ON A DECOUPLED INTEGRATED DESIGN APPROACH TO ROBOTIC MECHANISMS

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ABSTRACT

In this paper, we propose a new design process which is called decoupled integrated design process for robotic mechanisms or mechanisms with consideration of their multi-functions and constraints. The example of the functions includes path generation, function generation and rigid body guidance. The example of the constraints includes minimum negative side effects to the environment the mechanism interacts. The proposed design process is a decoupled one, inspired by axiomatic design theory. In particular, the first step of the design process is to analyze the design requirement and design parameter relation, which can be represented by a correspondence matrix. When the correspondence matrix is a triangular one, the corresponding design problem is called a decoupled design problem. Subsequently, we propose a decoupled integrated design process which is applied to a decoupled problem. Throughout the paper, we take a 2-DOF five-bar mechanism as an example; yet the approach is applicable to any kind of dynamic systems.

Keywords: concurrent design, decoupled design, inverse kinematics, path design problem, robotic mechanism.

1. INTRODUCTION

We consider a robotic mechanism as consisting of more than one motor (among which at least one is the servomotor). It is noted that there is a significant difference between a 1-DOF mechanism that has only one servomotor and a mechanism that has more than 1-DOF, among which at least one must be servomotor. For instance, for the 1-DOF four-bar mechanism, the servomotor is not able to change the shape of the path of the end-effector [Ouyang, 2002; Ouyang and Zhang, 2005]. In this paper, we consider a five-bar 2-DOF robotic mechanism to illustrate our idea without loss of generality (see Fig. 1).

There are three general mechanism design problems, namely path generation, function generation, and body guidance. A mechanism will also perform force transfer along with motion transfer. Therefore, there will be a load on the end-effector and motors on driving bodies. In the case of the mechanism in Fig. 1, point C serves as an end-effector and link 1 and link 4 serve as driving bodies. In this paper, we consider the path generation problem.

A mechanism under design is always subject to certain constraints, e.g., low cost, high quality, good robustness, a restricted operation space, less vibrations, user-friendliness, etc. Further, constraints have two types: those that have clear criteria to evaluate whether they are satisfied by the mechanism or not and those that do not have any clear criterion or that have some vague criteria (e.g., “low” cost).

Adapted from general design theory [Suh, 1990], design is to determine the structure to satisfy the functional and constraint requirements. The structure is represented by the parameter denoted by P. We can consider the constraint as a kind of function to be satisfied and denote R for the both functional and constraint requirement [Zhang, 2009]. The design is then a creation of the relation between P and R, denoted by P-R. Fig. 2 shows an instance of P-R where there
are three Ps and three Rs. Further a P-R relation can be represented by a P-R matrix as follows:

\[
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{21} & A_{22} & \cdots & A_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{n1} & A_{n2} & \cdots & A_{nn}
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_n
\end{bmatrix}
\]

(1)

where \(A_{ij}\) is 1 if \(P_j\) is associated with \(R_i\) and 0 if \(P_j\) is not associated with \(R_i\), and \(P_j\) is a parameter and \(R_i\) is a requirement. The above relation can be further written into a matrix form, i.e.,

\[
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{21} & A_{22} & 0 \\
0 & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix}
\]

(3)

The main concern of the present paper is the design process - how are all the P-R relations in a particular design (say the design instance in Fig. 2) manipulated? For example, do we actually determine \(P_1\) for \(R_1\) first, and then \(P_2\) for \(R_2\) or determine all Ps for all Rs simultaneously – an approach usually called “All-In-One” [Alyaqout et al., 2010] or concurrent design [Yan and Yan, 2009]? This paper attempts to argue that a design process should be tailored to a design problem. It is not always true to have a concurrent design process to every kind of design problem. In particular, for a decoupled problem (which will be defined later in this paper), a decoupled integrated approach (which will be proposed later in this paper) may better serve. In Section 2, we shall outline some relevant existing design process in the literature. In Section 3, we propose the decoupled design process. In Section 4, we present an example to illustrate how the decoupled design process works to the decoupled design problem. In Section 5, we discuss some implication of the proposed decoupled integrated design process. Finally, there is a conclusion.

2. THE EXISTING DESIGN PROCESS

Computationally, the design problem is usually formulated into a multi-objective optimization problem due to many Ps and Rs, and thus an optimization model is readily applied [Zhang et al., 1999; Li et al., 2001; Yan and Yan, 2009]. In such an optimization model, each objective represents one requirement in a design problem. Weights are used to be associated with individual objectives, reflecting the degree of importance of each objective in a total design problem. Often, weights are determined prior to determination of design parameters. In computation, all parameters and objectives are equally treated (i.e., updated to pursue an optimum of the overall objective function); see the work of Yan and Yan [2009]. However, in practice, the importance of a particular requirement is difficult to be assessed by the designer notwithstanding the determination of the importance of all requirements.

In fact, the above design process is only suitable to a fully coupled P-R relation in which no \(A_{ij}\) in Equation (1) is equal to zero. Another shortcoming with the design process is computational overhead, as the problem size is usually large, covering all Rs and Ps. For the convenience of later discussions in this paper, we shall call such a design process a full concurrent design (FCD) process or concurrent design process. In mechanism design, the approach proposed by Yan and Yan [2009] is a typical FCD process.

By recognition of the computational overhead with the FCD process, Alyaqout et al. [2010] proposed a sequential iterative design process for mechatronic systems, especially for two design goals or requirements: stability and robustness of the system. In their approach, a part of design parameters that have a strong connection with robustness goes first in optimization and then the remainder of design parameters goes for stability of the system. However, in their approach, there is no explicit notion of “decoupled” design problem which will be discussed later in this paper; in other words, their approach remains to be generalized.

In traditional mechanism design, it is a common design practice to fully cancel shaking force first and then to minimize shaking moment and driving torque subject to the condition of full cancellation of shaking force [Arakelian and Dahan, 2001]. The nature of such a design process is closer to what we shall propose in this paper, a decoupled integrated design process; yet, there has been no effort to generalize this design process to the general product or mechanism design, to our best knowledge. The proposed design approach can be readily extended to any product design; see the later discussion in Section 5.

3. THE DECOUPLED INTEGRATED DESIGN

We will first give definitions to several concepts and then we outline our approach.
**Definition 3.1** Coupled design problem. When the matrix (1) is neither diagonal nor triangular, the design problem is called coupled design. A full coupled design problem, as we mentioned before, is such that no $A_{ij}$ in Equation (1) is equal to zero.

**Definition 3.2** Uncoupled design problem. When the matrix (1) is a diagonal matrix, the design problem is an uncoupled design problem.

**Definition 3.3** Decoupled design problem. When the matrix (1) is a triangular matrix, the design problem is a decoupled design problem. By decoupled, it is meant that Rs will be satisfied by Ps in an order. For the upper triangular matrix, Equation (4) below, the order will be first for $R_n$ (with $P_n$), $R_{n-1}$ (with $P_{n-1}$), ..., and $R_1$ (with $P_1$).

$$
\begin{bmatrix}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{22} & A_{23} & \cdots & A_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{nn} & & & A_{nn}
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
\vdots \\
P_n
\end{bmatrix}
$$

(4)

**Definition 3.4** Integrated design process. Design considers all Rs as a whole but not necessarily in a FCD manner.

**Definition 3.5** Concurrent design process. Design will consider all Rs and Ps in one step – particularly in such a way that all Ps are updated for all Rs. The FCD process, as previously mentioned, refers to the concept of concurrent design process.

**Definition 3.6** Parallel design process. Design considers Rs with Ps independently from an order.

**Definition 3.7** Sequential design process. Design considers a particular order for Rs with Ps, with each time one R is satisfied.

Now we propose a decoupled integrated design (DID) process which can be stated as: taking a decoupled design process for a decoupled design problem. It is noted that DID is not FCD because DID has an order and DID is a specialized type of the sequential design process with its order decided by the R-P matrix which is a triangular matrix.

Take a design problem with two Rs and two Ps as an example to illustrate a DID process. Equation (5) below is a matrix representation of the R-P relation which is assumed to be a decoupled design problem.

$$
\begin{bmatrix}
R_1 \\
R_2
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} \\
0 & A_{22}
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2
\end{bmatrix}
$$

(5)

The DID process for this case has the following two steps:

1. **Step 1:** Optimize $P_2$ for $R_2$ with the relation of $R_2 = f_{22}(P_2)$;
2. **Step 2:** Optimize $P_1$ for $R_1$ with the relation of $R_1 = f_{11}(P_1, P_2)$;

(where $P_2^*$ is the optimized $P_2$ through Step 1 and is not changed at this step).

In the above design procedure, $f_{ij}$ means a mapping from $P_i$ to $R_j$, and $f_{jp}$ means a mapping from $P_j$ to $R_i$ while keeping other Ps unchanged. It is further noted that such a decoupled design process shall be straightforwardly extended to a decoupled design problem with more than two Ps and two Rs, as shown in Equation (4). The DID process will then involve a design steps that are executed in sequence; namely, from the bottom of Equation (5) toward the top. There are several remarks for DID further.

**Remark 3.1:** The DID process is also applied to a situation where a group-R corresponds to a group-P. In such a case, within group-R to group-P, the design problem may be a coupled design problem; however, for an entire design problem, the design problem is a decoupled design problem.

**Remark 3.2:** Before DID is applied, there is a pre-process which changes rows and / or columns of the R-P matrix and / or divides the matrix into groups so that the design problem based on the division matrix is a decoupled program.

### 4. EXAMPLE – 2-DOF FIVE-BAR MECHANISM

Suppose a design problem is to generate a trajectory of the end-effector (i.e., point C) of a 2-DOF five-bar mechanism. In particular the trajectory is represented by a set of points called required points (i.e., $(x,y,t)$), which is considered as R1. Further, we assume that the required points are measured as: $O(0.1102, 0.2416, 0)$, $A(-0.0245, 0.2158, 2)$, $B(0.0588, 0.1662, 4)$, $C(0.2366, 0.1504, 6)$, $D(0.2717, 0.1850, 8)$, and $E(0.2123, 0.2320, 10)$. The design constraint is that there should be no shaking force transmitted to the ground from the mechanism, which is considered as R2. Besides, at the required points, shaking moments should be zero, which is considered as R3.

The design parameters (Ps) are: the lengths (P1) of the links of the mechanism as shown in Fig. 1, servomotor’s control angle (P2) corresponding to the timing requirement in the trajectory, the mass and mass distribution of the links (P3) (see Fig. 3), and the servomotor’s motion planning (P4). Fig. 4 shows the R-P relation.

In Fig. 4, it is shown that R1 is satisfied by P1 and P2, the design pattern of which is also called redundancy design [Suh, 1990]. R2 is satisfied by P1 and P3. R3 is satisfied by P1, P3, and P4. The R-P relation is shown in Equation (6).

$$
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix}
$$

(6)

The above equation can be divided into groups as follows:

$$
\begin{bmatrix}
R_1 \\
R_2 \\
R_3
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix}
$$

(7)

The division matrix (7) shows that the design problem with respect to the division is a decoupled design problem. Therefore, the design steps in this case, according to DID, are as follows:
Step 1: To determine the lengths of the links by considering the constant angular velocities of link 1 and link 4, respectively, which can be achieved by the kinematic design in the traditional sense of mechanism design theory [Uicker et al., 2003]. Further, in this example, we assume the angular velocity of link 1 is $\pi/3$ and the angular velocity of link 4 is $2\pi/3$. The result of this step is shown in Table 1.

Step 2: To perform the inverse kinematics of the mechanism based on the lengths of links as shown in Table 1. This is to satisfy R1 by P2. The result of this step is shown in Table 2.

Step 3: To cancel the shaking force with the counter-weight method. The design equation can be found in Appendix A for the convenience of reader. This is to satisfy R2 by P3. The result of this step is shown in Table 3.

**Remark 4.1:** Although P1 is related to R2 (see Fig. 4). However, this does not necessarily imply that P1 needs to be determined or updated. If there is a design method to fulfill R2 with only P3 given P1, one can only update P3 for R2, which is the case of the current design example. If however there is no such a method available (i.e., one has to update both P1 and P3 for R2 and R1), the design problem (i.e., to meet R2 with P1 and P3 and to meet R1 with P1) is a coupled design and subsequently a design process that concurrently determines P1 and P3 for R3 and R1 has to be applied.

**Table 3** The result of mass and mass distribution for force balancing

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unbalanced linkage</th>
<th>CW linkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1(m)$</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$l_2(m)$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$l_3(m)$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$l_4(m)$</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$l_5(m)$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$r_1(m)$</td>
<td>0.0062</td>
<td>0.1000</td>
</tr>
<tr>
<td>$r_2(m)$</td>
<td>0.1250</td>
<td>0.1250</td>
</tr>
<tr>
<td>$r_3(m)$</td>
<td>0.1250</td>
<td>0.1500</td>
</tr>
<tr>
<td>$r_4(m)$</td>
<td>0.0286</td>
<td>0.1000</td>
</tr>
<tr>
<td>$m_1(kg)$</td>
<td>0.91</td>
<td>0.125</td>
</tr>
<tr>
<td>$m_2(kg)$</td>
<td>0.38</td>
<td>0.380</td>
</tr>
<tr>
<td>$m_3(kg)$</td>
<td>0.38</td>
<td>0.475</td>
</tr>
<tr>
<td>$m_4(kg)$</td>
<td>0.28</td>
<td>0.665</td>
</tr>
<tr>
<td>$l_1(kg\cdot m^2)$</td>
<td>0.00085</td>
<td>0.01159</td>
</tr>
<tr>
<td>$l_2(kg\cdot m^2)$</td>
<td>0.00400</td>
<td>0.00400</td>
</tr>
<tr>
<td>$l_3(kg\cdot m^2)$</td>
<td>0.00400</td>
<td>0.02804</td>
</tr>
<tr>
<td>$l_4(kg\cdot m^2)$</td>
<td>0.00063</td>
<td>0.00819</td>
</tr>
<tr>
<td>$\theta_1(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$\theta_2(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_3(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$\theta_4(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$q_3(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$q_4(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 4: To design a servomotor motion’s plan to cancel shaking moments at the selected points (O, A, B, C, D, E). The method for designing a servomotor motion’s plan is taken from [Sun et al., 2011, Sun et al., 2010]. This is to satisfy R3 by P4. The result of this step is shown in Table 4. Again, it is noted that both P1 and P3 are related to R3 as well but they are out of the design loop for R3 according to Remark 4.1.

---

**Table 2** The result of inverse kinematics

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unbalanced linkage</th>
<th>CW linkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1(m)$</td>
<td>0.08</td>
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<td>$l_2(m)$</td>
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<td>0.25</td>
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<td>$l_3(m)$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$l_4(m)$</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$l_5(m)$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$r_1(m)$</td>
<td>0.0062</td>
<td>0.1000</td>
</tr>
<tr>
<td>$r_2(m)$</td>
<td>0.1250</td>
<td>0.1250</td>
</tr>
<tr>
<td>$r_3(m)$</td>
<td>0.1250</td>
<td>0.1500</td>
</tr>
<tr>
<td>$r_4(m)$</td>
<td>0.0286</td>
<td>0.1000</td>
</tr>
<tr>
<td>$m_1(kg)$</td>
<td>0.91</td>
<td>0.125</td>
</tr>
<tr>
<td>$m_2(kg)$</td>
<td>0.38</td>
<td>0.380</td>
</tr>
<tr>
<td>$m_3(kg)$</td>
<td>0.38</td>
<td>0.475</td>
</tr>
<tr>
<td>$m_4(kg)$</td>
<td>0.28</td>
<td>0.665</td>
</tr>
<tr>
<td>$l_1(kg\cdot m^2)$</td>
<td>0.00085</td>
<td>0.01159</td>
</tr>
<tr>
<td>$l_2(kg\cdot m^2)$</td>
<td>0.00400</td>
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<td>0.00400</td>
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</tr>
<tr>
<td>$l_4(kg\cdot m^2)$</td>
<td>0.00063</td>
<td>0.00819</td>
</tr>
<tr>
<td>$\theta_1(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$\theta_2(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_3(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$\theta_4(^\circ)$</td>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>$q_3(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$q_4(^\circ)$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 4 The result of cancelling shaking moments at the selected points (O, A, B, C, D, E)

<table>
<thead>
<tr>
<th>Selected points</th>
<th>Shaking moment(Nm) $\times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.0869</td>
</tr>
<tr>
<td>A</td>
<td>0.5869</td>
</tr>
<tr>
<td>B</td>
<td>0.6218</td>
</tr>
<tr>
<td>C</td>
<td>0.2306</td>
</tr>
<tr>
<td>D</td>
<td>0.2365</td>
</tr>
<tr>
<td>E</td>
<td>0.6256</td>
</tr>
</tbody>
</table>

5. IMPLICATION OF THE DID AND RELATED WORK

The proposed decoupled integrated design approach to the decoupled design problem has a generalized implication to general product design. In the design process of any complex product, often an overall function needs to be decomposed into a set of sub-functions that are further to be fulfilled by sub-systems [Beitz et al., 2003]. Characterization of the design problem by coupling, decoupling and uncoupling is considered as one of the necessary steps in general product design, as implied by Suh [1990]. The proposed design approach is based on such characterization and further goes on with a decoupled design procedure to a decoupled design problem, and the approach is thus applicable to any general product design.

The concept of uncoupled, coupled and decoupled design problems was perhaps first described by Suh [1990]. However, there is no rationale given for the benefit of the decoupled design process in his work. There is no emphasis on the point that a decoupled design process should be applied to a decoupled design problem. Further, a general procedure to analyze the R-P matrix into a division matrix so as to extending the characterization of the R-P matrix upon a group of P (P-group) and a group R (R-group) is certainly first proposed in this paper.

6. CONCLUDING REMARK

In this paper, we proposed a decoupled integrated design approach to design of robotic mechanisms which have more than one motor and have at least one servomotor. We used a 2DOF five-bar mechanism as an example to illustrate the proposed design approach without loss of generality. The idea of such a decoupled integrated approach stems from the analysis of the design problem, which can be represented by the requirement-parameter matrix, inspired by a well known design theory called axiomatic design theory (ADT) [Suh, 1990], and a design process is then formulated by tailoring to a particular design problem.

It was further argued in the present paper that the contemporary thinking of the concurrent design process especially full concurrent design process has missed an important consideration – that is, the nature of design problem. It is easy to see the shortcoming with a full concurrent design process to an uncoupled design problem. Along this line of thinking, we argued that for a decoupled design problem, in fact, a decoupled integrated design process should be applied instead of a full concurrent design process.

It is noted that in a more general area of product design, the shortcoming of the concurrent design or engineering has been pointed out by others as well; e.g., AitShalia et al. [1995]. However, to our best knowledge, there is no published work that proposes any systematic process for an integrated.

Elsewhere, we presented a detailed comparison for a full concurrent design approach and a decoupled integrated approach to a particular decoupled design problem to show the benefit of such a decoupled integrated design approach [Sun et al., 2011]. The major contribution of the present paper lies in the full development of such a decoupled integrated design approach with a 2DOF five-bar robotic mechanism for a function generation problem as an example. Apparently, the proposed decoupled integrated design approach is applicable to designing any kind of products, albeit a mechanism design problem was used in this paper for the illustration purpose.

ACKNOWLEDGEMENT

The work has been partially supported by Natural Science and Engineering Research Council of Canada (NSERC) to the corresponding author. The authors also want to acknowledge the support for this research through a Donghua special professorship award program.

REFERENCES

Appendix A Equations for complete force balancing of a 2-DOF five-bar mechanism

Referring to the schematic of the 2-DOF five-bar mechanism, the equations for complete shaking force balancing are as follows:

\[ m_1r_1l_2 = l_1m_1r_2, \text{ and } \theta_1 = \theta_2, \]
\[ m_3r_3l_2 = l_3m_2r_2, \text{ and } \theta_3 = \theta_2 + \pi \]  
(A-1)
\[ m_1r_1l_3 = l_4m_3r_3, \text{ and } \theta_4 = \theta_3 \]

Appendix B The method for the servomotor motion’s plan [Sun et al., 2011]

Define:
\( q_i \): Inputs which correspond to the required positions of the end-effector; \( i = 1,2,\ldots, m \) (\( m \): the total number of the required positions at the end-effector).
\( I_{i,i+1} \): Servomotor’s plan for the segment \( i \) from to \( i+1 \).
\( P_d \): Polynomial function of time with the power being \( d \); for example, \( P_3: I=a_0+a_1t+a_2t^2+a_3t^3 \) \( (\text{where } I \text{: servomotor's plan; } t \text{: time}). \)

Take differential information such as \( \dot{q}_i \), \( \ddot{q}_i \) and so on at the control points is taken as an optimal variable for the plan to achieve objectives. Examples of the objective are minimization of shaking moment, driving torque, etc. In this paper, the objective is the minimization of the shaking moments at the selected points (O, A, B, C, D, E).

Consider three control points \( i-1, i, i+1 \) again. It is noted that \( q_i \) is known through inverse kinematics. Let us further consider the first order continuity at point \( t_i \). The constraint equations in this case are as follows:

\[
\begin{align*}
I_{i-1,i}(t_{i-1}) &= q_{i-1} \\
I_{i-1,i}(t_i) &= \tilde{q}_i \\
I_{i,i+1}(t_i) &= q_i \\
I_{i,i+1}(t_{i+1}) &= q_{i+1} \\
I_{i-1,i}(t_{i-1}) &= \dot{q}_{i-1} \\
I_{i-1,i}(t_i) &= \dot{q}_i \\
I_{i,i+1}(t_i) &= \dot{q}_i \\
I_{i,i+1}(t_{i+1}) &= \dot{q}_{i+1}
\end{align*}
\]  
(B-1)
(B-2)

If \( P_3 \) is applied to all the segments, there are 8 coefficients to be determined. From equations (B-1) and (B-2), there are 8 constraints. In this case, the 8 coefficients in the segment I can always be expressed as functions of \( q_j, \dot{q}_j \), where \( j=i-1, i, i+1 \). It is noted that \( q_j \) is known and \( \dot{q}_j (j = i-1, i, i+1) \) to be determined, and therefore, \( \dot{q}_j \) are taken as optimal variables. Let \( D \) be the order of continuity at a control point. One may happen to see that for \( P_3, m=2, D=1 \).

Let us consider a second order continuity with the method. In this case, we will have the following constraints:

\[
\begin{align*}
\dot{I}_{i-1,i}(t_{i-1}) &= \ddot{q}_{i-1} \\
\dot{I}_{i-1,i}(t_i) &= \ddot{q}_i \\
\dot{I}_{i,i+1}(t_i) &= \ddot{q}_i \\
\dot{I}_{i,i+1}(t_{i+1}) &= \ddot{q}_{i+1}
\end{align*}
\]  
(B-3)

For a most general situation (i.e., \( P_m, m, D \)), the total number of optimal variables with the method can be expressed by \( N_{\text{opt}} = N_{\text{h}} + N_{\text{c}} \), where \( N_{\text{opt}} \): the total number of optimal variables; \( N_{\text{h}} \): the total number of coefficients; \( N_{\text{c}} \): the total number of constraints; \( N_n \): the total number of D-order terms at the control points (\( D \geq 1 \)).

Further, the independent coefficients are selected from the coefficients which are associated with the higher power of time. Details of this along with the whole procedure as described herein should be referred to Sun et al. [2011].