Waveform-Diverse Moving-Target Spotlight SAR

Margaret Cheney\textsuperscript{a} and Brett Borden\textsuperscript{b}

\textsuperscript{a}Rensselaer Polytechnic Institute, Troy, NY, USA; \textsuperscript{b}Naval Postgraduate School, Monterey, CA, USA

ABSTRACT

This paper develops the theory for waveform-diverse moving-target synthetic-aperture radar. We assume that the targets are moving linearly, but we allow an arbitrary flight path and (almost) arbitrary waveforms. We consider the monostatic case, in which a single antenna phase center is used for both transmitting and receiving. This work addresses the use of waveforms whose duration is sufficiently long that the targets and/or platform move appreciably while the data is being collected.

Keywords: radar imaging, ambiguity function

1. INTRODUCTION

This paper considers waveform-diverse synthetic-aperture radar in the case when there are multiple moving targets in the scene. We consider a pulsed system traversing a circular flight path, and assume that the targets are moving linearly. We consider the monostatic case, in which a single antenna phase center is used for both transmitting and receiving. The problem is formulated in terms of forming an image in phase space, where the independent variables include not only position but also the vector velocity.

We include the case of waveforms whose duration is sufficiently long that the targets and/or platform move appreciably while the data is being collected. Figure 1 shows the regime of validity of the start-stop approximation for X-band. This figure plots $v \leq \lambda/T$, where $v$ is the relative velocity, $\lambda$ is the wavelength at the center frequency, and $T$ is the waveform duration. For shorter wavelengths, the curve moves towards the axes, and the region of validity is smaller. We see that the start-stop approximation is invalid for high-frequency systems or long-duration waveforms or high-velocity targets. For example, a target moving 30 m/sec (67 mph) moves 3 mm in 100 $\mu$sec and 3 m in .1 sec, distances that could easily be comparable to the system wavelength. The issue also arises when low-power, long-duration waveforms are used.

This paper is an extension of earlier work,\textsuperscript{2} which showed how to combine the temporal, spectral, and spatial attributes of radar data. In particular, the theory developed in this paper shows how to combine fast-time Doppler and range measurements made from different spatial locations. This approach can be used, for example, for SAR and ISAR imaging when relative velocities are large enough so that target returns at each look are Doppler-shifted. Alternatively, this theory shows how to include spatial considerations into classical radar ambiguity theory. In addition, this approach provides a connection between SAR and Moving Target Indicator (MTI) radar.

SAR for moving targets has been studied in previous work: Fienup\textsuperscript{4} analyzed the phase perturbations caused by moving targets and showed how the motion affects the image. Other work\textsuperscript{7} identifies ambiguities in four-dimensional space that result from attempting to image moving targets from a sensor moving along a straight flight path. A patent\textsuperscript{6} and other papers\textsuperscript{8,10,11} all use the start-stop approximation to identify moving targets from their phase history. Other work\textsuperscript{5} uses a fluid model to impose conservation of mass on a distribution of moving scatterers, and a Kalman tracker to improve the image adaptively.

In this paper, we outline a derivation for the phase-space point-spread function for a pulsed waveform-diverse spotlight SAR system traversing a circular flight path. We show that this point-spread function can be written in

Further author information: (Send correspondence to M.C.)
M.C.: E-mail: cheney@rpi.edu, Telephone: 1 518 276 2646
B.B.: E-mail: bhborden@nps.edu, Telephone: 1 831 656 2855
terms of the ordinary radar ambiguity function, evaluated at certain arguments. As an application, we compute the error that would be made if the processing were done instead assuming the start-stop approximation. We find that for rapidly-moving high-resolution systems, this error can be significant. A more complete exposition of this work will be published elsewhere.\textsuperscript{3}

2. MODEL FOR RADAR DATA

If the target reflectivity in its own reference frame is denoted by $Q(x)$, then the reflectivity of the target moving with velocity $v$ is $Q_v(x - vt) = Q(x - vt, v)$. We model the antenna as an isotropic point source.

2.1. The received field

The field received at the antenna, which is located at $x = \gamma(t)$, after scattering from a target $Q$ moving with speed $v$, is

$$
E_{B}^{sc}(t) = \int \frac{\delta(t - t' - |\gamma(t) - y|/c)}{4\pi|\gamma(t) - y|} \int Q(y - vt', v) \frac{\delta(t'' - t' - |y - \gamma(t'')|/c)}{4\pi|y - \gamma(t'')|} f(t'') dt'' dt' dy. \tag{1}
$$

Here $f(t'')$ is the transmitted waveform. The model (1) is more easily interpreted after we make the change of variables $y \mapsto z = y - vt'$, whose inverse is $y = z + vt'$. This change of variables converts equation (1) into

$$
E_{B}^{sc}(t) = \int \int \frac{\delta(t - t' - |\gamma(t) - (z + vt')|/c)}{4\pi|\gamma(t) - (z + vt')|} Q(z, v) \frac{\delta(t'' - t' - |z + vt' - \gamma(t'')|/c)}{4\pi|z + vt' - \gamma(t'')|} f(t'') dt'' dt' dz. \tag{2}
$$

The right side of (2) can be interpreted as follows. The part of the waveform $f$ that is transmitted at time $t''$ from location $\gamma(t'')$ travels to the target, arriving at time $t'$. At time $t'$, the target that started at $z$ is now at location $z + vt'$. The wave scatters with relative strength $Q(z)$, and then propagates to the receiver, arriving at time $t$. At time $t$, the receiver is at position $\gamma(t)$.

For multiple moving targets, we simply integrate over all the possible velocities.

We assume that the system transmits a train of pulses of the form

$$
\sum_{m} f_m(t - T_m) \quad m = 0, 1, 2, \ldots \tag{3}
$$

where the delay between successive pulses is sufficiently large so that successive pulses do not overlap.
2.2. Spotlight SAR

We now assume that the distance from the origin to the sensor position $\gamma$ is much larger than the distance from the origin to the target, and also much larger than the distance travelled by the target or sensor during any of the time intervals $T^T_{m+1} - T^T_m$. With these assumptions we can make the following expansions:

$$|z + vt' - \gamma(t)| = |z + vT^T_m + vt' - T^T_m| = |\gamma(T^R_m(t - T^R_m) + \cdots|

$$

$$= |\gamma(T^R_m - \gamma(T^R_m)z + vT^T_m + vt' - T^T_m) + \cdots|

$$

$$= |\gamma(T^T_m) - \gamma(T^T_m)z + v(t' - T^T_m) + \cdots|

$$

$$= |\gamma(T^T_m) - \gamma(T^T_m)z + v(t' - T^T_m) + \cdots|

$$

We use (4) in (2) and carry out the $t''$ and $t'$ integrations.

For the $m$th pulse, expression (2) then becomes

$$E^m_m(t) = -\sum_m \int \frac{f_m(\phi_m(t, z, v_m)) Q(z)}{(4\pi)^2|\gamma^T_m||\gamma^R_m(1 - \gamma^R_m v_m/c)(1 + \beta^T_m)|} d^3 z,

$$

where

$$\phi_m(t, z, v_m) \approx \frac{1}{1 + \beta^T_m} \left[ (1 - \beta^R_m)|t - R^R_m(z, v_m)/c| - R^T_m(z, v_m)/c \right] - T^T_m

$$

and

$$\beta^T_m = \gamma^T_m v_m/c

$$

$$\beta^R_m = \gamma^R_m v_m/c

$$

$$\alpha_{v_m, m} = \frac{1 + \gamma^T_m v_m/c}{1 - \gamma^R_m v_m/c}

$$

$$R^R_m(z, v_m) = |\gamma^R_m - \gamma^R_m(z + \Gamma_m - v_m T^T_m + \gamma^R_m T^R_m)

$$

$$R^T_m(z, v_m) = |\gamma^T_m - \gamma^T_m(z + \Gamma_m - v_m T^T_m + \gamma^T_m T^T_m)

$$

The quantities $\beta^T$ and $\beta^R$ are determined by the squint angle (angle relative to broadside) of the transmitter and receiver, respectively, and $\alpha$ is the Doppler scale factor.

Example: Circular SAR. For the case of circular SAR, where $R = |\gamma(T_m)|$, $\gamma_m = \gamma(T_m)/R$, and $R^o_m(z, v_m) = R - \gamma^R_m z$, we obtain

$$E^o_m(t) \propto \int f_m(\phi^o_m(t, z, v)) Q(z, v)d^3 z d^3 v_m,$

where the phase is

$$\phi^o_m(t, z, v) = \alpha_{v_m, m}(t - R^o_m(z, v)/c - R^o_m(z, v)/c - T_m

$$

where the Doppler scale factor is

$$\alpha_{v_m, m} = \frac{1 + \gamma^T_m v_m/c}{1 - \gamma^R_m v_m/c

$$

3. IMAGE FORMATION

In the circular SAR case, the estimate $I(p, u)$ (image) of the reflectivity $Q(p, u)$ at position $p$ and velocity $u$ is computed by weighted matched filtering:

$$I(p, u) \propto \sum_m \int f_m(\phi_m(t, p, u)) \alpha_{u, m}(1 - \gamma^R_m u/c) E^o_m(t) dt

$$
4. IMAGE ANALYSIS

Using the data model (8) in the image formation algorithm (11) gives rise to

\[ I(p, u) = \int K(p, u; z, v)Q(z, v)d^3z d^3v \]  

(12)

where the point-spread function \( K \) is

\[ K(u; p, z, v) = \sum_m \int f_m(\phi_m(t, p, u)) f_m(\phi_m(t, z, v)) \frac{1 - \gamma_m \cdot u/c}{1 - \gamma_m \cdot v/c} \alpha_{u, m} \ dt. \]  

(13)

If we make the change of variables \( t \to t' = \phi_m \), we obtain

\[ K(p, u; z, v) = \sum_m A_m \left( \frac{\alpha_{u, m}}{\alpha_{v, m}}, \Delta \tau_m(p, u; z, v) \right) \left[ \frac{1 - \gamma_m \cdot u/c}{1 - \gamma_m \cdot v/c} \right] \]  

(14)

where

\[ A_m(\sigma, \tau) = \int f^*_m(\sigma t - \tau) f_m(t) \ dt \]  

(15)

is the wideband ambiguity function \(^{13}\) and where

\[ \Delta \tau_m(p, u_m; z, v_m) = \frac{\alpha_{u, m}}{c} \left[ R^0_m(z, v) - R^0_m(p, u) \right] + R^0_m(z, v) - R^0_m(p, u) + T_m - T_m \]  

(16)

If the waveforms \( f_m \) have thumbtack ambiguity functions, then approximations to both downrange position and downrange velocity can be obtained at each look \( m \), and the variation in aspects as \( m \) varies (i.e., over the synthetic aperture) provides cross-range information.

5. APPLICATION: VALIDITY OF THE START-STOP MODEL

Almost all traditional SAR signal processing and analysis makes use of the start-stop approximation, also called the stop-and-shoot approximation, which is the assumption that neither target nor antenna is moving while each interacts with the wave.

5.1. The Start-Stop Model for Spotlight SAR Data

Under the start-stop assumption, (4) is instead

\[ |z + \nu' - \gamma(t)| \approx |z + \nu T_m - \gamma(T_m^R)| = |\gamma(T_m^R)| - \gamma(T_m^R) \cdot [z + \nu T_m] + \cdots \]

\[ |z + \nu' - \gamma(t')| \approx |z + \nu T_m - \gamma(T_m^T)| = |\gamma(T_m^T)| - \gamma(T_m^T) \cdot [z + \nu T_m] + \cdots . \]  

(17)

The start-stop signal model [(2) and (5)] reduces to

\[ E^{ss}_{s}(t) = - \int \frac{\delta(t - t' - |\gamma^R_m|/c + \gamma^R_m \cdot [z + \nu T_m]/c) \ Q(z)}{4\pi |\gamma^R_m|} \ dt' \frac{\delta(t' - t'' - |\gamma^T_m|/c + \gamma^T_m \cdot [z + \nu T_m]/c) \ f_m(t'' - T_m^T) \ dt'' \ dt' \ d^3 z}{4\pi |\gamma^T_m| |\gamma^R_m|} \]

\[ = \int f_m(t - (R_{m,ss}^T(z, v) + R_{m,ss}^R(z, v))/c - T_m^T) \ (4\pi)^2 |\gamma^R_m| |\gamma^T_m| \ Q(z, v_m) \ d^3 z \ d^3 v_m, \]  

(18)

where

\[ R_{m,ss}^T(z, v) = |\gamma^T_m| - \gamma^T_m \cdot [z + \nu T_m] \]

\[ R_{m,ss}^R(z, v) = |\gamma^R_m| - \gamma^R_m \cdot [z + \nu T_m] . \]  

(19)
5.2. Start-Stop Image Formation for Spotlight SAR

The image $I_{ss}(p, u)$ (estimate of the target reflectivity $Q$) is formed as

$$I_{ss}(p, u) = \sum_{m} (4\pi)^2 |\gamma_{m}^{R}||\gamma_{m}^{T}| \int f_{m}^{*} \left( t - T_{m}^{T} - (R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u))/c \right) \mathcal{E}_{ss}^{sc}(t) \, dt, \quad (20)$$

which gives rise to a point-spread function of the form

$$K_{ss}(p, u, z, v) = \sum_{m} \int f_{m}^{*} \left( t - T_{m}^{T} - (R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u))/c \right)$$

$$f_{m} \left( t - T_{m}^{T} - (R_{m}^{T,ss}(z, v) + R_{m}^{R,ss}(z, v))/c \right) \, dt
= \sum_{m} A_{m}(1, \Delta \tau_{m}^{ss}), \quad (21)$$

where

$$\Delta \tau_{m}^{ss} = (R_{m}^{T,ss}(z, v) + R_{m}^{R,ss}(z, v))/c - (R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u))/c. \quad (22)$$

A focussed image will be obtained when

$$0 = (R_{m}^{T,ss}(z, v) + R_{m}^{R,ss}(z, v)) - (R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u)) = B_{m} \cdot [(p - z) + (u - v)T_{m}'], \quad (23)$$

where we have written

$$B_{m} = \tilde{\gamma}_{m}^{T} + \tilde{\gamma}_{m}^{R}. \quad (24)$$

5.3. Mismatched Processing

If we incorrectly believe the start-stop data model to be accurate, we would form from $\mathcal{E}_{m}^{sc}$ of (5) an image via the same processing (20) as for the start-stop model, namely

$$I_{x}(p, u) = \sum_{m} (4\pi)^2 |\gamma_{m}^{R}||\gamma_{m}^{T}| \int f_{m}^{*} \left( t - T_{m}^{T} - \frac{R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u)}{c} \right) \mathcal{E}_{m}^{sc}(t) \, dt, \quad (25)$$

where $\mathcal{E}_{m}^{sc}(t)$ is given by (5). Substituting (5) into (25), we obtain

$$I_{x}(p, u) = -\sum_{m} \int f_{m}^{*} \left( t - T_{m}^{T} - \frac{R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u)}{c} \right) \int \frac{f_{m}(\phi_{m}(t, z, v_{m}))}{(1 - \tilde{\gamma}_{m}^{T} \cdot v_{m}/c)(1 + \beta_{m}^{T})} \, dz \, d^{3} v_{m} \, dt. \quad (26)$$

The point-spread function for mismatched processing is

$$K_{x}(p, u; z, v) = \sum_{m} \int f_{m}^{*} \left( t - T_{m}^{T} - \frac{R_{m}^{T,ss}(p, u) + R_{m}^{R,ss}(p, u)}{c} \right) \frac{f_{m}(\phi_{m}(t, z, v_{m}))}{(1 - \tilde{\gamma}_{m}^{T} \cdot v_{m}/c)(1 + \beta_{m}^{T})} \, dt
= \sum_{m} A \left( \frac{1 + \beta_{m}^{T}}{(1 - \beta_{m}^{R})\alpha_{v_{m},m}} \right) \frac{1}{(1 + \tilde{\gamma}_{m}^{T} \cdot v_{m}/c)(1 - \beta_{m}^{R})}, \quad (27)$$

where

$$\Delta \tau_{m}^{xx} = \left( T_{m}^{T} \left[ \frac{(1 - \beta_{m}^{R})\alpha_{v_{m},m}}{1 + \beta_{m}^{T}} - 1 \right] + \frac{R_{m}^{T,ss}(p, u)(1 - \beta_{m}^{R})\alpha_{v_{m},m}}{(1 + \beta_{m}^{T})c} - \frac{R_{m}^{T}(z, v_{m})}{(1 + \beta_{m}^{T})c} \right), \quad (28)$$
5.4. Error Analysis for Ideal High-Range-Resolution Waveforms

We expect that the start-stop approximation will be best in the case of a waveform whose ambiguity function is insensitive to its first argument, i.e., for a high-range-resolution waveform such as the ideal short pulse \( f(t) = \delta(t) \). For such an ideal high-range-resolution waveform, a focused image is attained when

\[
0 = \Delta r_m = T_m \left[ \frac{(1 - \beta R_m)\alpha_{v,m}}{1 + \beta R_m} - 1 \right] + \left( R_{m,ss}^T(p, u) + R_{m,ss}^R(p, u) \right) \frac{(1 - \beta R_m)\alpha_{v,m}}{1 + \beta R_m}c
\]

\[\] - \left( R_{m}^T(z, v_m) + R_{m}^R(z, v)\alpha_{v,m} \right) \frac{1}{(1 + \beta R_m)c}.

For the case of constant platform velocity, (29) becomes

\[
0 = B_m \cdot [T_m (z - p) + (v - u)T_m] / c
\]

\[\] - |\gamma|^T_m |B_m| (v - \gamma) / c^2 + R_{RT}^T B_m \cdot (v - \gamma) / c^2 - B_m \cdot [p + uT_m] |B_m| (v - \gamma) / c^2

\[\] + \tilde{\gamma}^T_m \cdot \gamma_m / c \left[ R_{RT}^T - B_m \cdot (z + \gamma T_m) \right] / c - \left( 1 - \tilde{\gamma}^T_m \cdot \gamma_m / c \right) \left[ \gamma^T_m \cdot \gamma_m / c - \tilde{\gamma}^R_m \cdot \gamma_m / c \right]

\[\] - \left[ \tilde{\gamma}^R_m \cdot (z + \tilde{\gamma}^R_m (T_m + |\gamma_m| / c)) |B_m| \cdot v / c^2 \right] \gamma^T_m \cdot \gamma_m / c - \tilde{\gamma}^R_m \cdot \gamma_m / c

\[\] + \left[ \tilde{\gamma}^R_m \cdot (z + \tilde{\gamma}^R_m (T_m + |\gamma_m| / c)) |B_m| \cdot v / c^2 \right].

The first line of (30) we recognize as the condition (23) for focusing under the start-stop approximation. The bottom line of (30) is of order \((v/c)^2\) and is neglected. Thus to leading order in \((v/c)\), the error made in using the start-stop approximation is

\[
|\gamma|^T_m |B_m| \cdot \gamma / c^2 - R_{RT}^T B_m \cdot \gamma / c^2 - B_m \cdot [p + uT_m] |B_m| (v - \gamma) / c^2
\]

\[\] + \tilde{\gamma}^T_m \cdot \gamma_m / c \left[ R_{RT}^T - B_m \cdot (z + \gamma T_m) \right] / c - \left( 1 - \tilde{\gamma}^T_m \cdot \gamma_m / c \right) \left[ \gamma^T_m \cdot \gamma_m / c - \tilde{\gamma}^R_m \cdot \gamma_m / c \right]

\[\] + \left[ \tilde{\gamma}^R_m \cdot (z + \tilde{\gamma}^R_m (T_m + |\gamma_m| / c)) |B_m| \cdot v / c^2 \right].

The third line of (32) is error due to the sensor having moved between the time of transmission and the time of reception. When this error is negligible, we have \( \gamma^T_m = \gamma_m \) and \( B = 2\tilde{\gamma} \), in which case the error (32) reduces to

\[
\frac{1}{c^2} \left( -4 (\gamma_m \cdot [p + uT_m]) \tilde{\gamma}_m \cdot (v - \gamma) - 2\gamma^T_m \cdot \gamma_m \left[ \tilde{\gamma}_m \cdot (z + \gamma T_m) \right] + \tilde{\gamma}_m \cdot (z + \gamma_m (T_m + |\gamma_m| / c)) 2\gamma_m \cdot v \right).
\]

If, in addition, there is no squint \( (\tilde{\gamma} \cdot \gamma = 0 = B \cdot \tilde{\gamma}) \), the error (33) reduces to

\[
\frac{(-B_m \cdot [p + uT_m] + \tilde{\gamma}_m \cdot z)}{c} (B_m \cdot v / c) = \left( -\frac{2\gamma_m \cdot [p + uT_m] + \tilde{\gamma}_m \cdot z}{c} \right) (2\gamma_m \cdot v / c),
\]

which vanishes for stationary targets. Thus for a stationary scene, we find that misfocusing due to the start-stop approximation can potentially take place only in a squinted system. Below we compute the magnitude of this effect.

**Effects on Image.** Resolution in the time delay \( \Delta \tau \) is related to range resolution by \( \Delta \tau = 2\Delta R / c \). Consequently, to compute the shift in the image due to the time-delay error (33), we multiply (33) by \( c / 2 \).
The image is affected only when the error due to the start-stop approximation is greater than a resolution cell, because in the backprojection process these contributions will not add correctly. It is well-known that the SAR cross-range resolution $\delta$ is determined by the aperture $\Delta \theta$ and center frequency $\lambda_0$ as

$$\delta = \frac{\lambda_0}{2\Delta \theta}. \quad (35)$$

The angular aperture $\Delta \theta$ is determined by the antenna path during which data is collected. Most systems are designed so that their down-range and cross-range resolutions are roughly equal.

We consider the specific example of a low-earth-orbit satellite whose altitude is 300 km and whose velocity is 8 km/sec, traversing a straight flight path and forming a spotlight SAR image of a scene 500 km from the satellite. We consider only stationary targets because the speed of most moving targets would be insignificant relative to the platform speed. We use the assumption $\gamma_T^m = \gamma_R^m$.

We assume data collection starts at pulse $m = 0$ and ends at pulse $m = M$. We denote the angle to the scene center relative to the antenna velocity vector by $\theta_0$ for pulse $m = 0$, and $\theta_M$ when for pulse $m = M$. See Figure 2.

![Figure 2. Spotlight SAR geometry](image)

The duration of the data collection interval is denoted by $T$, which means for this case that the synthetic aperture is of length $T|\gamma|$, and the system transmits a pulse every $T/(M - 1)$ seconds. To relate this pulse repetition interval, and hence the angular sampling interval, to the system’s cross-range resolution, we apply the law of sines to the triangle shown in Fig. 2:

$$\frac{\sin \Delta \theta}{T|\gamma|} = \frac{\sin \theta_0}{|\gamma_M|}. \quad (36)$$

Solving (36) for $T$, we find that

$$T = \frac{|\gamma_M| \sin \Delta \theta}{|\gamma| \sin \theta_0} \approx \frac{|\gamma_M| \lambda_0}{2|\gamma| \sin \theta_0}, \quad (37)$$

where we have used the small-$\Delta \theta$ approximation $\sin \Delta \theta \approx \Delta \theta$, which from (35) can be written in terms of the system resolution as $\lambda_0/(2\delta)$.

In our simulations, we computed the error (33) for each pulse, plotted as a function of the resolution $\delta$ and the starting angle $\theta_0$. In Figure 3, we show only the region in which the error is larger than a resolution cell.

We see from Figure 3 that for a high-resolution system, when the squint angle $\theta_0$ is large, the start-stop approximation could lead to significant image distortion and smearing.

6. CONCLUSION

We see from (14) that the phase-space point-spread function is a weighted sum of ordinary radar ambiguity functions, evaluated at arguments that depend on the difference in positions and velocities.
Figure 3. This shows the error as a function of squint and resolution for a target traveling 100km/hr in the direction opposite to the platform velocity (8 km/sec). We assume 3-cm wavelength (X-band).

ACKNOWLEDGMENTS

We are grateful to Air Force Office of Scientific Research\textsuperscript{*} for supporting this work under the agreements FA9550-06-1-0017 and FA9550-09-1-0013.

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