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Improving Conceptual and Procedural Knowledge: The Impact of Instructional Content Within A Mathematics Lesson

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Abstract

Background: Students, parents, teachers and theorists often advocate for direct instruction on both concepts and procedures, but some theorists suggest that including instruction on procedures in combination with concepts may limit learning opportunities and student understanding.

Aims: The current study evaluated the effect of instruction on a math concept and procedure within the same lesson relative to a comparable amount of instruction on the concept alone. Direct instruction was provided before or after solving problems to evaluate whether the type of instruction interacted with the timing of instruction within a lesson.

Sample: We worked with 180 second-grade children.

Methods: In a randomized experiment, children received a classroom lesson on mathematical equivalence in one of four conditions that varied in instruction type (conceptual or combined conceptual-and-procedural) and in instruction order (instruction before or after solving problems).

Results: Children who received two iterations of conceptual instruction had better retention of conceptual and procedural knowledge than children who received both conceptual and procedural instruction in the same lesson. Order of instruction did not impact outcomes.

Conclusions: Findings suggest that within a single lesson, spending more time on conceptual instruction may be more beneficial than time spent teaching a procedure when the goal is to promote more robust understanding of target concepts and procedures.

KEYWORDS: Conceptual knowledge; procedural knowledge; direct instruction; problem exploration; mathematics instruction

The Content of Instruction Within A Lesson Influences Conceptual and Procedural Knowledge in Mathematics

Proficiency in mathematics requires an understanding of central concepts and the ability to adapt solution procedures to solve novel problems (Blöte, Van der Burg, & Klein, 2001; Hiebert & Grouws, 2007). One factor that may influence such understanding is the type of mathematics instruction. Many researchers advocate for direct instruction on both concepts and procedures (Kirschner, Sweller, & Clark, 2006), but others argue that instruction should, at times, focus on concepts alone (Hiebert et al., 1996; von Glasersfeld, 1995).

The current study evaluates the effect of instruction on concepts and procedures within the same lesson relative to a comparable amount of instruction on concepts alone. The lesson targeted knowledge of mathematical equivalence (i.e., the values on both sides of the equal sign are the same amount). Understanding mathematical equivalence requires conceptual knowledge of the meaning of the equal sign as well as procedural knowledge for solving problems with operations on both sides of the equal sign. Unfortunately, elementary-school children often do not understand mathematical equivalence in symbolic form (Carpenter, Franke, & Levi, 2003; McNeil, 2008).

Prior Research on Conceptual and Procedural Instruction

Conceptual knowledge is one's mental representation of the principles that govern a domain and *procedural knowledge* is the ability to execute action sequences to solve problems (Baroody, Feil, & Johnson, 2007; Rittle-Johnson & Schneider, 2015). Similarly, *conceptual instruction* focuses on domain principles and *procedural instruction* focuses on step-by-step procedures. There is now broad agreement that both types of knowledge are important and that the relations between conceptual and procedural knowledge are usually bidirectional, with increases in one type of knowledge leading to subsequent increases in the other (see Rittle-Johnson, Schneider, & Star, 2015).

Empirical evidence and best practice guidelines emphasize the importance of instruction that supports both conceptual and procedural knowledge in mathematics, dating back many years (Brownell, 1947). Across a variety of teaching experiments, an emphasis on conceptual instruction with more limited procedural instruction was associated with greater conceptual knowledge and comparable procedural knowledge compared to typical classroom instruction that focused on procedural instruction (Blöte et al., 2001; Cobb et al., 1991; Fuson & Briars, 1990; Hiebert & Wearne, 1996; Kamii & Dominick, 1997). For example, over the course of three years, teaching assistants provided alternative instruction on place value and multi-digit arithmetic, focusing on the base-ten system and inventing procedures before instruction on standard procedures (Hiebert & Wearne, 1996). Students in these classrooms gained greater conceptual and procedural knowledge compared to students who received instruction from their regular classroom teacher that focused on standard procedures, with limited attention to underlying concepts. These studies illustrate the potential advantages of emphasizing conceptual instruction in an ecologically valid setting. However, it is difficult to draw conclusions about the isolated effects of conceptual and procedural instruction from these studies because the two instructional conditions varied on many dimensions.

Because of the many complexities of long-term classroom studies, randomized experimental studies on individual lessons provide important windows into the immediate impact of conceptual versus procedural instruction. Across several experimental studies, providing a conceptual lesson led to greater learning than a procedural lesson (Matthews & Rittle-Johnson, 2009; Perry, 1991; Rittle-Johnson & Alibali, 1999). For example, a conceptual lesson led many children to generate accurate solution procedures that they could appropriately adapt to solve transfer problems, whereas a procedural lesson was less effective in promoting procedural transfer (Perry, 1991). Focusing on concepts during instruction is thought to be particularly important for knowledge retention, because procedures learned in isolation are often difficult to recall after a delay (Baroody et al., 2007; Hiebert & LeFevre, 1986).

Prior research has not addressed *if* and *when* it is appropriate to provide procedural instruction *in addition to conceptual instruction* within the same lesson. Because instructional time is usually fixed, providing instruction on procedures often reduces the amount of time for instruction on concepts. Is this time well spent?

Indeed, many researchers advocate for instruction on both concepts and procedures (Cronbach & Snow, 1977; Klahr & Nigam, 2004; Mayer, 2004; Sweller, 2003). For example, Kirschner and colleagues (Kirschner et al., 2006) concluded that “novice learners should be provided with direct instructional guidance on the concepts and procedures…and should not be left to discover those procedures by themselves.” Theorists base their claim on research that contrasts conceptual-and-procedural instruction within a lesson with no direct instruction. Teachers and parents also advocate for instruction on procedures as well as concepts (Hiebert et al., 2003; Roelofs, Visser, & Terwel, 2003). However, little prior research has directly tested the impact of combining the two types of instruction within a lesson relative to conceptual instruction alone.

In contrast, some researchers have cautioned against the inclusion of procedural instruction with conceptual instruction. Memorization by rote, rather than with understanding, is a common concern surrounding procedural instruction (Brown & Burton, 1978). For example, Kamii and Dominick (1998) illustrated how procedural instruction was associated with rote implementation of algorithms that led to nonsensical answers. A second concern is that procedural instruction limits opportunities to explore problems and invent procedures (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Hiebert et al., 1996; Hiebert & Grouws, 2007). Limited experimental evidence suggests that including procedural instruction with conceptual instruction within the same lesson may be less effective than conceptual instruction alone. Perry (1991) found that providing conceptual and procedural instruction in a lesson prior to problem solving was less effective than providing only conceptual instruction in the lesson. In fact, the combination was no more effective than procedural instruction alone. She suggested that the procedural instruction distracted children from working to understand the conceptual instruction.

In most of this experimental research, instruction was provided prior to opportunities for children to solve related problems. Recent research has revealed that opportunities for problem exploration prior to direct instruction can increase the effectiveness of instruction (Kapur, 2010, 2011; Schwartz, Chase, Chin, & Oppezzo, 2011). In these studies, the instruction typically includes both concepts and procedures. For example, in Schwartz and colleagues (2011), middle-school students who received direct instruction on density concepts and procedures prior to problem solving (instruct-practice) focused on mapping the formula to subsequent problems. In contrast, students who solved the problems prior to receiving instruction (solve-instruct) better noticed the underlying structure of the problems and invented a variety of solution procedures. In turn, students in the solve-instruct condition were better able to transfer their knowledge to new problems after a three-week delay. Thus, when the lesson is provided after a problem-exploration phase, combining procedural instruction with conceptual instruction may help ensure that all students are exposed to correct procedures that apply to a wide range of problems. Prior research has not directly evaluated this possibility.

Current Study

We experimentally evaluated the effect of *instruction type* - direct instruction on concepts and procedures within the same lesson relative to a comparable amount of instruction on concepts alone - when instruction was provided in two different *instructional orders*: before problem solving (instruct-practice) or after problem solving (solve-instruct). We conducted this research with second graders learning about mathematical equivalence within their classrooms during small group work. We measured their conceptual and procedural knowledge, including on a retention test. We predicted an instruction type by instruction order interaction. In particular, we predicted that two iterations of conceptual instruction within a lesson would support greater conceptual and procedural knowledge than combined instruction when instruction was provided prior to problem solving (as in Perry, 1991). However, we predicted that combined instruction would support similar or greater knowledge when instruction was provided after problem solving.

Method

Participants

All second-grade children from 13 classrooms in three public schools serving predominantly middle- to upper-middle class populations in the United States were eligible to participate. Data from 21 children were excluded due to a missing posttest or retention test (n = 4), diagnosed disability (n = 15), or requiring off-script help during the intervention (n = 2). The final sample included 180 children (M age = 7.6 years, range = 6.8 – 8.9; 55% female; 81% White, 12% Black, 3% Asian, 3% Multiracial, 1% Hispanic).

Materials and Coding

Assessment. The assessment was adapted from a previously validated assessment and contained procedural and conceptual knowledge scales (Matthews, Rittle-Johnson, McEldoon, &

Taylor, 2012; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). See Table 1 for items and scoring criteria. Four conceptual knowledge items required an explanation or reconstruction of an equation. Two raters independently coded 20% of these responses, and agreement was very high (kappas = 0.95 - 0.96). A brief version of the assessment was created for use at pretest and midtest.

Intervention Problem-Solving Materials. A problem-solving workbook contained 17 problems to solve. The first 16 problems were presented in sets of four problems with similar addends, presented on the same page. The first three problem sets began with one or two easier problems meant to activate prior knowledge (e.g., 10 = 3 + ☐). The other 13 problems were four- and five-addend math equivalence problems with operations on both sides of the equal sign and the unknown immediately after the equal sign or at the end of the problem. On the final page, there was a math equivalence problem to solve as well as a prompt to define the equal sign. A few additional hints (e.g., think about what the equal sign means) were provided in the solve-instruct conditions to guide attention to important problem features and promote invention of procedures. All instructional materials are available in the online supplemental materials.

Intervention Instruction Materials. Conceptual instruction focused on the relational meaning of the equal sign in the context of non-standard equations. For each equation (e.g., 4 + 4 = 3 + 5), the experimenter identified the two sides of the equation, defined the equal sign as meaning the same amount as, and explained how the two sides of the equation were equal, gesturing to support noticing of important information and to ground verbal statements (Alibali & Nathan, 2012). For example, for the first equation, 3 + 4 = 3 + 4, part of the instruction was:

> There are two sides to this problem, one on the first side of the equal sign [sweeping gesture under left side] and one on the second side of the equal sign [sweeping gesture under right side]… The *equal sign* [pointing] means is that the things on both sides of the equal sign are equal or the same [sweeping hand back and forth].

On later items, children were prompted to identify the two sides. A single math equivalence problem was presented at the end of the instruction and the two sides of the equation and the meaning of the equal sign were reviewed. No procedure was discussed.

Procedural instruction focused on an add-subtract procedure, as in prior work in the area (Matthews & Rittle-Johnson, 2009; Perry, 1991; Rittle-Johnson & Alibali, 1999). Prior research suggests that instruction on this procedure is as effective as instruction on alternative procedures for promoting problem solving (Alibali, Phillips, & Fischer, 2009). On four math equivalence problems, instruction built on identifying and working with the two sides of the problem, with careful use of gestures. For example, for the problem 6 + 4 + 5 = 6 + ☐, the instruction included:

> "First, you combine the numbers on the one side of the equal sign. Then, you subtract the amount on the other side from what you got on the first side. So for this problem, first, you add up all the numbers on this side of the equal sign [sweeping gesture under left side]. What is 6 + 4 + 5? Right, 15. Then, subtract 6 on the second side [pointing]. What is 15 minus 6? So 9 is the number that goes in the box."

On later items, children were prompted to recall steps in the procedure. Children were told that there were multiple ways to correctly solve the problems, but they were only going to receive instruction on one way.

Design and Procedure

The experiment had a pretest–intervention–posttest design with a three-and-a-half week retention test (see Figure 1 for a diagram of the study design). All sessions occurred in children's classrooms during their mathematics class period and the intervention occurred during one 60-minute class period. Within each classroom, children were randomly assigned to one of four conditions based on crossing two factors: instruction type (conceptual or combined) and instruction order (instruct-practice or solve-instruct). Although the problem-solving materials were the same in the two instructional orders, we labeled the phase "practice" when it followed instruction as children could apply what they learned, and we labeled the phase "solve" when it preceded instruction because children had to generate their own procedures. Research assistants worked with small groups of 3-5 children assigned to the same condition and followed a script for that condition. This design allowed us to maintain the rigor of a true experimental design within a more typical, ecologically-valid learning environment. The students were in their classrooms, with their peers, and the session occurred during their normal math class. However, we were able to randomly assigned students in the same classroom to different conditions. All research assistants led all conditions, counterbalanced across classrooms, and completed a fidelity checklist after each session that contained a list of key aspects for the implemented condition (e.g., provided a relational definition of the equal sign). Adherence to critical features of instruction was close to 100%.

During the scripted instruction phase, children either received two iterations of conceptual instruction (conceptual conditions) or conceptual and procedural instruction (combined conditions) in the context of 9 equations. We equated length of total instruction, rather than length of conceptual instruction, because of the natural trade-off between time spent on procedural instructional and time spent on conceptual instruction in a classroom setting.

In the *conceptual instruction* conditions, children received two iterations of the conceptual instruction. In the *combined instruction* conditions, children received one iteration of conceptual instruction followed by one iteration of procedural instruction. In the combined condition, we chose to provide conceptual instruction before procedural instruction due to concerns that procedural instruction may distract children from working to understand the conceptual instruction and because this is the recommended order within mathematics education (NCTM, 2014). At the end of instruction, children in all conditions solved two math equivalence problems as a manipulation check. During the solve/practice phase, children completed the workbook individually at their own pace.

After completing the instruction and solve phases, all children were asked to check their answers from the solve phase using a purple pen, writing a new answer above their old answer if desired. For children in the instruct-practice conditions, the checking immediately followed the solve phase. For children in the solve-instruct conditions, the checking followed the instruction so children could use information from the instruction to check their previous performance in the solve phase.

Data Analysis

Two children were absent on the day of the pretest (one from the conceptual-instruction-then-practice condition and one from the solve-then-combined-instruction condition). Imputing missing independent variables leads to more precise and unbiased conclusions than omitting participants with missing data (Peugh & Enders, 2004). We used the expectation-maximization algorithm for maximum likelihood estimation via the missing value analysis in SPSS (Schafer & Graham, 2002) to impute the two missing pretest scores.

We worked with children in 52 small groups of 3-5 children each. To examine the amount of variability due to small group, we calculated intraclass correlations on the outcome measures, using the approach that allows for negative non-independence (Kenny, Kashy, Mannetti, Pierro, & Livi, 2002). As expected due to the scripted nature of the intervention, intraclass correlations were low, ranging from -0.027 to 0.022. Intraclass correlations were also low for classrooms, ranging from -0.026 to -0.001. This indicates that the data did not violate the assumption of independence needed for ANCOVA, so we report ANCOVA models.

Results

See Table 2 for raw scores and standard deviations by condition on each measure. At pretest, there were no significant differences between children in the four conditions in terms of prior knowledge, age, gender, or ethnicity, *p*s > .20.

Posttest and Retention Test

As shown in Figure 2, there was no interaction between instruction type and instruction order, contrary to expectations. Rather, the conceptual instruction condition usually outscored the combined condition, regardless of instructional order. We ran a multivariate ANCOVA on the four primary outcome measures: percent correct on procedural and conceptual knowledge scales at posttest and retention test. We included instruction type (conceptual or combined), instruction order (instruct-practice or solve-instruct) and their interaction as between-subject factors. We also included pretest scores and age as covariates. Preliminary analyses indicated that pretest scores and age did not reliably interact with condition, so these terms were not included in the model. There was a main effect of instruction type, $F(4, 171) = 2.73$, $p = .03$, with a medium effect size, $\eta_p^2 = .06$. There was no effect of order nor an interaction between the two, *F*s < 0.40, *p*s > .80 $\eta_p^2 = .01$. Pretest score was a powerful predictor of outcomes, $F(1, 171) = 19.86$, $p < .001$, $\eta_p^2 = .32$.

To examine the consistency of results across outcomes, we conducted separate ANCOVAs for each of the four dependent measures, focusing on the effect of instruction type. At posttest, there was no reliable effect of instruction type for conceptual knowledge, $F(1, 174) = 1.58$, $p = .21$, $\eta_p^2 = .01$, Hedges' $g = .18$, or for procedural knowledge, $F(1, 174) = 0.01$, $p = .91$, $\eta_p^2 = .00$, Hedges' $g = -.03$. At retention test, there was a significant effect of instruction type for both conceptual knowledge, $F(1, 174) = 6.24$, $p = .01$, $\eta_p^2 = .04$, Hedges' $g = .32$, and for procedural knowledge $F(1, 174) = 4.22$, $p = .04$, $\eta_p^2 = .02$, Hedges' $g = .26$. This reflected the fact that both procedural and conceptual knowledge significantly improved from posttest to retention test in the conceptual instruction condition, $F(1, 174) = 4.24$ and $F(1, 174) = 4.28$, $p = .04$, $\eta_p^2 = .02$, suggesting students were generalizing their knowledge over time. In contrast, knowledge tended to stay the same from posttest to retention test in the combined instruction condition, with no significant time differences for procedural or conceptual knowledge, ps > .05.

Intervention Activities

To explore whether the condition manipulations impacted performance during the intervention, we performed secondary analyses on intervention measures (see Table 2). As a manipulation check, all children solved two math equivalence problems after receiving instruction. All but one child (99%) in the combined instruction conditions solved at least one manipulation check problem correctly, compared to 81% of children in the conceptual conditions, $\chi^2 (1, N = 180) = 16.20$, $p < .001$.

During the solve/practice-phase, children solved problems, checked their answers and provided a definition of the equal sign. The type of instruction did not impact any of these measures, but the order of instruction did. Problem-solving accuracy, before answer checking, did not differ by instruction type, nor did instruction type interact with order, *Fs* < 1.50, *p*s > .23. There was an effect of order, such that children in the instruct-practice condition solved more problems correctly than children in the solve-instruct condition, $F(1, 174) = 28.76$, $p < .001$, $\eta_p^2 = .14$. Checking behavior (i.e., changing any answer or changing incorrect answers to correct answers) did not differ as a function of instruction type, nor did instruction type interact with order, *F*s < 0.84, *p*s > .36. Checking behavior did vary as a function of order. Children in the solve-instruct conditions made significantly more changes than children in the instruct-practice conditions, $F(1, 174) = 26.51$, $p < .001$, $\eta_p^2 = .13$, and changed an incorrect answer into a correct answer on more problems than children in the instruct-practice conditions, $F(1, 174) = 28.73$, $p < .001$, $\eta_p^2 = .14$. Finally, children in the instruct-practice conditions (75%) were much more likely to provide a relational definition of the equal sign than children in the solve-instruct conditions (who had not received instruction yet; 18%), χ^2 (1, $N = 180$) = 58.22, $p < .000$. Among children in the instruct-practice conditions, the number of children who provided a relational definition of the equal sign was similar in the conceptual (80%) and combined instruction (69%) conditions, χ^2 (1, $N = 91$) = 1.61, $p = .21$.

Finally, children completed a brief midtest between the solve and instruct phases, which assessed their knowledge of the structure of equations. There were no main effects of order or instruction type, nor was there an order by instruction type interaction, *F*s < 2.7, *p*s > .10.

Overall, as expected, order of instruction influenced problem-solving accuracy and checking behavior. However, instruction type did not influence problem-solving accuracy, checking behavior, providing a relational definition of the equal sign or midtest scores during the intervention.

Discussion

Reiterating direct conceptual instruction within a lesson – rather than spending a comparable amount of time providing procedural instruction after the conceptual instruction – has the potential to promote a more robust understanding of mathematical equivalence concepts and procedures. Children who received two iterations of conceptual instruction in a lesson gained more conceptual and procedural knowledge of math equivalence than children who received conceptual and procedural instruction. This finding was consistent across both instructional orders, suggesting that providing instruction after problem solving does not change the impact of these two types of instruction. There were no indicators from intervention performance that foreshadowed these effects. Even at posttest, differences due to instruction type were not reliable. Rather, it was retention of knowledge over a 3-week delay that revealed the advantages of two iterations of conceptual instruction in the current context. Focusing on conceptual knowledge is thought to be particularly important for knowledge retention (Baroody et al., 2007; Hiebert & LeFevre, 1986).

These findings contribute to a substantial body of research supporting the benefits of conceptual instruction for children's mathematics knowledge (Blöte et al., 2001; Cobb et al., 1991; Fuson & Briars, 1990; Hiebert & Grouws, 2007; Hiebert & Wearne, 1996; Kamii & Dominick, 1997; Matthews & Rittle-Johnson, 2009). Importantly, they suggest that these benefits may, under some circumstances, be more robust when direct conceptual instruction is provided in isolation rather than combined with procedural instruction within the same lesson. Students in the conceptual instruction condition, but not the combined instruction condition, improved from posttest to retention test, suggesting students were maintaining and generalizing their knowledge over time in this condition. Regular classroom instruction includes many instances of the equal sign (Powell, 2012), providing opportunities for students to generalize their knowledge. It is easier to remember things over time that make sense and that are integrated with and generalized to other knowledge (Anderson & Lebiere, 1998; Chi, 1978).

This stands in contrast to preferences by some educators and theorists for fully-guided lessons that include direct procedural instruction in conjunction with conceptual instruction (Hiebert et al., 2003; Kirschner et al., 2006; Klahr & Nigam, 2004; Mayer, 2004; Roelofs et al., 2003; Sweller, 2003). Theorists base their claim on research that contrasted combined conceptual and procedural instruction within a lesson with no direct instruction or procedural instruction alone. However, the benefits of combined instruction in these cases may be attributable to the lack of conceptual instruction in the control conditions. Little prior research has directly evaluated the impact of combining the two types of instruction within the same lesson relative to spending an equivalent amount of time on only conceptual instruction.

The current study highlights the need for future research to evaluate when and why spending an equivalent amount of time on conceptual instruction alone may be more effective than spending the time on combined conceptual and procedural instruction within the same lesson. One possibility is that students needed more time spent on conceptual instruction than occurred in our combined condition. If we had not controlled for instructional time, and included two iterations of conceptual instruction along with procedural instruction, the combined condition may have resulted in similar or even higher retention of knowledge. However, two pieces of evidence suggest this was not the case. First, children in the conceptual condition were not reliably better at defining the equal sign at the end of the solve phase than children in the combined condition, suggesting the repeated conceptual instruction was not exerting a powerful effect. Second, Perry (1991) found similar effects when the amount of conceptual instruction was the same across the conceptual-only and combined condition. Nevertheless, learning takes time, and re-iterating conceptual instruction is likely important, even within the same lesson.

A second possibility is that the order of instruction in the combined condition mattered. However, Perry (1991) found that a conceptual lesson alone was more beneficial for promoting transfer compared to a conceptual-then-procedural lesson and procedural-then-conceptual lesson. Thus, preliminary evidence suggests that the benefits of a lesson focused exclusively on conceptual instruction promotes better learning about mathematical equivalence whether the amount of conceptual instruction or the amount of total instruction is held constant and whether conceptual instruction is provided first or second when instruction is combined.

Together, these studies lead to the hypothesis that exposure to procedural instruction within a lesson can reduce the benefits of the conceptual instruction. For example, procedural instruction may guide children's attention and efforts towards practicing a taught procedure at the expense of reflecting on the instructed concept. Indeed, Perry (1991) found that combined instruction was no better than procedural instruction alone. Nevertheless, we predicted that the advantages of conceptual instruction alone would not generalize to a solve-instruct order. Because procedural instruction is thought to limit opportunities to explore problems and invent procedures (Carpenter et al., 1998; Hiebert & Grouws, 2007), providing a brief opportunity to explore problems and invent procedures prior to instruction might have neutralized such an effect. The fact that it did not suggests that order of instruction within a lesson may not be a constraint on when conceptual instruction alone is more effective. More broadly, evidence is mounting that instruction guides attention towards certain information and constrains children's thinking process (Bonawitz et al., 2011; Csibra & Gergely, 2009). It is critical to consider how procedural instruction may constrain children's thinking about other lesson content.

Future work is needed to test the generalizability of these results. For example, the current findings may generalize to other topics in which children invent correct procedures, such as single- or multi-digit addition, word problems and simple equations, but it seems unlikely that they would generalize to topics in which correct procedures are rarely invented (e.g., fraction division). The same may be true for children with mathematical learning difficulties if they struggle to invent correct procedures (Baker, Gersten, & Lee, 2002). In addition, the current findings likely would not generalize to settings in which the goal of a lesson is to promote computational fluency. Further, the findings may not generalize to less directive instructional contexts or more ecologically valid, teacher led lessons.

Importantly, the current findings do not indicate that procedures should never be taught. The current study focused only on the instructional content of a single lesson. Further, a variety of factors could increase the benefits of including procedural instruction in the same lesson as conceptual instruction. First, procedural instruction can be designed to encourage noticing of underlying concepts (Peled & Segalis, 2005). Second, explicit attention to why procedures work and integration of conceptual and procedural instruction are characteristics of exemplary mathematics teaching (Hiebert & Grouws, 2007). In the current study, procedural instruction built on the language and ideas introduced in the conceptual instruction, but it did not explicitly integrate the two types of instruction or fully explain why the procedure worked. Such integration should enhance the benefits of including procedural and conceptual instruction in the same lesson.

Although our results indicate that instruction type can affect children's knowledge of concepts and procedures, it is certainly not the only factor. For example, providing opportunities for problem exploration prior to direct instruction has also been shown to improve conceptual knowledge and/or procedural transfer (DeCaro & Rittle-Johnson, 2012; Kapur, 2010, 2011; Schwartz et al., 2011). We did not replicate this finding in the current study. The similarity in outcomes across the two instructional orders and the small effect sizes suggest this lack of condition difference was not due to low power. A majority of past research on a solve-instruct approach has been with adolescents, involved complex problems solved in pairs or small groups and occurred across multiple class periods (Kapur, 2010, 2011; Schwartz et al., 2011), features not present in the current study. In addition, research suggests that prior problem exploration is beneficial, in part, because it enhances students' ability to notice and encode key problem features (DeCaro & Rittle-Johnson, 2012; Schwartz et al., 2011), but such a benefit did not occur in the current study. The inclusion of additional features during the initial solve phase, such as highlighting the equal sign or including comparison prompts, may have enhanced attention to structure more effectively in the current study. Finally, the solve-instruct approach could be more effective for students with particular characteristics, such as mastery-goal orientation (DeCaro, DeCaro, & Rittle-Johnson, 2015). Other prior research has investigated the solve-instruct approach with elementary-school children in one-on-one tutoring contexts and has yielded mixed results (DeCaro & Rittle-Johnson, 2012; Fyfe, DeCaro, & Rittle-Johnson, 2014; Loehr, Fyfe, & Rittle-Johnson, 2014). Thus, future research is needed to evaluate the potential of a solve-instruct approach for supporting knowledge in elementary-school classrooms, including identifying critical features of such an approach.

Conclusion

The content of instruction within a lesson matters. Providing a lesson focused solely on key concepts facilitated children's knowledge of mathematical equivalence. Children who received two iterations of conceptual instruction had better retention of their conceptual and procedural knowledge than children who received both conceptual and procedural instruction. This was true whether the instruction was provided before or after problem solving. Thus, when the goal of a lesson is to promote understanding and retention of knowledge, a focus on concepts alone may be well worth the time.

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Table 1.

Procedural and Conceptual Knowledge Items on the Mathematical Equivalence Assessment

Item Type	Items	Scoring Criteria
Procedural		(n = 6; α = .83, .85)[a]
Familiar problems	8 = 6 + □ 3 + 4 = □ + 5 7 + 6 + 4 = 7 + □	Answer must be within 1 of correct answer
Transfer problems	□ + 2 = 6 + 4 8 + 5 – 3 = 8 + □ 7 – 2 + 3 = □ + 3	Same as above
Conceptual		(n = 7; α = .71, 78)[a]
Meaning of equal sign	Define equal sign	1 point for providing relational definition (e.g., "the same as" or "the two sides are equal" rather than "the total" or "equals")
	Which definition is the best definition of the equal sign?	1 point for choosing "two amounts are the same" as a best, over "add" and "the answer to the problem"
Structure of equations	Reproduce 4 + 3 + 9 = 4 + □ from memory [b]	1 point each for correctly reconstructing numerals, operators, equal sign and blank in correct location
	Reproduce 8 + 6 + 3 = □ + 2 from memory	Same as above
	Judge 3 = 3 and 7 = 3 + 4 as true or false	1 point for judging both equations as true
	Judge 31 + 16 = 16 + 31 and 7 + 6 = 6 + 6 + 1 as true or false	Same as above
	Decide if 6 + 4 = 5 + 5 is true and explain how you know	1 pt for correctly recognizing equation as true or false *and* providing a relational explanation (e.g., both sides are the same amount)

Note. [a] Internal consistency at posttest and retention test, respectively. [b] Reproducing information from memory measures how well people encoded the information, which is heavily influenced by their prior knowledge (Chi, 1978; McNeil & Alibali, 2004). Each equation was presented for 5 seconds.

Table 2.

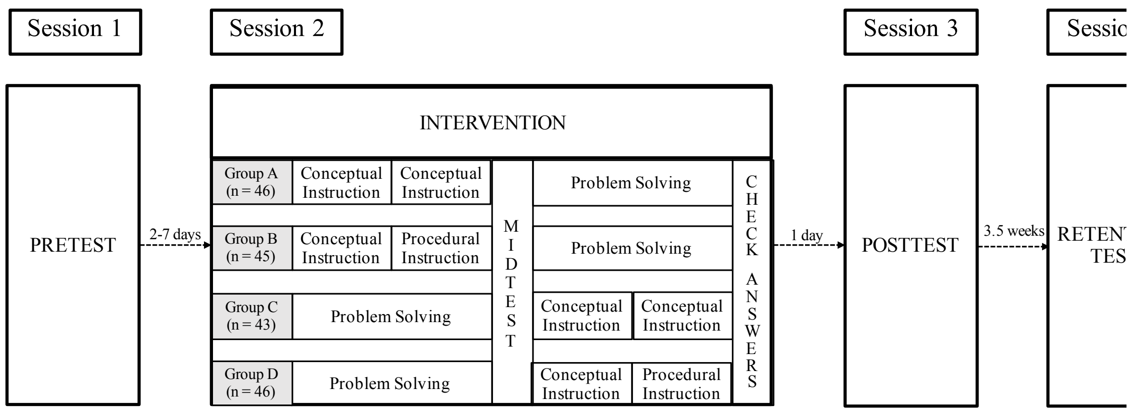
Performance on Pretest, Posttest, Retention Test and Intervention Measures by Condition

Measure	**Solve-Instruct Order**		**Instruct-Practice Order**	
	Conceptual Instruction	Combined Instruction	Conceptual Instruction	Combined Instruction
Pretest Knowledge: Percent correct	28 (23)	27 (27)	27 (28)	28 (26)
Posttest Procedural Knowledge: Percent correct	72 (34)	70 (28)	65 (36)	71 (32)
Posttest Conceptual Knowledge: Percent correct	64 (26)	58 (31)	63 (28)	61 (31)
[a]***Retention Test Procedural Knowledge:*** Percent correct	77 (35)	68 (32)	72 (36)	65 (33)
[a]***Retention Test Conceptual Knowledge:*** Percent correct	70 (26)	58 (34)	65 (28)	58 (35)
[b]***Instruction Problems:*** Percent solving at least one instruction problem correctly	77 (43)	100 (0)	85 (36)	98 (15)
[c]***Problem-Solving Accuracy:*** Percent correct on intervention problems before checking	59 (33)	61 (33)	77 (30)	86 (23)
[d]***Answer-Checking:*** Frequency of changing any answer (out of 17) during the check phase	3.7 (4.3)	4.2 (4.6)	1.5 (2.7)	1.0 (2.3)
[d]***Answer-Checking:*** Frequency of changing incorrect answers to a correct answer	2.4 (3.9)	3.0 (4.1)	0.3 (1.2)	0.4 (1.1)
[c]***Equal Sign Definition:*** Percent providing relational definition during intervention	21 (41)	15 (36)	80 (40)	69 (47)
Midtest: Percent correct	53 (34)	50 (31)	57 (31)	59 (35)

Note. Raw scores presented with standard deviations in parentheses. [a]Significant ($p < .05$) main effect of instruction type: Conceptual > Combined. [b]Significant ($p < .05$) main effect of instruction type: Combined > Conceptual. [c]Significant ($p < .05$) main effect of instruction order: Instruct-Practice > Solve-Instruct. [d]Significant ($p < .05$) main effect of instruction order: Solve-Instruct > Instruct-Practice.

Figure 1.

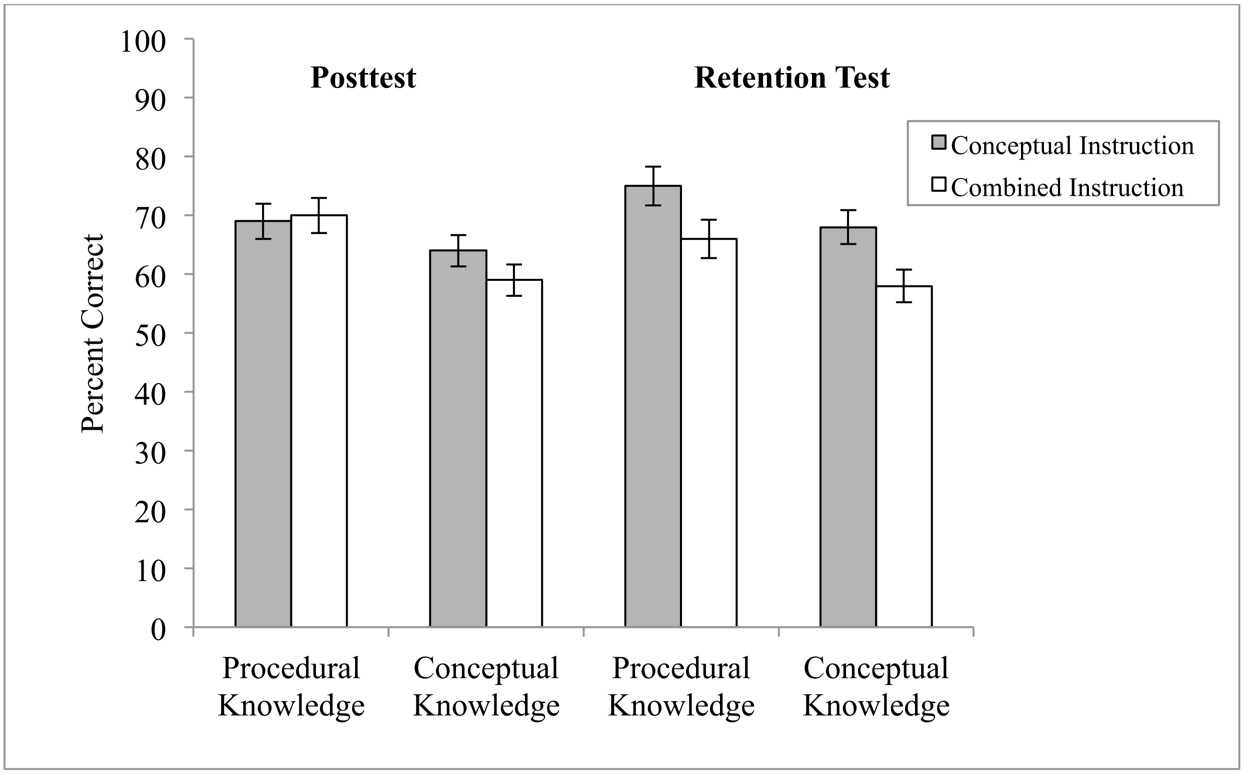
Diagram of Study Design Including Condition Differences.



Note. Group A = conceptual-instruction-then-practice condition. Group B = Combined-instruction-then-practice condition. Group C = Solve-then-conceptual-instruction conditio Group D = Solve-then-combined-instruction condition. For the retention test, the average was 26 days, *SD* = 1.5, *min* = 24, *max* = 34.

Figure 2

Effect of Instruction Type at Posttest and Retention Test



Note. Scores are estimated marginal means. Error bars represent standard errors.

Online Supplemental Materials

Description of the Intervention Materials
(presented in the order received for the solve-then-combined-instruction condition)

Problems presented in the problem-solving workbook.

1. 3 + __ = 10
2. 10 = 3 + __
3. 3 + 7 = __ + 5
4. 3 + 7 = __ + 6
5. 3 + __ = 8
6. 8 = __ + 5
7. 3 + 5 = 4 + __
8. 3 + 5 = 6 + __
9. 11 = 4 + __
10. 4 + 7 = 3 + __
11. 6 + 2 + 3 = __ + 8
12. 4 + 2 + 5 = 5 + __
13. 9 + 6 = __ + 5
14. 5 + 5 + 5 = __ + 6
15. 5 + 6 + 4 = 4 + __
16. 8 + 2 + 5 = __ + 5
17. 6 + 4 + 3 = 3 + __
18. What does the equal sign mean?

Problems presented on the midtest.

The first two problems were memory items: After viewing the problem for five seconds, children were instructed to write the problem exactly as they saw it.

1. 5 + 4 + 8 = 5 + __
2. 7 + 5 + 2 = __ + 3

The next six problems were true/false items: Children were instructed to decide whether the number sentence was true and to circle true, false, or don't know.

1. 8 = 8
2. 7 + 6 = 0
3. 31 + 16 = 16 + 31
4. 7 + 6 = 6 + 6 + 1
5. 8 = 5 + 10
6. 8 = 5 + 3

Script for conceptual instruction.

First, we're going to think about what the equal sign means and look at a few examples.
Let's look at a problem like this: *(hold up prompt)*

3 + 4 = 3 + 4

There are two sides to this problem, *(sweep gesture under side)* one on the first side of the equal sign and *(sweep gesture under side)* one on the second side of the equal sign.

The first side is 3 + 4 *(sweep side).*
The second side is 3 + 4 *(sweep side).*

The equal sign *(point)* means that the things on both sides of the equal sign are equal or the same *(sweeping hand back and forth*). So the first side of the equal sign always has the same amount as the second side of the equal sign.

You can answer all together. What is 3 + 4? *(Point to the left side. Wait for responses.)*

The first side of the equal sign is equal to 7.

And what is 3 + 4 on the second side? *(Wait for student responses)*

The second side of the equal sign is equal to 7, too.

We have 7 on this side *(gesture around left)* and 7 on this side *(gesture around right).* Because we get the same amount on both sides, we can say that they are equal. If both sides are not the *same amount*, then they aren't equal.

Let's look at another example. Take a look at this: *(hold up prompt).*

4 + 4 = 3 + 5

What is on the first side of the problem? *(Remind students can say it all together if needed. If a student doesn't respond, be sure to ask them what they think.)*

Right, the first side is 4 + 4. *(sweep gesture)*

What is on the second side of the problem? *(Wait for students to respond)*

The second side of the equal sign is 3 + 5. *(sweep gesture)*

Remember now, the equal sign always says that both sides have to equal the same amount.

So if we have 4 + 4 on the first side, how much is on the first side? *(Wait for response.)*

The first side has 8.

So how much has to be on the second side? *(Wait for response)*

The second side has to be 8!

What is 3 + 5? *(Wait for response)*

Right, 8.

Both sides have 8 so there should be an equal sign here. If they don't have the same amount, then the two sides aren't equal, and there shouldn't be an equal sign here *(point).*

So if we look at a problem like this *(hold up prompt):*

7 = 3 + 4

The equal sign still means that both sides are worth the same amount. The equal sign always means that the first side has the same amount as the second side. And it means that here, too.

What is on the first side? *(Wait for response. Switch to calling on non-participating students.)*

Right, 7.

We have 3 + 4 on the second side. How much is on the second side? *(Wait for response.)*

The second side has 7.

So, both sides are 7, and the equal sign tells us both sides have the same amount.

Now let's look at something else. For example, if you saw something like this, would it make sense to write an equal sign here? *(Wait for response.)*

2 + 3 ○ 3 + 6

If they say no: Good / *If they say yes:* Hmm, what do other people think?

How much is on the first side? *(wait for student response)*

Correct, the first side has 5.

How much is on the second side? *(wait for student response)*

Correct, the second side has 9.

Are they the same amount? *(Wait for student response)*

No, they aren't.

The equal sign means that the first side is the same amount as the second side. Since these are not the same amount, then they are not *equal*, so it would not make sense to write an equal sign.

Let's look at one last problem *(hold up prompt).*

5 + 4 + 3 = 5 + ☐

What is on the first side of the problem? *(Wait for student response)*

The first side is 5 + 4 + 3. *(sweep gesture)*

Now, what is on the second side. *(Wait for student response)*

The second side is *5* + box *(sweep gesture)*

Again, the equal sign means that the first side *(gesture)* needs to be *the same amount as* the second side *(gesture).*

Script for procedural instruction.

Now, we're going to go through a short lesson about how to solve problems like these.

The problems will look something like this *(hold up prompt):*

4 + 2 = 3 + ☐

Just like this problem, all of the problems we'll work on now will have an empty box and we need to figure out what number goes in the box. There is more than one way to solve this type of problem, but I'm going to show you one way to solve them today.

First, you combine the numbers on one side of the equal sign. Next, you subtract the number that's on the other side of the equal sign.

On this problem, first, you add up all the numbers on this side of the equal sign *(sweep gesture).*

What is 4 + 2? *(wait for response)*

Right, 6. *(draw a vee below 4+2 connecting them and write 6 at the point of the vee)*

Then, subtract 3 on the second side of the equal sign *(point to 3).* What is 6 – 3?

Great, so 3 is the number that goes in the box. *(write 3 in box)*

Let's look at another example *(hold up prompt):*

6 + 4 + 5 = 6 + ☐

First, you combine the numbers on the one side of the equal sign. Then, you subtract the amount on the other side from what you got on the first side.

So for this problem, first, you add up all the numbers on this side of the equal sign *(sweep side).*

What is 6 + 4 + 5? *(Wait for response.)*

Right, 15. *(draw a vee below the three numbers and write 15 at the point of the vee)*

Then, subtract 6 on the second side *(point to 6)*. 15 minus 6 is what? *(Wait for response).*

Great, so 9 is the number that goes in the box. *(write 9 in box)*

Okay, let's try another example *(hold up prompt).* You can use the same strategy to solve problems like this.

5 + 4 = ☐ + 3

Can you tell me what to do first? *(Wait for response.)*

We add up the numbers on this side *(sweep side)*.

If we add up the numbers on this side, what do we get? *(Wait for response.)*

We get 9. *(draw vee under 5 + 4 and write 9 at the point of the vee)*
Now we need to figure out what to subtract from this. What do we need to subtract?
On the other side we have a 3 *(pointing)*. That means we need to subtract 3 from 9. What is 9 minus 3?
So we know 6 goes in the box. *(write 6 in box)*

Now I want you to think about everything we have talked about so far, and let's look at one more problem together, okay? *(hold up prompt)*
2 + 1 + 4 = ☐ + 4
What do we do first? *(Wait for student to point)*
We start on the first side with 2 + 1 + 4 *(sweep side).*
What do the numbers on the first side add up to? *(Wait for response.)*
Right, so we know 2 + 1 + 4 = 7. *(draw vee under numbers and write 7)*
What do we do next? *(Wait for response.)*
To finish, we subtract the amount on the second side from what we got on the first side.
What is 7 minus 4? *(Wait for response.)*
7 minus 4 equals 3, so we know 3 goes in the box. *(write 3 in box)*

Manipulation check problems.

After the instruction in all conditions, children solved two problems. This was to check that children who received procedural instruction correctly learned to use the instructed procedure.

1. 3 + 6 + 2 = 3 + __
2. 2 + 8 + 3 = __ + 3