solid lines, closed edges by dashed lines. Wet nodes are indicated by circles.

Proof: Create two percolation graphs from Γ. Assign a random number z drawn from the uniform (0,1) distribution to each bond in the first graph. Assign the same number to the associated bond in the second graph. In the first graph, mark all bonds as open if $z < q$, closed otherwise. In the second graph, mark all bonds as open if $z < q'$, closed otherwise. By construction, these two graphs represent the two percolation processes. Also by construction, every bond that is open in the second graph is also open in the first graph. If node i, t is wet in the second graph, it must also be wet in the first graph. The same basic coupling argument can be used for Γ and $\mathbf{n}_0$.

It is also possible to characterize the limiting behavior of employment. A permanent state of full unemployment is possible in this model, and guaranteed for the finite case.

Proposition 3.2 *For any q and Γ:*

$$\Pr(n_{i,t} = 1 | q, \Gamma, \mathbf{n}_0 = \mathbf{0}) = 0 \qquad t \geq 0.$$

Proof: This can be verified by inspection.

Proposition 3.3 *If I is finite and $q < 1$, then for any initial condition $\mathbf{n}_0$ and network Γ:*

$$\lim_{t \to \infty} \Pr(n_{i,t} | q, \Gamma, \mathbf{n}_0) = 0.$$

Proof: From any state $\mathbf{n}_t$, the probability of moving in the next period to the state $\mathbf{n}_{t+1} = \mathbf{0}$ is strictly positive. By Proposition 3.2, the probability of moving from that state to any other state is zero. Therefore it is the only recurrent state, and the process will eventually settle there with probability one.

Many infinite networks have an associated critical value, which will be denoted q_c. Below this critical value, the unique long-run equilibrium is zero employment. Above this critical value a second long-run equilibrium with positive employment emerges. A critical value of this type is of interest for two reasons. First, above the critical value, the long-run outcome will be sensitive to initial conditions. Second, as will be shown in subsequent sections, the relationship between q and long-run outcomes can exhibit interesting nonlinearities at q_c.

By Proposition 3.1, q_c is a well defined quantity on the interval $[0, 1]$. In order for q_c to be of interest, it must be in the interior of that interval. Proposition 3.4 shows that $q_c > 0$.

Proposition 3.4 (Existence of subcritical phase) *For any network* Γ *there exists* $q_c > 0$ *such that* $q < q_c$ *implies* $\lim_{t\to\infty} \Pr(n_{i,t}|q, \Gamma, \mathbf{n}_0) = 0$.

Proof: By earlier assumption, the number of edges into a node is bounded above by some finite number d. The probability of an open path of length t is no greater than the expected number of such paths. The number of possible open paths of length t can be no greater than d^t. If edges are open with probability q, then the expected number of open paths of length t leading to i is no greater than $d^t * q^t$. If $q < 1/d$, (and d is finite by assumption) then this number goes to zero as t goes to infinity. This implies that $q_c \geq 1/d > 0$.

A positive-employment long-run equilibrium exists only if $q_c < 1$. The conditions under which this occurs can be best understood by looking at two special cases. In the perfect social mobility case, agents are matched randomly every period so there is no persistence over time in ties between successive workers. In the perfect social rigidity case, ties between successive workers are permanently fixed. A variety of intermediate cases in which social ties change slowly can also be constructed.

3.4 Long-run dynamics: Perfect social mobility

In the case of perfect social mobility, the social network shows no persistence over time. This benchmark case can be solved directly, and can be used to shed light on the properties of models with more general networks. The Γ with perfect social mobility can be constructed as follows. For each worker assign a connection from r randomly selected workers of the previous generation. Figure 3 shows an example of such a network. Define:

$$\bar{n}_t \equiv \frac{1}{I} \sum_{i=1}^{I} n_{i,t}. \tag{22}$$

$\bar{n}_t$ represents the fraction of the population employed at time t. Since all connections have equal probability, the probability of a randomly selected connection having a job at time t is always $\bar{n}_t$.

Proposition 3.5 *Conditional on* $\mathbf{n}_{t-1}$, $\bar{n}_t$ *has a binomial distribution with* n *equal to the number of agents and* p *equal to:*

$$1 - (1 - q\bar{n}_{t-1})^r.$$

Proof: Can be verified by inspection.

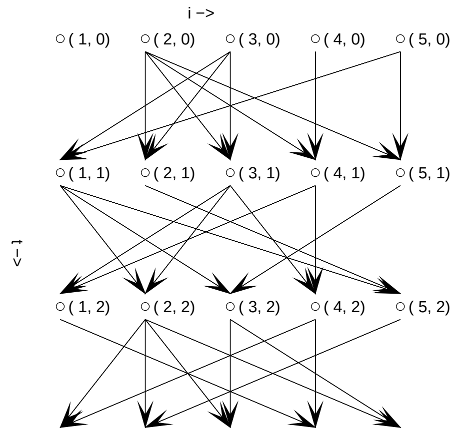

Figure 3: An example of a network with social mobility. Connections are assigned at random, but each agent has the same number of ties to the previous generation.

If the economy has a finite number of agents, then Proposition 3.3 implies that $\lim_{t\to\infty} \bar{n}_t = 0$ with probability one. Movement into this state, however, may take a very long time for a large economy. To investigate the behavior of a large economy, consider the case of an infinite economy. When I is infinite, the binomial distribution of Proposition 3.5 can be replaced with the following deterministic difference equation:

$$\bar{n}_{t-1} = 1 - (1 - q\bar{n}_{t-1})^r. \tag{23}$$

By inspection, equation (23) passes through the origin, and is continuous, differentiable, and concave (strictly concave for $r \geq 2$) in $\bar{n}_{t-1}$. Figure 4 shows equation (23) for different values of q. Because the difference equation passes through the origin, there is always a long-run equilibrium with zero employment. For low values of the offer rate q (keeping network density r fixed), this equilibrium is unique and stable. For higher values of q, the long-run equilibrium with zero employment becomes unstable and a stable long-run equilibrium with positive employment emerges.

Intuitively, this behavior can appear for the infinitely large economy because the size of the economy provides a form of insurance against a bad draw of employment. For a large but finite economy, simulation shows that the long-run equilibrium for the infinite economy is a good approximation over a fairly long time period. Figure 5 shows some simulation results for an economy with 1000 agents, as well as for an infinite economy. Note that the employment rate for the finite economy stays close to the equilibrium for the infinite case for a very long time. In contrast, when the economy is simulated with 10 agents, employment goes to zero within a few periods. These results suggest that this distinction between the finite and infinite economies can be carried over qualitatively to a distinction between small and large economies. A large but finite economy with a high enough value of q will likely have employment stay

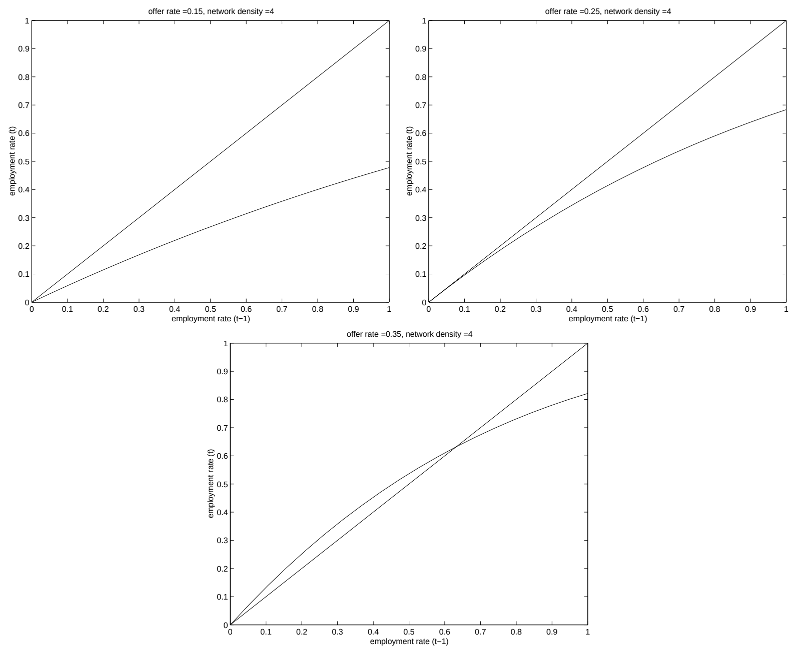

Figure 4: The difference equation $\bar{n}_t = 1 - (1 - q\bar{n}_{t-1})^r$ for $r = 4$ and $q = 0.15, 0.25, 0.35$.

above zero for a very long time. A small economy is much more likely to have bad luck in a single time period that kills off employment permanently. Empirical work suggests that residents of poor communities have more ties to family and close neighbors, whereas more affluent individuals tend to have a significantly greater dispersion of social ties both geographically and by employer. If the social structure in an economy can be thought of as a large connected network dispersed across the country (mainstream society) combined with a set of small isolated networks (ghetto communities) then it would not be surprising to see high stability of employment in the mainstream of society and high employment instability in more isolated communities even if all agents were identical with respect to number of connections, preferences, and ability.

Since the long-run equilibrium of the infinite economy is a good approximation to the behavior of large, finite economies, equation (23) can be used to investigate the relationship between long-run employment and the offer rate q. While equation (23) does not have a closed-form solution, q_c does. It is the q that solves:

$$\left. \frac{d\,\Pr(n_{i,t+1} = 1|\bar{n}_t)}{d\bar{n}_t} \right|_{\bar{n}_t = 0} = 1. \tag{24}$$

Solving equation (24) for q:

$$q_c = \frac{1}{r}. \tag{25}$$

Figure 6 shows this relationship graphically.[1]

[1]Note that this graph treats r as a continuous variable, whereas the previous discussion treats it as an integer. Let $r = r_0 + r_1$ where r_0 is an integer and r_1 is a real number in $[0, 1]$. Then each agent has r_0 contacts with certainty

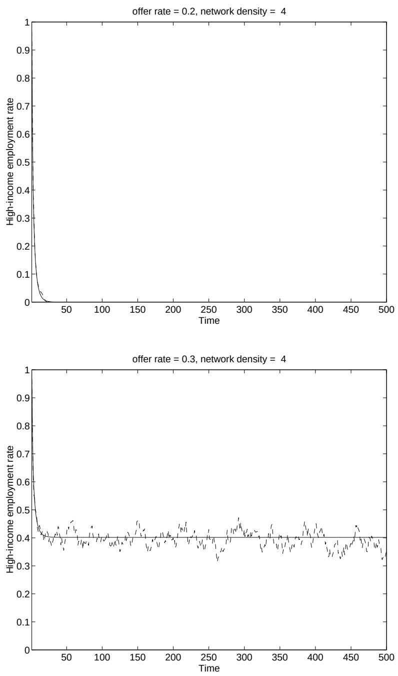

Figure 5: Simulated economies for the perfect social mobility case. Contact density $r = 4$, offer rate $q = 0.2, 0.25, 0.3, 0.4$. Solid line depicts infinite economy, dashed line depicts simulation results for 1,000 agents.

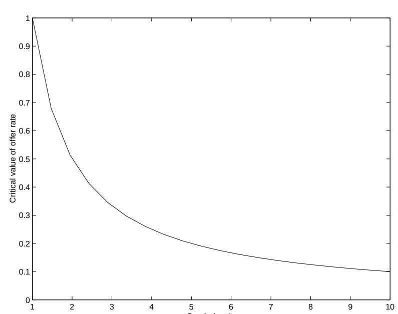

Figure 6: The critical value q_c as a function of graph density r for the case of perfect social mobility.

The relationship between q and long-run employment can be solved numerically. Figure 7 shows this relationship for several values of the network density r. Note that there is a threshold nonlinearity at q_c. Simulations for a variety of networks show that this nonlinearity is a robust property of the model, and the nonlinearity is the primary result of this paper.

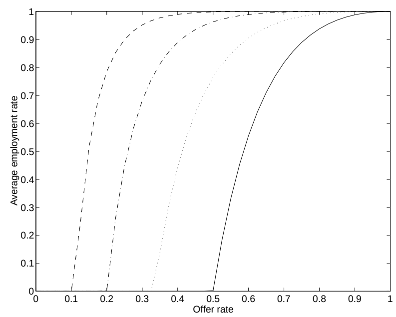

Figure 7: The relationship between long-run average employment $\bar{n}$ and offer rate q for the case of perfect social mobility with an infinite number of agents. Network density r is 2 (solid line), 3 (dotted), 5 (dash-dot) and 10 (dash).

As Figure 7 shows, a model of job networking can generate aggregate neighborhood patterns of the sort discussed by Wilson. Small changes in the employment prospects of the individuals in a neighborhood have impact on the employment prospects of others in the neighborhood, creating a social multiplier. If this social multiplier is strong enough, as in this model, there are large changes in neighborhood employment outcomes resulting from small changes in neighborhood composition. This type of nonlinearity has been an area of substantial recent interest in the theoretical literature on neigborhood effects and inequality. In contrast to this paper, the literature focuses on spillovers in the accumulation of human capital. These spillovers either affect human capital accumulation directly [2, 10] or by affecting the incentive to accumulate human capital [6, 18, 17, 23]. Education and human capital accumulation are clearly important for understanding intergenerational movements in income, and the developmental impact of living in an extremely poor neighborhood on children and adolescents is a justifiably active research area. However, these models have a significant problem in explaining the dynamics of neighborhood inequality observed since 1970. The majority of residents in almost any neighborhood are many years past school. Models of ghetto formation which rely on differences in human capital formation can only produce changes from one generation of workers to the next.

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and probability r_1 of having one more contact. Then

$$\Pr(n_{i,t} = 1|n_{t-1}) = 1 - (1 - q\bar{n}_{t-1})^{r_0}(1 - q\bar{n}_{t-1}r_1)$$

and $q_c = \frac{1}{r_0+r_1}$.

In contrast, the growth in the prevalence of social problems and unemployment in poor urban neighborhoods since 1970 has occurred rapidly and across age groups. In addition, there is substantial variation across cities in the formation of new high-poverty neighborhoods, with most new ghettos appearing in older cities of the Midwest and Northeast. For example, the number of high-poverty neighborhoods in Milwaukee increased from nineteen in 1980 to fifty-one in 1990 [14]. In a related empirical paper [15] I consider these issues in greater detail, and find some support for the existence of a threshold nonlinearity in the relationship between education and employment in United States Census tracts.

3.5 Long-run dynamics: Perfect social rigidity

Much of the previous analysis can be repeated for the case that there is some degree of intergenerational persistence in social ties. One convenient special case is that of perfect social rigidity, in which a connection from i, t to $j, t+1$ implies a connection between i, T and $j, T+1$ and there is always a connection from i, t to $i, t+1$. In this case Γ can be represented by a simpler graph Γ_0 in which the node set is just $\{1, 2, 3 \ldots, I\}$ and there is an edge from i to j if and only if there is an edge in Γ from i, t to $j, t+1$.

Most of the qualitative results in the previous section apply when the network is fixed. Unfortunately, this case is less tractable than the perfect mobility case. The following proposition proves the existence of a critical value under certain conditions:

Proposition 3.6 (Existence of critical value) *Suppose Γ_0 has an infinite Hamiltonian path to i. Then there exists $q_c \in (0,1)$ such that*

$$\lim_{t\to\infty} \Pr(n_{i,t}|q, \Gamma, \mathbf{n}_0 = \mathbf{1}) = 0 \qquad \textit{if } q < q_c$$

and

$$\lim_{t\to\infty} \Pr(n_{i,t}|q, \Gamma, \mathbf{n}_0 = \mathbf{1}) > 0 \qquad \textit{if } q > q_c.$$

Proof: Propositions 3.1 and 3.4 prove everything but that $q_c < 1$. It is known that $q_c < 1$ for the case that $\Gamma_0 = \{(i, i+1)\}$ (See Liggett [16]). Since Γ_0 contains this network, apply Proposition 3.1.

In the fixed network case, the long-run relationship between q and the employment rate as well as the corresponding critical value q_c is difficult to calculate. However, simulations can be used to estimate these quantities. Figure 8 shows this estimated relationship for four loop-style social networks. In an "r-loop" network structure each worker is connected to the r nearest neighbors in the next generation. For example, in a 2-loop, agent i, t is connected to agents $i, t+1$ and $i+1, t+1$. Connections wrap around so that I, t is connected to $1, t+1$. In a 3-loop agent i, t is also connected to agent $i-1, t+1$. As the figure shows, the relationship between neighborhood composition and neighborhood outcome is qualitatively similar to that seen in the perfect-mobility case (Figure 7). Again, the relationship exhibits a threshold nonlinearity in a neighborhood around the critical value.

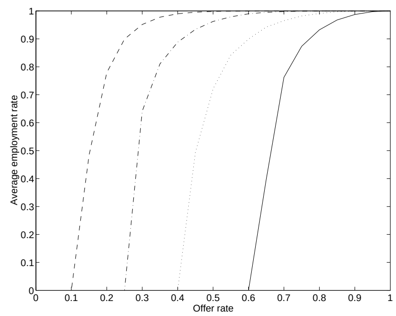

Figure 8: The relationship between long-run average employment $\bar{n}$ and offer rate q for the fixed network case. Relationships estimated by simulation of network with 1,000 agents for 2,000 periods. Networks are 2-loop (solid line), 3-loop (dotted), 5-loop (dot-dashed), and 10-loop (dashed).

3.6 The role of network structure

One advantage of the approach in this paper is that it allows for the analysis of a wide class of social networks. As pointed out by Page [20], a number of papers on social interactions are restricted to the use of simple grid or loop interaction structures, and may not be robust to more plausible structures. Computational modeling opens up a richer class of interaction structures which may be relevant to the problem at hand. For example the earlier results obtained for the special cases of perfect mobility and perfect rigidity have been verified for more complex networks.

One question of interest is the relative benefit or cost of social mobility. Figures 7 and 8 show that for a given network density and offer rate, the socially mobile network has a higher long-run employment rate than the socially rigid network. The reason for this is that the socially rigid network produces a more unequal distribution of employment contacts. In a socially rigid network, workers tend to have contacts with either many employed friends or with none. Since the marginal effect of an additional contact decreases with the number of contacts, a more unequal distribution of contacts will result in lower subsequent employment. Experiments in which a varying fraction of connections is changed each period verifies this conjecture, as the average employment rate is increasing in the rate of change in the connections.

The benefits of social integration in this model are also of interest. An orderly loop network in which workers only interact with those which are spatially close (low social integration) may perform differently from a network in which workers also interact with a few far-away workers (high social integration). In a recent paper, Watts and Strogatz [25] note that random networks tend to have shorter typical paths from one node to another than loops with the same number of edges. This property carries over to networks that are between loops and purely random graphs. Such a network can be constructed according to the following algorithm. Select two nodes i and j of the loop network at random. Then select one of i's connections, call it i', and one of j's connections, called j'. Delete the connection between i and i' as well as that between j and j'. Add a connection between i and j as well as one between i' and j'. Switch x percent of the total number of connections in the network in this manner. Call this an "x percent randomized" network. The relationship between social integration and employment outcomes is analyzed here by the following experiment. I estimate the employment-offer rate relationship for each of the various loop networks as depicted in Figure 8, then repeat the process for the 10% randomized version of that network, the 20% randomized version, etc. Surprisingly, this relationship is virtually identical no matter what the degree of randomization. In this model, social integration does not seem to matter.[2]

4 Conclusion and Further Directions

This paper develops a stylized equilibrium model of the labor market in which the employment probabilities of workers depend critically on personal ties to other workers, as well as on economy-wide initial conditions. An economy with these properties turns out to exhibit a number of interesting threshold effects in the relationship between long-run outcomes and model parameters. In this model, there is a critical level of neighborhood human capital below which long-run employment is low, and above which long-run employment is significantly higher. Small changes in effective human capital in a community, particularly a community starting with low human capital or weak social networks, can produce large changes in community employment rates.

The most likely extension of this work is to make human capital and the social network responsive to economic incentives. The current model treats these as exogenous in order to consider a wide variety of network structures and human capital distributions. In addition, this model suggests some empirical predictions, namely that the neighborhood-level relationship between average community human capital and unemployment rates exhibits a threshold nonlinearity. Preliminary empirical results [15] provide support for this prediction.

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[2] This is not such a surprise for the 2-loop, as all connected 2-regular graphs of a given size are isomorphic, or identical up to a relabeling. Thus by randomizing a nearest-neighbor loop, you get a nearest neighbor loop, or maybe two smaller loops. This is not the case for regular graphs with more connections per node.

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