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Effects of Computer Algebra System (CAS) with Metacognitive Training on Mathematical Reasoning

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Abstracts

The main purpose of the present study was to investigate the differential effects of Computer Algebra Systems (CAS) and metacognitive training on mathematical reasoning. Participants were 83 students (boys and girls) who studied algebra in four eighth-grade classrooms randomly selected to four instructional methods: CAS embedded within metacognitive training (CAS + META), metacognitive training without CAS learning (META), CAS learning without metacognitive training (CAS) and CONT learning without CAS and metacognitive training (CONT). Results showed that the CAS + META condition significantly outperformed the META and the CAS conditions, who in turn significantly outperformed the CONT condition on aspects of mathematical reasoning. No significant differences were found between the META and CAS conditions. In addition, the study found that the metacognitive students (CAS + META and META conditions) outperformed their counterparts (CAS and CONT) on their metacognitive knowledge. This paper describes research whose main focus is the use of Computer Algebra Systems (CAS) in mathematics classrooms and the didactical possibilities based on the IMPROVE method linked with its use.

Les effets du système d’algèbre par informatique (CAS) sur une formation métacognitive en raisonnement mathématique.

Le but principal de la présente étude était d’explorer les effets différenciels du CAS et d’une formation métacognitive en raisonnement mathématique. Les participants étaient 83 élèves (garçons et filles) qui étudiaient l’algèbre dans les classes (eighth-grade) choisies au hasard avec 4 méthodes d’instruction. CAS enchaîné à l’intérieur d’une formation métacognitive (CAS + META) métacognitive sans CAS (META), apprentissage CAS sans formation métacognitive (CONT) et apprentissage sans CAS + formation métacognitive (CONT). Les résultats ont montré que la formule CAS + META a surpassé de façon significative les formules META + CAS qui a leur tour ont surpassé la formule CONT quant aux aspects du raisonnement mathématique. Il n’y a pas de différences entre les formules META et CAS. De plus l’étude a montré que les élèves soumis aux conditions CAS + META et META ont obtenu de meilleurs résultats que leurs condisciples (CAS et CONT) quant à leurs connaissances métacognitives cet article décrit des recherches dont le but principal est l’utilisation du CAS dans les classes de mathématiques et les possibilités didactiques fondées sur la méthode IMPROVE à son usage.

Wirkungen des Computer Algebra Systems (CAS) mit metacognitivem Training auf mathematisches Denken

Introduction

Algebra has emerged as one of the central themes of the Principles and Standards for School Mathematics (NCTM, 2000). ‘To think algebraically, one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts’ (Friel, Rachlin, Doyle, Nygard, Pugalee Ellis and House, 2001, p. 2).

During the last decade the availability of Computer Algebra System (CAS) environments has increased dramatically. The interaction with the computerized system enabled practicing the algebraic language for symbolic and numerical computation. It encouraged understanding of algebraic reasoning, finding patterns, and reflecting on the solution process. But research (e.g., Monaghan, 1994; Lagrange, 1996) on using Computer Algebra System (CAS) indicated evidence of success but also of difficulties. Lagrange (1996) stated that CAS have a great potential to improve student learning in mathematics, but easier calculation did not automatically enhance students’ reflection and understanding. Their conclusion was that a great deal of fieldwork is required to establish in what conditions this potential of Computer Algebra can actualize.

In recent years, there has been much interest in the role of metacognition in mathematics education. Metacognition is interpreted as cognition about cognition, which means knowledge of thinking and regulation of learning processes (Flavell, 1979). Schoenfeld (1987) indicated that in the area of mathematics, metacognitive knowledge includes knowledge about one’s own thought processes (e.g., ‘How accurate are you in describing your own thinking?’), control or self-regulation (e.g., ‘How well do you keep track of what you’re doing when you’re solving problems/tasks?’), and beliefs and intuitions (e.g., ‘What ideas about mathematics do you bring to your work in mathematics?’).

Several previous studies have examined the effects of metacognitive training on mathematics achievement (e.g., Mevarech and Kramarski, 1997; Schoenfeld, 1985; 1987; Garofalo and Lester, 1985). A major common element of these programs is training students to do mathematics by asking and answering a series of self-addressed metacognitive questions before, during and after attempting a solution. The method of Mevarech and Kramarski, (1997), called IMPROVE emphasizes the importance of providing each student with the opportunity to construct metacognitive knowledge in mathematical learning via the use of self questioning. This questioning focuses on: (a) comprehending the problem (e.g., ‘What is the problem all about?’); (b) constructing connections between previous and new knowledge (e.g., ‘What are the similarities/differences between the problem at hand and the problems you have solved in the past? and why?’); (c) using of strategies appropriate for solving the problem (e.g., ‘What are the strategies/tactics/principles appropriate for solving the problem and why?’); and in some studies also (d) reflecting on the processes and the solution (e.g., ‘What did I do wrong here?’; ‘Does the solution make sense?’).

Evidence shows positive effects of metacognitive training employed in non-computerized environments (e.g., Mevarech and Kramarski, 1997; Kramarski, Mevarech and Liberman, 2001; Kramarski, Mevarech and Arami, 2002) as well as the effects of using such pedagogy in computerized environments (e.g., Teong, Threlfall, and Monaghan, 2001; Kramarski and Zeichner, 2001; Kramarski and Ritkof, 2002). Effects were found on mathematical reasoning, giving mathematical explanations and promoting mathematical discourse. Given these studies, there is reason to believe that providing metacognitive training within the CAS environment will improve students’ mathematical reasoning.

There is also evidence showing that students who were exposed to metacognitive training in classrooms improved their ability to use metacognitive knowledge more than students of the control groups (Masui and De Corte, 1999; Kramarski, Mevarech and Liberman, 2001). There is reason to suppose that providing metacognitive training within the CAS environment may affect students’ metacognitive knowledge differently. These students are expected to possess an improved capacity to reflect on using metacognitive knowledge as compared to students who are not exposed to such learning.

The main purpose of the present study is to investigate the differential effects of Computer Algebra System (CAS) and metacognitive training on mathematical reasoning and metacognitive knowledge. In particular to compare four instructional methods: Computer Algebra System learning embedded within metacognitive training (CAS + META), metacognitive learning in the whole class (META), Computer Algebra System learning with no metacognitive training (CAS), and learning in the whole class with no metacognitive training (CONT).
Method

Participants were 83 students (boys and girls) who studied in four eighth-grade classrooms randomly selected from four junior high schools.

All classrooms studied Algebra five times a week during a five-month period according to the mathematics curriculum suggested by the Israeli Ministry of Education.

Classrooms were randomly assigned to one of the following conditions:

**CAS + META condition**

Students in this condition were exposed to Computer Algebra System (20 hours in the computer lab, each weak one hour) embedded within metacognitive training. The CAS software enabled the students to practice the algebraic language for symbolic and numerical computation. In particular, the students practiced substitution of variables, simplifying algebraic expressions and solution of equations. The metacognitive training was based on the IMPROVE method. The acronym of IMPROVE represents all the teaching/learning stages of the metacognitive training: introducing the new topics to the whole class; metacognitive questioning; practicing; reviewing; obtaining mastery on higher and lower cognitive skills; verifying and enrichment. The metacognitive training provides each student with the opportunity to construct metacognitive knowledge by utilizing self-addressed metacognitive questions: Comprehension questions, strategic questions, connection questions and reflection questions.

In addressing **comprehension questions**, students had to read the problem/task, describe the concepts in their own words, and try to understand what the concepts meant. The **strategic questions** are designed to prompt students to consider which strategies are appropriate for solving the given problem/task and for what reasons.

**Connection questions** prompt students to focus on similarities and differences between the task at hand and task they had already solved.

**Reflection questions** prompt students to focus on the solution process and to ask themselves ‘what am I doing here?’ ‘does it make sense’ ‘what if’. The metacognitive questions were printed in Students’ Booklets, Teacher Guide, and on the hand held index cards that students used in problem solving. Students used the metacognitive questions orally in their small group/individualized activities, and in writing when they used their booklets.

**META condition**

In this condition, the metacognitive training was exactly the same as in the above condition, except that the metacognitive training was implemented only in the classroom.

**CAS condition**

Under this condition, students studied as in the CAS + META condition, but they were not exposed to the metacognitive training (20 hours in the computer lab, each weak one hour).

**Control condition**

Under this condition students were exposed to neither CAS environment nor to metacognitive training. This group served as a control group.

Measurements

Three measures were used in the present study to assess students’ mathematical reasoning and metacognitive knowledge: (a) a pre-test that focused on students’ mathematical knowledge prior to the beginning of the study; (b) a post-test that assessed students’ mathematics reasoning on five criteria: algebraic techniques, algebraic reasoning, patterns and change; and (c) a metacognitive questionnaire that assessed students’ metacognitive knowledge regarding their thoughts about using problem solving strategies before, during and after the solution process.
Mathematics: prior knowledge

A 21-item pre-test was administered to all students at the beginning of the school year. The test covered operations with positive and negative numbers, order of operations, the basic laws of mathematics operations, algebraic expressions, and open-ended computation problems.

Scoring

For each item, students received a score of either 1 (correct answer) or 0 (incorrect answer), and a total score ranging from 0 to 21. The Kuder Richardson reliability coefficient was $\alpha = .87$.

Mathematical Reasoning

The post-test included 17 items that are based on the following four criteria:

Algebraic techniques (e.g., solution of equations, operations with algebraic expressions); algebraic reasoning (e.g., what can you say about $r$ if: $r = s + t$ and $r + s + t = 30$); patterns (e.g., the following design is composed from white and black squares, if you know the number of the black squares can you find the number of the white squares? If the number of the black squares is K find the pattern for the number of the white squares); and changes (e.g., which changes faster: $2n$ or $n + 2$, explain).

Scoring

For each item, students received a score of either 1 (correct answer) or 0 (incorrect answer), and a total score ranging from 0 to 17. The Kuder Richardson reliability coefficient was $\alpha = .87$.

Metacognitive knowledge

The metacognitive questionnaire, adapted from the study of Montague and Bos (1990) and Kramarski, Mevarech and Liberman (2001) assessed students’ metacognitive knowledge regarding their thoughts about learning processes. The questionnaire includes 24 items: Eight items referred to strategies used before the solution process (e.g., ‘Before I solve a problem in mathematics, I try to say it in my own words’), eight items referred to strategies used during the solution process (e.g., ‘When I solve a problem in mathematics, I organize the data in a table’), eight items referred to strategies used at the end of the solution process (e.g., ‘After I solve a problem, I check whether the answer is logical’).

Scoring

Each item was constructed on a 5-point Likert type scale ranging from 1 (never) to 5 (always), and a total score ranging from 24 to 120. Cronbach’s alpha reliability coefficient was 83.

Results

Mathematical Reasoning

The first purpose of the present study was to investigate students’ mathematical reasoning under the four conditions described above. Table 1 presents the mean scores, adjusted mean scores, and standard deviations on mathematical reasoning by time and treatment.

One-way MANCOVA on the five topics of the mathematical reasoning post-test scores as dependent variables (algebraic techniques, algebraic reasoning, patterns and change) and the pre-test as a covariate indicated significant differences at the end of the study between the four conditions ($F(3, 78) = 7.19$, $p < .0001$). Further analysis (ANCOVA) on each dependent variable indicated significant differences between the learning groups on two criteria of mathematical reasoning: algebraic reasoning ($F(3, 78) = 9.43$, $p < .001$) and patterns ($F(3, 78) = 10.7$, $p < .001$).

Figure 1 presents the differences on mean scores of mathematical reasoning by treatment.

It was found that the CAS + META condition significantly outperformed the META and the CAS conditions, which in turn significantly outperformed the CONT condition on algebraic reasoning and discovering patterns. But no significant differences were found between the META and CAS conditions on mathematical reasoning.
Table 2 presents the mean scores, adjusted means and standard deviations on metacognitive knowledge by treatment.

Results indicated that prior to the beginning of the study no significant differences were found between the four groups on metacognitive knowledge (F(3, 78) = 0.79, p > .05). MANCOVA with the pre-scores as a
covariant indicated significant differences between the four groups on the post-test ($F(3,78) = 6.87; p < 0.001$). Further analysis (ANCOVA) on each dependent variable indicated significant differences between the learning groups on two measures of metacognitive knowledge: Using problem solving strategies before the solution process ($F(3, 78) = 9.48, p < 0.01$) and using problem solving strategies during the solution process ($F(3, 78) = 8.28, p < 0.01$).

Figure 2, 3 and 4 present the differences on mean scores of using metacognitive knowledge by time and treatment.

It was found that the CAS + META condition significantly outperformed the META condition, which in turn significantly outperformed the CAS condition and the CAS condition outperformed the CONT condition on the criteria: using problem solving strategies before the solution process. On the criteria using problem solving strategies during the solution process it was found that the two conditions that were exposed to metacognitive training (META + CAS and META) significantly outperformed the CAS condition, which in turn significantly outperformed the CONT condition.

No significant differences were found between the four conditions on the measure of using problem solving strategies at the end of the solution process ($F(3, 78) = 1.18, p > 0.05$).

**Discussion**

The study raises the question: How can computer technology be used to enhance cognitive development in mathematics?

<table>
<thead>
<tr>
<th></th>
<th>CAS + META N = 20</th>
<th>META N = 20</th>
<th>CAS N = 23</th>
<th>CONT N = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest M</td>
<td>85.4</td>
<td>81.1</td>
<td>84.8</td>
<td>85.1</td>
</tr>
<tr>
<td>S:</td>
<td>10.3</td>
<td>12.8</td>
<td>8.2</td>
<td>9.1</td>
</tr>
<tr>
<td>Posttest M</td>
<td>90.9</td>
<td>85.3</td>
<td>83.9</td>
<td>79.2</td>
</tr>
<tr>
<td>Adjusted M</td>
<td>90.4</td>
<td>86.7</td>
<td>83.5</td>
<td>78.6</td>
</tr>
<tr>
<td>S:</td>
<td>9.1</td>
<td>9.8</td>
<td>6.6</td>
<td>11.7</td>
</tr>
</tbody>
</table>

![Figure 2](image-url)  
*Mean scores on using metacognitive knowledge before solving the task/problem by time and treatment*
Our study investigates the differential effects of CAS environment and metacognitive training on algebraic thinking and metacognitive knowledge. Although all environments focus on promoting algebraic thinking, the study found that students who were exposed to CAS embedded within metacognitive training (CAS + META) showed more positive effects on different aspects of algebraic thinking than students who were exposed only to CAS or metacognitive training. The effects were observed on algebraic reasoning and discovering patterns. The study found no differences between the groups on manipulating algorithms and analyzing changes. In addition, the study found that the CAS + META students outperformed their counterparts on metacognitive knowledge.

**Figure 3** Mean scores on using metacognitive knowledge during solving the task/problem by time and treatment

**Figure 4** Mean scores on using metacognitive knowledge after solving the task/problem by time and treatment
Two factors may explain the findings of our study. First, students who are trained to reflect on their problem-solving processing probably focused on the information provided in the problem. The self-addressed comprehension questions (e.g., ‘What is the task/problem all about?’) probably guided students to look for all the relevant information. Also the connection questions (e.g., ‘How is this problem/task different/similar from what you have already solved?’) might lead students to pay attention to all the information and the structure of the task. Therefore, there is reason to believe that students who use the self-addressed questions succeeded in discovering patterns.

These conclusions support other research (Garofalo and Lester 1985; Schoenfeld 1987; Kramarski, Mevarech and Liberman, 2001) which indicates that students’ problem-solving failures are often due to the ineffective use of what they know, and not due to a lack of mathematical knowledge. Successful problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems. Such metacognitive knowledge is much more likely to be developed in a classroom environment that supports it.

Of particular interest here are the findings that students who were exposed to metacognitive training (CAS + META and META) were more effective in correctly using mathematical reasoning. There is reason to believe that metacognitive knowledge affects algebraic thinking. Perhaps the poor performance of the CAS and CONTROL students may have stemmed from the limited metacognitive knowledge employed in solving word problems. These conclusions support other research. Schoenfeld (1987) also reported that unsuccessful problem solvers, who exhibited limited metacognitive knowledge, mathematical problem solving is often seen as a system of taking one step at a time, without understanding the general principle of the problem.

Masui and De Corte (1999) and Kramarski, Mevarech and Liberman (2001) found that students who were exposed to metacognitive training had more metacognitive knowledge about orienting and self-judging themselves than students in the control groups. In these studies, both metacognitive knowledge and transfer behavior were positively related to academic performance. Further research based on systematic observations may identify other measures of metacognitive knowledge, so more empirical evidence can be collected regarding the relation between metacognitive knowledge and mathematical reasoning.

Conclusions and practical implications

Our findings indicate that effectively using technology to enhance students’ learning opportunities in mathematics classrooms depends on the didactic possibilities linked with its use. The study shows that features of self-questioning such as giving reasons before, during and the end of problem solving must be practiced and reinforced (Webb, 1991; Webb and Farivar, 1994; Kramarski, Mevarech and Liberman, 2001). Students’ exposure to metacognitive training enhances metacognitive knowledge that in turn affects algebraic thinking.

These findings call for the design of additional computer learning environments based on similar components. Future research based on interviews and observations may explore the development of general and specific metacognitive knowledge under different conditions.

References

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**Biographical notes**

Bracha Kramarski has a doctorate and specialized in mathematics and education in different learning environments. Her main interests are Teaching of Mathematics, Computers and Education, Cognition and Metacognition, Different Learning Environments and Teacher’s Training. She also develops programs for the teaching of mathematics by computers, the development and management of a project IMPROVE: Method for teaching mathematics in heterogeneous classrooms. She is currently Deputy Director of the School of Education and Head of the Teachers Training and Inservice Education.

Chaya Hirsh is a senior teacher in mathematics and computing. She holds a B.A. in Education and an M.A. in Technology and Communications in Education. She is currently working on a thesis whose subject is ‘The effects of a computerized algebra system (CAS), using a meta-cognitive approach, on problem solving and mathematical discourse’.

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