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Lg WAVES AND STRUCTURAL BOUNDARIES

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ABSTRACT

The propagation of *Lg* waves in complex media can be described either by means of a modal superposition scheme with numerical integration through the heterogeneity or by using ray diagrams. Rays are set off at equal horizontal intervals in a stratified zone adjacent to the heterogeneity, with phase velocities appropriate to particular modes. The constructive interference pattern in the stratified medium is modified in the horizontally varying region to give a graphic illustration of the propagation effects. The ray method agrees well with the modal calculations but may be conveniently applied in more general circumstances, e.g., to include surface topography or permanent changes in crustal structure.

Structural boundaries which involve sudden thinning of the crustal wave guide are particularly disruptive to the *Lg* train, as at a pinch in crustal thickness or the continent-ocean transition. The effects of localized thickening are more subtle. The relatively sharp cutoff for *Lg* waves in some structures, e.g. in the Tibetan Plateau, can be explained by the source being no longer able to couple into the crustal wave guide at the receiver.

INTRODUCTION

The most prominent regional phase generated by shallow events is the *Lg* phase identified by Press and Ewing (1952). On short-period seismograms, *Lg* has a complex structure with no clear onset, but a well-defined amplitude maximum with a group velocity of around 3.5 km/sec. It has a long coda with energy frequently visible to group velocities of 2.9 km/sec and in regions with thick sediments to even slower group speeds. There are two ways of describing the propagation of *Lg* waves in a stratified medium. First, it can be associated with a superposition of many higher modes of surface waves which interfere to give the complex observed waveforms. The important contributions come from the stationary portions of the group velocity curves for about the first 10 modes up to 1 Hz (Levshin, 1973; Knopoff *et al.*, 1973), and multi-mode synthetic seismograms can produce quite realistic records at short periods (see, e.g., Kennett, 1983, chapter 11). These modes correspond to waves trapped in the wave guide formed by the lower seismic velocities in the crust, compared to the mantle beneath. An alternative viewpoint is that the *Lg* train consists of the interference of multiply reflected *S* waves bouncing back and forth between the crust-mantle boundary and the free surface, as can be seen in the theoretical seismograms presented in Figure 1 of Kennett (1985). The *Lg* phase has recently received considerable attention because of potential use in discriminating between earthquakes and small explosions at regional distances (see, e.g., Blandford, 1981). It has also been used to establish a widely used magnitude measure m_{bLg} (Nuttli, 1973; North, 1985).

However, it has long been recognized that *Lg* is sensitive to changes in structure along its path, which can have deleterious effects on any role as a discriminant or magnitude measure. For example, only 100 to 200 km of oceanic path is sufficient to block the transmission of *Lg* (Ewing *et al.*, 1957). A number of studies have used the sensitivity of *Lg* to structural effects to map regions of anomalous propagation and try to associate them with crustal structure. Thus, Ruzaikin *et al.* (1977) and Ni and Barazangi (1983) have considered *Lg* propagation across the complex

Himalayan, Tibetan, Tien Shan, Pamirs regions with emphasis on paths to the north and south, respectively. A similar qualitative study using WWSSN records was made by Kadinsky-Cade *et al.* (1981) for the Turkish and Iranian Plateaus. In all of these studies, the density of available paths was limited, so only major features could be mapped. Where a dense coverage of paths is available, it is possible to produce a relatively detailed map of apparent heterogeneity. This has been done for the North Sea region by Kennett *et al.* (1985), building on the earlier work of Gregersen (1984). Over 150 different paths were used to stations all around the North Sea basin, and the transmissivity for *Lg* waves was mapped onto 1° by 1° cells, clearly outlining the known graben structures in the centre of the basin.

Despite the successful application of the character of *Lg* waves to the delineation of crustal structure, quantitative descriptions of the interactions are limited. Clarke (1982) modified methods used in acoustics and fibre optics to consider the transmission of guided *SH* waves by the introduction of coupling between different modes of the crustal wave guide system. Kennett (1984) used a similar description of the wave field, but was able to extend the treatment to reflected and transmitted waves for both Love and Rayleigh modes. This approach was subsequently applied by Kennett and Mykkeltveit (1984) to explain the severe attenuation of *Lg* waves crossing the Central Graben zone of the North Sea by a combination of thick sediments and a raised Moho.

In this paper, we investigate the effect of a variety of classes of structural disruptions of the crust on the propagation of *Lg* waves. We will use the modal decomposition technique of Kennett (1984), complemented by ray diagrams to give a graphic representation of the structural effects.

MODELING THE EFFECT OF STRUCTURE ON *Lg*

A description of the *Lg* phase in complex media must take into account the nature of the phase as an interference phenomenon either between higher surface-wave modes or between multiple *S*-wave reflections.

The method introduced by Kennett (1984) relies on the description of the displacement field as a superposition of the higher modes of Love and Rayleigh waves for a reference, stratified, medium. At a fixed frequency, both forward- and backward-traveling waves for each of the trapped modes have to be included to give a full representation of the field. The variation of the modal amplitude coefficients with position on a laterally heterogeneous region is determined by a set of coupled differential equations. Reflection and transmission problems can be handled by welding the heterogeneous zone onto the reference structure at each side. Direct calculations can be made for the redistribution of energy between modes corresponding to reflection and interconversion by solving a set of matrix Ricatti equations. Although the technique is quite complicated, quantitative measures of surface-wave propagation through a wide variety of heterogeneous structures can be readily made once the modal eigenfunctions have been determined. The sensitivity of the various modes to heterogeneity depends on the shape of the eigenfunctions; for example, at higher frequencies, low-order modes tend to have energy concentrated in any sedimentary layers and so are most affected by changes in this region.

This modal approach can, in principle, be adapted to a slowly varying reference model but is most computationally attractive with a stratified reference. This means that it is well suited to looking at velocity perturbations without changes in the depths of interfaces. However, where boundaries are modified, they must return to their original level.

The modal interaction scheme works at fixed frequency, and so defines a characteristic phase velocity for each mode of the reference structure; e.g., at 1 Hz, there are 18 modes in Figure 1 with phase velocity less than 5.0 km/sec. This combination of phase velocity and frequency is such that reflections from the free surface interfere constructively with the reflections from the structure. For the phase velocities involved in *Lg* propagation (around 4.60 km/sec or less), propagating modes are confined to the crust, and the dispersion curves are controlled by the variation with phase velocity, of the crustal phase delay for *S*-wave propagation for both Love and Rayleigh waves (Kennett and Clarke, 1983).

With each modal phase velocity, we can associate a system of *S* rays with an inclination to the vertical in each layer determined by Snell's law. If these rays are set off at equal increments in horizontal position at the top of the stratified structure, they will establish a repetitive pattern of reflections corresponding to the constructive interference condition. If, however, a set of such rays is allowed to impinge on a horizontally varying structure, the rays no longer follow such simple propagation patterns, and the distortions of the ray paths give a clear indication of the way in which different portions of the *Lg* train are affected (see Figures 2 to 4). The constructive interference argument is fraught with difficulties in laterally heterogeneous media (Gregersen, 1976), and so we will assume that we have a well-established "coherent" field arriving from a reference, stratified, medium attached to the left-hand side of the model. For illustrative purposes, we have taken a simple uniform velocity throughout the crust (3.4 km/sec) overlying a uniform mantle of 4.6 km/sec. All perturbations to the crustal wave guide are defined by the shape of the boundaries. However, there is no difficulty in incorporating more complex velocity distributions into the ray tracing scheme.

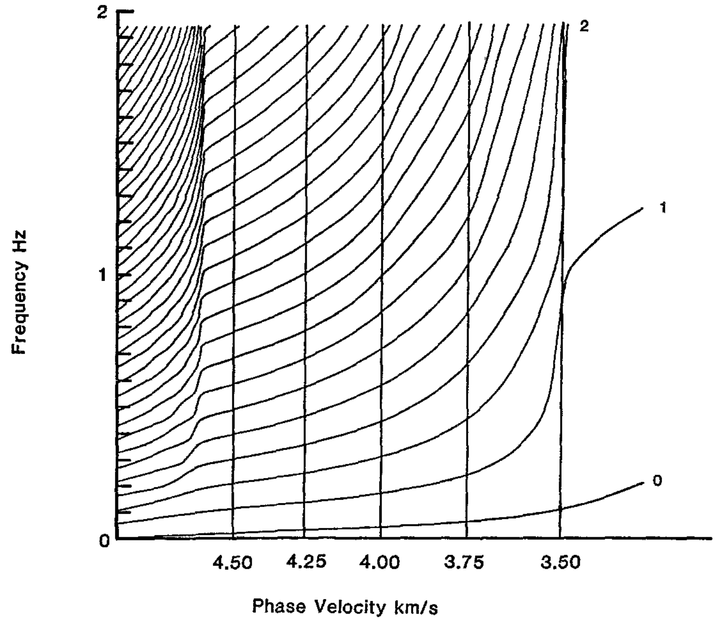

FIG. 1. Dispersion curves for a typical crustal-mantle model with 2 km sedimentary cover (Bouchon, 1982) showing how the secular equation defines phase velocities corresponding to different modes at a sequence of increasing frequencies. Each such combination leads to constructive interference between surface-reflected waves and those reflected back from the stratified medium.

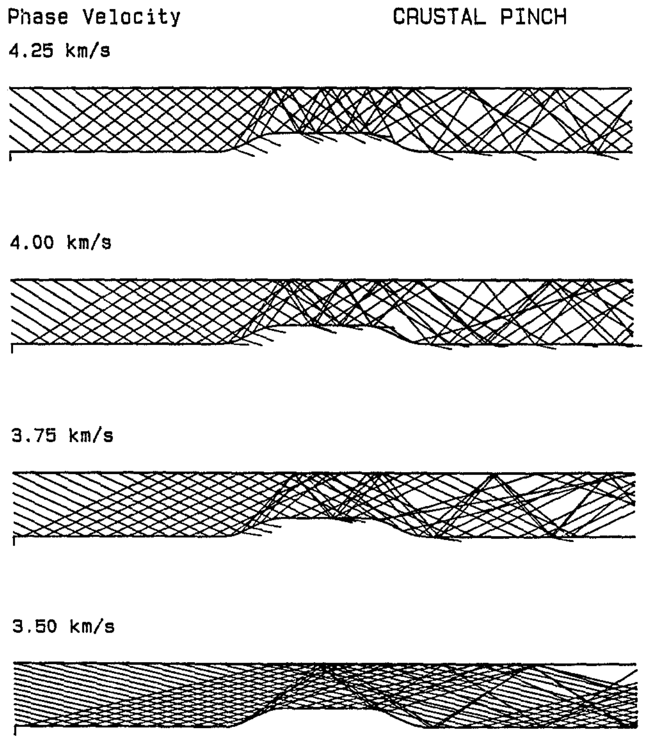

FIG. 2. Ray pattern for a range of phase velocities, indicated in Figure 1, impinging on a simple model of a crustal pinch, where the Mohorovičić discontinuity is shallow. The diagram is drawn to true scale. Ray segments below the crust-mantle boundary indicate leakage out of the crustal wave guide.

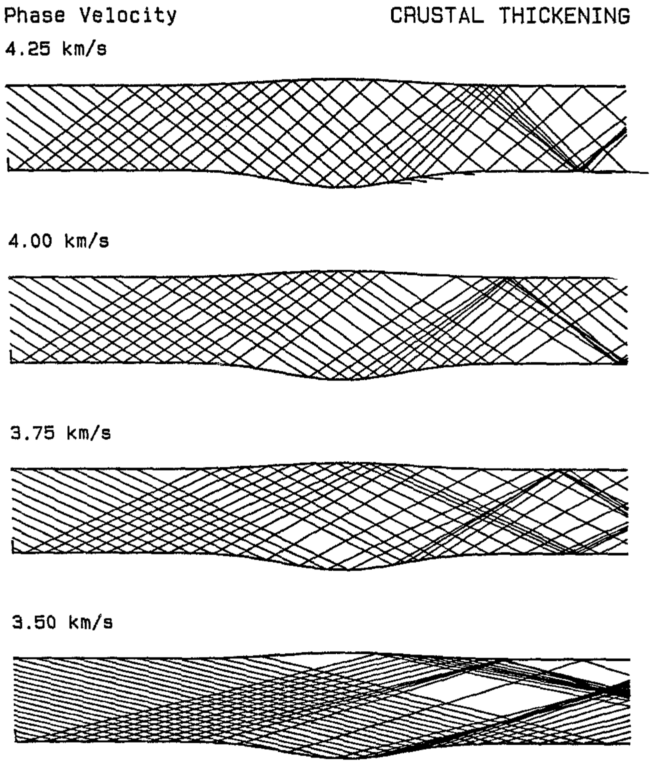

FIG. 3. Ray patterns for a localized zone of crustal thickening for the same set of phase velocities as in Figure 2, drawn to true scale.

The ray pattern is dictated by the imposed phase velocity and may, therefore, be associated with a sequence of modes at different frequencies (Figure 1). Alternatively, we can view the change in propagation characteristics with phase velocity as confining attention to an individual mode at different frequencies. With the ray scheme, we are restricted to a graphic presentation of most of the results, but are no longer so tied to the reference structure. It is, therefore, easier to incorporate surface topography (Figures 3 and 4) or to consider cases where there is a permanent horizontal change in structure (Figure 4). In order to allow a comparison between the modal decomposition method and the ray technique we have just introduced, we will start by considering structures with horizontally limited zones of deviation from stratification.

A crustal pinch. The first case we will study is one in which there is a significant elevation of the crust-mantle boundary over a zone about 100 km wide in a 30-km-thick crust (Figure 2). Ray diagrams are presented for a range of phase velocities spanning the range of importance for *Lg* propagation. As the phase velocity increases, corresponding to increasing mode number at fixed frequency, the angle of propagation steepens with a consequent change of the appearance of the regular interference pattern in the stratified section to the left. Once the waves enter the pinch, the ray patterns are significantly altered.

At the smallest phase velocity, corresponding to a low-order mode at low to moderate frequencies, the propagation angle is sufficiently shallow that much of the wave field ignores the pinch. However steeper traveling rays are also produced. These correspond to conversions to higher order modes at the same frequency. As the phase velocity increases, not all of the waves are perfectly reflected at the crust-mantle boundary and leakage from the crustal wave guide into the mantle occurs, corresponding to conversion of *Lg* to *Sn.* This effect is indicated in Figure 2 by short ray segments into the mantle, starting from the point of reflection at the angle of retraction. A feature which is common to all of the ray diagrams of Figure 2 is that once the pinch zone is entered, propagation occurs at a wide range of angles. These are usually both steeper and shallower than the incident waves, indicating conversion to a wide range of higher order and lower order modes at the same frequency.

These deductions are borne out by the results of detailed calculations using the modal decomposition approach. The particular levels of mode interconversion depend on the details of the shape of the crust-mantle boundary, but in all cases there is a redistribution of energy into a swathe of modes with orders above and below the incident mode. For low order modes, nearly all transmission occurs in the original mode. The higher *Lg* modes interact strongly with others, with conversion to lower order crustal-guided modes and also to the Sn-type modes whose energy is concentrated in the mantle. This example illustrates the considerable disruption which can be produced by constricting a wave guide. For frequencies around 1 Hz, much of the *Lg*-type energy would be associated with phase velocities around 4.25 km/sec and so be particularly susceptible to the effects of the pinch. For each mode, the conversions to other modes will give rise to new wave features moving at both higher and lower group velocities than before. The result of the scattering will be a tendency to increase the time duration of the *Lg* phase.

Crustal thickening. As a contrast to the previous example, we now consider a situation in which there is a localized increase in crustal thickness. For the ray diagram in Figure 3, the surface topography is based on a generalized profile of the Tien Shan Mountains, and a modest crustal root is included beneath. The changes to the ray patterns as the heterogeneity is entered are less severe for the higher phase velocities than for the crustal pinch, but are tending toward establishing a set of caustics associated with rays initially impinging on the far side of the crustal bump. For the lowest phase velocity (3.5 km/sec), the combination of the surface and Moho topography leaves rays traveling close to their original angles but imposes concentrations of rays with significant variations in amplitude some distance to the right of the crustal feature. The relatively modest change in ray inclinations means that coupling is confined to the nearest neighbor modes. A similar pattern holds for the higher phase velocities, with amplitude fluctuations to be expected for the *Lg* mode contributions to the right of the obstacle, together with reduced transmission in the original mode. The shape of the crustal root shown here does not produce very much leakage into the mantle, but in some cases this can be quite significant.

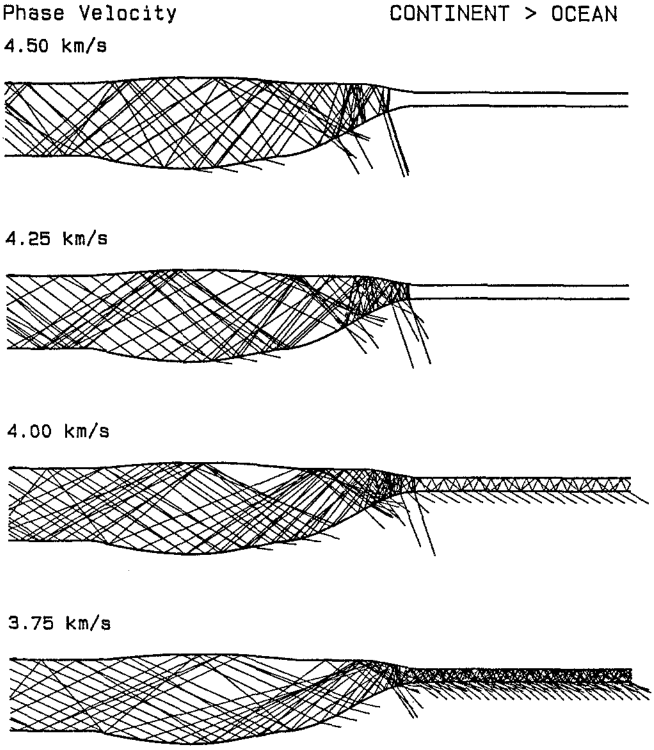

FIG. 4. Ray patterns for a continental to oceanic crust transition drawn to true scale. The sudden termination of the thicker wave guide gives rise to very significant reflections. Even when energy is injected into the oceanic crustal wave guide, it is associated with strong leakage into the mantle.

Once again, the ray picture agrees well with the results of full modal calculations. The lower order modes are transmitted well with some coupling to the nearest neighbors, whereas for the higher order modes, coupling into the neighboring modes is almost as important as transmission in the original mode. The level of *Lg* mode to Sn mode conversion is strongly dependent on the shape of the obstacle, but for that shown in Figure 3 is relatively weak.

Continental to oceanic crust transition. This is an example of a situation that is not readily amenable to full modal analysis, but which can be tackled well by the ray scheme. The true scale section shown in Figure 4 is based on the transition at the southeast Australian margin. We see that the rapid decrease in the size of the wave guide has the effect of inducing strong reflections at the continental margin, with only limited transmission into the oceanic crustal wave guide at the lower phase velocities. Even that energy which enters the oceanic crust is not well confined, and leakage into the mantle occurs with each successive reflection at the crust-mantle boundary. Because of the small thickness of the guide, such reflections occur frequently even with shallow incident angles on the margin zone. As a result, there is a very rapid loss of *Lg*-type energy in the oceanic zone, with up to 90 per cent of the energy incident on the left of the model lost after 100 km of oceanic path even for phase velocities of 3.75 km/sec. At frequencies above 0.2 Hz, most of the *Lg* energy is associated with phase velocities of 4.0 km/sec or higher which have even greater difficulties in matching into the oceanic guide. These calculations fit in well with the observations that only 100 to 200 km of oceanic path are needed to eliminate *Lg* (Ewing *et al.*, 1957) and confirm that the mismatch in the wave guides is likely to be the dominant effect.

At the continental margin, we see that there is strong coupling between the *Lg*-type waves and the mantle. If we invoke the principle of seismic reciprocity, this would imply that incident *Sn* energy in the mantle could readily be fed into the crustal wave guide in the form of *Lg*. This is in accord with the observations reported by Isacks and Stephens (1975) of an earthquake near Bermuda recorded at stations in the United States.

DISCUSSION

We have just seen how the simple device of ray diagrams can provide considerable insight into the nature of the propagation process as an *Lg* wave train travels into a heterogeneous medium, in circumstances where it is rather expensive and inconvenient to use direct numerical methods. In particular, we have observed the very considerable disruption associated with a sudden thinning of the wave guide both at the crustal pinch and at the continental margin. The effects of increasing crustal thickness are more subtle but none the less significant. However, it requires a considerable zone of varying crustal thickness to produce the same effect as that produced by halving the thickness of the crustal wave guide. In the figures, we have considered waves heading straight for the structural features so that interactions are maximized. With obliquely traveling waves, the apparent horizontal scale is stretched and so the effects of the boundaries are reduced, but the waves spend a longer time in a heterogeneous zone and so may suffer other distortions.

The ray diagrams show clearly the important role played by station position in assessing the effect of structure from observed *Lg* trains. In the study of Ruzaikin *et al.* (1977), the station at Talgar (TLG) which lies on the northern flank of the Tien Shan records clear, if somewhat complex, *Lg* waves from events in the Altyn Tagh range. These paths lie almost perpendicular to the trend of the Tien Shan and are thus in a similar configuration to that shown in Figure 3. A station just to the right of the thickened section would see only a mildly distorted field (as observed) whereas greater complications could occur further away from the range.

A feature noted by Ruzaikin *et al.* (1977) and Ni and Barazangi (1983) is that there appears to be relatively sharp boundaries separating events for which *Lg* is observed or not observed at particular stations. Such boundaries lie further away from the stations than the obvious structural features. For observations to the north, the *Lg* cutoff occurs behind the Altyn Tagh range. For observations to the south, the cutoff boundary is placed by Ni and Barazangi (1983) behind the crest of the Himalayas beyond the Indus-Tsangpo suture. The crustal thickness under the Tibetan Plateau (around 70 km) is very much greater than that to the north in the Tarim depression or to the south beneath India. In each case, we have a relatively rapid transition in crustal thickness represented approximately to true scale in Figure 5.

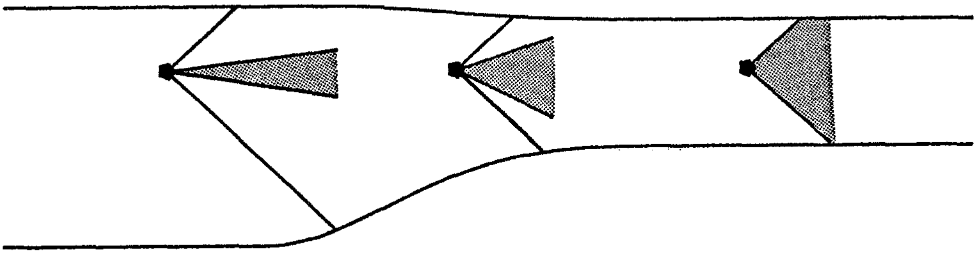

FIG. 5. Effect of source position on the potential *Lg* wave generation in a true scale model of the Himalayan front. From each source, a cone is drawn corresponding to a phase velocity of 4.6 km/sec, which indicates most of the rays which can give rise to *Lg* for a source in the thinner crust to the *right*. As the source position moves to the *left*, the ray zone which couples into the thinner wave guide is reduced and is indicated by the *shaded region*. Eventually, this zone is too narrow to correspond to any propagating modes in the thinner crust.

We have already seen the havoc wrought by thinning the crustal wave guide, so it is not difficult to envisage that events lying well within the Tibetan Plateau will be unlikely to generate significant *Lg* waves at stations outside the plateau. However, the situation is rather different for sources near the transition zone. For an event in the thinner crust, the waves which constitute the *Lg* phase are generated in a ray cone defined by a phase velocity of 4.6 km/sec, indicated by the shading. When the source lies in the transition zone, only part of the potential *Lg* energy can couple into the thinner wave guide, and this is indicated by the shaded inner cone. As the source moves further into the plateau, the effective ray cone diminishes further and once the angles correspond to less than around 3.45 km/sec, *Lg* is extinguished. It is now only possible to feed energy into the lowest order modes, which at high frequencies have energy confined to the sediments. The result is that as the events move into the plateau, the characteristic frequency of *Lg* drops, as is indeed observed, and a sharp cutoff arises once it is no longer possible to excite the entire crustal wave guide leading to the receiver.

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