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Mixed Convection and Nonlinear Radiation in the Stagnation Point Nanofluid flow towards a Stretching Sheet with Homogenous-Heterogeneous Reactions effects

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Abstract

The effect of homogeneous-heterogeneous reactions on the steady two dimensional mixed convection stagnation point boundary layer flow of an incompressible, viscous, electrically conducting, nonlinear radiative heat transfer and chemically reacting copper-water nanofluid toward a linear stretching sheet are numerically investigated in this paper. The spectral local linearization method (SLLM) is used to find the solution for the fluid velocity, fluid temperature and species concentration. Such type of nanofluid flow model are useful in many engineering processes such as insulation system, heat exchanger devices and chemical catalytic.

1. Introduction

Exploration of the flow characteristics of viscous, incompressible fluids with suspended nano-sized solid particles has now attracted many researchers due to the application of such fluids in heat transfer devices. Nanofluids, due to their higher thermal conductivity and convective heat transfer rates are used in wide variety of engineering applications, such as in advanced nuclear systems Buongiorno and Hu [1]. Most common fluid such as water, oil and ethylene glycol mixture are poor heat transfer fluids, because the thermal conductivities of these fluids are low. The thermal conductivity of these fluids may be enhanced by suspending nanoparticle materials in the liquid to form a nanofluid. Choi [2] was the first to use the term nanofluids to refer to the fluid with suspended nanoparticles. Thermophysical properties of nanofluids such as thermal conductivity, diffusivity and viscosity have been studied by Kang et al. [3] and Rudyak et al.[4]. Due to various industrial applications, researchers are interested to consider the stagnation point flow of incompressible nanofluid over stretching/shrinking sheet. Bachok et al. [5] studied the two-dimensional stagnation point flow of a nanofluid over a stretching/shrinking sheet. Narayana and Sibanda [6] studied the effects of

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laminar flow of a nanoliquid film over an unsteady stretching sheet. They investigated that the effect of the nanoparticle volume fraction is to reduce the axial velocity and free-stream velocity and compare with Cu-water nanoliquid and Al$_2$O$_3$-water nanoliquid. The study of the MHD flow of an electrically conducting fluid due to stretching/shrinking sheet is important in modern metallurgy and metal-working processes. Pavlov [7] studied the magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a surface. Anderson [8] studied the analytical solution of two-dimensional Navier-Stokes’ equation over a stretching sheet considering the effects of external magnetic field. Jafar et al. [9] studied the effect of MHD flow and heat transfer over a stretching/shrinking sheet with an external magnetic field, viscous dissipation and Jule effects. Kameswaran et al. [10] studied the hydromagnetic nanofluid flow due to a stretching/shrinking sheet with viscous dissipation and chemical reaction effects.


The aim of the present work is to investigate numerically the effect of homogeneous-heterogeneous reactions on the steady two dimensional mixed convection stagnation point boundary layer flow of an incompressible, viscous, electrically conducting, nonlinear radiative heat transfer and chemically reacting copper-water nanofluid toward a linear stretching sheet. Spectral Local Linearization Method (SLLM) is being used to solve the nonlinear coupled partial differential equations describing the model.

2. Formulation of the problem

We consider a steady two dimensional mixed convection boundary layer flow of an incompressible, viscous and electrically conducting radiative nanofluid in a region $y > 0$, driven by a stretching/shrinking surface at $y = 0$ which is having a fixed stagnation point. The fluid flow is also affected by a thermal radiation through the nonlinear Rosseland diffusion approximation. The wall is stretched as a result of two equal and opposite forces are applied along the sheet, keeping the position of the origin fixed. It is assumed that the stretching/shrinking velocity is $U_w = cx$ while, $U_{\infty} = dx$ is the free stream velocity, where $c > 0$ and $d > 0$ are constants and $x$ is a coordinate measured along the stretching surface. The fluid is permeated with a uniform transverse magnetic field $B_0$. We are neglecting the induced magnetic field as it is very low in comparison to the applied one. This presumption is valid for the fluid with low magnetic Reynolds number [18]. Also, the effect of polarization is negligible as applied or polarized voltages do not exist [19].

In our model, fluid under consideration is a water based nanofluid containing copper (Cu) nanoparticles. Thermal equilibrium is maintained between base fluid and the nanofluid and there is no slip occurs between them. The thermophysical properties of the nanofluid are given in Table 1. It is assumed that a simplest model for the mutual interaction between a homogeneous reaction and a heterogeneous reaction involving the two chemical species $A$ and $B$ in a boundary layer flow proposed by Chaudhary and Merkin [14] in the following form:

$$A + 2B \rightarrow 3B, \quad \text{rate} = k_c ab^2,$$  \hspace{1cm} (1)

$$A \rightarrow B, \quad \text{rate} = k_a a,$$  \hspace{1cm} (2)

where $a$ and $b$ are the concentrations of the chemical species $A$ and $B$ respectively, $k_c$ and $k_a$ are the rate constants. It is assumed that both the reaction processes are isothermal. Under the above assumptions, usual boundary layer and Rosseland diffusion approximations, the MHD free convective nanofluid flow, with heat and mass transfer are
governed by the following equations, (Nield and Kuznetsov [20] and Kuznetsov and Nield [21])

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}
\]

\[
\frac{u}{\partial x} + v \frac{u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{nf}} (u - U_\infty) + g\beta_{nf} (T - T_\infty), \tag{4}
\]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{nf}} \frac{\partial q_r}{\partial y}, \tag{5}
\]

\[
\frac{\partial a}{\partial x} + v \frac{\partial a}{\partial y} = D_A \frac{\partial^2 a}{\partial y^2} - k_{cA} b^2, \tag{6}
\]

\[
\frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} = D_B \frac{\partial^2 b}{\partial y^2} + k_{cB} a^2, \tag{7}
\]

where \( u \) and \( v \) are velocity component of the nanofluid in \( x \) and \( y \) directions, respectively, \( D_A \) and \( D_B \) are the respective diffusion species coefficients of \( A \) and \( B \), \( T \) is the temperature of the nanofluid, \( g \) is the acceleration due to gravity, \( \beta_{nf} \) is the coefficient of thermal expansion, \( \mu_{nf}, \sigma, \rho_{nf}, k_{nf}, (c_p)_{nf} \) are respectively, viscosity of the nanofluid, electrical conductivity, nanofluid density, nanofluid thermal conductivity, specific heat at constant pressure of the nanofluid and \( q_r \) is the radiative heat flux. Here the subscripts \( nf, f, \) and \( s \) refer to the thermophysical properties of the nanofluid, base fluid and nano solid particles, respectively Brinkman [22].

The boundary conditions for the problem are

\[
u = U_w(x) = c x, \quad v = 0, \quad T = T_w, \quad D_A \frac{\partial a}{\partial y} = k_{cA}, \quad D_B \frac{\partial b}{\partial y} = -k_{cA} \text{ at } y = 0; \tag{8}
\]

\[
u \to U_{\infty}(x) = dx, \quad T \to T_\infty, \quad a \to a_0, \quad b \to 0 \text{ as } y \to \infty, \tag{9}
\]

Introducing the following transformation

\[
\psi(x, y) = \sqrt{v_f U_{\infty} f(x, \eta)}; \quad \theta = \frac{(T - T_w)}{(T_w - T_\infty)}; \quad a = a_0 f(\eta), \quad b = a_0 h(\eta) \text{ where } \eta = \sqrt{\frac{U_{\infty} y}{v_f}}, \tag{10}
\]

where \( \psi \) is the dimensionless stream function, and the above transformation is chosen in such a way that \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \).

Using the above transformation (10), the equation of continuity (3) is automatically satisfied and we obtain from Eqs. (4), (5), (6), and (7), respectively as

\[
f'''' + \phi_1 f f''' - f_1 f'' - M\phi_2 (f - 1) + \lambda \phi_3 \theta + \phi_1 = 0, \tag{11}
\]

\[
\frac{4\phi_4}{3N_R} \left[ 1 + (\omega_0 - 1)^2 \right] \theta'''' + \frac{4\phi_4}{N_R} \left[ 1 + (\omega_0 - 1)^2 \right] (\omega_0 - 1)^2 \theta'''' = 0, \tag{12}
\]

\[
\frac{1}{Sc} g'''' + f g' - Khg' = 0, \tag{13}
\]

\[
\frac{1}{Sc} h'''' + f h' + Khg' = 0, \tag{14}
\]

where

\[
M = \frac{\sigma B_0^2}{\rho_f d}, \quad \lambda = \frac{G_r_s}{Re_x}, \quad G_r_s = \frac{g^2 B_f T x^3}{\nu_f^2}, \quad Re_x = \frac{U_{\infty} x}{\nu_f}, \quad N_R = \frac{Ku^*}{4\sigma^* T_\infty^3}, \quad Pr = \frac{v_f}{\alpha_f}, \quad Sc = \frac{v_f}{\nu_f}, \quad D_A = \frac{\delta}{D_A}, \quad \delta = \frac{D_B}{D_A}, \tag{15}
\]

\[
K = k_a a_0, \quad \phi_1 = (1 - \phi)^2.5 \left[ \{1 - \phi + \phi \left( \frac{\partial a}{\partial y} \right) \} \right], \quad \phi_2 = (1 - \phi)^2.5, \quad \phi_3 = (1 - \phi)^2.5 \left[ \{1 - \phi + \phi \left( \frac{\partial b}{\partial y} \right) \} \right], \tag{15}
\]

\[
\phi_4 = \frac{k_s + 2k_f + 2\phi(k_f - k_s) \left[ \{1 - \phi + \phi \left( \frac{\partial a}{\partial y} \right) \} \right]}{k_s + 2k_f - 2\phi(k_f - k_s) \left[ \{1 - \phi + \phi \left( \frac{\partial a}{\partial y} \right) \} \right]}, \quad \phi_5 = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}, \quad \text{and } \theta_w = \frac{T_w}{T_\infty}. \tag{15}
\]
The parameters defined above are the magnetic parameter $M$, the local mixed convection parameter $\lambda$, the local Grashof number $Gr_x$, the thermal radiation parameter $NR$, the Prandtl number $Pr$, the Schmidt number $Sc$, the ratio diffusion constants $\delta$, the homogeneous reaction rate $K$ and the temperature ratio parameter $\theta > 1$. The functions $\phi_1, \phi_2, \phi_3, \phi_4,$ and $\phi_5$ are also nondimensional and are depending upon the thermophysical properties of the nanoparticles and base fluid. It is here noted that the mixed convection parameter defined is not fully non-dimensionalized, however for suitably chosen constant values of it will give us a local similar solutions which are of practical use.

The boundary conditions (8) and (9), subject to the transformation (10), are given by

$$f(\eta) = 0, \quad f'(\eta) = \epsilon, \quad \theta(\eta) = 1, \quad g' = K_s g(1 - g)^2 = 0;$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad g(\eta) \to 1, \quad h(\eta) \to 0 \text{ as } \eta \to \infty,$$

where $\epsilon = \frac{U_w}{U_\infty}$ is the stretching/shrinking parameter ($\epsilon > 0$ for stretching and $\epsilon < 0$ for shrinking) and $K_s = \frac{k_s}{D_s} \sqrt{\frac{\nu}{\delta}}$ is the strength of the heterogeneous reaction.

It is expected that the diffusion coefficients of chemical species $A$ and $B$ are of a comparable size which leads us to make a further assumption that the diffusion coefficients $D_A$ and $D_B$ are equal, i.e. $\delta = 1$ [14]. This assumption leads to the following relation

$$g(\eta) + h(\eta) = 1.$$

The Eqs. (13) and (14) under this assumption reduce to

$$g'' + Scfg' - ScKg(1 - g)^2 = 0,$$

and are subject to the boundary conditions

$$g'(0) = K_s g(0), \quad g(\eta) \to 1 \text{ as } \eta \to \infty.$$

The problem now reduces to the problem of solving Eqs. (11), (12), and (19) subject to the conditions provided in Eqs. (16), (17) and (20).

The other physical quantities of significance, which need to be investigated are the local skin friction coefficient $C_{fx}$ and the local Nusselt number $Nu_{x}$. They may be obtained in the form as

$$C_{fx} Re_x^{1/2} = \frac{1}{\phi_2} f''(0),$$

$$Nu_{x}/Re_x^{1/2} = -\phi_5 \left[1 + \frac{4}{3NR} \left(1 + (\theta_w - 1)\theta(0)\right)^3\right] \theta'(0).$$

The reduced local Nusselt number can be written as

$$Nu = Nu_{x}/Re_x^{1/2} = -\phi_5 \left[1 + \frac{4}{3NR} \left(1 + (\theta_w - 1)\theta(0)\right)^3\right] \theta'(0).$$

3. Solution technique - The Spectral Local Linearization Method (SLLM)

Spectral Local Linearization Method (SLLM) (see Motsa [23]) is employed to solve the Eqs. (11), (12), and (19) subject to the boundary conditions (16), (17) and (20). Just like the SRM, the SLLM also imports the Gauss Seidel idea for decoupling the system into a sequence of subsystem. This method then relies on the Taylor Series expansion for purposes of Locally linearizing each subsystem. This gives rise to a new decoupled system of linear equations.
4. Results and discussion

The combined effects of homogeneous-heterogeneous reactions, mixed convection, magnetic field, and thermal radiation on the flow of a viscous, incompressible, and electrically conducting stagnation point nanofluid flow due to a linear stretching sheet are studied. The base fluid is water containing copper (cu) nanoparticle. To study the effect of stretching/shrinking parameter, mixed convection, thermal radiation, temperature ratio at the surface on the fluid velocity $f'(\eta)$, fluid temperature $\theta(\eta)$ and species concentration $g(\eta)$. The profile of these physical quantities are presented graphically in Figs. 1 to 4 taking $Sc = 1$ and $Pr = 6.7850$.

It is noticed from fig. 1 that the fluid velocity and species concentration are increased with an increase in the stretching rate of the sheet while the fluid temperature decreases, i.e., the stretching rate of the sheet has tendency to reduce the fluid temperature and enhanced the fluid velocity and species concentration. In increase in the value of stretching parameter the fluid velocity is becoming increasingly grater than the free stream. The momentum and concentration boundary layers get thicker while the thermal boundary layer gets thinner with an increase in the stretching parameter. It is revealed from from fig. 2 that with an increase in mixed convection parameter the fluid velocity and species concentration are increased whereas fluid temperature decreases. It is concluded that in case of cu-water nanofluid, the fluid velocity and species concentration are enhanced while fluid temperature is reduced with an increase in mixed convection parameter. Thus the influence of mixed convection is to increase the thickness of momentum and concentration boundary layers whereas to decrease the thickness of thermal boundary layer. Fig. 3 presents the variation in the profiles of fluid velocity, fluid temperature and species concentration for various values of thermal radiation. From equation (12), it is noted that the effect of radiation is inversely proportional to the thermal radiation parameter $N_R$. Thus, small value of $N_R$ signifies a large radiation effect whereas $N_R \rightarrow \infty$ signifies to the no radiation. It is found from fig. 3 that fluid velocity, fluid temperature and species concentration are decreases with an increase in the thermal radiation parameter. Fig. 4 shows the influence of temperature ratio parameter $\theta_w \left( \frac{T_w}{T_\infty} > 1 \right)$ on the fluid velocity, fluid temperature and species concentration. It is revealed from figure that there are increments on fluid velocity, fluid temperature and species concentration with an increase in temperature ratio parameter $\theta_w$. It can be concluded that fluid velocity, fluid temperature and species concentration are getting enhanced by temperature ratio parameter.

Table 2 shows the effect of magnetic field, nanoparticle volume fraction, stretching rate of sheet, mixed convection, thermal radiation, temperature ratio on the local coefficient of skin friction, reduced local Nusselt number and Sherwood number taking $Pr = 6.7850$ and $Sc = 1$. It is evident from Table 2 that the local coefficient of skin friction increases with an increase in the magnetic field, nanoparticle volume fraction, stretching rate and thermal radiation while its opposite affects are seen by mixed convection and temperature ratio. On the other hand the rate of heat transfer getting enhanced with an increase in nanoparticle volume fraction, mixed convection, thermal radiation, temperature ratio and stretching rate whereas it has a reverse effect on magnetic field. It is also observed from the table that Sherwood number decreases with an increase in $M$, $N_R$ and $K$ where as it increases with increase in $\phi$, $\epsilon$, $\lambda$, $\theta_w$ and $K_S$. Table 3 shows the comparison of our work with Bachok et al.[24] in the absence of $M$, $\lambda$ and very large $N_R$. It is observed that the accuracy of the method used for our model is quite excellent in agreement with the previous data available in the literature.

5. Conclusion

The effect of nonlinear thermal radiation and homogeneous-heterogeneous chemical reactions on the steady two dimensional mixed convection stagnation point boundary layer flow of an incompressible, viscous, electrically conducting copper-water nanofluid towards a linear stretching sheet are investigated numerically in this model. The important findings are:

- The fluid velocity, fluid temperature and species concentration are decreases with an increase in the thermal radiation parameter.
- In increase in the value of stretching parameter the fluid velocity is becoming increasingly grater than the free stream velocity.
- The fluid velocity, fluid temperature and species concentration are getting enhanced by temperature ratio parameter.
• The rate of heat transfer getting enhanced with an increase in nanoparticle volume fraction, mixed convection, thermal radiation, temperature ratio and stretching rate.

Table 1: Thermo-physical properties of water and nanoparticles

<table>
<thead>
<tr>
<th></th>
<th>ρ (kg/m³)</th>
<th>C_p (J/kgK)</th>
<th>k (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>8933</td>
<td>385</td>
<td>401</td>
</tr>
</tbody>
</table>

Table 2: The effect of various parameters on coefficient of local skin-friction, reduced local Nusselt number and reduced local Sherwood number when Pr = 6.7850 and Sc = 1.

Table 3: Comparison of value of $C_p Re_x^{1/2}$ for different values of $\epsilon$ and $\phi$ when $M = \lambda = 0$ and $NR$ is large.
Fig. 2: The effect of the mixed convection parameter $M$ on (a) the fluid velocity $f'$, (b) the fluid temperature $\theta$ and (c) species concentration $g$ when $M = 2$, $\phi = 0.2$, $\epsilon = 2$, $N_R = 10$, $\theta_w = 2$, $K_s = 0.5$, $K = 0.5$, $Pr = 6.7850$ and $S_c = 1$.

Fig. 3: The effect of the thermal radiation parameter $N_R$ on (a) the fluid velocity $f'$, (b) the fluid temperature $\theta$ and (c) species concentration $g$ when $M = 2$, $\phi = 0.2$, $\epsilon = 2$, $\lambda = 1$, $\theta_w = 2$, $K_s = 0.5$, $K = 0.5$, $Pr = 6.7850$ and $S_c = 1$.

Fig. 4: The effect of the temperature ratio parameter $\theta_w$ on (a) the fluid velocity $f'$, (b) the fluid temperature $\theta$ and (c) species concentration $g$ when $M = 2$, $\phi = 0.2$, $\epsilon = 2$, $\lambda = 1$, $N_R = 10$, $K_s = 0.5$, $K = 0.5$, $Pr = 6.7850$ and $S_c = 1$.

References