A Two Level Feedback System Design to Provide Regulation Reserve*

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Abstract—Demand side management has gained increasing importance as the penetration of renewable energy grows. Based on a Markov jump process model of a group of thermostatic loads, this paper proposes a two level feedback system to govern the interactions between a independent system operator (ISO) and a number of regulation reserve providers such that two objectives are achieved: (1) the ISO can optimally dispatch regulation signals to multiple providers in real time in order to reduce the requirement for expensive spinning reserves, and (2) each regulation provider can control its thermostatic loads to respond to the ISO signal. It is also shown that the amount of regulation reserve that can be provided is implicitly restricted by a few fundamental parameters of the provider itself, such as the allowable set point choice and its thermal constant. An interesting finding is that the regulation provider’s ability to provide a large amount of long term accumulated regulation and short term signal tracking restrict each other. Simulation results are presented to verify theoretical results and illustrate the performance of the proposed framework.

I. INTRODUCTION

Wind energy is increasing its penetration around the world. Europe, Asia, and North America will have an exponential yearly increase in the amount of added wind power capacity by 2020 [1]. The increase in wind energy makes the generation side less controllable due to the intermittent nature of wind. As a consequence, building level demand side management (DSM) has become a crucial part of DSM controlling the grid that can be achieved generally through either a price-based signalling control [4], [7], [14] or a non-priced direct load control (DLC). Compared with pricing mechanism, the operators in DLC will have a better understanding of the loads’ response and thereby be able to provide high resolution regulation reserve [6].

DLC approaches for thermostatic loads have been studied extensively during the past decade for different purposes, including load shifting, load smoothing, and load following. An early investigation of load shifting has been studied with a state queuing model to illustrate the effect of set-point change [12]. It was shown that even a simple step change will result in a complicated load profile due to instantaneous change of loads’ status. Taking into account both the appliances duty cycle as well as a probabilistic model of consumer behaviour, a steady state appliances population distribution during a load shifting demand response is analysed in [8]. More recently, the possibility of integrating DLC into load smoothing or following was investigated. The notion of packetized DLC was proposed in [15], [16] to smooth electricity consumption with the idea of energy quantization. Load following can be found in [5], [10], [13] where various control approaches are used, including switched control, set point control, and model predictive control. Results show that loads, by acting as both a positive and negative generation sources, are promising to respond to reserve signals and maintain grid balance.

Despite the comprehensive study, there are two fundamental issues that remain to be answered. The first is to investigate the regulation reserve limitation and solve for the maximum reserve that can be provided given the provider’s features. The second is the information exchange between the independent system operator (ISO) and the reserve providers. Instead of a one way signalling from the ISO to the providers, a information feedback should be established on the other direction to benefit the grid operation.

The contribution of this paper is to develop a two level feedback system to solve the two issues mentioned above. We focus primarily on smart buildings that are operated by a building operator who has the authority to shift the set point of thermostatic appliances within the building. Consumers are assumed to authorize the operator to control their set point once they provide their preferred allowable set point range. It will be shown that the building operator in the lower feedback level can design a feedback controller to track the regulation signal within a certain limit. The higher feedback loop enables a two-way communication between the ISO and buildings where the former dispatches signals and the latter sends information back to characterize their capability to respond. We derive analytically the maximum regulation reserve that can be provided based on two considerations – the capability to provide long term accumulated and short term ramping reserve. It is further shown that two fundamental parameters determining the limitation are the building thermal constant $\tau$ and appliances temperature gain $T_g$. Large $\tau/T_g$ enables the building to provide larger long term, but smaller short term regulation reserve. Small $\tau/T_g$ behaves oppositely. The overall two level system has good performance in real time operation as it reduces the amount of spinning reserves required from high-cost generators.

The paper proceeds as follows. Sec.II develops the state space model for which the feedback system is designed. Sec.III gives the overall design of the two level feedback system in which we first introduce the lower level controller with feedback linearization. This is followed by an illustration of regulation reserve limitation. We then consider higher level information feedback that enables the ISO to optimally dispatch real time signals to buildings. Sec.IV proposes an

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observer to reduce the states estimation error. Sec.V provides simulation and Sec.VI concludes.

II. MARKOV JUMP THERMAL PROCESS MODELLING

We model the system as a continuous time, discrete state Markov jump process where a state $i$ is defined as a pair of temperature values and thermostat values $(T_i, \text{on/off})$. Previous literature has considered similar definitions [8], [12], [13]. The departure of our model from previous ones is that we consider the state in a Markov setting and establish the implicit relations between thermal and the Markov jump processes. Denote the comfort band as $\{T_{\text{min}}, T_{\text{max}}\}$ and discretize temperature in the band into bins of width $\delta$, with the number of temperature bins being $N = \Delta_{\text{band}}/\delta$, where $\Delta_{\text{band}}$ is the width of the comfort band. We say that an appliance is in state $i$ for $i = 1, \ldots, N$ if $T_i$, the ambient temperature that the appliances regulates, satisfies $T_i \in [T_{\text{min}} + (i - 1)\delta, T_{\text{min}} + i\delta]$ with status off, and $i$ for $i = N+1, \ldots, 2N$ if $T_i \in [T_{\text{max}} - (i - N - 2)\delta, T_{\text{max}} - (i - N - 1)\delta]$ with status on.

First we model the Markov transition of the uncontrolled thermal process with $u = 0$. Denote by $\alpha$ the transition rate when the thermostat is off, and $\beta$ the rate when the thermostat is on; see Fig.1(a). In the duty off process, the probability that an appliances will be in state $i$ at time $t+1$ given that it is in state $j$ at time $t$ is given by,

$$p_{i,j} = \begin{cases} 
\alpha \Delta t + o(\Delta t) & \text{if } i = j + 1 \\
1 - \alpha \Delta t + o(\Delta t) & \text{if } i = j \\
0 & \text{otherwise,}
\end{cases}$$

(1)

where $o(\Delta t)$ means $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. We eliminate the probability of Markov jump between non-adjacent states with small $\Delta t$. The above equation can be interpreted that the transition probability between adjacent states linearly increases with small $\Delta t$.

![Fig. 1. Markov jump process model. (a) Markov chain transition rate diagram. (b) Transition from state $i$ to $i+1$. With temperature rise $\Delta T$ (length of arrow), appliances whose temperatures are inside the dotted area (A, C, E) change state and those that are outside (B, D) remain in the same state.](image)

From a thermal point of view, the temperature change $\Delta T$ within a small $\Delta t$ is proportional to its warming rate $r_{\text{off}}$ due to inside-outside temperature difference: $\Delta T = r_{\text{off}} \Delta t$. Assuming that an appliance’s actual temperature is uniformly distributed among its bins, then the probability that the Markov jump happens is equal to the probability that the appliance temperature is in the dotted area in Fig.1(b). The probability of being in the dotted area is given by,

$$p_{\text{dot}} = \frac{r_{\text{off}} \Delta t}{\delta} = \frac{\Delta_{\text{band}} \Delta t}{\delta \Delta t} = \frac{N \Delta t}{\Delta t},$$

(2)

where the second equality follows from the relation between warming rate and duty cycle. Since $p_{i,i+1} = p_{\text{dot}}$, comparing (1) with (2) we have $\alpha = N/\Delta t$. Similarly, $\beta = N/\Delta t$ for duty on process. These two equalities are the implicit relation between the duty cycle and the Markov model.

When control is applied to shift the set point at a rate of $r_{\text{set}}$ (the unit of $r_{\text{set}}$ is the same as $r_{\text{on}}$ or $r_{\text{off}}$), there is also a transition between adjacent states within $\Delta t$. Intuitively, when we change the set point, the absolute temperature in each individual room does not change instantly, but its relative position to the comfort band changes as the comfort band shifts. When the set point rises, namely $r_{\text{set}} > 0$, we can show that the resulting change in the transition rate is $u = r_{\text{set}}/\delta$. The transition rate is the same with $r_{\text{set}} < 0$. The combined transition rate by the thermal process and set point shifting process is the sum of these two individual processes as shown in Fig.1(a). Note that the allowable control set is $L_u = \{u \mid -\beta \leq u \leq \alpha\}$ to maintain a non-negative Markov rate. When $u > \alpha$ or $u \leq -\beta$, the system fails to be a Markov chain.

Similar to (1), we can write for non-zero $u$,

$$p_{i,j} = \begin{cases} 
(\alpha - u) \Delta t + o(\Delta t) & \text{if } i = j + 1 \\
1 - (\alpha - u) \Delta t + o(\Delta t) & \text{if } i = j \\
0 & \text{otherwise.}
\end{cases}$$

(3)

Let $x(t)$ be a vector whose $i^{th}$ component is the probability that an appliance is found in the $i^{th}$ state in the Markov chain. It can be shown [17] that the dynamics of $x(t)$ is

$$\dot{x}(t) = [A + Bu(t)]x(t),$$

(4)

where $A$ and $B$ are given by

$$A = \begin{pmatrix} 
-\alpha & 0 & \cdots & 0 \\
0 & -\alpha & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & -\beta \\
\end{pmatrix},$$

$$B = \begin{pmatrix} 
1 & 0 & \cdots & 0 \\
0 & 0 & \cdots & -1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & -1 \\
\end{pmatrix}.$$
respectively. The output of the system, namely the aggregated consumption, is
\[ y(t) = Cx(t), \]
where \( C = [0, \ldots, 0, N_c, \ldots, N_c] \), and \( N_c \) is the total number of appliances in the pool.

III. Two Level Feedback System Design

Based on the state space model developed in Sec.II, we are able to design our two level feedback system as shown in Fig.2. In the building level feedback loop, we design a state feedback law such that the building can track the required regulation signal within certain limits. In the higher level feedback, each individual building sends to the ISO its information to characterize the building’s ability to respond to regulation signals. The ISO, after receiving all information from each provider, dispatches real time signals after solving an optimization problem. The next sub-section will briefly discuss the building level controller design based on feedback linearization.

A. Building Level Feedback Linearization Design

To solve the building level controller for the non-linear system described in (4) and (5), the feedback linearization method can be readily applied [11]. Note that the system has relative degree 1 as the first derivative of \( y \),
\[ \dot{y} = C\dot{x}(t) = C[A + Bu(t)]x(t), \]
deeps on the control \( u \) if
\[ x(t) \in \{ x(t) \in \mathbb{R}^{2N} | CBx(t) \neq 0 \}, \]
\[ x(t) \in \{ x(t) \in \mathbb{R}^{2N} | x_N(t) + x_N(t) > 0 \}, \]
which can be guaranteed as long as the Markov chain is irreducible and ergodic. (7) means the number of controllable appliances near the comfort band is positive, and the set point shift can change the aggregated consumption.

To design a tracking controller for the relative degree 1 system where \( R(t) \) is the regulation signal given by the ISO, we need \( \dot{R}(t), \ddot{R}(t) \) to be available and bounded for all \( t > 0 \). This will be assured since the ISO regulation signal is bounded: \( R(t) \in [-R_t, R_t] \), where \( R_t \) is amount of reserve sold to the market, and \( \dot{R}(t) \) is updated every \( \Delta t = 4 \) or 6 seconds prior to consumption for the building to obtain \( \dot{R}(t) \)
\[ \dot{R}(t) = \frac{R(t + \Delta t) - R(t)}{\Delta t}. \]
The time derivative of the tracking error,
\[ \dot{e}(t) = \dot{y}(t) - \dot{R}(t) = C[A + Bu(t)]x(t) - \dot{R}(t), \]
will asymptotically approach 0 if \( u(t) \) is chosen such that
\[ \dot{e}(t) = Ke(t), K > 0. \]
The controller
\[ u(t) = -\frac{CAx(t)}{CBx(t)} + \frac{1}{CBx(t)}[-K(\xi(t) - R(t)) + \dot{R}(t)] \]
satisfies the requirement because substituting (10) into (8) yields (9).

B. Long and Short Term Reserve Limitation

In this section we answer the first question proposed in Sec.I, namely what is the regulation reserve limitation that a building can provide given its model parameters and customers’ specified settings. This question has two parts: (1) in the long term, we need the accumulated shift of set point to be within the allowable range given by the customers, and (2) in the short term, we need the signalling change to be within certain limits such that the controller can track it. The first proposition will discuss the long term limitation.

Proposition 1 For a given allowable set point range \( [T_{set} - \frac{1}{2}\Delta_{set}, T_{set} + \frac{1}{2}\Delta_{set}] \) from customers, the accumulated amount of reserve that a building can provide up to time \( t \), \( S(t) = \sum_{i=0}^{t} R(i) \), is bounded by
\[ S(t) \leq \frac{N_c \Delta_{set}}{2T_{g}}. \]

Proof. Denote the base consumption level as \( R_b \), then the total consumption up to time \( t \) for a given sequence of \( R(i) \) is given by,
\[ P_{cool} = tR_b + \sum_{i=0}^{t} R(i) = tR_b + S(t), \]
with an average consumption level of
\[ P_{ave} = R_b + \frac{S(t)}{t}. \]
Providing the above amount of regulation reserve is equivalent to maintaining at a consumption level of \( P_{ave} \) to provide sustained response up to time \( t \). The relation between the average consumption and non-zero control is,
\[ P_{ave} = N_c \frac{t_{on}}{t_{on} + t_{off}} = N_c \frac{N}{\alpha - u} \cdot \frac{\beta + u}{\beta + u} = N_c \frac{\alpha - u}{\alpha + \beta} \]
(14)
For an uncontrolled process at average consumption,
\[ R_b = N_c \frac{\alpha}{\alpha + \beta}. \] (15)

From the above three equations,
\[ S(t) = \frac{N_c}{\alpha + \beta} \frac{u}{t} \] (16)
resulting a control value of
\[ u = -\frac{S(t)(\alpha + \beta)}{tN_c}. \] (17)
The set point shift after time \( t \) by the control in (17) is,
\[ T(t) = T(0) + tr_{set} = T(0) + tu \delta \]
\[ = T(0) - \frac{S(t)}{N_c} \left( \frac{N}{t_{off}} + \frac{N}{t_{on}} \right) \frac{T_{band}}{N} \]
\[ = T(0) - \frac{S(t)}{N_c} (r_{on} + r_{off}) \]
\[ = T(0) - \frac{S(t) T_g}{\Delta_k}. \] (18)

The third equation is based on the relation between transition rate and duty cycle. The last equation says that the sum of cooling and warming rate equals the fraction \( T_g/\tau \) because \( r_{off} = r_{amb} \) and \( r_{on} = r_{app} - r_{amb} \), where \( r_{amb} \) is the warming rate caused by the ambient temperature that is higher than room temperature, and \( r_{app} = T_g/\tau \) is the cooling rate caused by the operation of the air conditioner compressor [15]. To maintain the set point shift within the allowable band with initial condition \( T(0) = T_{set} \), we need
\[ T(t) \in [T_{set} - \frac{1}{2} \Delta_{set}, T_{set} + \frac{1}{2} \Delta_{set}]. \] (19)

Substituting (18) into (19) yields (11). ■

**Remark.1** The long term reserve limitation is proportional to three parameters: \( N_c, \Delta_{set}, \) and \( \tau/T_g \). The intuition for the first two parameters is that a large appliance population and allowable set shift authorized by customers enable the operator to provide more reserve. As for the fraction \( \tau/T_g \), we note that a large value of \( \tau \) impedes, and \( T_g \) facilitates, the change of room temperature to the boundary of allowable set point range.

To consider the short term limitation, the possible ramp rate of consumption is limited by the state \( x(t) \) because we are shifting the set point rather than directly turning appliances on or off. Aside from \( x(t) \), we are interested in finding a few fundamental parameters that characterize the short term limitation similar to those found above. The desired result is the following.

**Proposition.2** For a given temperature band around the set point \( [T_{set} - \frac{1}{2} \Delta_{band}, T_{set} + \frac{1}{2} \Delta_{band}] \), the amount of reserve that a building can provide for the short term is limited by,
\[ -Nx_{2N} \frac{N_c T_g}{\tau \Delta_{band}} \leq \Delta R \leq Nx_{2N} \frac{N_c T_g}{\tau \Delta_{band}}. \] (20)

**Proof.** In the dynamic operation when the tracking error is 0, the feedback controller is given by,
\[ u = \frac{1}{N_c (x_{N} + x_{2N})} [N_c (\alpha x_{N} - \beta x_{2N}) - \Delta R]. \] (21)

Since the allowable control set is \( u \in [-\beta, \alpha] \), then \( \Delta R \) is restricted by
\[ -N_c (\alpha + \beta) x_{2N} \leq \Delta R \leq N_c (\alpha + \beta) x_{N}. \] (22)

Using the relation between transition rate and duty cycle developed in Sec.II and the fact
\[ \Delta_{band} = t_{off} - r_{on} = t_{on} - r_{off}, \]
the following inequality is seen to hold:
\[ -N_{x_{2N}} \frac{N_c (r_{on} + r_{off})}{\Delta_{band}} \leq \Delta R \leq N_{x_{2N}} \frac{N_c (r_{on} + r_{off})}{\Delta_{band}}. \] (23)

Using the fact that \( r_{on} + r_{off} = T_g/\tau \) yields (20). ■

**Remark.2** The short term reserve ability is proportional to \( N_c \), and inversely proportional to \( \Delta_{band} \) and \( \tau/T_g \). The proportionality to \( N_c \) shares the same explanation as in remark 1. The two inverse proportionalities can be explained that small value of \( \Delta_{band} \) makes the Markov transition faster, and large value of \( T_g/\tau \) makes the thermal transfer faster. These two factors in turn facilitate the state transition to provide larger instant-by-instant reserve.

**Remark.3** The advantage of using small \( \Delta_{band} \) is that we can provide large short term reserve and make room temperatures stick to the set point with the potential that we shorten the appliances’ duty cycle. This trade off between system performance and appliances’ duty cycle functioning is consistent with what we find in [15] where electricity consumption can be smoothed by shortening appliances duty cycle.

**Remark.4** The fraction \( \tau/T_g \) affects both long and short term reserve limitation. Based on (11) and (20), we find that a building able to provide large amount of short term ramping regulation is more incapable of providing long term accumulated regulation. The opposite also holds. Thus the two capabilities restrict each other.

### C. Buildings in the Regulation Reserve Market

Based on the above propositions, we determine the maximum regulation reserve that a building can provide to the power market. To become a qualified provider in the U.S., a building needs to pass the T-50 qualifying test [2]. Fig.3 shows the test signal from PJM. The dotted line is the ideal consumption response of the building to the test signal. Two key requirements are needed to pass the test:
- Rate of response: the building needs to reach the maximum (minimum) consumption level within \( k \) minutes.
- Sustained response: the building needs to remain at the maximum (minimum) consumption level for 5 minutes.

The following corollary gives an upper bound on the regulation reserve that a building can provide.

**Corollary.** To pass the T-50 qualifying test with a response rate in \( k \) minutes \((k \leq 5)\), the maximum regulation reserve \( R_{t,\text{max}} \) that a building can sell to the market is given by,
\[ R_{t,\text{max}} = \min \left\{ \frac{N_c \tau \Delta_{set}}{20T_g}, \frac{kN_c}{\max(t_{on}, t_{off})} \right\}. \] (24)

**Proof.** The proof follows from the first two propositions. We refer to [17] for details.■
From the above two inequalities, from the provider.

are the maximum and minimum reserve capability threshold where

\[ R_{\text{c}} = \min\{r_{\text{off}}(x_N - x_{2N}), r_{\text{on}}(x_N + x_{2N})\}, \]

\[ R_{\text{min}} = N_c \left\{ (r_{\text{off}} x_N - r_{\text{on}} x_{2N}) + \min \frac{T_{\text{set}} - T_{\text{set min}}}{\Delta t}, r_{\text{on}}(x_N + x_{2N}) \right\}, \]

\[ R_{\text{max}} = N_c \left\{ (r_{\text{off}} x_N - r_{\text{on}} x_{2N}) - \min \frac{T_{\text{set max}} - T_{\text{set}}}{\Delta t}, r_{\text{off}}(x_N + x_{2N}) \right\}, \]

are the maximum and minimum reserve capability threshold from the provider.

Proposition 3 Denote \( \Delta P \) as the stochastic demand change, then the spinning reserve \( P_{\text{spin}} \) needed to maintain grid balance is given by,

\[ P_{\text{spin}} = \left( \Delta P - \Delta R_{\text{max}} \right) I_{\Delta P > \Delta R_{\text{max}}} + \left( \Delta P - \Delta R_{\text{min}} \right) I_{\Delta P < \Delta R_{\text{min}}}, \]

where \( I_{\{\}} \) is the indicator function, and

From (21), the instant respond can be expressed in terms of the control \( u \),

\[ \Delta R = N_c \left\{ (\alpha x_N - \beta x_{2N}) - u(x_N + x_{2N}) \right\} \]

Substituting (27) into (28) yields \( \Delta R \in [\Delta R_{\text{min}}, \Delta R_{\text{max}}] \), where \( \Delta R_{\text{min}} \) and \( \Delta R_{\text{max}} \) take values in (26). For the grid balance, we have,

\[ \Delta P = \Delta R + P_{\text{spin}}. \]

We wish to use as little spinning reserve as possible, then \( P_{\text{spin}} \) will take value in (25). The indicator function in (25) gives the condition that \( \Delta P \) is outside the range of \( \Delta R \).

E. Real Time Regulation Signal Dispatch

In power market, the ISO purchases reserves from a number \( m \) of providers. Assuming that feedback signals are available between all providers and the ISO to exchange real time information. Upon receiving information, the ISO knows their individual capability limitations, and then dispatches regulation signals to each of the provider. The signals are dispatched such that the minimum spinning generation is used and that they should be within the capability the providers to respond. The question is how can the ISO dispatch signals in an optimized way at time \( t \). One criteria is to maximize the regulation response capability at time \( t + 1 \). From proposition 3, the \( i \)th building can provide regulation reserve with range \( \Delta R_i \in [\Delta R_{\text{min}}, \Delta R_{\text{max}}] \). The width of the closed interval of regulation reserve is given by

\[ W_d^i = (\Delta R_{\text{max}} - \Delta R_{\text{min}}) \]

\[ = N_c^i (x_N^i (t + 1) + x_{2N}^i (t + 1)) \]

\[ \left\{ \min \left( \frac{\Delta T_{\text{set}}}{\Delta \delta}, r_{\text{on}}^{i} \right), \right. \]

\[ \left. \min \left( \frac{\Delta T_{\text{set}}}{\Delta \delta}, r_{\text{off}}^{i} \right) \right\}, \]

Then the objective becomes

\[ \max \sum_{i=1}^{m} W_d^i - MP_{\text{spin}}^2, \]

which is to say that we are maximizing the sum of \( m \) regulation reserve range widths from the buildings at time \( t + 1 \), minus the spinning generation penalty \( P_{\text{spin}} \) with positive \( M \). The maximization problem is subject to the following constrains:

State dynamics:

\[ x_N^{i+1}(t+1) = x_N^i(t) + \Delta t(\alpha x_N^i - u^i x_N^i) + \Delta t(\alpha x_{2N}^i - u^i x_{2N}^i) \]

\[ x_{2N}^{i+1}(t+1) = x_{2N}^i(t) + \Delta t(\beta u^i x_N^i - \beta u^i x_{2N}^i) \]

Feedback controller design:

\[ N_c^i \Delta t(x_N^i(t) + x_{2N}^i(t))u^i + \Delta R^i = N_c^i \Delta t(\alpha x_N^i(t) - \beta x_{2N}^i(t)) \]
Suppose demand balance:
\[
\sum_{i=1}^{m} \Delta R_i^t + P_{\text{spin}} = \Delta P. \tag{34}
\]

Allowable control set:
\[-\beta^i \leq u_i^t \leq \alpha^i. \tag{35}\]

Allowable regulation range:
\[R_b - R_t \leq Cx + \Delta R_i^t \leq R_b + R_t. \tag{36}\]

Allowable set point range:
\[T_{\text{set}}^{\min,i} \leq T_i^t(t + 1) = T_{\text{set}}^i(t) + u^t \Delta t \delta \leq T_{\text{set}}^{\max,i}. \tag{37}\]

The above problem cannot be solved with standard techniques because the objective function has the $\min(\cdot, \cdot)$ operator. We transform it into a quadratic program. Let
\[
m_1^i = \min \left( \frac{T_{\text{set}}^i(t + 1) - T_{\text{set}}^{\min,i}}{\Delta t \delta}, r_{\text{on}} \right),
m_2^i = \min \left( \frac{T_{\text{set}}^{\max,i} - T_{\text{set}}^i(t + 1)}{\Delta t \delta}, r_{\text{off}} \right). \tag{38}\]

Then the objective becomes
\[
\max \sum_{i=1}^{m} N_i^2 (x_i^b(t + 1) + x_{2N}^i(t + 1))(m_1^i + m_2^i) - MP_{\text{spin}}^2. \tag{39}\]

If we add the following constraints,
\[
m_1^i \leq r_{\text{on}}, m_1^i \leq \frac{T_{\text{set}}^i(t + 1) - T_{\text{set}}^{\min,i}}{\Delta t \delta},
m_2^i \leq r_{\text{off}}, m_2^i \leq \frac{T_{\text{set}}^{\max,i} - T_{\text{set}}^i(t + 1)}{\Delta t \delta}, \tag{40}\]

and solve the $QP$,
\[
\max \quad (39)
\text{s.t.} \quad (32) - (37), (40), \tag{41}\]

we will obtain the same optimal solution as solving the original problem. This is because when reaching the optimal solution, one of the inequality constraints for both $m_1^i$ and $m_2^i$ in (40) will be strict, otherwise the optimal solution is not reached since we can increase the value of $m_1^i$ or $m_2^i$ to increase the value of objective function due to positive coefficient $N_i^2(x_i^b(t + 1) + x_{2N}^i(t + 1))$ in (39). Then the optimal solution for (41) will satisfy (38) and optimizing over (41) is equivalent to solving (31) subject to (32)-(37). Note that (41) has quadratic objective with linear constraints so that we can solve it efficiently.

\section*{IV. Time Varying Observer Design}
In controller (10), it is assumed that $x(t)$ is measurable. If this is not true, especially when the temperature band is finely discretized, i.e. $\delta$ is small so that the sensor cannot provide the required precision, we need an observer to estimate the state. Considering the following dynamics of the observer,
\[
\dot{x} = Ax + Bxu + L(t)(y - C\hat{x}), \tag{42}\]

where $L(t)$ is the time varying column vector to be designed. The last term is similar to the innovation term in a Luenberger filter [9]. Define the estimation error, $e = x - \hat{x}$. Then
\[
\dot{e} = [A + Bu(t) - L(t)C]e. \tag{43}\]

In the observer design, we restrict $u(t)$ to (44) to prevent control saturation with small constant positive margin $\bar{e} > 0$,
\[
u(t) \in [-\beta + \bar{e} \leq u(t) \leq \alpha - \bar{e}]. \tag{44}\]

This guarantees an irreducible and ergodic Markov chain. If the control signals is allowed to stay saturated for a long time, then the Markov chain breaks and some of the states become isolated, see Fig.1(a) when $u = -\beta$ or $u = \alpha$. Estimation error of the isolated states may not be reduced. The desired observer design is shown below.

**Proposition 4** If the control is restricted to (44), then
(1) There exists a time varying $L(t) = [0, \ldots, 0, L_{2N}(t)]^T$
\[
\text{such that } e(t)^T[A + Bu(t) - L(t)C]e(t) < -\epsilon(t)^T[e(t)]^2 \text{ for all } e(t)
\]
and (45)
(2) The estimation error converges to zero asymptotically, i.e. $\lim_{t \to \infty} \epsilon(t) = 0$.

**Proof.** (1). It is equivalent to show
\[
e(t)^T[A + Bu(t) - L(t)C + e(t)]e(t) < 0. \tag{45}\]

If $\hat{A} = A + Bu(t) - L(t)C + e(t)I$, then $\hat{A}(t)$ can be expressed by the following equation based on the special structure of the state space matrices $A, B$ and $C$ derived in section II.

\[
\hat{A} = \begin{pmatrix}
-\alpha + u & 0 & \cdots & \beta + u \\
\alpha - u & 0 & \cdots & 0 \\
0 & -\alpha + u & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & -\beta - u \\
0 & 0 & \cdots & -\beta + u \\
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\end{pmatrix}
\tag{46}
\]

Denote $\hat{A}_i$ as the $i-$by$-i$ square matrix from the upper left part of $\hat{A}$. In order to show that $\hat{A}$ is negative definite, it is equivalent to show
\[
(-1)^i \det(\hat{A}_i) > 0, \text{ for } i = 1, \ldots, 2N. \tag{47}\]

For $i = 1, \ldots, 2N - 1$, $\hat{A}_i$ is a triangular matrix whose determinant is the product of its diagonal elements. From
Since (46), the diagonal elements $\tilde{A}_{i,i}$ are given by,
\[
\tilde{A}_{i,i} = \begin{cases} 
-\alpha + u + \epsilon(t) & \text{if } i = 1, \ldots, N \\
-\beta - u + \epsilon(t) & \text{if } i = N + 1, \ldots, 2N - 1.
\end{cases}
\]
\hspace{1cm} (48)
Since $\epsilon(t) = \gamma \min[\beta + u(t), \alpha - u(t)]$, we have $A_{i,i} \leq (\gamma - 1) \min[\beta + u(t), \alpha - u(t)] < 0$ for $i = 1, \ldots, 2N - 1$. This yields
\[
(-1)^i \det(\tilde{A}_i) = (-1)^i \prod_{j=1}^{i} \tilde{A}_{j,j} > 0, \text{ for } i = 1, \ldots, 2N - 1.
\]
\hspace{1cm} (49)
As for $i = 2N$, $\det(\tilde{A})$ is a linear function of $L_{2N}$ as the observer parameter only appears in the last row of the matrix, so there exists $L_{2N}(t)$ such that $\det(\tilde{A}) > 0$. This completes the proof of negative definiteness of $\tilde{A}$ and therefore (45) holds.

(2). Let
\[
y(t) = e^{\int_0^t \epsilon(\tau)d\tau} \|e(t)\|^2,
\]
then the derivative of $y(t)$ is given by,
\[
\dot{y}(t) = \epsilon(t) e^{\int_0^t \epsilon(\tau)d\tau} \|e(t)\|^2 + 2e^T(t)\dot{A}e(t) e^{\int_0^t \epsilon(\tau)d\tau}
\]
\[
= e^{\int_0^t \epsilon(\tau)d\tau} [e(t)\|e(t)\|^2 + 2e^T(t)\dot{A}e(t)]
\]
\[
< e^{\int_0^t \epsilon(\tau)d\tau} [e(t)\|e(t)\|^2 - 2e(t)\|e(t)\|^2]
\]
\[
= -e^{\int_0^t \epsilon(\tau)d\tau} e(t)\|e(t)\|^2
\]
\[
= -\epsilon(t)y(t),
\]
\hspace{1cm} (51)
The above equation indicates that $y(t)$ is stable at zero. On the other hand, from (50) we have $\lim_{t \to \infty} e^{\int_0^t \epsilon(\tau)d\tau} = \infty$ and $\lim_{t \to \infty} y(t) = 0$. Necessarily we need $\lim_{t \to \infty} \epsilon(t) = 0$ to satisfy these two conditions, namely the estimation error approaches zero asymptotically. 

**Remark.5** $\epsilon(t) = \gamma \min[\beta + u(t), \alpha - u(t)] \geq \gamma \dot{\epsilon}$ is the lower (conservative) bound of the convergence rate we can achieve. In real time control when the signal is distributed across the allowable set being away from saturation more than $\dot{\epsilon}$, the convergence rate is larger than the conservative bound. We note that as the duty cycle increases, the boundary value of $\alpha = N/t_{on}$ and $\beta = N/t_{off}$ decreases, which means we have smaller allowable control set. Consequently, the control with large duty cycle appliance would be closer to the boundary than the control with small duty cycle appliances given the same ISO signal. It is therefore anticipated that the error convergence will be faster for system having short duty cycle appliances.

**Remark.6** The restriction in (44) is also conservative. A less restrictive statement is the following:

Let $S$ be the set of time that the control is bounded away from saturation with some constant value $\dot{\epsilon} > 0$, i.e.,
\[
S = \left\{ t : -\beta + \dot{\epsilon} \leq u(t) \leq \alpha - \dot{\epsilon} \right\}.
\]
\hspace{1cm} (52)
then Proposition.4 holds if the measure of the set is infinity: $\mu(S) = \infty$ as $t \to \infty$.

V. SIMULATION

We illustrate selected results. For comprehensive simulation and discussion, please refer to [17].

A. Long and Short Term Reserve Limitation

Fig. 4 shows how the long and short term reserve limitation affect the performance in the T-50 test according to (24). Fig.4(a) is an example of the inability to provide long term reserve due to limited allowable set point range $\Delta_{set}$. The set point hits the lower bound of the allowable set point range after between 15 to 20 minutes. As a consequence, the building cannot provide a sustained high consumption by further adjusting the set point. Similarly, the set point hits the upper bound after 40 minutes that prevents the building from providing sustained lower level reserve. Fig.4(b) is an example of the inability to provide short term regulation reserve. Although the set point shift is within the allowable set, the response rate to provide positive and negative reserve is smaller than the required rate for a given maximum level $R_t$ sold to the market. This is because the thermostatic appliances have large duty cycle that heavily restricts the instant-by-instant response.

![Fig. 4. Reserve provider fails to pass the T-50 qualifying test due to either a bounded allowable set point range or a limited ramping capability.](image-url)

(a) Long Term Regulation Reserve

(b) Short Term Regulation Reserve
B. Optimal Regulation Signal Dispatch

We use real time PJM data [3] to verify the performance of the two level feedback system containing lower level control and upper level optimization. Fig.5 and Tab.1 illustrates the comparisons between the proposed two level feedback system and the system having only lower level controller without higher level information feedback. We find that the real time aggregated spinning generation is reduced approximately by 50% using optimized dispatch signals. The spinning generation’s standard deviation, maximum, and minimum values are reduced by 30%, 30%, and 15% respectively. Clearly the higher information feedback results in significantly reduced operation costs.

![Fig. 5. Real time spinning generation before/after optimization. Spinning generation is more concentrated around 0 with reduced variation after regulation signals are optimally dispatched.](image)

Fig. 5. Real time spinning generation before/after optimization. Spinning generation is more concentrated around 0 with reduced variation after regulation signals are optimally dispatched.

<table>
<thead>
<tr>
<th>Unit/kW</th>
<th>Spin. Gen.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max</th>
<th>Min</th>
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</thead>
<tbody>
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<td>Opt. Sig.</td>
<td>10027</td>
<td>102.16</td>
<td>27.20</td>
<td>100.27</td>
<td>-210.59</td>
</tr>
<tr>
<td>N-Opt. Sig.</td>
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<td>0.44</td>
<td>146.19</td>
<td>146.19</td>
<td>-247.65</td>
</tr>
</tbody>
</table>

C. Observer Performance

Fig.6 shows the convergence of the error norm \(|e(t)|\) when appliances with different duty cycles receive the same ISO signal. We find in both cases that convergence speed is quicker than the conservative bound in remark 5. The norm converges to zero after 20 minutes for appliances with 10 minutes duty cycle, and converges to zero after 35 minutes for appliances with 20 minutes duty cycle. This is because smaller duty cycle enables a larger allowable control set that facilitates convergence, i.e the system has large values of \(\alpha\) and \(\beta\) to prevent control saturation.

![Fig. 6. Convergence speed of the estimation error vector e(t) depends on the duty cycle.](image)

VI. CONCLUSIONS AND FUTURE WORK

This paper proposes a two level feedback system design to provide regulation reserve. The lower level building feedback controller allows the operator to track a given signal within certain limits, and the higher level information feedback allows the ISO to optimally dispatch real time regulation signals to multiple providers. We derive analytically the building’s limitation in providing both long and short term regulation reserve and provide intuitive explanations. Finally we propose an observer to reduce the state estimation error. Simulation is presented to illustrate the theoretical findings. Future work will include our electricity packet framework [15], [16] in the two level feedback system design.

REFERENCES