Annals of the Association of American Geographers

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/raag20

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Published online: 03 Oct 2013.


To link to this article: http://dx.doi.org/10.1080/00045608.2013.834236

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Reliable Space–Time Prisms Under Travel Time Uncertainty

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Time geography is a powerful framework for analyzing human activities under various space–time constraints. At the core of time geography is the concept of the space–time prism, which delimits an individual’s potential activity locations in space and time. The classical space–time prism, however, admits only deterministic travel speeds and ignores the stochastic nature of travel environments. In this article, the classical space–time prism model is extended to congested road networks with travel time uncertainty. A reliable space–time prism is proposed to consider explicitly an individual’s on-time arrival probability concerns in the face of travel time uncertainty. The reliable space–time prism is defined as the set of space–time locations where an individual can participate in an activity and return to his or her destination with a given on-time arrival probability. To construct such a reliable space–time prism in a road network, a solution algorithm is developed. A case study using real-world traffic information is carried out to demonstrate the applicability of the proposed prism model. The results of the case study indicate that the proposed prism model can represent well individuals’ space–time taking into account various on-time arrival probability concerns. Key Words: reliability, reliable space–time prism, time geography, travel behavior, travel time uncertainty.

时间地理学是在各种空间—时间限制下分析人类活动的有效框架。时间地理学的核心是时空棱镜的概念，该概念限制了一个人在空间与时间中的可能活动场所。但传统的时空棱镜仅能容纳确定性的旅行速率，而忽略了旅行环境的随机本质。本文将传统的时空棱镜模型延伸至拥挤的道路网络与随之而来的旅行时间不确定性。本文将提出一个可信赖的时空棱镜，确切地考量在面对旅行时间不确定性时，个人对于准时抵达的可能性之考量。可信赖的时空棱镜定义为一组时空地点，其中可以参与某项活动，并在给定的准时抵达可能性中，回到他或她的目的地。为了在道路网络中建构此可信赖的时空棱镜，本文将建立一个求解算法。本文将进行一个运用真实世界交通资讯的案例研究，证实所提出的棱镜模型的适用性。案例研究结果显示，本文所提出的棱镜模型可纳入各种准时抵达可能性的考量，充分再现个人的空间—时间。 关键词: 可信赖性, 可信赖的时空棱镜, 时间地理学, 旅行行为, 旅行时间不确定性。

La geografía-tiempo es un potente marco para analizar las actividades humanas sometidas a varias limitaciones del espacio–tiempo. En el núcleo de la geografía-tiempo está el concepto del prisma espacio–tiempo, que delimita las localizaciones de la actividad potencial de un individuo en el espacio y el tiempo. Sin embargo, el prisma espacio–tiempo clásico solamente admite velocidades de desplazamiento deterministas e ignora la naturaleza estocástica de los entornos del viaje. En este artículo, el modelo del prisma espacio–tiempo clásico es extendido para incluir las congestionadas redes de carreteras con incertidumbre del tiempo de viaje. Se propone un prisma espacio–tiempo confiable para considerar explícitamente las preocupaciones de un individuo con la probabilidad de llegar a tiempo frente a la incertidumbre del tiempo de viaje. El prisma espacio–tiempo confiable se define como un conjunto de localizaciones espacio–tiempo donde un individuo puede participar en una actividad y regresar a su destino dentro de una probabilidad dada de llegar a tiempo. Para construir tal prisma espacio–tiempo confiable en una red de carreteras, se desarrolló un algoritmo de solución. Un estudio de caso fue realizado utilizando información del tráfico en el mundo real para demostrar la aplicabilidad del modelo de prisma propuesto. Los
Time geography is a powerful framework for analyzing human activities under various space–time constraints (Hägerstrand 1970). At the core of time geography is the concept of a space–time prism model. The space–time prism is a postulated prism-shaped structure delimiting the set of locations that can be physically reached by an individual from specified locations within a given time budget (Miller 1991). The space–time prism model has been widely adopted in accessibility analysis (Kwan 1998; Miller 1999; Li et al. 2011; Delafontaine, Neutens, and Van de Weghe 2012), travel behavior modeling (Wang and Law 2007; Fang et al. 2011; X. Chen and Kwan 2012; Ronald, Arentze, and Timmermans 2012; Farber et al. 2013), and tracking data modeling and visualization (Kuijpers and Othman 2009; Winter and Yin 2011; Downs and Horner 2012).

In early time–geographic studies, the space–time prism was constructed based on the uniform speed assumption, meaning that movements occur at a constant speed everywhere in a planar space. This uniform speed assumption can oversimplify the complexities of real-world travel environments and therefore cannot well delineate individuals’ activity space–time extents in real-world cases (Miller 1991). Recognizing that people in urban environments are generally traveling within road networks, Miller (1991) proposed a network-based space–time prism to bound individuals’ activity space–time extents in road networks. Following this seminal work, other researchers have improved the space–time prism model by considering the complexities of travel conditions in road networks, including spatial and temporal variations in traffic congestion (Y. H. Wu and Miller 2001; Weber and Kwan 2002). Miller and Bridwell (2009) introduced a concept of velocity field in the space–time prism to consider the effects of heterogeneous travel speeds within the space. Several computational algorithms have been developed to construct space–time prisms in road networks (Miller 1991; Kwan 2000; Kim and Kwan 2003; Neutens et al. 2008; Yu and Shaw 2008; Shaw and Yu 2009). The analytical foundation and measurement theory have also been established to represent the space–time prism both in planar spaces and in road networks (Miller 2005).

Although great strides have been made in constructing space–time prisms within road networks, most existing studies assumed that travel speeds (or link travel times) are deterministic. In reality, link travel times in congested road networks are highly stochastic due to demand fluctuations and supply degradations (B. Y. Chen et al. 2011; B. Y. Chen, Lam, Sumalee, Li, and Li 2012). Studies based on field observations have found that link travel times in congested road networks follow either normal or log-normal distributions (Kaparias, Bell, and Belzner 2008; Rakha, El-Shawarby, and Arafeh 2010; Du, Peeta, and Kim 2012).

Under travel time uncertainty, individuals might not know the exact space–time extent that they can reach within a given time budget. Many empirical studies have found that individuals indeed consider travel time uncertainty as a risk and become risk-averse, especially when planning for important activities (e.g., job interviews; T. C. Lam and Small 2001; Noland and Polak 2002; Tam, Lam, and Lo 2008; Carrion and Levinson 2012). It is apparent that large travel time variations can cause late arrivals and the subsequent imposition of high penalties on travelers. Consequently, risk-averse individuals tend to budget extra travel time (called safety margin; Hall 1983) to ensure a desirable on-time arrival probability (also known as travel time reliability; Bell and Iida 1997; Chen, Lam, Sumalee, and Li 2012). As a cost of this risk-averse behavior, the accessible space–time extent of risk-averse individuals is reduced due to the reservation of a safety margin (Hall 1983). To represent individuals’ space–time extents in complex travel environments with travel time uncertainty, individuals’ concerns about the probability of on-time arrival should therefore be included in the space–time prism model.

In recent years, the issue of uncertainty has become increasingly recognized in geography and geographical information science fields (Hunter and Goodchild 1997; J. X. Zhang and Goodchild 2002; Shaw 2003; Gober et al. 2010; Kwan 2012). In the time–geographic study, Hall (1983) pointed out that individuals’ space–time extents can be affected by three types of uncertainties: travel time uncertainty, anchor uncertainty, and random coupling constraint. Anchor uncertainty is defined as the measurement error in the locations of origin and destination. Random coupling constraint indicates the uncertainty about an individual’s optimal arrival time. Research efforts have been devoted to improving the classical space–time prism models by considering anchor uncertainty (Kuijpers...
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Kobayashi, Miller, and Othman (2011) developed analytical methods for assessing error propagation in space–time prisms and their intersections under scenarios of anchor uncertainties and random coupling constraints.

The space–time prism under travel time uncertainty has received much attention. In his seminal work, Hall (1983) discussed the effects of travel time uncertainty and random coupling constraint on the space–time prism. No analytical formulation and computational solution algorithm was provided, however, to construct the space–time prism under travel time uncertainty. Based on rough set theory, Neutens et al. (2007) proposed a rough space–time prism model in a planar space. In their proposed model, a lower and an upper bound are constructed to represent, respectively, an individual’s minimum and maximum space–time extents under travel time uncertainty. The space–time locations inside the lower bound are certainly reachable, whereas the area outside the upper bound represents space–time locations that are certainly un reachable. The space–time locations between these lower and upper bounds are roughly labeled as possibly reachable locations. Delafontaine, Neutens, and Van de Weghe (2011) extended this rough space–time prism model to a constrained planar space with obstacles. The rough space–time prism can be useful in delimiting individuals’ minimum and maximum space–time extents under travel time uncertainty. Nevertheless, individuals’ on-time arrival probability concerns were not considered in this rough space–time prism model. Extending such a model in the planar space to a road network is also required for real transportation applications.

Along the line of previous work (Hall 1983; Miller 1991; Kwan 1998; Neutens et al. 2007), this study investigates the space–time prism in road networks under travel time uncertainty with explicit consideration of individuals’ on-time arrival probability concerns. The space–time prism considering on-time arrival probability is hereafter referred to as the reliable space–time prism to distinguish it from previous space–time prism models.

In this study, an analytical model is proposed to delineate individual space–time extents under travel time uncertainty. In the proposed analytical model, the reliable space–time prism is defined as the set of space–time locations where an individual can participate in an activity and arrive at the destination with at least α on-time arrival probability. In this way, individuals’ space–time extents are determined not only by travel speeds but also by the predetermined on-time arrival probability α. Accordingly, the proposed model can represent individuals’ space–time extents under various on-time arrival probabilities (i.e., ∀α ∈ (0, 1)). From this perspective, the proposed model can be regarded as a generalization of the work of Neutens et al. (2007), which only describes the minimum and maximum space–time extents (i.e., α ≈ 0 and α ≈ 1).

To construct the reliable space–time prism in a road network, a solution algorithm is developed in this study based on the recently developed reliable shortest path algorithm (B. Y. Chen, Lam, et al. 2013), which determines the reliable shortest path in a road network to ensure a given on-time arrival probability. A case study using real-world traffic information is carried out to demonstrate the applicability of the proposed model. The results indicate that the proposed reliable space–time prism can well represent individuals’ space–time extents under various on-time arrival probability values.

The rest of the article is organized as follows. The next section briefly introduces the classical space–time prism model to provide necessary background. We then present the reliable space–time prism model in planar space. The following section extends the reliable space–time prism model to the road network and introduces a computational algorithm to construct a network-based reliable space–time prism. We then report a case study to illustrate the applicability of the proposed reliable space–time prism. We conclude the article with possible directions for future research.

Classical Space–Time Prism

This section provides necessary background on the classical space–time prism concept. In time geography, activities can be broadly divided into fixed and flexible activities according to the ease of rescheduling or relocating their times and locations (Hägerstrand 1970). Fixed activities (e.g., work) are generally conducted at key locations such as the workplace, forming the basis of individuals’ daily activity schedules. Flexible activities (e.g., shopping) are scheduled around the fixed activities and are easy to relocate and reschedule. The space–time prism delineates the individual’s space–time extent for scheduling a flexible activity between two fixed activities.

Figure 1 illustrates the classical space–time prism model in a planar place. As shown in Figure 1, one fixed activity has been completed by an individual at origin r and time instance t₁, and another fixed activity will be conducted at destination s and time instance t₂. Between these two fixed activities, a flexible
activity might be scheduled at location $x$ and time instance $t_x$ with a minimum duration $c_{\text{min}}$. Therefore, the time budget for traveling, denoted by $b$, can be calculated as:

$$b = t_s - t_r - c_{\text{min}}$$  \hspace{1cm} (1)

Let $t_r^x$ be the path travel time from origin $r$ to location $x$, and let $t_s^x$ be the path travel time from location $x$ to destination $s$. These two path travel times in the planar place can respectively be calculated as:

$$t_r^x = \|r - x\|/v$$  \hspace{1cm} (2)

$$t_s^x = \|x - s\|/v$$  \hspace{1cm} (3)

where $\|r - x\|$ is the Euclidean distance between origin $r$ and location $x$, $\|x - s\|$ is the Euclidean distance between location $x$ and destination $s$, and $v$ is the deterministic travel speed. Based on Equations 1 through 3, the set of geographic locations reachable by an individual within the given travel time budget forms a potential path area (PPA), which can be defined as:

$$\text{PPA} = \{x | t_r^x + t_s^x \leq b\}$$  \hspace{1cm} (4)

As illustrated in Figure 1, the space–time prism is a three-dimensional (3D) entity capturing all potential space–time locations for an individual between two fixed activities. The space–time prism $\text{STP}(t)$ can be obtained as the intersection of a forward cone, a backward cone, and a cylinder (Miller 2005):

$$\text{STP}(t) = \text{FC}(t) \cap \text{BC}(t) \cap \text{C}(t)$$  \hspace{1cm} (5)

$$\text{FC}(t) = \{x | t_r^x \leq t - t_r, t \leq t_i\}$$  \hspace{1cm} (6)

$$\text{BC}(t) = \{x | t_s^x \leq t_i - t, t \geq t_r\}$$  \hspace{1cm} (7)

$$\text{C}(t) = \{x | t_r^x + t_s^x \leq b, t_r \leq t < t_i\}$$  \hspace{1cm} (8)

where $\text{FC}(t)$ is the forward cone composed of all space–time locations that can be reached from origin $r$ by elapsed time $t - t_r$; $\text{BC}(t)$ is the backward cone encompassing all space–time points where an individual can return to destination $s$ in the remaining time budget $t_i - t$; and $\text{C}(t)$ is the cylinder delimiting all geographic locations within the potential path area.

As shown in Figure 1, the potential path area is the projection of the space–time prism in two-dimensional (2D) geographic space. For each location $x \in \text{PPA}$, the earliest arrival time $t_a^x$ from origin $r$ is bounded by the forward cone as $t_a^x = t_r + t_r^x$, and the latest departure time $t_d^x$ to destination $s$ is bounded by the backward cone as $t_d^x = t_i - t_r^x$. The space–time prism $\text{STP}(t)$ is formed by the intersection of these cones and the cylinder, capturing all potential space–time locations reachable within the given time budget $b$. The set of geographic locations reachable by an individual is defined as the potential path area (PPA), which is the intersection of the forward cone, backward cone, and cylinder.
cone as $t_s^d = t_s - t^{xs}$. The height of the space–time prism at location $x$ indicates the time duration $c_x$ that the individual can stay at location $x$, which can be written as:

$$c_x = t_s^d - t_s = c_{\text{min}} + b - (t^x + t^{xs}) \quad (9)$$

To participate in a flexible activity at location $x$, the time duration should satisfy the $c_x \geq c_{\text{min}}$ (or $t^x + t^{xs} \leq b$) constraint.

In the classical space–time prism just described, travel times are assumed to be deterministic. It is well recognized, however, that travel times in congested urban road networks are highly stochastic due to demand fluctuations and interruptions caused by traffic accidents, adverse weather, and other factors. The next section describes the extension of the classical space–time prism to incorporate travel time uncertainty.

**Reliable Space–Time Prism in Planar Space**

This section presents a reliable space–time prism model in planar space with travel time uncertainty. Unlike the classical space–time prism, travel speed in Equations 2 and 3 is represented as a random variable (denoted by $V$). In this case, the path travel time distribution from location $x$ to destination $s$, denoted by $T^{xs}$, can be calculated as:

$$T^{xs} = \|x - s\|/V \quad (10)$$

Given a departure time $t^d_x$ at location $x$, the arrival-time distribution at the destination can be expressed as:

$$T_s = t^d_x + T^{xs} \quad (11)$$

The probability of arriving at the destination before the preferred arrival time $t_s$ (denoted by $\alpha \in (0, 1)$) can be expressed as

$$\alpha = \Phi_{T_s}(t_s) = \int_0^{t_s} f(t)dt, \quad \alpha \in (0, 1) \quad (12)$$

where $f(t)$ and $\Phi_{T_s}(t_s)$, respectively, are the probability density function (PDF) and the cumulative distribution function (CDF) of the arrival-time distribution $T_s$. This on-time arrival probability $\alpha$ is related to how individuals determine the importance of an activity. For example, one might wish to arrive at a destination with 99 percent probability for a job interview, whereas one might not care about the probability of arriving at a coffee shop to have a cup of coffee. In the literature, this on-time arrival probability $\alpha$ is also regarded as a measure of the individual's risk-taking behaviors toward the risk of being late:

- If $\alpha > 0.5$, then the individual is risk-averse for on-time arrival.
- If $\alpha = 0.5$, then the individual is risk-neutral for on-time arrival.
- If $\alpha < 0.5$, then the individual is risk-seeking for on-time arrival.

Many empirical studies have found that individuals have heterogeneous risk-taking behaviors in the face of travel time uncertainty (T. C. Lam and Small 2001; W. H. K. Lam, Shao, and Sumalee 2008; Tam, Lam, and Lo 2008; B. Y. Chen et al. 2011; B. Y. Chen, Lam, et al. 2013). Based on a stated-preferences survey, de Palma and Picard (2005) observed that the groups of risk-averse, risk-neutral, and risk-seeking commuters account for approximately 60 percent, 6 percent, and 33 percent, respectively, of total commuters. T. C. Lam and Small (2001) empirically found that women and commuters with a higher income level are substantially more risk-averse to travel time uncertainty. Tam, Lam, and Lo (2008) found that business air passengers give a significantly higher value to on-time arrival probability than nonbusiness air passengers. Therefore, individuals can assign different values of on-time arrival probability depending on their activity purposes and socio-economic characteristics.

Given a predetermined on-time arrival probability $\alpha$, an individual’s latest departure time from location $x$ can be determined as:

$$t^d_s = t_s - \Phi_{T_s}^{-1}(\alpha) \quad (13)$$

where $\Phi_{T_s}^{-1}(\alpha)$ is the inverse CDF of the path-time distribution $T^{xs}$ at confidence level $\alpha$. Using $t^d_s = t_s - \Phi_{T_s}^{-1}(\alpha)$ as departure time, the individual can arrive at the destination at least with $\alpha$ probability of on-time arrival. Similarly, the path travel time $T^{rs}$ from origin $r$ to location $x$ is also a random variable:

$$T^{rs} = \|r - x\|/V \quad (14)$$

The earliest arrival time from origin $r$ to location $x$ with on-time arrival probability $\alpha$ can be determined
as:

\[ t_x^d = t_r + \Phi_{T_x}^{-1}(\alpha) \]  

(15)

where \( \Phi_{T_x}^{-1}(\alpha) \) is the inverse CDF of the path-time distribution \( T_x \) at confidence level \( \alpha \). With the earliest arrival time and the latest departure time at location \( x \) known, the time duration \( c_x \) that the individual can stay at location \( x \) can be calculated as:

\[ c_x = t_x^d - t_x^a = c_{\text{min}} + b - (\Phi_{T_x}^{-1}(\alpha) + \Phi_{T_x}^{-1}(\alpha)) \]  

(16)

To participate in the flexible activity at location \( x \), the time duration should satisfy the constraint \( c_x \geq c_{\text{min}} \), which can also be expressed as:

\[ \Phi_{T_x}^{-1}(\alpha) + \Phi_{T_x}^{-1}(\alpha) \leq b \]  

(17)

Therefore, all geographic locations satisfying this constraint form a reliable potential path area \( \text{RPPA}(\alpha) \), which can be described as:

\[ \text{RPPA}(\alpha) = \{ x \mid \Phi_{T_x}^{-1}(\alpha) + \Phi_{T_x}^{-1}(\alpha) \leq b \} \]  

(18)

The reliable potential path area delineates all geographic locations where an individual can participate in a flexible activity and return to destination \( s \) with a probability equal to or greater than the desirable probability of on-time arrival.

Figure 2 illustrates the reliable potential path area concept using a simple example. As shown in Figure 2A, an individual departs from his office (i.e., origin \( r \) at 12:30 and plans to arrive at an exhibition center (i.e., destination \( s \)) at 14:00. Before his arrival at the exhibition center, this individual might go to a restaurant (also called a point of interest [POI]) to have lunch. Three restaurants (i.e., \( x_1, x_2, \) and \( x_3 \)) with different cuisines might be chosen by the individual. It is assumed that travel times (or travel speeds) are uncertain. The individual’s probabilities of arriving at the exhibition center on time after having lunch at each of these three restaurants are shown in Figure 2B.

Figure 2A shows the individual’s spatial extent under various predetermined on-time arrival probabilities \( \alpha \). When the individual is risk-neutral (\( \alpha = 0.5 \)), he or she schedules the activity based only on mean travel time. In this case, the reliable potential path area (i.e., \( \text{RPPA}(0.5) \)) includes both \( x_1 \) and \( x_2 \), from either of which the individual can return to the office with at least 50 percent probability of on-time arrival. If the individual is risk-averse (\( \alpha = 0.8 \)), he or she is more concerned about the risk of being late and tends to reserve a safety margin to ensure 80 percent probability of on-time arrival. Compared to the risk-neutral scenario, this reliable potential path area (i.e., \( \text{RPPA}(0.8) \)) is significantly reduced and only includes \( x_1 \). For the risk-seeking scenario (\( \alpha = 0.3 \)), the risk-seeking individual tends to explore a larger space by taking the risk of being late. In this case, the reliable potential path area (i.e., \( \text{RPPA}(0.3) \)) includes all three restaurants (i.e., \( x_1, x_2, \) and \( x_3 \)) but can guarantee only 30 percent probability of on-time arrival. Therefore, the size of the reliable potential path area varies with the predetermined on-time arrival probability \( \alpha \).

By analogy to the classical space–time prism, the reliable space–time prism (denoted by \( \text{RSTP}(\alpha, t) \)), which delimits the space–time extent for an individual to conduct a flexible activity with a given on-time arrival probability \( \alpha \in (0, 1) \), can be defined as:

\[ \text{RSTP}(\alpha, t) = \text{RFC}(\alpha, t) \cap \text{RBC}(\alpha, t) \cap \text{RC}(\alpha, t) \]  

(19)

\[ \text{RFC}(\alpha, t) = \{ x \mid \Phi_{T_x}^{-1}(\alpha) \leq t - t_r, t \leq t_f \} \]  

(20)

\[ \text{RBC}(\alpha, t) = \{ x \mid \Phi_{T_x}^{-1}(\alpha) \leq t - t_r, t \geq t_f \} \]  

(21)

\[ \text{RC}(\alpha, t) = \{ x \mid \Phi_{T_x}^{-1}(\alpha) + \Phi_{T_x}^{-1}(\alpha) \leq b, t_r \leq t < t_f \} \]  

(22)

where \( \text{RFC}(\alpha, t) \) is the reliable forward cone including all space–time locations that can be reached from origin \( r \) by elapsed time \( t - t_r \) with desirable on-time arrival probability \( \alpha \); \( \text{RBC}(\alpha, t) \) is the reliable backward cone encompassing all space–time points where an individual can return to destination \( s \) in the remaining time budget \( t_f - t_r \) with desirable on-time arrival probability \( \alpha \); and \( \text{RC}(\alpha, t) \) is the reliable cylinder encompassing the space–time locations within the reliable potential path area.

Figure 3 illustrates the reliable space–time prism concept using the same example as in Figure 2. As shown in Figure 3, given a predetermined \( \alpha \) value, the reliable space–time prism is a 3D entity in space and time, representing an individual’s space–time constraint for conducting flexible activities under travel time uncertainty. In addition to travel environments, the volume of the reliable space–time prism is determined by the individual’s on-time arrival probability concern (i.e., the \( \alpha \) parameter). As discussed earlier, with a larger \( \alpha \) value, the individual becomes more risk-averse and
tends to reserve a larger travel time safety margin to ensure a higher probability of on-time arrival. In this case, the individual’s potential space–time extent is reduced due to the reservation of a safety margin. When $\alpha = 0.5$, the reliable space–time prism is equivalent to the classical space–time prism concept, which considers only the mean travel time and ignores travel time variability.

Note that the volume of the reliable space–time prism is defined by an upper bound and a lower bound. The upper bound is approached as the on-time arrival probability $\alpha$ approaches zero (e.g., $\alpha = 0.01$). The space–time locations outside the upper bound are not reachable by the individual for conducting flexible activities. Conversely, the lower bound is approached as the $\alpha$ value approaches one (e.g., $\alpha = 0.99$). This
lower bound delineates all space–time locations that are certainly reachable. Between these upper and lower bounds, the individual's space–time extent under any desirable on-time arrival probability can be explicitly represented. From this observation, the proposed reliable space–time prism can be regarded as an extension of the work of Neutens et al. (2007), which represents only two extreme scenarios (i.e., $\alpha = 0.01$ and $\alpha = 0.99$) using rough set theory.

**Reliable Space–Time Prism in the Road Network**

In this section, the reliable space–time prism model is further extended to the road network, and a solution algorithm is presented to construct the network-based reliable space–time prism. Let $G = (N, A, \Psi)$ be a directed road network consisting of a set of nodes $N$, a set of links $A$, and a set of turning movements $\Psi$. Each link $a_{ij} \in A$ has a tail node $i \in N$, a head node $j \in N$, and a random travel time $T_{ij}$. The mean and standard deviation of the link travel time are denoted by $\bar{t}_{ij}$ and $\sigma_{ij}$, respectively. Each node $i$ has a set of successor nodes $SCS(i) = \{j : a_{ij} \in A\}$ and a set of predecessor nodes $PDS(i) = \{w : a_{wi} \in A\}$. A movement $\psi_{ijk} = (a_{ij}, a_{jk}) \in \Psi$ represents an allowed movement at node $j$ (e.g., through movement or right turn). A movement $\psi_{ijk} \notin \Psi$ means that this turning movement is restricted at node $j$ (e.g., no left turn or no U-turn).

Unlike planar space, an individual's spatial extent in the road network is confined by network links and turn restrictions. A path in the road network between two nodes is a set of consecutive links. Let $p_{rux}$ be a path from origin $r$ to node $x$ going through a link $a_{ux}$. The path travel time distribution, $T_{rux}$, can be calculated as:

$$T_{rux} = \sum_{a_{ux}} T_{ij} \delta_{riju}(23)$$

where $\delta_{riju}$ is the path-link incidence variable; $\delta_{riju} = 1$ means that link $a_{ij}$ is on path $p_{rux}$, and $\delta_{riju} = 0$ otherwise. The mean and standard deviation of the path travel time are denoted by $\bar{t}_{rux}$ and $\sigma_{rux}$, respectively. When the link travel times are statistically independent and follow normal distributions, $\bar{t}_{rux}$ and $\sigma_{rux}$ can be calculated as:

$$\bar{t}_{rux} = \sum_{a_{ux}} \bar{t}_{ij} \delta_{riju}(24)$$

$$\sigma_{rux} = \sqrt{\sum_{a_{ux}} \sigma_{ij}^2 \delta_{riju}}(25)$$

Based on the normality assumption, the effective travel time $\phi^{-1}_{rux}(\alpha)$ required to ensure on-time arrival
probability $\alpha$ can be expressed as:
\[
\Phi_{\tau_i x}^{-1}(\alpha) = \bar{r}_{i x} + z_{\alpha} \sigma_{i x}, \tag{26}
\]
where $z_{\alpha}$ is the inverse CDF of a standard normal distribution at confidence level $\alpha$.

In planar space, the optimal path between two locations is simply a straight line (see Equations 10–14). In the road network, however, the optimal path between two locations is the reliable shortest path. Let $P_{\tau x} = \{p_{1,wx}, ..., p_{m,wx}\}$ be a set of paths from origin $\tau$ to node $x$ going through the same link $a_{wx}$. The reliable shortest path $p_{\tau,wx}$ to link $a_{wx}$ is defined as the path with the minimum effective travel time $\Phi_{\tau,wx}^{-1}(\alpha)$ required to satisfy a given on-time arrival probability $\alpha$ (A. Chen and Ji 2005). Because of its nonlinear structure, the effective travel time $\Phi_{\tau,wx}^{-1}(\alpha)$ in Equation 26 cannot be calculated by summing the related link costs $\Phi_{\tau,wx}^{-1}(\alpha)$. In this case, the reliable shortest path problem cannot be easily solved by classical link-based shortest path algorithms on the basis of the additive property (Gutierrez and Medaglia 2008). Recently, B. Y. Chen, Lam, et al. (2013) proposed a multicriteria label-setting algorithm to efficiently determine the reliable shortest path when link travel times follow independent normal distributions. This multicriteria label-setting algorithm is modified in this study to determine the reliable forward- and backward cones. The detailed steps of the reliable forward-cone algorithm (called RFC-LS) and the reliable backward-cone algorithm (called RBC-LS) are given in the Appendix.

Given an origin–destination (OD) pair, a departure time $t_d$, a preferred arrival time $t_a$, an activity duration $c_{\text{min}}$, and an on-time arrival probability $\alpha$, the solution algorithm for constructing a network-based reliable space–time prism can be described as follows.

In the first step, a reliable forward cone (RFC($\alpha$, $t_d$)) is constructed using the RFC-LS algorithm (see Appendix). This algorithm can determine the reliable shortest paths from origin to destination using the network links that can be traversed with the desirable on-time arrival probability $\alpha$. Therefore, for each reachable network link $a_{ij}$, the reliable shortest path $p_{\tau,ij}^{\alpha}$ with minimum effective travel time $\Phi_{\tau,ij}^{-1}(\alpha)$ can be calculated. Through the reliable shortest path $p_{\tau,ij}^{\alpha}$, the earliest arrival time at the head node $j$ going through link $a_{ij}$ can be obtained as $t_j^d = t_d + \Phi_{\tau,ij}^{-1}(\alpha)$. Along the reliable shortest path $p_{\tau,ij}^{\alpha}$, the effective travel time at tail node $i$ can be also obtained as $\Phi_{\tau,ij}^{-1}(\alpha)$, and therefore the earliest arrival time at the tail node $i$ is $t_i^a = t_d + \Phi_{\tau,ij}^{-1}(\alpha)$.

In the second step, the reliable backward cone (RBC($\alpha$, $t_a$)) is constructed using the RBC-LS algorithm (see Appendix). This algorithm can generate reliable shortest paths at network links, by which an individual can return to destination $s$ with desirable on-time arrival probability $\alpha$. Thus, for each reachable link $a_{ij}$, the reliable shortest path $p_{T,ij}^{\alpha}$ with minimum effective travel time $\Phi_{T,ij}^{-1}(\alpha)$ can be determined. Hence, the latest departure time from tail node $i$ going through link $a_{ij}$ can be calculated as $t_i^d = t_a - \Phi_{T,ij}^{-1}(\alpha)$. Along the reliable shortest path $p_{T,ij}^{\alpha}$, the effective travel time to the head node $j$ can also be determined as $\Phi_{T,ij}^{-1}(\alpha)$. Thereby the latest departure time from the tail node $j$ is $t_j^d = t_a - \Phi_{T,ij}^{-1}(\alpha)$.

In the third step, the reliable potential path area (RPPA($\alpha$)) is constructed to determine the set of links $a_{ij}$ satisfying $c_i = t_i^d - t_i^a \geq c_{\text{min}}$ and $c_j = t_j^d - t_j^a \geq c_{\text{min}}$.

In the last step, the reliable space–time prism (RSTR($\alpha$, $t_d$)) is constructed. Let $(i_x, i_y)$ be the coordinates of the tail node $i$, and let $(j_x, j_y)$ be the coordinates of the head node $j$. For each link $a_{ij} \in \text{PPA}($, a space–time polygon is created as $\{(i_x, i_y, t_d^i), (i_x, i_y, t_d^i), (j_x, j_y, t_d^j), (j_x, j_y, t_d^j)\})$. If the link $a_{ij}$ has several intermediate nodes, their corresponding earliest arrival times and latest departure times are interpolated from those of the tail and head nodes.

Figure 4 illustrates a reliable space–time prism in the road network. The origin and destination nodes are set as Node 1 and Node 6, respectively, and the on-time arrival probability is set to $\alpha = 0.9$. Seven space–time polygons are constructed for describing individual space–time extents on these network links. The height of the space–time prism at each network

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Reliable space–time prism in the road network.}
\end{figure}
node is the maximum time duration through all links connecting to this node.

Next, the complexity of the reliable space–time prism algorithm described earlier was analyzed. In the worst case, the RFC-LS algorithm (the first step) must generate reliable shortest paths from the origin to all network nodes. Therefore, the RFC-LS algorithm has the same complexity as the reliable shortest path algorithm (Chen, Lam, et al. 2013) running in $O(|A||P| + |N|\log|N|)$, where $|N|$ and $|A|$ are the number of network nodes and links, respectively, and $|P|$ is the number of nondominated paths maintained at the network nodes. Similarly, the RBC-LS algorithm (the second step) also has complexity $O(|A||P| + |N|\log|N|)$. The third and last steps only run in $|A|$. Therefore, the complexity of the reliable space–time prism algorithm is $O(|A||P| + |N|\log|N|)$. Theoretically, the reliable space–time prism algorithm has a nonpolynomial complexity, because $|P|$ grows exponentially with network size. In practice, this complexity is pseudo-polynomial, because $|P|$ is generally very small in congested road networks (Nie and Wu 2009; Chen, Lam, et al. 2013).

**Numerical Example**

**Case Study**

This section presents a numerical example using real-world traffic data to demonstrate the applicability of the proposed reliable space–time prism model. In this study, the reliable space–time prism algorithm presented earlier was implemented using the Visual C# programming language. A prototype system was developed and integrated with the ArcGIS software (Zeiler 2001). The ArcScene module of the ArcGIS software was used to represent and visualize the network-based reliable space–time prism. By using $z$ values for the time dimension, the space–time framework of time geography can be simulated in 3D space and visualized in ArcScene (Kwan 2000; Yu and Shaw 2008).

Real-world traffic information collected in Wuhan, China, was used for this case study. As shown in Figure 5, the Wuhan road network consists of 19,306 nodes and 46,757 links. The link travel time distributions in the Wuhan road network were estimated using floating-car data (FCD) collected by a real floating-car system.
Figure 6. Coefficient of variation of link travel time distributions in Wuhan network. CV = coefficient of variation.

in Wuhan. The floating-car system consists of 11,248 taxi vehicles equipped with Global Positioning System (GPS) devices to record their traveling locations every forty seconds. The collected GPS sampling points are then matched with the road network to generate GPS trajectories using a map-matching algorithm (B. Y. Chen, Yuan, et al. forthcoming). Consequently, travel time distributions can be estimated based on these generated GPS trajectories of taxi vehicles (Shi, Chen, and Fang 2012). Using this method, FCD were collected on a typical weekday (17 September 2009, a Thursday) during the morning peak period (7 a.m.–10 a.m.) to estimate link travel time distributions for this case study.

Figure 5 illustrates the traffic status of the Wuhan road network during this typical morning peak period with respect to mean link travel speeds (i.e., link distance divided by means of link travel time distributions). In Figure 5, links in green, yellow, and red represent uncongested (> 40 km/h), slightly congested (20–40 km/h), and congested (< 20 km/h) links, respectively. It can be seen that 17.6 percent of the links in the Wuhan road network are congested during the morning period. Most of these congested links are located around the central business districts (CBDs) of Han-Kou (HK), Han-Yang (HY), and Wu-Chang (WC).

Figure 6 shows the link travel time variations in the Wuhan road network using the concept of coefficient of variation (CV; i.e., the ratio of the standard deviation to the mean). A larger CV value indicates larger link travel time variations. It can be seen that 58 percent of the links are highly stochastic, with CV values greater than 0.5. Therefore, the assumption that link travel times are deterministic (i.e., CV value equal to 0), as used in the classical space–time prism model, is erroneous in this typical congested urban road network.

The prototype system provided interfaces to create reliable space–time prisms in road networks under a set of user-specified space–time parameters. Consider an individual departing from Wuhan University at \( t_i = 8:00 \) a.m. and preferring to arrive at Hongshan Gymnasium at \( t_f = 9:00 \) a.m. The origin and destination locations are shown in Figure 5. During this period, the individual might have a cup of coffee with an activity duration \( c_{\text{min}} = 30 \) min. The maximum distance from a coffee shop to its nearest network link was set to 150 m and the off-network walking time from the coffee shop to the network link was set to 5 minutes. Therefore, the time budget for traveling in the road network was \( b = 20 \) minutes.

Figure 7 illustrates a 3D view of the generated reliable space–time prism with an on-time arrival probability of \( \alpha = 0.99 \). The reliable space–time prism is indicated as a violet 3D entity and the reliable potential area by red links. The blue points represent the origin and destination, and the green points are identified coffee shops. As illustrated, the reliable space–time prism consists of a reliable forward cone, a reliable backward cone, and a reliable cylinder. For a coffee shop, the space–time point on the reliable forward cone indicates the individual's earliest arrival time at this shop with a 99 percent on-time arrival probability. The space–time
point on the reliable backward cone represents the individual’s latest departure time from the coffee shop to the destination with a 99 percent on-time arrival probability. The height of the reliable space–time prism at the coffee shop indicates how long the individual can stay to have coffee and return to the destination with 99 percent probability of on-time arrival.

The effects of on-time arrival probability on reliable space–time prism generation were investigated. The same set of parameters was used as in Figure 7, except for different values of the $\alpha$ parameter (i.e., $\alpha = 0.3, 0.5, 0.7, 0.9, 0.95,$ and $0.99$). Figure 8 shows the 3D view of the generated reliable space–time prisms for both $\alpha = 0.5$ and $\alpha = 0.99$. The inner prism is the one with $\alpha = 0.99$. It can be observed from Figure 8 that the $\alpha$ parameter can significantly affect the volume of the reliable space–time prisms. For the easy observation, Figure 9 illustrates the generated reliable space–time prisms in a 2D view (i.e., using the reliable potential path area). Following the work of Miller (1991), the size of the reliable potential path area was quantified by the number of network links and number of POI (i.e., coffee shops) within the area.

It can be easily observed in Figure 9 that the size of the reliable potential area is significantly affected by on-time arrival probability (i.e., $\alpha$). When $\alpha = 0.5$ (Figure 9B), the individual is risk-neutral and schedules the activity based only on mean travel time. In this case, 1,068 links and 230 coffee shops are identified in the reliable potential area. This result is equivalent to that of a classical space–time prism model based on deterministic link travel times. When $\alpha = 0.3$ (Figure 9A), the risk-seeking individual prefers to take the risk of arriving late to explore a larger space–time extent for
conducting activities. In this scenario, an additional 149 links and six coffee shops are identified in the reliable potential path area. Only 30 percent probability of on-time arrival can be guaranteed for going to these additional coffee shops, however. The majority of individuals who are risk-averse for late arrival tend to budget an additional safety margin to ensure a high on-time arrival probability. Due to the safety margin, the size of the reliable potential path area is reduced. For example, when $\alpha = 0.9$ (Figure 9D), the size of the reliable potential path area is reduced to 806 links and 198 coffee shops, but the guaranteed on-time probability is increased to 90 percent. When $\alpha = 0.99 \approx 1$ is chosen (Figure 9F), the minimum size of the reliable potential area is reached, with 609 links and 184 coffee shops. Individuals having a cup of coffee at these coffee shops can certainly return to their destinations on time. Therefore, the proposed reliable space–time prism can represent individuals’ space–time extents under various on-time arrival probabilities ($\forall \alpha \in (0, 1)$).

**Computational Performance**

Figure 10 reports the computational times required to construct the reliable space–time prisms shown in Figure 9. Also illustrated in Figure 10 is the computational performance of the reliable space–time prism algorithm under various travel time budgets from $b = 10$ min to $b = 120$ min. All experiments were conducted on a desktop computer with an Intel dual-core 3.1 GHz CPU (only a single processor was used), 8 GB RAM, running on the Windows 7 operating system. It can be seen from Figure 10 that the computational time significantly increased with the travel time budget $b$. For example, when $b = 20$ min and $\alpha = 0.9$, the proposed solution algorithm required only 11.7 milliseconds. When
Figure 9. Reliable potential path area under various on-time arrival probabilities: (A) $\alpha = 0.3$; (B) $\alpha = 0.5$; (C) $\alpha = 0.7$; (D) $\alpha = 0.9$; (E) $\alpha = 0.95$; (F) $\alpha = 0.99$. (Color figure available online.)
It can be observed from Figure 10 that the proposed solution algorithm runs faster as individuals become more risk-neutral ($\alpha = 0.5$). For example, to construct reliable space–time prisms under $b = 120$ minutes, the algorithm required 326.1 milliseconds when $\alpha = 0.3$, 303.4 milliseconds when $\alpha = 0.5$, and 729.4 milliseconds when $\alpha = 0.99$. This result is expected; as mentioned earlier, the proposed solution algorithm uses the reliable shortest path algorithm to determine the optimal path between network nodes instead of the classical shortest path algorithm. This reliable shortest path algorithm requires more computational time than the classical shortest path algorithm, because multiple non-dominated paths must be maintained at network nodes. The closer the $\alpha$ value is to 0.5, the fewer the generated non-dominated paths, and therefore the proposed reliable space–time prism algorithm achieves better computational performance. When $\alpha = 0.5$, the proposed solution algorithm has the same computational performance as the classical deterministic space–time prism algorithm (Kuijpers and Othman 2009).

It can also be seen from Figure 10 that the computational times required to construct all the reliable space–time prisms were within one second. This result demonstrates that the proposed solution algorithm is capable of constructing reliable space–time prisms in large-scale road networks, even with large travel time budgets and $\alpha$ values.

Conclusions

This study investigated the problem of modeling space–time extents for scheduling individuals’ activities in a congested road network with travel time uncertainty. A reliable space–time prism model was introduced to extend the classical space–time prism model by explicitly considering individuals’ various attitudes toward the risk of being late. Based on previous empirical findings, an individual’s risk attitude was represented by a predetermined on-time arrival probability parameter (denoted by $\alpha \in (0, 1)$). $\alpha > 0.5$, $\alpha = 0.5$, and $\alpha < 0.5$, respectively, represented risk-averse, risk-neutral, and risk-seeking attitudes. Given a predetermined $\alpha$ value, the reliable space–time prism was defined as the set of space–time points where an individual could participate in activities and return to a destination with at least $\alpha$ on-time arrival probability. The proposed reliable space–time prism model has several unique features.

1. In addition to travel environments, the reliable space–time prism is determined by individuals’ on-time arrival probability concerns (i.e., the $\alpha$ parameter). When $\alpha = 0.5$, the reliable space–time prism is equivalent to the classical space–time prism, which considers only the mean travel time and ignores travel time uncertainty. With higher $\alpha$ values, the individual becomes more risk-averse, resulting in a reduction of the individual’s potential space–time extent due to the reservation of a travel time safety margin.

2. The size of the reliable space–time prism is defined by an upper bound and a lower bound. The upper bound is reached when the on-time arrival probability $\alpha \approx 0$. The space–time locations outside the upper bound are not reachable by the individual for conducting flexible activities. Conversely, the lower bound, reached when $\alpha \approx 1$, delineates all space–time locations that are certainly reachable. From this observation, the proposed reliable space–time prism can be regarded as an extension of the work of Neutens et al. (2007), which represented only these upper and lower bounds using rough set theory.

To construct a reliable space–time prism in the road network, a solution algorithm has been proposed in this study. The proposed algorithm was implemented and integrated into the ArcGIS software to develop a prototype system. A case study using real traffic information was carried out on the Wuhan road network.
to demonstrate the features of the proposed reliable space–time prism. The results show that more than half the network links are highly stochastic, with coefficients of variation greater than 0.5. This indicates that the commonly used assumption that link travel times are deterministic might be erroneous in congested urban road networks. The results also demonstrated that the proposed reliable space–time prism model can represent individuals’ space–time extents well under various levels of on-time arrival probability concerns (i.e., \( \forall \alpha \in (0, 1) \)). Computational experiments indicated that the proposed solution algorithm is capable of constructing reliable space–time prisms in large-scale road networks even with large time budgets.

In this article, individuals’ risk-taking behaviors have been formulated with respect to on-time arrival probability. There is another line of research in the literature that uses utility functions with early and late arrival penalties to model individuals’ risk-taking behaviors (Noland and Polak 2002; Ettema and Timmermans 2007; Carrion and Levinson 2012). Researchers have recently found that this utility-based formulation of risk-taking behaviors is equivalent to the on-time arrival probability (Fosgerau and Karlström 2010; X. Wu and Nie 2011). Therefore, the proposed reliable space–time prism model can also be used to delimit individuals’ space–time extents in terms of early and late arrival penalties.

Floating-car data were used in the case study to generate travel time distributions. Great strides have recently been made in intelligent transportation system technologies toward a data-rich era (Chang et al. 2013). Accurate travel time distributions can be estimated using other traffic data collection techniques, including loop detectors, cellular phone tracking, automatic vehicle identification, electronic license plate matching, and video imaging techniques (J. P. Zhang et al. 2011). Travel time distributions can also be indirectly estimated by means of the traffic assignment models used for transportation planning (B. Y. Chen et al. 2011).

Several directions for future research are worth noting. First, in the proposed algorithm, the assumption of independent link travel times was made to simplify the problem of constructing reliable space–time prisms in the road network. Empirical studies have found that travel times in the congested road network are strongly correlated among neighboring links (Chan, Lam, and Tam 2009). A possible way to relax this assumption would be to use a spatially dependent reliable shortest path algorithm (B. Y. Chen, Lam, Sumalee, and Li 2012) in the proposed reliable space–time prism algorithm. Second, in this study, link travel time distributions are assumed to be stable for the period of interest. In reality, link travel time distributions vary with time of day (B. Y. Chen, Lam, Sumalee, Li, and Tam forthcoming). How to incorporate this stochastic time-dependent nature of link travel times into the proposed reliable space–time prism model needs further investigation. Third, it was assumed in this study that the locations of the origin and destination are perfectly fixed or known. In practice, the locations of the origin and destination might have a substantial degree of measurement error or inherent flexibility (Kuijpers et al. 2010). The incorporation of such anchor uncertainty into the proposed reliable space–time prism model would be an interesting extension. Last but not least, another possible further research direction would be to apply the proposed reliable space–time prism model to accessibility analysis and choice set formation by taking account of individuals’ on-time arrival probability concerns (Kwan 1998; Miller 1999; Ashiru, Polak, and Noland 2003; Ettema and Timmermans 2007; X. Chen and Kwan 2012; Delafontaine, Neutens, and Van de Weghe 2012). The effects of individuals with different socioeconomic characteristics (e.g., elderly, young, and disabled) could also be considered.

Acknowledgments

The authors are thankful to the editor and anonymous referees for their comments and suggestions that improved the quality of this article. The work described in this article was jointly supported by research grants from the National Science Foundation of China (No. 41231171, 41201466, 41071285, 41021061), the Research Grant Council of the Hong Kong Special Administration Region (RGC No. PolyU 5196/10E), China Postdoctoral Science Foundation (No. 2012M521469, 2013T60741), Shenzhen Scientific Research and Development Funding Program (No. ZDSY20121019111146499), and Shenzhen Dedicated Funding of Strategic Emerging Industry Development Program (No. JCYJ20121019111128765).

References


Appendix

This appendix presents two solution algorithms, named RFC-LS and RBC-LS, for generating forward and reliable backward cones, respectively. First we describe the reliable forward cone algorithm (i.e., RFC-LS algorithm). This algorithm is to determine reliable shortest paths from origin to network links that can be reached with the desirable on-time arrival probability \( \alpha \). Such an algorithm is modified from the multicriteria label-setting algorithm proposed by B. Y. Chen, Lam, et al. (2013), which searches the reliable shortest path from an origin to a destination.

The multicriteria label-setting algorithm relies on the M-B dominance condition to generate a set of M-B nondominated paths at each node \( x \). Then, the reliable shortest path from origin to node \( x \) can be obtained as one of the generated M-B nondominated paths at node \( x \). This node-based labeling technique cannot take into account the turn restrictions in real road networks. In view of this, a link-based M-B dominance condition is proposed as follows.

Given an on-time arrival probability \( \alpha \) and two paths \( p_{ux}^{r,wx} \neq p_{ux}^{r,wx} \in P_{ux}^{r,wx} \), \( p_{ux}^{r,wx} \) M-B dominates \( p_{ux}^{r,wx} \), if \( p_{ux}^{r,wx} \) and \( p_{ux}^{r,wx} \) satisfy \( \overline{t}_{ux}^{r,wx} \leq \overline{t}_{ux}^{r,wx} \) and \( \Phi_{1}^{-1} \overline{t}_{ux}^{r,wx}(\alpha) < \Phi_{1}^{-1} \overline{t}_{ux}^{r,wx}(\alpha) \). Accordingly, a path \( p_{ux}^{r,wx} \in P_{ux}^{r,wx} \) is said to be an M-B nondominated path at link \( a_{wx} \), if and only if \( p_{ux}^{r,wx} \) is not M-B dominated by any path \( p_{ux}^{r,wx} \in P_{ux}^{r,wx} \).

Figure A.1 illustrates the concept of link-based M-B nondominated paths by means of a small network. As shown in Figure 10A, there are three paths (\( p_{1}^{4,56}, p_{2}^{4,56}, \) and \( p_{3}^{4,56} \)) from Node 4 to Node 5 passing through the same link \( a_{56} \). The travel time distributions of these three paths are given in Figure 10B. According to the preceding definition, given an on-time arrival probability \( \alpha = 0.9 \), \( p_{1}^{4,56} \) M-B dominates \( p_{3}^{4,56} \), because \( \overline{t}_{1}^{4,56} \leq \overline{t}_{3}^{4,56} \) and \( \Phi_{1}^{-1}(\overline{t}_{1}^{4,56}(\alpha)) < \Phi_{1}^{-1}(\overline{t}_{3}^{4,56}(\alpha)) \). The dominated path \( p_{3}^{4,56} \) can be discarded without further consideration in the reliable shortest path search to link \( a_{57} \). As the paths \( p_{1}^{4,56} \) and \( p_{2}^{4,56} \) cannot be M-B dominated by each other, they are M-B nondominated paths at link \( a_{56} \). Both these two paths have to be kept at link \( a_{56} \), because either of them can be a subpath of the reliable shortest path at link \( a_{57} \).

Based on the preceding link-based M-B nondominated condition, the RFC-LS algorithm for constructing the reliable forward cone is described next. In the RFC-LS algorithm, a set of M-B nondominated paths \( P_{r,ij} = \{ p_{r,ij}^{1}, \ldots, p_{r,ij}^{m} \} \) is maintained at each link \( a_{ij} \). Nondominated paths from all links are maintained in a scan eligible set, denoted by \( SE = \{ p_{r,ij}^{1}, \ldots, p_{r,kw}^{m} \} \). All nondominated paths in the priority queue are ordered based on their effective travel time \( \Phi_{1}^{-1} \overline{t}_{r,ij}(\alpha) \). At each iteration, a nondominated path \( p_{r,ij}^{m} \) at the top of \( SE \) (with minimum \( \Phi_{1}^{-1}(\overline{t}_{r,ij}(\alpha)) \)) is selected from \( SE \) for path extensions. A temporary path is constructed by extending the selected path \( p_{r,ij}^{m} \) to its successor link \( a_{ik} \), denoted by \( p_{r,ik}^{m} := p_{r,ij}^{m} \oplus a_{ik} \). The dominant relationship between the newly generated path \( p_{r,ik}^{m} \) and the set of nondominated paths \( P_{r,ik} \) at link \( a_{ik} \) is determined. If newly generated path \( p_{r,ik}^{m} \) is a nondominated path at link \( a_{ik} \), this path is then inserted into \( P_{r,ik} \) and \( SE \). The newly generated path \( p_{r,ik}^{m} \) might also dominate a set of paths in \( P_{r,ik} \), denoted by \( P_{D,ik}^{m} \). Such dominated paths in \( P_{D,ik}^{m} \) can be eliminated from \( P_{r,ik} \) and \( SE \) without further consideration. The algorithm continues the path extension process until \( SE \) becomes empty or effective travel time \( \Phi_{1}^{-1} \overline{t}_{ik}(\alpha) \) is larger than the predetermined travel time budget \( b \).

**Figure A.1.** An illustrative example for the M-B nondominated paths: (A) a simple network; (B) path travel time distributions. (Color figure available online.)
RFC-LS Algorithm

**Inputs:** Origin \( r \), travel time budget \( b \), and on-time arrival probability \( \alpha \)

**Returns:** The reliable forward cone

**Step 1. Initialization:**
For each link \( a_j \) emanating from origin node \( r \)
- Create a path \( p^{r,j} \) at link \( a_j \) and set \( \tau^{r,j} := 0 \), \( (\sigma^{r,j})^2 := 0 \) and \( \Phi_{\tau^{r,j}}^{-1}(\alpha) := 0 \).
- Set \( P^{r,j} := \{p^{r,j}\} \) and \( SE := SE \cup \{p^{r,j}\} \).
End for

**Step 2. Label selection:**
- If \( SE = \emptyset \), then Stop; otherwise, continue.
- Select \( p^{r,j}_u \) at the top of \( SE \) and set \( SE := SE \setminus \{p^{r,j}_u\} \).
- If \( \Phi_{\tau^{r,j}}^{-1}(\alpha) > b \), then Stop; otherwise continue.

**Step 3. Path extension:**
- For each turning movement \( \psi_{ijk} \) emanating from selected link \( a_{jk} \)
  - Generate a new path \( p^{r,jk}_u := p^{r,j}_{u} \oplus a_{jk} \) and set \( \tau^{r,jk}_u = \tau^{r,j}_{u} + \bar{t}_{jk} \), \( (\sigma^{r,jk}_{u})^2 = (\sigma^{r,j}_{u})^2 + \sigma_{jk}^2 \) and \( \Phi_{\tau^{r,jk}_{u}}^{-1}(\alpha) = \tau^{r,jk}_{u} + \zeta_{\alpha} \sigma_{u}^{r,jk} \).
  - If \( p^{r,jk}_u \) is an M-B nondominated path at link \( a_{jk} \), then insert \( p^{r,jk}_u \) into \( P^{r,jk} \) and \( SE \) and remove all paths in \( P^D_{r,jk} \) dominated by \( p^{r,jk}_u \) from \( P^{r,jk} \) and \( SE \).
End for

Go to Step 2.

RBC-LS Algorithm

The RBC-LS algorithm is to construct the reliable backward cone. This algorithm generates reliable shortest paths at all links, from which an individual can return to destination \( s \) with the desirable on-time arrival probability \( \alpha \). The RBC-LS algorithm is similar to the preceding RFC-LS algorithm. The only difference is that the RBC-LS algorithm runs from the destination downward along the time dimension, whereas the RFC-LS algorithm runs from origin upward along the time dimension. The detailed steps of the RBC-LS algorithm are described next.

**Inputs:** Destination \( s \), travel time budget \( b \), and on-time arrival probability \( \alpha \)

**Returns:** The reliable backward cone

**Step 1. Initialization:**
- For each link \( a_{ij} \) merging into destination \( s \)
  - Create a path \( p^{i,s}_{j} \) at link \( a_{ij} \) and set \( \tau^{i,s}_{j} := 0 \), \( (\sigma^{i,s}_{j})^2 := 0 \) and \( \Phi_{\tau^{i,s}_{j}}^{-1}(\alpha) := 0 \).
  - Set \( P^{i,s}_{j} := \{p^{i,s}_{j}\} \) and \( SE := SE \cup \{p^{i,s}_{j}\} \).
End for

**Step 2. Label selection:**
- If \( SE = \emptyset \), then Stop; otherwise, continue.
- Select \( p^{i,s}_{j} \) at the top of \( SE \) and set \( SE := SE \setminus \{p^{i,s}_{j}\} \).
- If \( \Phi_{\tau^{i,s}_{j}}^{-1}(\alpha) > b \), then Stop; otherwise continue.

**Step 3. Path extension:**
- For each turning movement \( \psi_{ijk} \) merging into selected link \( a_{jk} \)
  - Generate a new path \( p^{i,s}_{j,k} := a_{ij} \oplus p^{i,s}_{j,k} \) and set \( \tau^{i,s}_{j,k} = \bar{t}_{ij} + \bar{t}_{jk} \), \( (\sigma^{i,s}_{j,k})^2 = (\sigma_{ij}^2 + (\sigma_{jk}^2) \) and \( \Phi_{\tau^{i,s}_{j,k}}^{-1}(\alpha) = \tau^{i,s}_{j,k} + \zeta_{\alpha} \sigma^{i,s}_{j,k} \).
  - If \( p^{i,s}_{j,k} \) is an M-B nondominated path at link \( a_{jk} \), then insert \( p^{i,s}_{j,k} \) into \( P^{i,s}_{j,k} \) and \( SE \) and remove all paths in \( P^D_{i,s,k} \) dominated by \( p^{i,s}_{j,k} \) from \( P^{i,s}_{j,k} \) and \( SE \).
End for

Go to Step 2.