MobiMaths: An approach to utilising smartphones in teaching mathematics

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Abstract

The teaching of mathematics at second level is well known to be a challenging issue. An overemphasis on didactic teaching, lack of encouragement to explore possible alternative solutions to problems, an overemphasis on procedure and the separation of mathematical procedures from real world problems are just some of the factors which are put forward as contributing to the difficulties in math education. Through its inherent ability to support collaboration, and contextualised learning, mobile technology offers the potential to address at least some of the issues in mathematics education. This paper describes the approach we are following to create a set of tools, learning applications and teacher supports, which exploit smartphone technology to aid in the teaching and learning of mathematics. The work is underpinned by a social constructivist pedagogy with an emphasis on collaborative problem solving and the contextualisation of learning. This paper discusses issues in mathematics education before going on to describe the broad approach being followed in our research. The underlying technical architecture is described along with the first two activities we have developed. The preliminary results from a user evaluation study are reported upon.

Keywords

mathematics; smartphones; contextualised learning.

1. INTRODUCTION

There is considerable disquiet that students leave secondary school systems with a fragmented view of mathematics and are unable to put their knowledge to constructive use in the workplace (Goos, 2004; Goss, 2009; Grossman Jr, 2001). It is argued that a number of related factors are responsible for this including: an overemphasis on didactic teaching, in which the teacher is commonly seen by students as an absolute authority on the subject whose role is to transmit the knowledge that is needed to master the problem so that students are discouraged from exploring possible alternative solutions or finding their own (Conway & Sloane, 2005; Muis, 2004); a behaviourist approach to learning in which complex problems are commonly presented as aggregations of one-dimensional tasks which are then mastered discreetly; an overemphasis on procedure, in which mathematics is presented as a 'highly fragmented set of rules and procedures rather than a complex highly interrelated conceptual discipline' (Garofalo, 1989). Most importantly from the point of view of this work is the decontextualised way in which mathematics is often taught. Students rarely are exposed to real world data, situations or problems and have extreme difficulty relating the de-contextualised material they are exposed to any aspect of their lives.

Mobile technology and smartphones in particular, offer a means whereby at least some of the issues in mathematics education can be addressed. The potential for mobile technology to support collaborative, constructivist, contextualised learning is well documented in the literature both in terms of conceptual frameworks (Kukulska-Hulme A., Sharples M., Milrad, Arnedillo-Sánchez I., & Vavoula G., 2009; Patten B., Arnedillo-Sánchez I., & Tangney B., 2006; Sharples M. (Ed), 2006) and practical examples of prototypes in action – for example (Facer, 2004) and (Wijers M., Jonker V., & Kerstens K., 2008). In particular smartphone capabilities in terms of location awareness; peer to peer communication between devices; anytime anywhere internet access; accelerometers; touch screens; image and video capabilities; and data capture mean they can be used as the basis for scaffolded learning scenarios which open up the exploration of mathematical concepts in innovative ways.

For example, and as described in the body of this paper, the accelerometer in a smartphone can be used to create a tool which measures an angle of elevation. This allows a series of activities to be designed to measure the height of a building or structure. The extent to which the learning is scaffolded is up to the teacher and can range from giving detailed step by step instructions to following a much more open discovery learning approach. With little or no prompting learners should realise that by measuring the angle of elevation, and the horizontal distance to the base of the structure in question, the TAN function can used to determine the height. The exercise becomes even more mathematically interesting when learners are encouraged to estimate distances, heights and angles before measuring them and to compare these to the actual values measured. Issues to do with margins of error in measurement can be explored as well as means of calibrating their answers against known heights. A second activity described below is based upon manipulating Cuisenaire Rods on the screen of the smartphone in order to engage in a learning activity concerned with fractions. The activity can be made collaborative by configuring the initial allocation of rods to learners so that they are required to exchange rods with each order in order to arrive at a solution.

Both of these activities form the basis for rich contextualised, collaborative, constructivist learning experiences which we argue address many of the concerns of mathematics educators. Our research goal is not to create sample tools and scenarios but rather to produce a set of activities which cover a substantial block of curricular material in a comprehensive and cohesive manner so that teachers can be facilitated in making extensive use of mobile learning in a pragmatic way in their day to day teaching. To this end we are systematically working going through Grade 7 of 2006 NCTM Principles and Standards for School Mathematics (NCTM, 2006) in order to devise a set of suitable learning activities which address the main areas of: Data Analysis; Measurement and Geometry; Number, Operations and Algebra.

2. Issues in Mathematics Education

Internationally there is growing concern about the participation levels and success rates within the study of mathematics across the developed world (Noyes & Sealey, 2009). No single factor is responsible for this, rather a combination of interlinked factors are at play some of which have been mentioned earlier. Other factors include curriculum and assessment constraints (Entwistle, 2000); teaching standards and methodologies (Lyons, Lynch, Close, Sheerin, & Boland, 2003) and students perceptions and attitudes (Brown, Brown, & Bibby, 2008; Burghes & Hindle, 2004).

In particular mathematics curricula tend to concentrate on the abstract concepts of mathematics rather than application or discover of concepts. This leads to students' perception of mathematics to be an arid subject irrelevant to the practical world outside the classroom (Breen, 2009; Burghes & Hindle, 2004; Smith, 2004). (Greeno & Collins, 2008) argue that the most fundamental problem of the mathematics curriculum in the US is that students learn a large volume of information with practically no understanding of how that information may be used in the 'real world'. They concentrate upon abstract concepts and place emphasis on procedural drills rather than concrete application. This type of curriculum is assessed in a format which tends to test substantial volumes of factual recall and the use of standard algorithms rather than understanding. This in turn impacts upon the way in which mathematics teachers approach their teaching (Brown et al., 2008).

(Nardi & Steward, 2003) also identify a "mystification through reduction" effect whereby teachers attempt to make mathematics simpler by reducing it to a list of rules. With the emphasis on modus operandi rather than its application, this notion of mathematics teaching is in stark contrast to teaching higher order thinking skills where students are encouraged to apply methods and concepts to situations that were previously unfamiliar to them (Donovan & Bransford, 2005).

By not highlighting the practical applications of mathematics within mathematics education in the curriculum, teaching methods and assessment, students are not challenged to develop higher order thinking skills. Thus students often learn mathematics without being able to solve problems in 'real world' situations (Schoenfeld, 1988). According to (Boaler, 1993) attempts to address this failing by introducing contextualised learning have had limited success for a number of reasons. Many of the 'real world' problems students come into contact with derive from textbooks which create pseudo-real-world problems that require students to ignore facts pertinent to the real life version of the task. Therefore they are 'school problems with a thin veneer of real world' (Boaler, 1993, p.4). Often the contextualised problems are sourced from the adult world rather than that of the students, impacting on the students' interest in solving the problem. This runs contrary to Piaget's argument (Piaget, 1958) that individuals construct new knowledge on the basis of their interest. If there is an over emphasis on the 'described situation' within the textbook as a method of contextualising mathematics rather than creating learning scenarios that are contextually realistic, interesting and of educational benefit, the use of contextualised learning becomes futile.

2.1 Mobile Technology and Mathematics Education

Many digital tools produced for mathematics education are essentially electronic versions of textbooks, drill and practice sometimes disguised as games (Bottino & Kynigos, 2009).

To date most of the mathematics learning applications for smartphones are versions of applications, or are at least very similar in style, to ones which are available for desktops. Puzzles and games are popular as well as graphic calculators. For example, many mathematics orientated "apps" on the Android Market are geared towards drill and practice of number and operation and are very similar in content and function, e.g. *ChoiceMath*, *MathPractice* and *MathSkill*. These "apps" are typically geared towards a single user and have no element of context or collaboration with other users. There are a small number of "apps" which are geared towards data collection – for example *TennisMath* allows users record events during a tennis match and then output a detailed analysis of the data collected. A number of "apps" for mobile phones such as *Pocket Autograph* and *Maths4Mobile* replicate the function of a graphing calculator on a keypad phone and have not been aimed at smartphones. On the other hand *TouchMaths* is a collection of mathematical tools which has been produced specifically for touchscreen phones but it assumes a high level of mathematical understanding by the user.

By and large smartphone "apps" for mathematics do not fully exploit the affordances of mobile devices to support contextualised, collaborative, constructivist learning nor do they attempt to explore the contexts in which learning takes place - the later being of crucial significance given the concern about the decontextualised nature of math learning.

More interestingly (Wijers M. et al., 2008) describe an interactive location based game which explores the construction and deconstruction of quadrilaterals in a hybrid virtual/real-world space. They follow the Realistic Mathematics Education philosophy (Gravemeijer K.P.E., 1994), in which "learning activities should be 'experientially' real", and in which social interaction is a key component of the learning experience. In their case this is achieved by requiring the learners to navigate in the physical space in order to construct and de-construct geometric shapes in an overlaid virtual world.

A hybrid approach made up of a combination of a location specific learning experience followed up by in-class activities is advocated by Spikol & Milrad (Spikol D. & Milrad M., 2009). In their case students use a mobile device to assist in measuring and estimating the height, area and volume of buildings as part of a data gathering exercise and then in-class use tools such as Sketch Up to design their own buildings. Not only does the learning activity integrate in-class and out-of-class learning it is also a good example of a technology supported cross curricula learning activity which helps to show the relevance of mathematics.

3. MobiMaths

We argue that it is time for Mobile Learning to move beyond the development of innovative prototype applications and activities which make for engaging one-off (albeit sometimes of long duration) learning experiences. For mobile learning to be successfully integrated into the classroom in any meaningful large scale fashion it must be applicable across a number of elements of the curriculum and come with an appropriate amount of support for the teacher so that they can not only see the benefits of mobile learning but also a clear path to how they can incorporate it into their daily classroom practice. Because of its widespread applicability, among a large set of teachers, we have chosen to focus on Grade 7 of the USA NCTM Principles and Standards for School Mathematics and the Curriculum Focal Points (NCTM, 2006). This has given rise to a focus on the areas of: Data Analysis; Measurement and Geometry; Number, Operations and Algebra. We are working through each of these areas to create learning activities according to the pedagogical underpinnings outlined below.

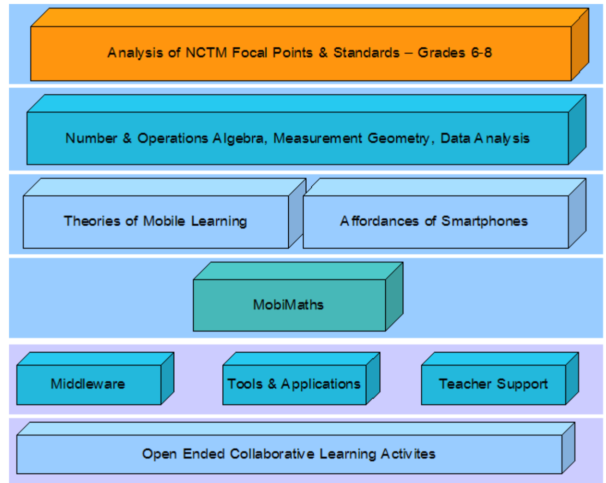

Figure 1: The MobiMaths Approach

Of the applications described previously MobiMaths is closest in spirit to the work of (Wijers M. et al., 2008) and (Spikol D. & Milrad M., 2009). We follow a broadly social constructivist pedagogy to mobile learning (Patten B. et al., 2006). In order to overcome the issues in mathematics education identified in the literature tools, applications and learning activities should: encourage learning and problem solving activities which occur (where possible) in real-life contexts; take place in an environment which is rich in information; involve performing authentic tasks in ill-structured domains; involve interactions with others. Finally there should be an emphasis on learning processes rather than solutions.

MobiMaths (Tangney B. et al., 2009) aims to provide an integrated toolkit encompassing all aspects from hardware through to lesson plans. From the hardware perspective learners will be provided with smartphones which can communicate with each other and with the teacher's console machine. The toolkit will include a range of neutral tools (Somekh, 1997) which can be applied broadly across the curriculum (e.g. an in-class voting response system) and a range of "Mindtool" applications (Jonassen, 2006) which are purpose defined by the curriculum and serve to amplify conceptual understanding, extend thinking and enhance problem solving (e.g. the Cuisenaire Rod application for fraction addition described below).

Using these tools and applications teachers are free to create innovative learning activities as suits their approach to teaching. MobiMaths support for teachers will also include a detailed set of activity sheets which will correlate to keys skills and topics within the relevant curricular area. Each activity sheet will also provide at least one open ended "challenge" to engage learners in solution strategy development and mathematical reasoning across a wider curriculum area.

We do not underestimate the issues to do with technical maintenance of phones and school policies on phone usage and ownership. Such issues are outside the scope of this paper but we assume that smartphones will be allocated to students (or groups of students) for at least the duration of the learning activity. Schools may follow schemes very similar to those already adopted to manage student laptops with each student having their own smartphone or the teacher may have access to a mobile cart of charged phones which are given to students for the duration of a learning activity.

Finally we are following an interdisciplinary design methodology with the team being made up of software engineers, educational technology researchers and experienced maths teachers. The core team is augmented with graphics design expertise as needed. An incremental prototyping approach is being followed. All tools and applications are being tested in authentic school settings with feedback flowing back through the design and prototyping process as appropriate.

4. Technical Architecture

This technical architecture is depicted in Figure 2. A four layered architecture separates core middleware functionality from behavior specific components.

The platform abstraction layer is the fundamental layer that provides essential device-specific functionalities. These include sensor readings, (e.g., GPS, accelerometer and compass), communication, (e.g., access to Wifi and 3G) and basic GUI functionality. Although we are currently developing for Android phones[1] this layer facilitates the porting of the educational activities to a variety of smartphone devices by providing abstractions from device-specific implementations.

The middleware layer implements generic functionality such as group communication primitives, GUI support, activity coordination, persistent storage, location determination and access to sensors.

Communication is crucial to enable collaborative problem solving. MobiMaths communication is web service based with the service residing on a remote web server accessed via the hypertext transfer protocol (HTTP). MobiMaths web services use Apache Axis technology to generate service descriptions using the Web Services Description Language (WSDL) and to generate appropriate Simple Object Access Protocol (SOAP) responses to client requests. These XML based messages are sent back and forth between the smartphones and the server. KSoap is a SOAP web service client library for constrained Java environments such as mobile phones. Requests are generated on the device based on application and tool requirements. The MobiMaths server services client requests and generates SOAP responses to return the required information. The SOAP response is then parsed by the KSoap client on the smartphone.

The component layer contains a set of components that provide functionality used in the development of MobiMaths applications. Each component provides a specific behaviour e.g., messaging, group management, etc. The group management component allows for the assignment of students into groups and the matching of groups with tasks. The messaging component provides messaging functionality within learning applications using communication functionality provided by communication primitives in the middleware layer. Above the component layer, the application layer includes MobiMaths applications and tools. Each application draws on behaviour provided by the lower layers to create applications supporting curriculum based activities. Each application is specific to a learning activity and will contain data and an application-specific GUI.

A teacher management system (TMS) enables teachers to manage and monitor learning activities. The management component is used to organise students into appropriate groups. Application specific data is generated by the teacher and pushed to student devices. This allows for varying levels of difficulty according to student ability. The TMS's other primary role is monitoring. On completion of a learning activity students send an acknowledgment of completion including any application specific results and metrics. These are recorded and can be accessed via the TMS to monitor progress and to customise future activities for a particular student.

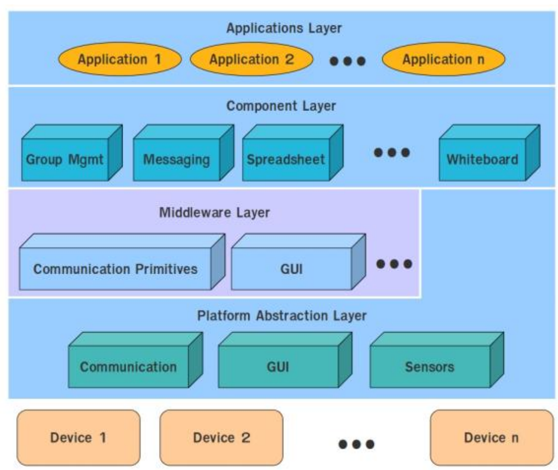

Figure 2: MobiMaths Technical Architecture

[1] The Android operating system is managed by the Open Handset Alliance, a consortium of 65 hardware, software, and telecom organisations and is currently used by various handset manufacturers including Ericsson, Motorolla, Samsung and Google.

5. Initial Tools and Applications

A sample application and tool are described below, the first for trigonometry and the second for fractions. They show how the toolkit can be used in different ways to support different aspects of the curriculum and hence meet the objectives outlined previously.

5.1 Angle Tool

The Angle Tool uses the phone's accelerometer to produce a visual readout of the angle at which the smartphone is being held. The tool displays the angle of elevation of the device and records that reading on user instruction, i.e. by tapping the screen. A running average of the previous five recorded angle readings is automatically maintained.

Learning activities based around the Angle Tool are mapped to the geometry and trigonometry section of the curriculum. One of the many criticisms of trigonometry is that it is taught in a context free fashion which leads to students having problems applying concepts to everyday experiences. Activities based upon using the Angle Tool facilitate the introduction of context into students' learning by having them apply theory to real world environments, situations and scenarios.

In the simplest case students can be given a task to measure the height of a nearby structure. Unlike many problems that students encounter in text books, there are no sub-steps for the posed problems to act as "marker points" for finding the right answer. It is envisioned that students develop their own sub steps, e.g. measuring the distance from the structure to the point of angle measurement, measuring from a number of different distances to calculate an average, comparing estimations with calculated answers.

To support teachers a detailed lesson plan is provided which explains the use of the tool and maps out clearly where in the curriculum it can be used. A number of scaffolding activities are also suggested to help promote discussion and reasoning among students. These also encourage students to consider the real world applications of the tool and how the calculated data could be used.

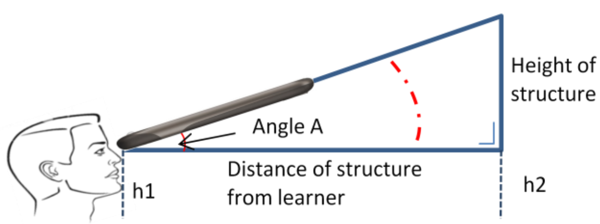

Figure 3: How to measure height

5.2 Cuisenaire Rods

Cuisenaire Rods - sometimes known as "Algebricks" or "Integer Bars" - are named after Georges Cuisenaire and were made popular as a tool for teaching fractions by Caleb Gattegno. The rods consist of rectangles of different colour and length which represent different fraction values and they can be used to teach the basic concepts of fraction addition, subtraction and equivalence.

We have developed an application to explore the addition of fractions. The activities which use this application also promote the development of reasoning, estimation, communication, and collaboration. Learners receive a number of fractions that are displayed as coloured rods on the phone screen. The challenge is to fill the empty unit space with some combination of the rods they have received. To avoid learners adopting a 'guess and test' strategy, both the time taken to complete the challenge and the number of moves made are recorded. The complexity of the activity increases as students are required to use a greater number of rods to fill the unit space.

Rods can be traded by selecting "Swap Rod" from the touchscreen menu, choosing the destination player and touching the rod to trade. This generates a web service request to send the chosen rod to the server. This rod is then held on the server and sent to the destination device. When a learner completes the activity (by filling the unit space) they generate a request that sends an acknowledgment of solution to the server with the number of moves, number of swaps involved and the time taken.

Collaborative learning experiences via paired or larger groups can be organised by configuring the allocation of rods to learners so that in order to fill the unit space on their phones learners must "trade" or "swap" with other players to acquire the correct solution set. Skills of estimation and reasoning are promoted and learners must communicate using the appropriate mathematical language and justify how a "trade" can benefit both parties involved.

Figure 4: Screenshot showing sample set of rods

The teacher can parameterize the algorithm used to create the sets of rods given to each learner and the overall level of difficulty of the activity can also be controlled.

6. User Study

An initial exploratory evaluation of an activity using the Angle Tool has been carried out and while the size of the study is too small to draw any substantive conclusions the initial findings are positive. A class of 24 second level students, working in 6 groups of 4, used the tool in an exercise to measure the heights of 3 structures in their school – a school building and sets of floodlights and goalposts on the sports field. The activity took place over 3 consecutive 40 minute classes on a Friday afternoon followed by a reflection and feedback session in a single 40 minute period on the following Monday. During this time students also filled in a post activity questionnaire.

As is typical in one off interventions such as this the participants found the experience engaging and enjoyed the collaborative aspect. In the post activity questionnaire the following attitudes were reported: 81% of the students found the tool easy to use; 62% said that the activity changed their ideas on trigonometry (in a positive way) with the remaining 38% being neutral; 85% reported that the activity made trigonometry easier to understand; 80% said they felt the activity aided in establishing concepts that the teacher had covered in class; 90% reported that they would like to use more of these types of activities in learning about mathematics. The students did however find the somewhat open ended specification of the task challenging. Students were asked to justify the approach to solving the task which they adopted and were not given readymade diagrams to follow. As one student reported "*the hardest part was working out the way you had to do it....*". They were also asked to estimate all readings and measurements before they took them and to estimate the heights of the structures before they calculated them. The later yielded some interesting results which are very much in keeping with the concerns about mathematics education raised earlier. Initial estimates of heights were off by as much as 170% (estimated 30m actual height 11m). More worryingly some groups did not connect their calculated height for one structure to their estimation of the (not too dissimilar) height of the next structure. Equally so some groups ignored the discrepancies between their estimated and calculated heights rather than probing further to see in which the error was made. The teacher was able to pick up on these discrepancies and explore students' grasp of estimation and the significance of number in the post activity reflective session.

While the study is too small to draw any definitive conclusions it does indicate that suitably constructed and scaffolded learning experiences along the lines proposed could lead to richer in class conversation about mathematical concepts and help the learners achieve a deeper engagement with and understanding of the topics in question.

7. Current Studies and Future Work

At the time of writing we are continuing to implement the Technical Architecture. We are also working through Grade 7 of the "NCTM Principles and Standards for School Mathematics and the Curriculum Focal Points" to devise suitable learning activities so that we can build the applications and tools needed support those activities. As new tools, applications and activities become available they will be tested by students in local schools.

Although our evaluation is still at an early stage we argue that for mobile learning to be of real benefit in schools it should be used to support learning activities which are integrated into the curriculum. Mobile technology is ideally suited to support collaborative and contextualised learning activities but the design of these to be relevant to the curriculum is not easy. For both of these reasons extensive support, in the form of both detailed lesson plans and ideas for open ended activities, must be provided to scaffold the teacher in the adoption of both sophisticated technology and potentially unfamiliar teaching and learning strategies.

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