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Modular localization and the holistic structure of causal quantum theory, a historical perspective

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Dedicated to the memory of Jürgen Ehlers (1929–2008)

ABSTRACT

Recent insights into the conceptual structure of localization in QFT (modular localization) led to clarifications of old unsolved problems. The oldest one is the Einstein–Jordan conundrum which led Jordan in 1925 to the discovery of quantum field theory. This comparison of fluctuations in subsystems of heat bath systems (Einstein) with those resulting from the restriction of the QFT vacuum state to an open subvolume (Jordan) leads to a perfect analogy; the globally pure vacuum state becomes upon local restriction a strongly impure KMS state. This phenomenon of localization-caused thermal behavior as well as the vacuum-polarization clouds at the causal boundary of the localization region places localization in QFT into a sharp contrast with quantum mechanics and justifies the attribute "holstic". In fact it positions the E–J Gedankenexperiment into the same conceptual category as the cosmological constant problem and the Unruh Gedankenexperiment. The holistic structure of QFT resulting from "modular localization" also leads to a revision of the conceptual origin of the crucial crossing property which entered particle theory at the time of the bootstrap S-matrix approach but suffered from incorrect use in the S-matrix settings of the dual model and string theory.

The new holistic point of view, which strengthens the autonomous aspect of QFT, also comes with new messages for gauge theory by exposing the clash between Hilbert space structure and localization and presenting alternative solutions based on the use of stringlocal fields in Hilbert space. Among other things this leads to a reformulation of the Englert–Higgs symmetry breaking mechanism.

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1. Preface

The subject of this paper grew out of many discussions about Jordan's discovery of quantum field theory (QFT) which I had with the late Jürgen Ehlers. They focussed in particular on events between the publication of Jordan's thesis on quantum aspects of statistical quantum mechanics in 1924 (Jordan, 1924a), and his discovery of QFT around 1925 which was published in one section of the famous 1926 "Dreimännerarbeit" (Born, Heisenberg, & Jordan, 1926) together with Born and Heisenberg. This paper was in fact the second paper after Heisenberg's discovery of quantum mechanics (QM). The resistance of Born and Heisenberg against Jordan's section has its natural explanation in that these two authors felt that Jordan was burdening the conceptual struggle to understand the new quantum mechanics with something which may distract from this project.

I met Jürgen Ehlers the first time around 1957 at the University of Hamburg when he was Jordan's assistant and played the leading role in Jordan's general relativity seminar. Our paths split, after I wrote my diploma thesis on a topic of particle theory at the time when particle physics moved away from the university physics institute to the newly constructed high energy laboratory at DESY. Contacts with Ehlers and the relativity group became less frequent and ended when both of us took up research associate positions at different universities in the US.

Only 40 years later, when Ehlers moved to Potsdam/Golm in the 1990s as the founding director of the new Albert Einstein Institute (AEI), we met a second time. After having done important research on problems of general relativity and astrophysics he became increasingly interested to understand some of Jordan's famous early work on quantum field theory about which we knew

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little at the time of Jordan's weekly relativity seminar.[2] Ehlers was in particular interested to understand some subtle points in a dispute between Jordan and Einstein concerning Einstein's use of statistical mechanics fluctuation arguments for black body radiation (Einstein, 1925). The ensuing dispute around this purely theoretical argument in favor of the existence of photons has been more recently referred to as the *Einstein–Jordan conundrum* (Duncan & Janssen, 2008).

As the terminology reveals, the E–J conundrum was a poorly understood relation between fluctuations caused by restricting the vacuum state to the observables in a subvolume in Jordan's newly discovered field quantization and Einstein's use of statistical mechanics within the old Bohr–Sommerfield quantum setting. This led him to identify a particle-like component in the fluctuation spectrum of a black body radiation ensemble (which he termed "Nadelstrahlung") with his 1905 interpretation of the photo-electric effect as a manifestation of the corpuscular nature of light.

The E–J conundrum has sometimes been seen as an illustration of the particle–wave dualism of quantum mechanics, but with the hindsight of modern QFT its real significance points into a much deeper level. This was certainly Ehler's view when he drew my attention to what he considered its real significance. Coming from general relativity and cosmology he thought that this problem is analogous (Ehlers, Hoffmann, & Renn, 2007) to the problems related to vacuum polarization used to explain the origin of the cosmological constant in terms of fluctuations of the quantum field theoretic vacuum. He hoped that with my experience of 40 years of QFT I could be of some help to obtain a better understanding.

I learned recently through John Stachel that conjectures about possible connections between thermal aspects of the subvolume fluctuations in QFT as they occur in the E–J conundrum and Hawking–Unruh problems already existed in the 1980s (Stachel, 1986). In fact it will become clear in the course of the present work that it indeed can and should be viewed this way.

For some time this problem remained out of my range of interest; I did not want to loose time on something which would draw me into opaque historical problems away from my research on new foundational insights into to QFT via "modular localization"[3] (Schroer, 1999). During a 2 year stay (2002/2003) in Brazil, a CNPq supported research project "The Modular Structure of Causal Quantum Physics" provided the chance to extend this research. Around 2007 I suddenly realized that the complete understanding of the E–J conundrum can be obtained with the help of precisely those newly gained insights. One just had to apply the *principle of modular localization*, which assigns a certain number of unexpected properties to localized subalgebras. Whereas the global vacuum state is pure, the restriction to a causally localized subalgebra renders it impure; in fact its impurity can be described as a thermodynamic KMS state (Haag, 1996) with respect to a "modular Hamiltonian". This is a general result of the application of the so-called Tomita–Takesaki modular theory of local operator algebras to the subalgebra which observables localized in a spacetime region (whose causal completion remains smaller than Minkowski spacetime) generate.

This reduced vacuum state is entangled in a more radical sense than the entanglement of particle states in Schrödinger's QM of particle states under a binary split of the system into spatial inside/outside subsystems. Entanglement in quantum mechanics resulting from binary inside/outside splits of degrees of freedom resulting from the reduction to the inside and the ensuing loss of the outside information is a well-known phenomenon; it has been observed in quantum optical experiments and the results led to a Nobel prize. But the quantum mechanical "vacuum" (the mathematical reference state which one needs for the "second quantization" multiparticle description of QM) remains completely inert against entanglement. In fact *the singular vacuum entanglement caused by localization in QFT is characteristic for the enormous conceptual distance between the two quantum theories*. The terminology E–J "conundrum" refers to the fact that for a long time this aspect of the vacuum remained outside theoretical comprehension.

In fact most theoretical physicists became for the first time aware of the KMS nature of the QFT restricted vacuum state in connection with the Unruh's "Gedankenexperiment" in which the localization region is a spacetime wedge. This aspect of vacuum entanglement also points at the "fleeting" nature of this effect; it remains many orders of magnitude below the measured quantum optical entanglement of quantum mechanical particle states. But even if it will always remain a "Gedanken" concept,[4] it is at the heart of QFT and follows directly from the *quantum adaptation of the* Faraday–Maxwell "action at the neighborhood" which Einstein converted into the Minkowski spacetime *causality principle*. Its quantum counterpart is of a radically different nature whose physical manifestations are somewhat unexpected. It will be referred to as *modular localization*; a terminology which relates its mathematical formulation with its physical implications. In the present work it will be shown that its conceptual range is not limited to shed light into dark corners of QFT's history as the before mentioned E–J conundrum, but it also plays an important role in an ongoing conceptual reformulation of QFT (which includes gauge theories and the recently much discussed "Higgs mechanism").

The two components in Einstein's statistical mechanics fluctuation properties are indeed, as Jordan claimed, also present in the physical vacuum state after restricting it to the ensemble of observables which are localized in a subvolume. It is important to not impose boundary restrictions (box quantization) but remain within the realm of "open systems". Here it is irrelevant whether Jordan's calculation treated this aspect correctly (Duncan & Janssen, 2008); many important observations in the history of quantum physics have been made within doubtful calculations.

When I was about to explain my findings (Schroer, 2011c, 2013, 2012) in 2008 to Ehlers, I learned that he passed away shortly before my return to Berlin.

The main aim of this paper, which I dedicate to the memory of Jürgen Ehlers, is to explain my findings and their relation to the ongoing research in QFT in more details and a larger context than previously in Schroer (2013).

I remember that Ehlers, in his capacity as the founding director of the AEI in Potsdam, took an interest in string theory (ST). He was however annoyed by the fact that he was unable to bridge the gaps between his understanding of spacetime properties of gravity and the (sometimes bizarre) claims of members of the ST group at the AEI; notwithstanding the fact of the enormous amount of mathematical sophistication and the professional reputation of some of the protagonists of ST.

The work on modular localization also led me to string-localized fields and their improved short distance property which promised a radical extension of renormalization theory to interaction between fields with higher spins. The reason why I mention

[2] After WWII Jordan's interest was mainly focussed on general relativity and philosophical implications of quantum theory. Since he never mentioned his early work on QFT, we remained quite ignorant about it.

[3] Here modular localization stands for an intrinsic formulation of causal localization which is independent on what quantum field "coordinatization" one uses in order to describe the particular model of QFT.

[4] The situation becomes less "fleeting" if the horizon of the localization region is an (Unruh observer-independent) black hole "event horizon".

this here is that this new concept of string-localized fields in Hilbert space also revealed that string theory (ST) and its derivatives (embeddings, dimensional reductions, the AdS–CFT isomorphism) has no relation to causal localization in spacetime; it is rather the result of a fundamental misunderstanding on these issues. Hence Ehlers' problems with the ancient Einstein–Jordan conundrum and his new problems with ST were interconnected in a curious way. His death in 2008 prevented me from conveying this insight.

It is the purpose of these notes to explain the constructive (Schroer, 2013) as well as critical (Schroer, 2014) power in a historical context.

Usually a historical paper revisits the past about already closed subjects; typical examples are research papers on the discovery and the conceptual development of QM. In contrast to such subjects, which are closed from a foundational physical point of view (but sometimes still lead to bitter philosophical feuds), the situation of the problems addressed in this paper is very different. Most of them, although some having been present in QFT from its historical beginnings, were only properly understood recently and have not yet been addressed by philosophers. In contrast to QM, QFT is still far from its conceptual closure not to mention its philosophical exploration. The present paper attempts to give an account of the present situation.

The Einstein–Jordan conundrum was often misunderstood as a confirmation of the particle–wave duality which, since de Broglie's matter–wave idea and Schrödinger's wave equation, was an integral part of QM. But the E–J dispute addresses a much deeper issue which, before the appearance of modular localization concept in QFT, had little chance to be properly understood.

My posthumous thanks for introducing me to a fascinating topic from the genesis of QFT which, far from being a closed part of history, exerts its conceptual spell over actual particle theory, naturally go to Jürgen Ehlers. The present exploration of the foundational principle of modular localization did not only change the view about hitherto incompletely understood problems at the dawn of QFT (Schroer, 2013), but also promises to have an important say about its future (Schroer, 2014).

2. Introduction

A dispute between Einstein and Jordan (referred to as the E–J conundrum, Duncan & Janssen, 2008) led Jordan to propose the first quantum field theoretical model with the purpose to show that there exists a quantum analog of Einstein's thermal subvolume fluctuations in open subvolumes (intervals) of two-dimensional quantized Maxwell waves in a global vacuum state. For this purpose Jordan invented the simplest QFT which in modern terminology is the model generated by a conformal chiral current. A brief sketch of the pre-history which led to the E–J conundrum may be helpful.

- Einstein 1917 in Einstein (1917): calculation of mean square fluctuations in an open subvolume in statistical mechanics of the thermal black body radiation shows presence of two components: wave- and particle-like (Nadelstrahlung) fluctuation structure which Einstein interpreted as a theoretical evidence for photons (after his 1905 paper based on the observational support coming from the photoelectric effect).
- Jordan in his PhD thesis (1924, Jordan, 1924b) argued that the particle-like component $\overline{E}_\nu \sim h\nu$ is not needed for attaining equilibrium.
- Einstein's reaction (Einstein, 1925) consisted in a publication in which Jordan's argument is conceded to be mathematically correct but physically flawed (the absorption is incorrectly described). However he praised Jordan's statistical innovations (Stosszahlansatz).
- Einstein's paper caused Jordan's radical change of mind; he fully accepted Einstein's view by demonstrating that he can obtain the same wave- and particle-like fluctuation components by restricting a "two-dimensional quantized Maxwell field" (modern terminology: $d=1+1$ chiral current model) to a subinterval. In this way he discovered field quantization, probably without understanding *why* a vacuum in QFT behaves radically different from a quantum mechanical "no particle state", in particular why the reduced vacuum shares the kind of impurity with that of a KMS statistical mechanics state.

Shortly after this episode Jordan published his first field quantization in a separate section in the famous 1926 "Dreimännerarbeit" (Born et al., 1926). Gaps in Jordan's computation and his somewhat artistic treatments of infinities caused some ruffling of feathers with his coauthors Born and Heisenberg (Duncan & Janssen, 2008). From a modern point of view the picture painted in some historical reviews, namely that this was a typical case of a young brainstorming innovator set against a scientific establishment (represented by Born), is not quite correct. Born and Heisenberg had valid reasons to consider Jordan's fluctuation calculations as incomplete, to put it mildly. Conceding this does however not lessen Jordan's merits as the protagonist of QFT .

One reason why this discovery of QFT was not fully embraced at the time was that, although a free field on its own (staying with its linear properties) is a rather simple mathematical object, the problem of energy fluctuations in open subvolumes is anything but simple. To understand why subvolume fluctuations in the vacuum state of QFT are similar to Einstein's statistical mechanics thermal fluctuations is a deep conceptual problem which could not have been solved solely by calculations; especially because before the arrival of the concept of modular localization such calculations could only have been done in terms of conceptually uncontrolled approximations. But now it can be satisfactory answered with the help of a new view of QFT which generically relates the restriction of the vacuum to the observables of a spacetime subvolume with thermal properties and vacuum polarization ("split inclusions" of modular localized algebras, Haag, 1996); this is precisely what "modular localization" achieves. One may safely assume that Born and Heisenberg perceived that this new quantum field model of Jordan with infinitely many oscillator degrees of freedom did not quite fit into their quantum mechanical project which Heisenberg started a short time before; in particular Jordan's nonchalant way of handling infinities led to critical comments (Duncan & Janssen, 2008).

Nevertheless Heisenberg, who in comparison to Jordan understood less about statistical mechanics at the time of the E–J conundrum, later became aware of vacuum polarization (which is absent in QM) probably still under the influence of Jordan's fluctuation problem. A letter he wrote to Jordan before he published his famous vacuum polarization paper mentions a logarithmic divergence $\lim_{\varepsilon\to\infty}\log\varepsilon$, with ε describing the "fuzziness" at the interval ends of Jordan's one-dimensional model (Duncan & Janssen, 2008). Indeed vacuum polarization and thermal manifestations of vacuum entanglement from causal localization are opposite sides of the same coin.

One note of caution. Since the terminology "particles" and "waves" played an important role in the Einstein–Jordan dispute, the reader may think that it refers (as mentioned before) to the quantum mechanical particle–wave dualisms (the two equivalent descriptions of QM); in this way its real significance, namely the thermal aspects of vacuum entanglement through causal localization of quantum matter, is sometimes overlooked.

The important distinction between the global quantum mechanical nature of infinitely many oscillators and their holistic role in the implementation of causal localization in a quantum theory of local fields had to wait almost five decades before being understood on a foundational level. For some time QFT was even suspected to be afflicted by internal inconsistencies which lead to ultraviolet divergencies (the "ultraviolet catastrophe"). Even after discovering the covariant renormalized perturbation theory for quantum electrodynamics (and finding an impressively successful agreement of low order perturbation with experimental observations) some of these doubts lingered on. Renormalized perturbation theory remained for a long time a collection of recipes about how to extract finite time-ordered correlation functions from the quantization rules starting with classical Lagrangians. What convinced people despite the weakness was the internal consistency of the finite results.

The quantization parallelism to the classical field theory of Faraday and Maxwell as embodied in the Lagrangian or functional integral quantization prevented for a long time an awareness about some radical differences resulting from quantum causal localization as compared to its classical counterpart. One manifestation of such a difference was that quantum fields, in contrast to smooth causally propagating classical functions, were rather singular operator-valued Schwartz distributions. They require testfunction smearing in order to attain the status of (generally) unbounded operators with which one then can construct operator algebras of bounded operators which are causally localized in spacetime regions. The other surprise was that these operator algebras have properties which were somewhat unexpected from the conceptual viewpoint of QM. Causal localization causes the global vacuum state to become impure upon restriction to a local operator subalgebra $\mathcal{A}(\mathcal{O})$ generated by covariant fields $A(x)$ smeared with $\mathcal{O}-$supported test functions. These impure "partial" states fulfill the so-called KMS property (Haag, 1996) with respect to a *modular Hamiltonian* which is intrinsically determined by the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ of local algebra and vacuum state vector. In fact all physical (i.e. finite energy) states restricted to a local algebra behave like statistical mechanics states.

The mathematical theory of operator algebras which highlights such properties is the *Tomita–Takesaki modular operator theory* which is omnipresent in QFT thanks to its causal localization structure. The presentation of QFT in terms of a net of operator algebras and their properties was proposed by Haag shortly after Arthur Wightman published his characterization of covariant fields in terms of properties of their correlation functions (Streater & Wightman, 1964). Haag's textbook on "local quantum physics" (LQP) (Haag, 1996), based on an operator-algebraic approach to QFT, appeared only many decades after he gave a first account of this new formulation; for a historical review see (Haag, 2010). The terminology LQP in the present article is used whenever it is important to remind the reader that the arguments go beyond the view about QFT which he meets in most textbooks (which are usually restricted to a formulation of perturbation theory within the setting of Lagrangian quantization and its functional integral formulation).

The mathematical property which guaranties the applicability of the T–T modular operator theory is the so-called *standardness* of the pair $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ i.e. the property that the operator algebra acts on Ω_{vac} (more generally on all finite-energy state vectors) in a cyclic ($\overline{\mathcal{A}(\mathcal{O})\Omega_{vac}} = H$) and separating ($\mathcal{A}(\mathcal{O})$ *contains no annihilators* of Ω_{vac}) manner. The cyclicity of the vacuum is closely related to the positivity of the energy of the representation of the Poincaré group, whereas the separating property results from spacelike commutativity of observables and is equivalent to the fact that the commutant, which contains the algebra of the causal complement $\mathcal{A}(\mathcal{O})' \supseteq \mathcal{A}(\mathcal{O}')$, acts also cyclic on Ω_{vac} as long as the spacelike complement $\mathcal{O}'$ is non-void. This physicists know under the name of the "Reeh–Schlieder property" (Haag, 1996), whereas the operator algebraists call this the "standardness" of the pair $(\mathcal{A}(\mathcal{O}), \Omega)$. This property is not shared by QM and accounts for the significant differences between these two QT (Schroer, 2010b).

For a structural comparison it is convenient to rewrite (the Schrödinger form of) QM into the Fock space setting of "second quantization" which converts wave functions into fields. As mentioned before in this reformulation the newly introduced vacuum remains, as opposed to its active role in QFT, completely inert with respect to the action of the Schrödinger "quantum field" (no vacuum entanglement leading to vacuum polarization). Instead of the cyclic action the local algebra at a fixed time[5] corresponding e.g. to a spatial region $\mathcal{R} \subset \mathbb{R}^3$, one obtains a subspace and a tensor factorization of H

$$H(\mathcal{R}) = \overline{\mathcal{A}(\mathcal{R})\Omega_{QM}} \subset H = H(\mathcal{R}) \otimes H(\mathcal{R}^{\perp})$$
$$\mathcal{A}(\mathcal{R}) = B(H(\mathcal{R})), \mathcal{A} \equiv \mathcal{B}(\mathcal{H}) = \mathcal{A}(\mathcal{R}) \otimes \mathcal{A}(\mathcal{R}^{\perp}) \quad (1)$$

of with a factorizing vacuum Ω_{QM}. This inertness against entanglement of the quantum mechanical vacuum is very different from the "vacuum polarizability" of Ω_{vac} in QFT which is connected to the lack of tensor factorization (despite the fact that by definition the commutant $A(\mathcal{O})'$ contains all operators which commute with $\mathcal{A}(\mathcal{O})$). In terms of structural properties of operator algebras these remarkable differences in the mathematical structure amount to the existence of two non-isomorphic factor algebras which are met in QFT: the global $\mathcal{B}(\mathcal{H})$ algebra of all bounded operators on a Hilbert space (the unique type I_∞ factor) and the local *monad* algebras $\mathcal{A}(\mathcal{O})$ which are all isomorphic to the unique hyperfinite type III_1 factor algebra in the Murray–von Neumann–Connes classification of factor algebras (Haag, 1996).

The choice of terminology reveals the intention to see the new local quantum physical view of QFT in analogy to the way Leibnitz understood *reality in terms of relations between monads*. In this extreme relational view, a monad by itself is nearly structureless, similar to a point in geometry. Indeed in the local quantum physical description of QFT, all properties of quantum matter, including the Poincaré covariance of its localization in spacetime and its possible localization-preserving inner symmetries, can be shown to arise from the abstract (non-geometric) modular positioning of a finite number of copies (depending on the spacetime dimension) of the monad within a shared Hilbert space (Section 3); the Poincaré group can be extracted from the modular groups of the contributing algebras and the concept of modular inclusions (Kaehler & Wiesbrock, 2001).

Together with the thermal KMS property of the locally restricted vacuum, there is the formation of a vacuum polarization cloud at the causal boundary of localization which accounts for a *localization entropy*, a special type of entanglement entropy. By replacing the boundary by a thin shell of size ε the localization entropy can be described in terms of a function of the dimensionless area $\alpha = \text{area}/\varepsilon^2$ which diverges in the limit $\varepsilon \to 0$. This relation between the increasing sharpness of localization and the increasing localization entropy is the *substitute of the lost quantum mechanical Heisenberg uncertainty relation*. The position operator $\mathbf{x}_{op}$ is, as all quantum mechanical observables, of global nature; it does not belong to the observables obeying the causal localization principle of LQP but may be used in the (non-covariant) effective description of wave-function propagation. The divergence in the sharp localization limit $\varepsilon \to 0$ shows another aspect in which QFT differs from QM.

The entanglement between the wedge-localized algebra and its opposite (that of the spacelike separated wedge) is always infinite

[5] In LQP such an algebra at a fixed time $\mathcal{A}(\mathcal{R})$ is defined as the intersection of all spacetime algebras $\mathcal{A}(\mathcal{O})$ with $\mathcal{R} \subset \mathcal{O}$.

in the sense that it is not possible to describe the associated state as density matrix (accounting for the singular nature of vacuum entanglement); indeed there are no pure states nor density matrix states on monad algebras; all states are impure in a very radical way. This is not a disease of QFT but rather its conceptual heart; without it there would be no relativistic QFT. In quantum statistical mechanics this kind of KMS state is only met in the thermodynamic limit of density matrix Gibbs states diverge and pass to KMS states on a monad algebra. In this case the QFT generated by the commutant describes a "shadow world" outside the localization concept (Schroer & Wiesbrock, 2000). Local algebras $\mathcal{A}(\mathcal{O})$ in QFT are monads and have no density matrix or pure states[6] at all; every global state restricted to such an algebra will be rather singular. In fact all physical (i.e. finite energy) states restrict to singular KMS states (i.e. one which cannot be written as a density matrix state).

The reduced vacuum state assign a *probability* to the ensemble of local observables contained in $\mathcal{A}(\mathcal{O})$; this is a consequence of the KMS (statistical mechanics-like) nature of the impure reduced vacuum state. Unlike the probability interpretation, which Born added to QM and which Einstein rejected (God does not throw dice), the ensemble viewpoint of probability as in statistical mechanics (which Einstein accepted) is intrinsic to QFT. KMS states on the ensembles of $\mathcal{O}-$localized observables are like thermal states of statistical mechanics and not "Gedanken-ensembles" as in case of Born's assignment of probabilities to individual mechanical systems of QM which refers to the statistics of repeated measurements. Einstein had no problems with probability of real ensembles in statistical mechanics, but it is the at that time unknown modular localization aspect which permits to recognize the ensemble aspect of local observables.

There have been attempts to improve Jordan's approximations (Duncan & Janssen, 2008) since the subvolume fluctuation problem is not solvable in closed form. The characterization of the algebra of operators localized in a subvolume is a *holistic problem*; the enclosure of the subsystem in a quantization box is not the same as reducing the vacuum to the subvolume algebra. Dealing with open subsystems is an "holistic" challenge in which the knowledge of the global oscillators is of not much help. Standard QFT does not provide a clear mathematical concept in order to characterize the ensemble of operators which is localized in a subvolume $\mathcal{O}$. On way of doing this would be to smear the quantum fields with $\mathcal{O}-$supported testfunctions and use the algebra which they generate. Even then one needs some knowledge about the "modular Hamiltonian" which is related to the kind of statistical mechanics associated with the KMS state corresponding to the restricted vacuum. In certain cases one can guess it in the form of a geometric transformation which leaves $\mathcal{O}$ invariant. For a noncompact wedge region in Minkowski spacetime e.g. $W_3 = \{x; x_3 > |x_0|\}$ this would be the wedge-preserving Lorentz subgroup $\Lambda_{W_3}(\chi)$, for Jordan's model (a chiral subalgebra on a lightlike interval, see Section 4) it is the interval-preserving dilation subgroup of the Möbius group; but in the generic case one has to refer to modular theory. What is important in the historical review is not whether Jordan got this right, but rather that in his attempt to counter Einstein he invented QFT.

In order to avoid any misunderstandings it should be emphasized that in saying that the concept of probability enters QFT in a more natural way than in QM, one is not implying that this is changing the epistemic aspects of the measurement theory in QT. All the conceptual aspects of entanglement (including Bell's inequality) remain valid. What QFT adds is a more radical realization of these phenomena on a much smaller scale; as already mentioned the scale of localization-caused vacuum entanglement is that of the Unruh effect and Hawking radiation. The reality of entanglement of particle states with respect to binary subdivisions in QM is experimentally accessible in terms of quantum optical arrangements, whereas the KMS impurity of the spacetime-restricted vacuum (e.g. the Unruh effect) will presumably always remain experimentally inaccessible (including even high energy nuclear experiments).

Part of the problem is that it is nearly impossible to describe precisely in terms of existing hardware how a perfect causal localization can be realized; even for noncompact spacetime regions as Unruh's Rindler wedges, the effect depends on the state of uniform acceleration of the observer; observer-independent manifestations appear only in the context of metric-induced event horizons of black holes. Fortunately foundational principles do not need to permit *direct* observational verification; they only have to be conceptually consistent, incorporate the reality which existed before their inception, and lead to new observable consequences. In this respect, QFT, which only shares with QM the Hilbert space and $\hbar$ but not the causal locality principle, has been and promises to continue to be the most inclusive successful physical theory.

One can entertain wonderful dreams of what may have happened if important concepts would have appeared decades earlier. But in the real world big conceptual jumps against the prevalent ideas of the time (the Zeitgeist) are virtually impossible; even for getting from inertial systems in Minkowski spacetime to General Relativity it took Einstein many years and the same can be said about the development of QM out of the old semiclassical Bohr–Sommerfeld ideas. The problem for the case at hand is aggravated by the fact that, up to the middle of the 1960s, there did not even exist a mathematical framework of operator algebras in which ideas about localization could have been adequately formulated.

It is interesting to note that modular operator theory and its physical counterpart of modular localization is the only theory to whose discovery and development mathematicians (Tomita, Takesaki, Connes) and physicists (Haag, Hugenholz and Winnink) contributed on par. They first realized this at a 1965 conference in Baton Rouge,[7] with statistical mechanics of open systems and the role of the KMS property representing the physical side (Haag, 1996). The study of the relation between modular operator theory and causal localization in LQP started a decade later (Bisognano & Wichmann, 1976), and its first application consisted in a more profound understanding (Sewell, 1982) of the Unruh Gedankenexperiment (Unruh, 1976). The terminology "modular localization" is more recent and marks the beginning of a new constructive strategy in QFT based on the modular aspects of localization of states and algebras (Brunetti, Guido, & Longo, 2002; Schroer, 1999). In mathematics the theory was the decisive instrument which led to Connes closure of the Murray–von Neumann project of classifying von Neumann factor algebras.

The E–J conundrum represents in fact a precursor of the Unruh Gedankenexperiment and, as the latter, can be fully resolved in terms of the principle of modular localization. In fact in the special case of Jordan's chiral current model (the historically first and simplest model of a QFT), the solution of the E–J conundrum amounts to a unitary *isomorphism* between a system defined by the vacuum state restricted to the algebra $\mathcal{A}(I)$ localized in an

[6] A state is a normalized linear positive functional on an algebra and only if this algebra consists of all bounded operators in a Hilbert space $B(H)$, states can be represented by vectors (modulo phase factors).

[7] The mathematicians worked on the generalization of the modularity of Haar measures (unimodular) in group representation theory whereas the physicists tried to understand quantum statistical mechanics directly in the thermodynamic infinite volume limit (open system statistical mechanics) by using the KMS identity instead of approaching this limit by tracial Gibbs states.

interval I and an associated global statistical mechanics system at finite temperature. Such isomorphic relations are referred to as describing an "inverse Unruh effect" (Schroer & Wiesbrock, 2000) and the Jordan model is the only known illustration. However in both cases the KMS temperature is not something which one can measure with a thermometer or use for "egg-boiling" (and there is also no "boiling soup" of particle/anti-particle pairs) (Buchholz & Solveen, 2013).

The attribute "holistic" will be used quite frequently in connection with modular localization. This terminology has been previously introduces by Hollands and Wald (2004) in connection with their critique of calculations of the cosmological constant in terms of simply occupying global energy levels (with a cutoff at the Planck mass). In previous papers (Schroer, 2013), it refers to the intrinsicness of localization which is connected with the cardinality of phase space degrees of freedom and their subtle local interplay. This distinguishes physical localization of quantum matter from mathematical/geometrical concepts. In fact it presents a strong barrier against attempts of geometrization of QFT and explains why the Atiyah–Witten attempt of the 1970s to "geometrize" QFT did not lead to the breakthrough which many people (including the author) hoped for.

The simplest illustration of the meaning of holistic consists in the refutation of the vernacular: "(free) quantum fields are nothing more than a collection of oscillators" which often students are told in courses of QM. Knowing continuous families of oscillators in the form of creation and annihilation operators $a^{\#}(\mathbf{p})$ does not reveal anything about free quantum fields and their associated local operator algebras. The free Schrödinger field and a free scalar covariant field share the same global oscillator creation/annihilation operators

$$a_{QM}(\mathbf{x},t)=\frac{1}{(2\pi)^{3/2}}\int e^{i\mathbf{px}-(\mathbf{p}^2/2m)}a(\mathbf{p})\,d^3p,\quad [a(\mathbf{p}),a^*(\mathbf{p}')]=\delta^3(\mathbf{p}-\mathbf{p}')$$

$$A_{QFT}(x)=\frac{1}{(2\pi)^{3/2}}\int\left(e^{-ipx}a(\mathbf{p})+e^{ipx}a^*(\mathbf{p})\right)\frac{d^3p}{2\sqrt{\mathbf{p}^2+m^2}},\quad p=(\mathbf{p},\sqrt{\mathbf{p}^2+m^2}) \tag{2}$$

In both cases the global algebra is the irreducible algebra of all operators $B(H)$, generated by the shared creation/annihilation operators. But the local algebras[8] generated by testfunction-smearing with finitely supported Schwartz functions supp$f(\mathbf{x})\subset\mathcal{R}$ of the fields and its canonical conjugate at a fixed time in a spatial region $\mathcal{R}$ are very different in both cases. In the relativistic covariant case they are identical to the algebras $\mathcal{A}(\mathcal{O}_\mathcal{R})$, $\mathcal{O}_\mathcal{R}=\mathcal{R}''$ the causal spacetime completion of $\mathcal{R}$ (which is also generated by smearing with $\mathcal{O}_\mathcal{R}-$supported spacetime smearing functions). According to what was stated before, these algebras are of "monad" type and the $\mathcal{A}(\mathcal{O}_\mathcal{R})-$restricted vacuum state is a KMS state; in the case of the Schrödinger field the associated subalgebra $B(H(\mathcal{R}))$ is of the same type as the global algebra; the QM vacuum continues to be an inert state in the "smaller" factor Hilbert space $H(\mathcal{R})$.

Whereas the global QM algebra is simply the tensor product of its factor algebras, the relation of the net of local algebras to its $\mathcal{A}(\mathcal{O})$ "pieces" is a more holistic relation; although together with its complement it generates the global algebra $\mathcal{A}(\mathcal{O})\vee\mathcal{A}(\mathcal{O})'=B(H)$, the global algebra $B(H)$ is not a tensor product of the two. The most surprising property which underlines the terminology "holistic" is the fact that the full net of local operator algebras which contains all physical informations can be obtained by "modular tuning" *of a finite number of copies of a monad* in a shared Hilbert space[9]; the reader who is interested in the precise formulation and its proof is referred to Kaehler and Wiesbrock (2001), see also Schroer (2010b). The fact that the global oscillator variables are the same in both cases (2) does not reveal these fundamental holistic differences of spacetime organization of quantum matter which have very different physical consequences. The present quantization formalism (Lagrangian, functional integral) does not shed light on those properties of QFT which solve the Einstein–Jordan conundrum in a clear-cut way. If it comes to ensemble properties of localized observables, the global aspects of generating covariant fields (which have no definite localization region) on which covariant perturbation theory is founded are of lesser importance than the local operator algebras $\mathcal{A}(\mathcal{O})$ which are generated by all smeared fields $A(f)$ with sup $pf\subset\mathcal{O}$. The emphasis changes from covariance properties of fields to properties of relative localization of operator algebras and this change finds its appropriate mathematical form in the LQP (local quantum physics) setting of QFT (Haag, 1996).

It is precisely this holistic aspect which renders any calculation of the subvolume fluctuation difficult; the simplicity of global oscillators is of no help here. A calculation in closed form is (even in the absence of interactions) not possible, and the imposition of covariance, which was the important step for obtaining the modern form of perturbation theory, also does not provide guidance. For renormalized perturbation theory one has clear recipes which were extracted from the imposition of covariance, but this is of not much help when one wants to find appropriate description of localized fluctuation in open subsystems. Saying that the global aspects can be described in terms of oscillators is almost as useless as trying to understand the holistic structure of a living body in terms of its chemical composition (in this analogy the chemical substances correspond the global operators whereas the nature of live corresponds to the organization of global oscillators into algebras of local observables).

Although modular localization theory asserts the existence of "modular Hamiltonians", in its present state it does not provide a generic method to explicitly construct them. Jordan's chiral model is an exceptional case for which, similar to the Unruh Gedankenexperiment, an explicit identification of the modular Hamiltonian in terms of the spacetime symmetries of the model is possible. Actually one may view Jordan's fluctuation problem as a predecessor of the Unruh effect in other words: QFT was born with the "thermal" localization aspects of the E–J conundrum which includes a completely intrinsic pre-Born notion of ensemble-probability; however the proximity of its date of birth to that of QM prevented an in-depth understanding of differences beyond the shared $\hbar$ and the Hilbert space.

This begs the question how, with the understanding of foundational properties of QFT still being that incomplete, it was possible to achieve the remarkable progress in renormalized perturbation theory. To phrase it in a more provocative historical context: how could one arrive at the Standard Model without having first solved the 1925 Einstein–Jordan conundrum? The answer is surprisingly simple: to get from the old Wenzel–Heitler formulation of perturbation theory, in which the vacuum polarization contributions were still missing, to the Tomonage–Feynman–Schwinger–Dyson perturbation theory for quantum electrodynamics (QED), one only needed to impose covariance and "exorcise" some ultraviolet divergences by finding plausible recipes. It was the internal consistency of the result and not its derivation from Lagrangian

[8] Technical points as the connection between fields and the algebras they generate are not important in the present context and therefore will be omitted.

[9] This number n is two for the simplest case of a chiral algebra, whereas for a net in four spacetime dimension the correct modular positioning can be achieved in terms of $n=7$ copies. The emergence of the spacetime symmetries in Minkowski spacetime as well as possible inner symmetries of quantum matter is a consequence of this holistic tuning.

quantization which made renormalized perturbation theory successful.

Many years later there was also derivation of these renormalization rules by starting from invariant free field polynomials (without using Lagrangians quantization[10]) and invoking spacelike commutativity in an inductive way (the causal perturbation setting of Epstein & Glaser, 1973). But such conceptual refinements (of reducing prescriptions to an underlying principle) had little impact on the Zeitgeist; in any case it would not have helped to obtain the foundational insight into modular localization which is required in order to solve the E–J conundrum.

This lucky situation of making progress by playfully pushing ahead and working once way through a yet conceptual incomplete formalism with the help of consistency checks did not extend much beyond Lagrangian quantization and renormalized perturbation theory. As will be shown in Section 6, it is precisely this setting which determined the fate of QFT for more than half a century which is now being replaced by a more general setting based on modular localization. The latter has not only removed unnecessary restrictions from renormalization theory, but also led to a different view about on-shell constructions (Section 5). When, in the aftermath of the Lehmann–Symanzik–Zimmermann (LSZ) scattering theory and the successful adaptation of the Kramers–Kronig dispersion relations, the first attempts of S-matrix based on-shell construction were formulated, the conceptual difficulties of analytic aspects of on-shell properties were underestimated. As one knows through more recent progress about modular localization, an important aspect of the S-matrix, namely its role as a relative modular invariant of wedge-localization, was missing. As a result, the true nature of the particle crossing property was misunderstood by identifying it with Veneziano's dual model crossing which was then passed to string theory (ST).

The correct formulation of the on-shell crossing property within a new S-matrix-based construction project and the solution of the E–J conundrum are interconnected via the principle of modular localization. It is the aim of this paper to show the power of the latter by presenting the solution to these two problems.

The first attempts to formulate particle physics and obtain an constructive access outside of quantization and perturbation theory was the S-matrix in Mandelstam's project (Mandelstam, 1968). As we know nowadays, and as it will be explained in detail in the present work, this failed as a result of the insufficient understood on-shell analytic properties. Their connection to the causality principle are much more subtle than those to the off-shell correlation functions. In retrospect it is clear that with the scant understanding of the central crossing property (and more generally the conceptual origin of on-shell analyticity properties), there was no chance in 1970s for Mandelstam's S-matrix based particle theory project to succeed.

In retrospect it is also clear why this happened precisely when Veneziano's mathematical construction of a crossing symmetric meromorphic function in two variables was accepted as a model realization of particle crossing for elastic scattering amplitudes. It is appropriate in an paper, whose intention is to shed light on still ongoing misunderstandings, to explain their origin in a historical context.

The importance of the E–J conundrum in the development of QFT can be best highlighted by *following Galileio's example and imagine a dialog between Einstein and Jordan about subvolume fluctuations but placing it in the year 1927, after Max Born added his probability interpretation to Heisenberg's and Schrödinger's quantum mechanics.*

Einstein: Dr. Jordan, I appreciate that you finally accepted my invitation to come to Berlin and I am very interested to understand why, after first criticizing my fluctuation calculations in my statistical mechanics thermal blackbody radiation model, you now claim that you find similar fluctuation components in your new wave quantization at zero temperature.

Jordan: Thank you Professor Einstein for taking so much interest in my work. The appearance of such a fluctuation spectrum in my new setting of quantized waves in a vacuum state is indeed surprising. Although my wave quantization of 2-dimensional Maxwell waves generalizes Heisenberg's quantization in some sense, the fluctuation properties obtained by restricting the vacuum to a subinterval leads to a very different situation from that expected in his and Born's formulation of QM. It seems that my quantized Maxwell waves cannot be subsumed into a quantum mechanics of systems with an infinite number of oscillators.

Einstein: As you remember, I have some grave reservation against associating a probability to an individual measurement on a quantized mechanical system which I occasionally expressed in the formulation "the Dear Lord does not throw dice". But I never had any problem with probability in statistical mechanics, in fact my calculation of the Nadelstrahlung-component in the black body fluctuation spectrum, which led me to the particle nature of light on pure theoretical grounds, is based on the probability of quantum statistical mechanics. Does the result of your subvolume fluctuation calculation in the pure ground state of your field quantization mean that this state appears impure if analyzed in the setting of an open subsystem?

Jordan: Professor Einstein, I am glad that you raised this question. I have been breaking my head over these unexpected consequences of my new quantized field theory and I would be dishonest with you, if I claim to understand these conceptual implications. But since the main difference to mechanics is the causal propagation (which was already implicit in the Nahewirkungsprinzip of Faraday and Maxwell and which you then succeeded to generalize into your new relativity principle in a Minkowski spacetime), I am inclined to suspect that the ensemble aspect, which one needs in order to avoid the assignment of a probability to an individual mechanical system (as proposed by my adviser Prof. Max Born), has its origin in the quantum realization of causal localization. Somehow this principle creates a natural ensemble associated with its causal completion of a localization region, namely the ensemble of all local observables attached to that spacetime region. This is in contrast to QM which deals with individual mechanical systems for which the association to an ensemble is a useful mental construct for the interpretation of QM. I tried to convince Prof. Born and my colleague Werner Heisenberg, who despite their initial resistance finally agreed to permit me to present my idees in a separate section of a joint paper which was published 2 years ago. But I was not able to remove their doubts. It would be very helpful for me to obtain some support from your side.

Einstein: I need some time to think about this new situation. Your conjecture seems to suggest that your new theory of quantum fields, which is certainly much more fundamental than Heisenberg's and Schrödinger's quantized mechanics, comes with an intrinsic notion of localized ensembles of observables and an associated statistical mechanics type of probability. If one could better understand how the less fundamental global quantum mechanics can be related as a limiting case to your new fundamental quantum field theory in such a way that Born's postulated probability is a relict of your local ensemble probability, this may change my view and perhaps even influence my quantum physical Weltanschauung. Let us remain in contact and keep me informed about future clarifications on the points raised in our conversation.//

In this imagined dialog, which could have radically changed the history of QFT, I avoided the use of advanced mathematical concepts of modular localization for which there was no mathematical support

[10] The free fields do not have to fulfill Euler–Lagrange equations.

in the 1920s. The E–J conundrum is best understood as a progenitor of an Unruh-like Gedanken experiment.

The organization of this paper is as follows. In the next section the vacuum polarization on the boundary of causal localization is derived for the "partial charge", which is a modern formulation of Heisenberg's original observation. Section 3 sketches the issue of modular localization and its KMS property with special emphasis on the difference between a KMS (Carnot) temperature and that measured by a thermometer. In Section 4 the KMS property is used for the explicit construction of an isomorphism between the thermal subvolume (interval in Jordan's chiral model) fluctuations in Jordan's model with a corresponding statistical mechanics model representing Einstein's side. Section 5 explains modular localization and its relation with the Tomita–Takesaki modular operator theory. The ongoing impact of modular localization on on-shell constructions of QFT, with particular emphasis on the connection of particle crossing with the KMS identity, is addressed in Section 7.

The most important consequence of modular localization for the ongoing research in particle theory is the generalization of renormalized perturbation to interactions involving arbitrarily high spin through the use of string-localized fields in Section 6. In the case of spin $s=1$ it leads to a much deeper understanding of why the gauge theory requires the indefinite metric Krein space setting and how modular localization allows a formulation which remains throughout in the Hilbert space.

The same ideas which lead to unexpected progress also permit to expose the misunderstandings which led to the dual model and ST as presented in Section 7. In contrast to the stringlocal fields in higher spin QFT the localization which string theorist attribute to it is that of a chain of quantum mechanical oscillators (Born's localization) which bears no relation to causal localization in spacetime. Section 8 addresses some old and in the maelstrom of time lost insights about the connection between the cardinality of phase space degrees of freedom and causal localization. This includes problems concerning dimensional changes which came from ST but which can also be formulated in the setting of QFT. The critique of the Maldacena conjecture, concerning the nature of the AdS–CFT correspondence, addresses one of those problems. The concluding remarks in the last section attempt to position the present situation in the particle theory within the historical context and the expectations about its future.

3. Vacuum polarization, area law

In 1934, Heisenberg (1934) finally published his findings about vacuum polarizations (v.p.) in the context of conserved currents which are quadratic expressions in free fields. Whereas in QM they lead to well-defined partial charges associated with a volume V

$$\partial^\mu j_\mu = 0, \quad Q_V^{clas}(t) = \int_V d^3x j_0^{clas}(t,\mathbf{x})$$
$$Q_V^{QM}(t) = \int_V d^3x j_0^{QM}(t,\mathbf{x}), \quad Q_V^{QM}(t)\Omega^{QM} = 0 \tag{3}$$

there are no such sharp defined "partial charges" Q_V in QFT, rather one finds (with g_T a finite support smooth interpolation of the delta function) (Requardt, 1976)

$$Q(f_{R,\Delta R}, g_T) := \int j_0(\mathbf{x},t) f_{R,\Delta R}(\mathbf{x}) g_T(t)\, d\mathbf{x}\, dt,$$
$$f_{R,\Delta R} = \begin{pmatrix} 1, & \|x\| \le R \\ 0, & \|x\| \ge R+\Delta R \end{pmatrix}$$

$$\lim_{R\to\infty} Q(f_{R,\Delta R}, g_T) = Q, \ \|Q(f_{R,\Delta R}, g_T)\Omega\| = \begin{cases} F_2(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_2 \ln\left(\dfrac{R}{\Delta R}\right) \\ F_n(R,\Delta R) \overset{\Delta R\to 0}{\sim} C_n \left(\dfrac{R}{\Delta R}\right)^{n-2} \end{cases} \tag{4}$$

where the logarithmic divergence corresponds to $n=2$.

The *dimensionless* partial charge $Q(f_{R,\Delta R}, g_T)$ depends on the "thickness" (fuzziness, roughness) $\Delta R = \varepsilon$ of the boundary and becomes the f and g-independent (and hence t-independent) conserved) global charge operator in the large volume limit. The deviation from the case of QM is caused by v.p. Whereas the latter fade out in the $R\to\infty$ limit, they grow with the dimensionless area $(R/\Delta R)^{n-2}$ for $\Delta R \to 0$. The simplest calculation is in terms of the two-point function of conserved current of a zero mass scalar free field. In the massive case the leading term in the limit $\Delta R\to 0$ remains unchanged. We leave the elementary calculations (not elementary at the time of Heisenberg) to the reader.

The presence of v.p. causes relativistic quantum fields to be more singular than Schrödinger fields and requires the formulation in terms of the Schwartz distribution theory as used in the above smearing of the current with smooth finitely supported test function. The LQP setting on the other hand avoids the direct use of such singular objects in favor of local operator algebras. In such a description the singular nature of vacuum polarization is not directly perceived in the individual operators but rather shows up in ensemble properties of operator algebras. It turns out that under rather general conditions there exists between two monad algebras a distinguished (by modular theory) intermediate type I_∞ algebra N (Haag, 1996)

$$\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}) \supset N \supset \mathcal{A}(\mathcal{O}_{\mathcal{R}}), \quad H \overset{V}{\to} H(N) \otimes H(N'), \quad \eta \equiv V(\Omega\otimes\Omega)$$
$$VAB'\Omega = A\Omega \otimes B\Omega, \quad A \in \mathcal{A}(\mathcal{O}_{\mathcal{R}}), \quad B' \in \mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R}),$$
$$VNV^* = B(H) \otimes \mathbf{1} \tag{5}$$

i.e. there exists a unitary operator V which permits to write the full Hilbert in terms of a tensor product such that $\mathcal{A}(\mathcal{O}_{\mathcal{R}}) \subset N$, $\mathcal{A}(\mathcal{O}_{\mathcal{R}+\Delta R})' \subset N'$ where the "split vacuum" η is a state in the original Hilbert space which corresponds to the tensor product of vacua.

In QM the unitary V would be simply the identity operator expressing the fact that the vacuum is an auxiliary mathematical state which remains physically inert under splitting, i.e. the QM vacuum is not entangled under spatial subdivisions. In QFT it is a state which on $N\otimes N'$ is nontrivially entangled in the sense of quantum information theory. However in the sharp localization limit $\Delta R\to 0$ the "quantum mechanical" type I_∞ converge towards the monads $A(\mathcal{O}_R), A(\mathcal{O}_R')$ which commute but do not tensor-factorize. The limiting entanglement is of a very singular kind which has no counterpart in quantum information theory and is characteristic for subalgebras which do not admit density matrix states as the monad. The situation is analogous to that encountered in finite temperature statistical mechanics in the thermodynamic infinite volume limit when the tracial nature (the Gibbs formula) of the state is lost and only the KMS property remains.[11]

The above described nontrivial behavior under splitting leads to a nontrivial ΔR dependent *localization entropy* which is consistent with the KMS impurity of the restricted vacuum. In fact, since the vacuum polarization happens in a layer of size ΔR (the

[11] Whereas the thermodynamic limit monad is approximated from the inside, the split property approximates the local monad from the outside.

"fuzzy" boundary) the entropy is a function of the dimensionless area

$En(R, \Delta R) = \text{split localization entropy}$

$$En|_{\Delta R \to 0} \simeq ca, \quad a = \frac{\text{area}}{(\Delta R)^{d-2}} \quad \text{for } d > 2 \tag{6}$$

where the second line is the leading order in the sharp localization limit which one expects if the "polarization clouds", which determine the singular behavior of smeared fields as Heisenberg's partial charges (4), are the same as those which appear in the above entropy argument.

Note that in contradistinction to the treatment in the literature where the connection with the model of local QFT is lost by introducing an imagined and ill-defined momentum space cut-off,[12] the implementation of the split property is a construction *within* the QFT model.

The logarithmic behavior for $d=2$ split entropy can actually be rigorously derived (Schroer, 2011b) and is well known to condensed matter physicists. For Jordan's chiral current model used in the E–J conundrum, the entropy can be directly obtained from the isometry with a chiral statistical mechanics model (Section 4). This situation is very special and has been termed "the inverse Unruh effect (Schroer & Wiesbrock, 2000). For $d=1+3$ 't Hooft has obtained the area behavior in terms of the "brickwall picture" ('tHooft, 1996), but a rigorous derivation, solely based in the split property of modular localization, is not yet available. Bekenstein's area law results if one relates ΔR with the Plank length.

There exists a conjecture that even in the general case there could remain a weak form of the "inverse Unruh effect" (Schroer & Wiesbrock, 2000) in which the spatial volume factor is replaced by the "volume factor" of a box with two spacelike and one lightlike direction. In that case the two spacelike extensions would account for the dimensionless area factor and the lightlike contribution would be (as in the chiral Jordan model) logarithmic (Schroer, 2011b) so that the net result is a logarithmically modified area law.

Either behavior of localization-entropy shows that although there are genuine infinities in QFT; they are limited to sharp localization within a model and not a predicate of QFT; in case of quantum fields they are controlled in terms of testfunction smearing. Unlike the misunderstood ultraviolet divergencies in the old formulation of perturbation theory, they have no relation to the "ultraviolet catastrophe" i.e. they threaten in no way the consistency of QFT; to the contrary, they are a direct consequence of its most foundational modular localization property. In a certain sense the divergence of thermodynamic infinite volume limit correspond to the infinity obtained in the sharp boundary limit (increasing sharpness of the boundary) $\varepsilon \to 0$.

With the notion of "localization temperature" and energy one has to be much more careful than with the dimensionless localization entropy. When one naively interprets the Unruh temperature as that measured by a thermometer, one enters a conceptual mine field. The equality of the thermometer (local) temperature (related to the zeroth thermodynamic law with the "Carnot temperature" of the second fundamental law of an KMS equilibrium state is only correct in an inertial system, but the "egg-boiling local temperature of the Unruh effect refers to an accelerated observer. In fact the thermometer *temperature in a vacuum state remains zero*; it is a "local temperature" which does not depend on the Unruh trajectory (Buchholz & Solveen, 2013). The same holds for other situations described by modular theory (next section); although there is always a dimensionless modular Hamiltonian and a dimensionless temperature $\beta = 2\pi$ associated with modular KMS states. The still ongoing hot topic about "firewalls" (Papadodimas & Raju) is dangerously close to the Unruh "cooking temperature" and more investigations about possible differences between causal horizons (Unruh) and event horizons of black holes are necessary for clarification.

Another useful conceptual warning in passing from classical fields to quantum fields is to avoid to attribute a direct physical meaning to fields, but rather to view them in a similar role as that which coordinates play in the description of geometry. This is facilitated by the fact that quantum fields are not directly measured (no experimentalist has measured a nuclear field); rather the notion of a quantum field serves as a *device to describe particles* which are related to a particular subset of quantum field i.e. the same particles can be interpolated by many different fields. It has turned out that to view fields in their role as coordinatizing or generating local algebras is the most useful way of keeping track of the differences between description-dependent fields from intrinsic particles. In this way particles do not correspond to individual fields but rather to local field classes which carry the same superselection charges. All structural properties of LQP and the resulting general theorems can be expressed in terms of local nets of operator algebras, but the present formulation of renormalized perturbation theory still needs generating fields.

Note that the well-known entropy conjecture by Bekenstein, based on equating a certain area behavior in classical general relativity with quantum entropy, results formally from the above area law by equating ΔR with the Planck length. Quantum gravity is often thought of as that still elusive theory which explains why and how the quanta of gravity can escape the consequences of modular localization for sharp localization which are responsible for the singular short distance aspects of causal localization. If Bekenstein's conjecture really describes quantum aspects of gravity (and not just quantum matter in curved spacetime), then modular localization cannot be extended to quantum gravity.

As mentioned before the relation between ΔR and the entropy is reminiscent of Heisenberg's quantum mechanical uncertainty relation in which the uncertainty in the position is replaced by the split distance ΔR within which the vacuum polarizations can attenuate, so that outside the vacuum returns to play its usual role (if tested with local observables in the causal complement of $\mathcal{O}_{R+\Delta R}$).

It should be stressed again that the probability interpretation, which Born had to add to Heisenberg's and Schrödinger's formulation of QM, is completely intrinsic to LQP. It is a consequence of the "thermal" KMS property of ensembles of operators contained in a localized algebra $\mathcal{A}(\mathcal{O})$ in $\mathcal{O}-$restricted physical (finite energy states). As such it is not different from the statistical mechanics probability, which Einstein used in his fluctuation arguments in terms of which he challenged the physical content of Jordan's thesis. It is only with the modern concept of modular localization and the hindsight of more than eight decades of QFT that one realizes how close the E–J conundrum came to the intrinsic probability coming from the quantum formulation of the Faraday–Maxwell–Einstein causal locality principle in Minkowski spacetime. Einstein's problem was the assignment of a probability to an individual mechanical system (which requires to *imagine* it as a member of an ensemble for which the probabilistic nature is seen in repeated measurements). The fact that probability is intrinsic to QFT and that the vacuum entanglement of sharp localization is more singular than that of quantum information theory influences the discussions around Bell's inequalities but does not invalidate them. The effects of the (more radical form of) vacuum entanglement in QFT remain orders of magnitudes below the quantum mechanical entanglement of particle state which can be directly measured in terms of quantum optical methods.

A particular radical illustration of the conceptual differences between QFT and QM is the reconstruction of a net of operator

[12] One can cut off integrals but to cut off a model of QFT is ill defined.

algebras from the relative modular position of a finite number of copies of the monad (Schroer, 2010b). For chiral theories on the lightray one needs two monads in a shared Hilbert space in the position of a *modular inclusion*, for $d=1+2$ this "modular GPS" construction needs three and in case of $d=1+3$ seven modular positioned monads are sufficient to create the full reality of a causal quantum matter world, including its Poincaré symmetry (and hence Minkowski spacetime) from the abstract modular groups (Kaehler & Wiesbrock, 2001). This possibility of obtaining concrete models by modular positioning of a finite number of copies of an abstract monad (indecomposable constructs without inner structure) in a shard Hilbert space is the strongest "holistic outing" of QFT; the reader is encouraged to look at this application of modular theory (Kaehler & Wiesbrock, 2001). For $d=1+1$ chiral models the modular positioning leads to a partial classification of chiral theories as well as to their explicit construction of large classes of models (Section 5).

Apart from $d=1+1$ factorizing (integrable) models, where modular properties in the form of *nuclear modularity* were used for existence proofs of models (Lechner, 2008), QFT has not yet reached the state of maturity where such holistic properties can be applied for classifications and existence proofs of families of models and their mathematically controlled approximation. An extension to curved spacetime would be very interesting; the simplest question in this direction is the modular construction of the local diffeomorphism group on the circle in the setting of chiral theories.

4. Modular localization and its thermal manifestation

The aim of this section is to present the concept of *modular localization* which is the backbone of LQP and represents the intrinsic formulation of causal quantum localization. Since, as mentioned before, subalgebras $\mathcal{A}(\mathcal{O})$ localized in spacetime regions $\mathcal{O}$ with $\mathcal{O}'' \subsetneq R^4$ are known to in a act cyclic and separating manner on the vacuum (the Reeh–Schlieder property Haag, 1996), the "standardness" condition for the validity of the Tomita–Takesaki modular theory is always fulfilled for local subalgebras. This leads to a uniquely defined Tomita operator $S_{\mathcal{O}}$ whose properties will be the main subject of this section.

It has been known for a long time that the algebraic structure underlying free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the functorial images of *subspaces of the Wigner wave function spaces* (second quantization[13]).

Before presenting some mathematical details, it is useful to recall some philosophical points. LQP avoids the parallelism to the classical field theory which characterizes the Lagrangian quantization approach of QFT and the closely related functional integral representation. Accepting that QFT is more fundamental than the classical field theory, the content of QFT should reveal itself in terms of its own principles without the detor of a "quantization parallelism" to the classical field theory.

In contrast to QM, the LQP setting of QFT de-emphasizes individual operators in QFT in favor of *ensembles of operators* which share the same spacetime localization region. These ideas also follow more closely the situation in the laboratory, where the experimentalist measures coincidences between events in spacetime. All the measured particle properties, including the nature of spin and internal quantum numbers, were obtained by repetitions and refinements of observations based on counters which are placed in a compact spatial region and are maintained activated for a limited time. Their detailed internal structure is generally not known what matters is their localization in spacetime and the sensitivity of their response. However without a precise mathematical backup which matches these physical concepts, LQP remains in the philosophical realm.

The role of covariant quantum fields in LQP is that of generators of a net of local operator algebras $\{\mathcal{A}(\mathcal{O})\}_{O \in R^4}$ which act in a fixed Hilbert space. In the Wightman setting a field is a covariant operator-valued distribution $A(x)$ which is globally defined for all $x \in R^4$. From its global definition one passes to (unbounded) $\mathcal{O}-$localized operators, formally written as $A(f) = \int A(x) f(x) d^4x$, $\mathrm{supp} f \subset \mathcal{O}$, which according to Wightman's axioms define a system of polynomial (generally unbounded) operator *-algebras $\mathcal{P}(\mathcal{O})$. Formally these unbounded operators can be associated with an aforementioned net of (mathematically easier manageable) bounded operators forming von Neumann algebras, which is the starting point of Haag's LQP setting. The advantage is that one obtains access to the well-developed mathematical theory of operator algebras (omitting from now on "bounded"). Certain causality aspects allow a more natural definition and more profound understanding in the LQP setting. The mathematical details, which allow to pass between Wightman's description to the algebraic local nets of observables in the LQP setting and vice versa, are tedious and still technically incomplete (Haag, 1996), but this had little effect on progress.

Whereas both settings are different formulations of closely related physical concepts, there is a significant distinction between these settings and constructions based on Lagrangian or (the closely related) functional integral based quantization methods. Quantization is not a physical principle; whereas classical descriptions often help to find a perturbative description (quantization) of a QFT, there is no general correspondence. The fact that the less fundamental QM which lacks causal localization and its holistic consequences is capable to maintain an almost (up to ordering prescriptions of operators) unique connection to classical mechanics does not imply that such a close relation must continue to hold in QFT. The strong link between classical mechanics and its quantum counterpart finds its best-known expression in the fact that Lagrangian quantization (canonical quantization) and functional quantization (path integrals) enjoy solid mathematical support from measure theory but not in QFT.

All this breaks down in interacting QFT with realistic short distance behavior.[14] Apart from $d=1+1$ integrable models (Section 5), for which rigorous methods of LQP led to existence proofs (Lechner, 2008, 2000), there is of course renormalized perturbation theory; but since perturbative expansions in the coupling strengths (which are consistent on the level of polynomial relations) inevitably lead to divergent series, they are not the right objects for a mathematically controlled approach to QFT. In fact there exists not even a mathematical argument that they define an asymptotic approximation in the limit of vanishing coupling to an existing model of QFT, although the use of low order perturbative results led in many cases to quite spectacular agreements with observations. Whereas the setting of QM has reached its closure a long time ago, the conceptual/mathematical flanks of QFT remain open.

The *causal perturbation* setting of Epstein & Glaser (1973) avoids the ultraviolet divergencies of the Lagrangian or functional setting by implementing causal locality in terms of time-ordered products in an inductive way. A specific model is defined in terms of its free field content and the starting point is a first-order

[13] Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

[14] Free field short distance behavior of polynomially coupled scalar fields is still in the reach of measure-theoretical functional methods (Glimm & Jaffe, 1972).

interaction density in form of a Lorentz-invariant (scalar) Wick-polynomial. The scaling degree of the interaction density is determined in terms of the scaling degrees of the participating fields and their derivatives. If the scaling degree of the interaction defining first-order polynomial in terms of free fields does not surpass $d_{s.d.}^{int}=4$ one obtains a renormalizable model in which the short distance dimensions of quantum fields remain bounded, independent of the number of iterative steps (order of perturbation).

The problem with this setting is its limitation with respect to the spin of pointlike free fields in a Hilbert space setting. The short distance dimension of massive pointlike free fields in Hilbert space increases with spin as $d_{s.d.}=s+1$. Hence a $m>0$, $s=1$ Proca potential with $d_{s.d.}=2$ does not admit any renormalizable interaction in Hilbert space and the infrared divergencies of its $m=0$ limit are well-known additional obstacles of perturbation theory. Wigner's 1939 classification of particles in terms of positive energy representations led to a clear statement about the field content of covariant $(m=0, s\geq 1)$ representations: there are covariant pointlike field strengths[15] but no covariant pointlike potentials. This is the famous *clash between Hilbert space positivity and pointlike localization*. The conventional way out is that of keeping the pointlike structure and allowing indefinite metric so-called Krein spaces instead of Hilbert spaces.

This problem is not present in the classical Maxwell theory; in that case the use of vectorpotentials contains a redundancy which affects the connection of Cauchy data and their causal propagation and is conveniently taken care of in terms of the concept of gauge transformations and gauge invariance (the return to field strengths and currents). Lagrangian quantization and functional integral prescriptions for gauge theories lead out of the Hilbert space; in fact pointlike interaction-free massless vector potentials are well known to require a Krein space formulation (the Gupta–Bleuler formalism and its BRST extension). Since the Hilbert space setting is the foundational pillar of QT, *quantum gauge theory* in the presence of interactions of massive or massless vectormesons is an undesired but inevitable compromise which is suggested by Lagrangian quantization. Since classical field theory does not know anything about Hilbert space positivity, there is a serious obstacle to quantization for interacting $s\geq 1$ interactions and gauge theory is a compromise which only describes the vacuum sector which is generated by the subalgebra of gauge invariant pointlike local fields acting on the vacuum states) and leaving the important charge-creating operators and the physical particle-like states they create from the vacuum outside the physical range of the quantum gauge setting.

This makes it desirable to turn to another description which the previously mentioned alternative suggests: *abandon pointlike localization and keep instead the Hilbert space*. Since this is inconsistent with the quantization of pointlike classical gauge theory, it is not surprising that such an alternative requires a radical change of the Epstein–Glaser causal perturbative setting (Epstein & Glaser, 1973). Although this formalism does not depend on quantization of a classical field structure,[16] it still uses pointlike generating fields in an essential way. The safest procedure is to try to extract an information from the foundational localization principles of LQP by asking the following structural questions: *what is the tightest localization which can be derived solely from the mass gap property*?

[15] Massive pointlike potentials and their associated field strengths have the same $d_{s.d.}=s+1$, but whereas the zero mass limit of field strengths exists, that of potentials does not.

[16] In particular it does not depend on whether the quantum fields are solutions of Euler–Lagrange equations.

The type of models for which such a question could be relevant is interacting massive vectormesons. As mentioned before pointlike interactions of such fields are nonrenormalizable, and since the new Hilbert space setting shows that the concept of renormalizability is intimately related to the short distance aspects of localization. The weakening from point- to string-localization is the result of the restrictive Hilbert space positivity which is absent in the Krein space setting of gauge theory.

Interestingly it is not necessary to use weaker than string-localized fields in order to describe a QFT; this is part of a theorem by Haag (1996): all LQP with a mass-gap (which are known to admit scattering theory) can be generated by spacelike semi-infinite stringlocal fields. Covariant generating stringlocal fields $\Psi(x,e)$, $e^2=-1$ are localized on $x+\mathbb{R}_+e$ and commute for spacelike separated strings (appropriately modified for Fermions). In Section 6 the string-extended E-G perturbation theory will be exemplified in massive gauge theories. Whereas the local observables (field strengths, currents) remain pointlocal and the interacting physical matter fields are stringlocal, the S-matrix turns out to be e-independent. Massive vectormesons also permit a coupling to neutral matter (scalar Hermitian fields H).

These couplings reveal what was known to some researchers for a long time: the Higgs mechanism about a mass-creating symmetry breaking is not supported by QFT; the intrinsic property of all couplings of massive vectormesons to matter (independent of whether the latter is charged or neutral) is the "Schwinger–Higgs screening" of the Maxwell charge which is directly related to the field strength of the massive vectormeson. Although this is consistent with the BRST gauge setting, the new Hilbert space setting using renormalizable couplings of stringlocal massive vectormesons lead to these results without having to rely on unphysical Krein space methods (Section 6). Computations need not any more be based on successful but (from the quantum viewpoint) somewhat miraculous descriptions. A surprising new structure which results from the Hilbert space positivity for renormalizable interactions of massive $s\geq 1$ stringlocal fields is the appearance of lower spin "escort" fields. In the case of massive vectormesons this is a stringlocal neutral scalar ϕ field which share many properties with the Hermitian H fields of the Higgs model apart from the fact that they have no relation to any mass-generating symmetry breaking (Section 6).

The fundamental idea which is behind the ongoing radical changes for interaction involving stringlocal fields is a much deeper understanding of *quantum* causal locality in the algebraic operator setting of modular localization. Individual quantum fields never played a similar distinguished physical role as they do in classical field theory. As mentioned before they are not directly measured (measuring a hadronic field?) and the particles which are identified with counter events are always associated with an infinite class of (composite) fields which carry the same superselected charge and are relatively local with respect to each other. Whereas in QM it makes sense to distinguish in terms of elementary particles and their bound states, such a hierarchy is rather meaningless in QFT; the omnipresence of vacuum fluctuations only respects the superselected charges but couples all states which have the same such charge. The fields within one superselected class are distinguished by their short distance scale dimensions, and the renormalizable Lagrangian couplings highlight fields with low $d_{s.d.}$. but the particle field relation is based on infinite timelike separations (time-dependent scattering theory) for which low $d_{s.d.}$ values are irrelevant. QFT is a quantum theory in which *everything which according to the superselection rules can be coupled is actually coupled* (there is always a process in which this coupling is activated). This explains why methods of quantum mechanics are rather useless in QFT but at the same time this is the prize to be paid for a fundamental theory. Modular localization theory brings all these foundational properties (which still remain somewhat hidden in the perturbation theory in terms of individual fields) into the forefront.

The central issue in LQP refers to two physically motivated requirements on the local net of operator algebras

$$[\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0, \quad \mathcal{O}_1 >< \mathcal{O}_2 \text{ Einstein causality}$$
$$\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'') \text{ causal completeness}$$
$$\mathcal{A}(\mathcal{O}') = \mathcal{A}(\mathcal{O})' \text{ Haag duality} \tag{7}$$

The first line is a condensed notation for the commutativity of operators from spacelike separated regions; it is only required for observable fields. The commutation property for *non-observable* operators, as those coming from spinor fields or fields carrying superselected charges, is determined by the local representation properties of the observables (the superselection theory to their associated observable subalgebras, Haag, 1996).

The *causal completeness property* (7) is a local adaptation of the old time-slice property (Haag & Schroer, 1962). In classical relativistic field theory the field values in the relativistic "causal shadow" (or causal completion) V'' (two-fold causal complement of V) is the region in which the classical field values are uniquely determined in terms of the (properly defined) initial values in a finite volume V at fixed time. Its quantum adaptation in the LQP setting is the algebraic causal completeness property. Often particle theoreticians only consider the simpler Einstein causality property and ignore causal completeness. But there are situations which are consistent with Einstein causality but violate causal completeness.[17] In fact in Haag and Schroer (1962) the simplest model, a so-called generalized free field with a suitable continuous mass distribution was used as an illustrative example for a physically unacceptable Einstein-causal field. Whereas in the setting of Lagrangian quantization causal completeness is the formal consequence of the quantization of causally propagating relativistic fields, this property needs special attention in situations in which classical analogs are not available as e.g. ideas coming from string theory.

This affects in particular relations between QFTs in different spacetime dimensions. The fact that in some cases they are backed up by a mathematical isomorphism does not imply that they are physically acceptable. One such trend-setting case is the Maldacena conjecture which originally arose in string theory. Its mathematical basis is an algebraic isomorphism (Rehren, 2000) which extends the well-known equality of the spacetime conformal symmetry of a conformal field theory (CFT_n) in n spacetime dimensions with the spacetime symmetry of an anti-de-Sitter space in $n+1$ dimensions (AdS_{n+1}) to a mathematical isomorphism which between suitably chosen local subalgebras on both sides. But it turns out that this relation only preserves Einstein causality but violates the causal completeness requirement; if one starts from an AdS theory which fulfills both the resulting conformal field theory fulfills Einstein causality but violates causal completeness and a similar problem exists if one uses the isomorphism in the opposite direction; a physical correspondence requires more than a mathematical isomorphism between certain localized subalgebras.

Unfortunately the knowledge about these important properties (the relevance of causal completeness) which was attained in the early 1960s (Haag & Schroer, 1962) has been lost within the string-theory community, otherwise Maldacena would not have been able to convince a world-wide community that the mathematical $AdS_{n+1} \longleftrightarrow CFT_n$ isomorphism can be lifted to a physical correspondence. Only holographic projections onto a $n-1$ *null-surface* lead to a right "thinning out" of degrees of freedom (loss of information). As a consequence of a loss of some informations one cannot return to the original theory; nevertheless most informations are in the holographic projection.

There exist however situations in certain quantum field theories, which contain massless $s \geq 1$ in which for multiply connected spacetime regions the Haag duality is violated in a specific way; the prototype is the quantum Aharonov–Bohm effect for the net of algebras generated by the quantum electromagnetic field strength (Schroer, 2011a). In the case of zero mass field strengths for $s \geq 1$ this is directly related to the clash between pointlike localization of potentials and the positivity of Hilbert space and its resolution in terms of stringlocal potentials (Schroer-b).

Mathematically it is very easy to construct Einstein-causal theories which violate causal completeness and as a consequence (apart from the aforementioned topological exceptions) lead to pathological physical properties with respect to their "degrees of freedom" behavior. Well-known cases in addition to the mentioned Maldacena conjecture arise from embedding lower dimensional quantum field theories and its reverse: Kaluza–Klein dimensional reductions.

As a result of a subtle relation between the cardinality of phase-space degrees of freedom with localization (split property, causal completeness, etc.), the nuclearity property (introduced by Haag, 1996) in conjunction with modular theory (modular nuclearity) became an important concept for the classification and nonperturbative construction of models of QFT (Lechner, 2008; Schroer, 2013).

After having presented some of the physical requirements of the LQP formulation of QFT, we now pass to a brief description of its main mathematical support: the Tomita–Takesaki modular operator theory. This theory has its origin in the operator-algebraic aspects of group representation algebras from which Tomita took the terminology "modular" (originally referring to properties of Haar measures). A conference in the US (Baton Rouge, 1967), which was attended by mathematicians (Tomita, Takesaki, Kadison, etc.) and mathematical physicists (Haag, Hugenholz, Winnink, Borchers, etc.), marks the beginning of the Tomita–Takesaki modular operator theory as a joint project (Borchers, 2000). The participating physicists had already obtained important partial results of that theory through their project of formulating quantum statistical mechanics directly in the thermodynamic limit (statistical mechanics of *open systems*) (Haag, 1996). In their new way of thinking, the Kubo–Martin–Schwinger property (originally an analytic shortcut for computing Gibbs traces) assumed a conceptual role in the new formulation of thermal equilibrium states for *open quantum systems*. Although these ideas originated independently, this conference united them; there is hardly any area in which the contribution of mathematicians and physicists have been that much on par as in modular operator theory/modular localization.

One reason for this perfect match was that the area of physical application of modular theory widened the scope of statistical mechanics and, combined with *causal localization*, became the most important mathematical/conceptual tool of LQP. The basic fact which led to this new connection was the Reeh–Schlieder theorem (Haag, 1996) which secures the validity of the "standardness" requirement for the applicability of the Tomita–Takesaki theory. Standardness of a pair $(\mathcal{A}, \Omega)$ (algebra and state) means that the action of the operator algebra $\mathcal{A}$ on the state vector Ω generates the Hilbert space (cyclicity of Ω) and that there are no annihilators of Ω in $\mathcal{A}$ (Ω is separating)

$$\text{cycl.}: \overline{\mathcal{A}\Omega} = H, \quad \text{separ.}: A\Omega = 0 \curvearrowright A = 0, \quad A \in \mathcal{A}$$

The Reeh–Schlieder theorem guaranties the validity of this property for any pair $(\mathcal{A}(\mathcal{O}), \Omega), \mathcal{O}'' \subset \mathbb{R}^4$; in fact this even holds if the vacuum is replaced by any finite energy state. The importance of

[17] In quantum physical terms a completeness violating situation exhibits a "poltergeist" behavior: new degrees of freedom (which were not present in $\mathcal{A}(\mathcal{O})$) enter $\mathcal{A}(\mathcal{O}'')$ from "nowhere".

the relation between localization and the T–T theory was noted a decade after then Baton Rouge conference by Haag (1996); these authors found that in the context of localization in a wedge region $\mathcal{O}=W$ the Tomita–Takesaki theory makes contact with known geometrical/physical objects.

The general T–T theory is based on the existence of an unbounded antilinear closable involution S with a dense domain $domS$ in H which contains all states of the form $A\Omega$, in case of a standard pair (Doplicher & Longo, 1984; Summers). Whereas the cyclicity secures the existence of its dense domain, the absence of annihilators of Ω in $\mathcal{A}$ guaranties its uniqueness

$$S_{\mathcal{O}}A\Omega = A^{*}\Omega, \quad A\in\mathcal{A}\subset B(H, \; S = J\Delta^{1/2} = \Delta^{-1/2}J$$
$$J \text{ antiunit.}, \; \Delta^{it} \text{ mod. unitary}, \quad \sigma_t(A) = Ad\Delta^{it}A \tag{8}$$

The existence of a polar decomposition in terms of a antiunitary J and a positive generally unbounded operator Δ follows from the closability of S (in the following S stands for the closure). The modular unitary gives rise to a *modular automorphism* group of the localized algebra $\mathcal{A}$.

The physical interpretation in massive theories is only known for $\mathcal{O}=W=$ wedge regions, which are Poincaré transforms of the standard t–z wedge $W_0=\{z>|t|;\mathbf{x}\in\mathbb{R}^2\}$. In that case the modular objects are the unitary transformation representing the W-preserving Lorentz (boost) subgroup $\Delta_W^{it}=U(\Lambda_W(-\pi t))$ and the reflection on the edge of the wedge J which is, up to a π-rotation within the edge, equal to the TCP operator. Since in a theory with a complete particle interpretation (to which the considerations of this paper are restricted, unless stated otherwise) the interacting TCP operator and its incoming (free) counterpart are known to be related by the scattering operator S_{scat} (Jost, 1963), we obtain for all J for arbitrary W (Schroer, 1999)

$$J_W = S_{scat}J_{W,in} \quad \text{for all } W$$

This expresses a property of S_{scat} which turns out to be indispensable for the constructive use of modular localization in QFT, namely S_{scat} *is a relative modular invariant between the interacting and the associated free* (*particle*) *wedge algebra*. This property was recently used within a more physical derivation (Mund, 2001) of the Bisognano–Wichmann theorem which reduces the interacting case in theories with mass gaps and a complete particle interpretation to that of free fields (see below).

The relative modular invariance of S_{scat} is the crucial property which accounts for the analyticity of on-shell objects as S_{scat} and the related formfactors. These on-shell analytic properties find their important manifestation in the *particle crossing property*. It is also the starting point of the algebraic construction of integrable QFT (Schroer, 1999). The connection between algebraic and analytic properties is much more subtle for on-shell objects as the S-matrix and formfactors than for off-shell correlation function. Since most of these properties were not understood in the 1960s, it is not surprising that Mandelstam's project of formulating particle physics as a quantization-avoiding on-shell project failed on the lack of understanding of the relevant on-shell analytic properties.

The misunderstandings about the particle crossing property in the construction of the *dual model*, which later entered string theory, have their origin in confusions about the meaning of localization in QFT as opposed to QM. In Section 7 these misunderstandings will be analyzed in the light of recent progress.

Since it is not possible to present a self-consistent complete account of the mathematical aspects of modular localization and its physical consequences in a history-motivated setting as the present one, the aim in the rest of this section will be to raise awareness about their physical origin.

It has been known for a long time that the algebraic structure associated to free fields allows a functorial interpretation in which operator subalgebras of the global algebra $B(H)$ are the *functorial images of certain real subspaces* of the Wigner space of one-particle wave functions (the famous so-called "second quantization"[18]), in particular the spacetime localized algebras are the images of localized real subspaces. This means that the issue of localization to some extend can be studied in the simpler form of localized subspaces of the Wigner particle representation space (unitary positive energy representations of the $\mathcal{P}-$group).

These localized subspaces can be defined in a intrinsic way (Brunetti et al., 2002) i.e. without quantization, only using operators from the positive energy representation U of the proper Poincaré group $\mathcal{P}_+(det=+1)$ on the direct sum of two copies of the Wigner representation u of the connected component (proper orthochronous $\mathcal{P}_+^{\uparrow}$) on the one-particle space H_1. For simplicity of notation the transformation formulas are limited to the case of a spinless charged particle

$$H_1 : (\varphi_1,\varphi_2) = \int \overline{\varphi}_1(p)\varphi_2(p)\frac{d^3p}{2p_0}, \quad \hat{\varphi}(x) = \frac{1}{(2\pi)^{3/2}}\int e^{ipx}\varphi(p)\frac{d^3p}{2p_0} \tag{9}$$

$$U(g)(\varphi_1\oplus\varphi_2) = u(g)\varphi_1\oplus u(g)\varphi_2, \quad u(a,\Lambda)\varphi(p) = e^{ipa}u(\Lambda^{-1}p)$$
$$\Theta \equiv TCP, \quad \Theta(\varphi_1\oplus\varphi_2) = C\varphi_2\oplus C\varphi_1, \quad C\varphi(p) = \overline{\varphi(p)} \tag{10}$$

Any $\mathcal{P}_+$ transformation can be generated from $U(g)$ and Θ. For representations with $s>0$ the Lorentz group acts through Wigner rotations (Wigner's "little group") on the "little Hilbert space" which in the massive case is the $2s+1$ component representation space of rotations. The massless case leads to a 2-dimensional Euclidean "little space" whose degenerate representation (with trivially represented "little translations") form a two-component little helicity space, whereas faithful representation acts in an infinite dimensional Hilbert space (infinite spin) (Mund, Schroer, & Yngvason, 2006). The Lorentz transformations as well as Θ act also (through representations of the little group) on the little Hilbert space.

It is precisely through the appearance of this little Hilbert space that the problem of causal localization of states (wave functions) cannot be simply solved by Fourier transformation and adding positive frequency contributions of particles with those of negative frequency from antiparticles. Whereas in the case of the two classes of finite little spaces (the massive and zero mass finite helicity class) of positive energy Wigner representation, their "covariantization" was easily achieved in terms of group theoretic methods (Weinberg, 2000) and led to local pointlike generating wave functions and fields, this third infinite spin class posed a series obstacle. Attempts to convert its members into covariant pointlike wave functions and corresponding fields remained unsuccessful and there was no understanding of the origin of this failure.[19] Weinberg (2000) dismissed this large positive energy representation class by stating that nature does not make use out of it. Since all important physical properties are connected to aspects of localization which are precisely those properties which at that time remained poorly understood, this dismissal could be premature, in particular in times of dark matter.

The localization problems of the infinite spin class were finally solved (Brunetti et al., 2002) with the help of modular localization, a concept which for different problems was already used in Schroer (1999). In fact the main theorem in that paper states (Brunetti et al., 2002) that all positive energy wave functions are

[18] Not to be confused with quantization; to quote a famous saying by Ed Nelson: "quantization is an art, but second quantization is a functor".

[19] Reference Yngvason (1970) is an exception in that certain aspects of the localization problem were already noted.

localizable in noncompact spacelike cones and only the first two classes permit the sharper localization in double cones (the causal shadow of a 3-dim. sphere). Since the (topological) core of arbitrarily small double cones is a point and that of arbitrary narrow spacelike cones a semi-infinite spacelike string, the remaining problem consisted in the actual construction of the generating fields of these representation; this was achieved in Mund et al. (2006). The result can be described in terms of operator-valued distributions $\Psi(x,e)$ which depend in addition to the start x of the semi-infinite string also on the spacelike direction $e, e^2=-1$. They are covariant under simultaneous transformations of x and e and fulfill Einstein causality for spacelike separated strings

$$[\Psi_1(x_1,e_1),\Psi_2(x_2,e_2)]_{gr}=0, \quad x_1+\mathbb{R}_+e\rangle\langle x_2+\mathbb{R}_+e_2 \tag{11}$$

where gr stands for graded (fermionic strings anticommute).

The modular localization of states uses the following construction. With a wedge $W=(x|x_3>|x_0|)$ there comes a wedge-preserving one-parametric group of Lorentz-transformation $\Lambda_W(\chi=-2\pi\tau)$ where χ is the hyperbolic boost parameter and Θ_W denotes the x_0-x_3 reflection. The latter differs from the total reflection Θ by a π-rotation r_W around the x_3 axis (in the x_1-x_2 plane) and therefore acts on the wave functions as $J_W=U(r_W)\Theta$. Both transformations Λ_W and J_W commute. Since the generators of one-parametric strongly continuous unitary groups are self-adjoint operators, there exists an "analytic continuation" in terms of positive unbounded operators with dense domains which decrease with the increase of distance from the real axis. This forces the W-localized wave functions to have certain analyticity properties in the momentum space rapidity θ, $(p_0,p_3)=\sqrt{m^2+p_\perp^2}(ch\theta, sh\theta)$ which relate the analytic continuation of particle wave function to the complex conjugate of the antiparticle wave function.[20] Using the notation $\Delta_W^{i\tau}\equiv U(\Lambda_W(-2\pi\tau))$, the commutation with the antiunitary J_W leads to

$$\begin{aligned} &S_W=J_W\mathfrak{d}_W^{1/2}=\mathfrak{d}_W^{-1/2}J_W, \quad S_W^2\subset 1 \text{ acts on } H_1\oplus H_1\\ &S_W\psi=\overline{\psi}, \quad K_W\equiv\{\varphi\in domS_W; S_W\varphi=+\varphi\}, \quad S_Wi\varphi=-i\varphi\\ &K_W \text{ "is standard"}: K_W\cap iK_W=0, \quad K_W+iK_W \text{ dense in } H_1\oplus H_1 \end{aligned} \tag{12}$$

where $\overline{\psi}$ denotes the localization-independent S-conjugate wave function (the complex conjugate for the case at hand).[21] The properties are straightforward consequences of the commutation between the boost and the associated reflection (Brunetti et al., 2002). The important point here is that S relates wave functions to their conjugates in a way which involves analytic continuation where the analyticity came from spacetime W-localization.

The properties in (12) result simply from the commutativity of $\Lambda_W(\chi)$ with the reflection J on the edge of the wedge; since J is antiunitary it commutes with the unitary boost, there will be a change of sign in its action on the analytic continuation of u. Hence it has all the properties of a modular Tomita operator. The K-spaces $K(\mathcal{O})$ for causally closed subspaces localized in $\mathcal{O}$ can be obtained by intersections i.e. $\cap_{W\supset\mathcal{O}}K(W)$; this intersection may however turn out to be trivial (see below) if the region is "too small".

The surprise resides in the fact that the transformation of wave functions to their S-conjugate ((12), second line) does not only encode the information about two geometric objects: a one-parametric modular group leaving a wedge invariant and a reflection on that wedge into its opposite, but (and at this point the positive energy property of the Wigner representation becomes relevant Brunetti et al., 2002) it also contains the information about the spacetime localization of the wave function. This is certainly something which has no counterpart in QM; it points to an incomplete understanding of the foundations of QFT which becomes fully revealed in the relation between localized subalgebras and modular operator theory in the presence of interactions.

The connection with causal localization is of course a property which only appears in the physical context. The general setting of modular real subspaces is a Hilbert space which contains a real subspace $K\subset H$ which is standard in the sense of (12). The abstract S-operator is then defined in terms of K and iK.

The above application to the Wigner representation theory of positive energy representations[22] also includes the *infinite spin representations* which lead to semi-infinite string-localized wave functions. In this case there are no pointlike covariant wave function-valued distributions which generate these representations; they are genuinely string-localized (which the superstring representation of the Poincaré group is not; so beware of misleading terminology!). The application of the above-mentioned second quantized functor converts the modular localized subspaces into a net of $\mathcal{O}-$indexed interaction-free subalgebras $\mathcal{A}(\mathcal{O})$. Interacting field theories can clearly not be obtained in this way. The relation between particles and fields becomes much more subtle in the presence of interactions and this applies even to models which have a complete particle interpretation i.e. in which the particles related to fields via the LSZ large time behavior of fields (the LSZ scattering formalism, Haag, 1996) lead to the identification of the Hilbert space as a WignerFock particle space (Section 7).

The algebraic setting in terms of modular localization also gives rise to a physically extremely informative type of inclusion of two algebras which share the vacuum state, the so-called *modular inclusions* $(\mathcal{A}\subset\mathcal{B},\Omega_{vac})$ where modular means that the modular group of the bigger $\Delta_{\mathcal{B}}^{it}$ compresses (or extends) the smaller algebra (Kaehler & Wiesbrock, 2001). A modular inclusion automatically forces the two algebras to be of the monad type. The above-mentioned "GPS construction of a QFT" from a finite number of monads positioned in a common Hilbert space uses this concept in an essential way. It is perhaps the most forceful illustration of the holistic nature of QFT.

There are two properties which always accompany modular localization and which are interesting in their own right. Both are related to the statistical mechanics nature of impure $\mathcal{A}(\mathcal{O})-$restricted vacuum:

- *KMS property*: By ignoring the world outside $\mathcal{O}$ one gains infinitely many KMS modified commutation properties with modular Hamiltonians $\hat{K}$ associated to the $\hat{\mathcal{O}}$ restricted vacuum

$$\langle AB\rangle=\left\langle Be^{-K}A\right\rangle, \quad \Delta=e^{-K}, \quad A,B\in\mathcal{A}(\mathcal{O})$$
$$\text{infinitely many } \widehat{K} \text{ for } \widehat{\mathcal{O}}\supset\mathcal{O}$$

In contrast to the inert factorizing vacuum of QM in the Fock space (second quantization) description, the spatially restricted

[20] If there exists an operator creating a particle, the negative frequency part associated with the antiparticle annihilation must be related to the positive frequency part of the antiparticle creation of its hermitian adjoint.

[21] Although the action of S_W is diagonal, the definition of the J_W needs the antiparticle doubling of the Wigner space.

[22] The positive energy condition is absolutely crucial for obtaining the prerequisites (12) of modular localization.

QFT vacuum fulfills infinitely many KMS relations associated with modular Hamiltonians of larger spacetime regions:

- Area law for localization-entropy, see (6)

  $\mathrm{Entr} = f\left(\frac{\mathrm{area}}{\varepsilon^2}\right), \quad \varepsilon = \text{split size}$

As mentioned in the previous section, one needs to invoke the so-called split property in order to approximate the singular KMS state by a sequence of density matrix states; this is similar to the construction of the thermodynamic limit state in statistical mechanics. In contrast to the approximation of the latter in terms of box-quantized finite volume Gibbs states, the split formalism for open subsystems is a part of the (presently computational rather inaccessible) modular localization theory. It is in particular not clear whether the density matrix from the split property leads to a plain dimensionless area law $f \simeq \mathrm{area}/\varepsilon^2$[23] as in (6) or to a logarithmically modified area law (Schroer, 2011b). For chiral conformal theories on the lightray there is a rigorous derivation of the localization entropy for an interval with vacuum attenuation length ε (surface fuzziness) from the well-known linear length $l \to \infty$ behavior (the "one-dimensional volume factor" l). They are related as $\ln \varepsilon^{-1} \sim l \times kT$. This *inverse Unruh effect* plays an important role in the full understanding of the E–J conundrum and will be presented in the next section.

Great care needs to be taken in identifying the modular localization "temperature" with that measured with a thermometer. This is because the notion of thermometer temperature is based on the zeroth thermodynamic law (the *local temperature* in Buchholz & Solveen, 2013), whereas the KMS temperature refers to the second law according to which it is impossible to gain energy from equilibrium states by running a Carnot cycle (the absolute temperature). In inertial systems those two definitions coalesce (after proper normalization), whereas in a accelerated systems (used e.g. in the Unruh Gedankenexperiment to achieve the Rindler-wedge localization) this is not the case.

A closer examination shows (Buchholz & Solveen, 2013) that the conclusion about "egg-boiling" and particle radiation claimed to be observed by an accelerated observer are incorrect, a fact which has been ignored in the literature on the Unruh effect. The correct local temperature, different from the Carnot temperature, does not depend on the acceleration and since it vanishes at spacelike infinity, it vanishes everywhere. Although the black hole situation is different, the application of Einstein's equivalence principle suggests caution about the relation of a rescaled modular temperature with that measured by a thermometer. This includes also the presently very popular ideas about *firewalls* which are allegedly created by restricting generically locally normal states to a causal/event horizon.

In order to facilitate the reader's accessibility to philosophical and historical aspects, but also to maintain a lighthearted touch in dealing with issues which by some are considered to be controversial, the following will be presented in the form of Galileio's famous dialogs between Sagredo and Simplicio. Since fundamental properties of nature are expected to be based on simple physical principles, the role of the presenter of foundational viewpoints in this dialog is Simplicio.

Sagredo: Dear friend Simplicio, I noticed that you have some critical opinions about the topic of extra dimensions and dimensional reductions. Can you explain your arguments against these extremely popular ideas?

[23] This is suggested by the vacuum polarization clouds of smeared fields in the limit of a sharp cut-off smearing function (see previous section).

Simplicio: Kaluza and Klein observed that in classical field theories and quasiclassical approximations one may relate models in different spacetime dimensions by appropriately reinterpreting the field content. In this way the combined gravitation + electromagnetism may be obtained by dimensional reduction from a five-dimensional pure gravity theory. However the recent more foundational understanding of the issue of causal localization in its precise form of *modular localization of quantum matter* reveals that the localization aspects are a characteristic part of quantum matter and one confronts grave problems if one tries to reduce spacetime dimensions. A first indication comes from Wigner's theory of positive energy representations of the Poincaré group which has a functorial relation to quantum matter in the absence of interactions. The latter depend in an essential way on the representation theory of Wigner's "little group" which changes with spacetime dimension. The fact that dimensional regularization can be used as a technical trick in renormalization theory and that in case of spinless matter Wilson's dimensional ε-expansion led to reasonable approximate results for critical indices should not be taken as an indication that causal quantum matter can be "transplanted" by an imagined dimensional reduction.

Sagedo: But there *are* rigorous relations between theories in different spacetime dimensions, as the famous $AdS-CFT$ correspondence.

Simplicio: The $AdS_{n+1}-CFT_n$ correspondence is indeed a mathematical isomorphism between the algebraic structure of QFT on two different spacetimes which extends the prior known equality of their symmetry groups; in fact it is the only known case in which two spacetime manifolds in different dimensions share the same symmetry group. What prevents this mathematical isomorphism from defining a physical correspondence is that it does not preserve an important aspect of causality. Starting from a causal AdS theory the corresponding CFT maintains the spacelike Einstein causality but violates the causal completeness property. There are more degrees of freedom in the algebra of the causal closure $\mathcal{A}(\mathcal{O}'')$ than there are in $\mathcal{A}(\mathcal{O})$. For an observer living in such a world there are degrees of freedom in $\mathcal{O}''$ which according to the causal completeness property should have been already present in $\mathcal{O}$. A QFT in which new degrees of freedom come apparently from nowhere is physically not acceptable. In the opposite direction, i.e. started from a causal CFT, it was shown by Rehren (2000) that the resulting AdS theory does not have enough degrees of freedom in order to support the existence of nontrivial algebras of observables for compact localization regions; in such situations nontrivial algebras only exist for noncompact spacetime localization regions in the AdS spacetime.

The intuitive picture behind this violation of causal completeness is that the cardinality of degrees of freedom of causal quantum matter depends on the spacetime dimensionality and hence the concept of causal quantum matter cannot be separated from spacetime. The algebraic stuff which the above isomorphism generates from physical matter is not the expected causal quantum matter after having applied the isomorphism. This shows that Maldacena's conjecture, claiming that the isomorphism connects two physical theories, is not correct. This failure of causal completeness is symptomatic for all attempts of relating QFTs via dimensional reduction.

The AdS–CFT isomorphism shown that even under optimal mathematical conditions such ideas run into serious problems with causal localization. It is worthwhile to mention that there is only one relation between QFTs to which the present critique does not apply; this is the holographic projection onto null-surfaces (Schroer, 2011b). The important point here is that in an projection (instead of an isomorphism) the cardinality of degrees of freedom is reduced to that which is appropriate for the lower dimensional null-surface.

Sagredo: The classical Kaluza–Klein dimensional reduction idea entered particle theory when it became clear that the high dimensional solutions of string theory remain academic unless one finds a way to extract properties which are relevant for the real world. The attempts to adjust the dimensional reduction in classical field theory to the requirements of QFT led to the idea to compactify a spacetime coordinate and "curl it up" to a tiny circle so that the resulting QFT appears as one which lives on a reduced spacetime. Therefore my question: is a such a dimensional curling up also flawed?

Simplicio: It is correct that for QFTs which permit a Euclidean description one can formally compactify a coordinate. Physically this means that one passes from the vacuum expectation values to expectation values in a thermal state whose inverse temperature is proportional to the radius of the circular compactified coordinate. What is however not correct is to relate this thermal QFT with increasing temperature to a Klein–Kaluza reduction. There is simply no classical analog of increasing thermal fluctuations; passing to a thermal state with a high temperature has little to do with a dimensional reduction a la Kaluza–Klein.

The continued uncritical use of the dimensional reduction idea is more of a sociological problem; as long as the protagonists and leading defenders of string theory accept dimensional reduction as a way which allows to obtain properties of real particle theory from theories with extra dimensions, the members of the string theory community will continue to use it with the result that the understanding of local quantum physics will becomes increasingly metaphoric.

Of course particle physics at its foundational frontiers was always speculative and errors are sometimes unavoidable, but the old "Streitkultur" between equals at the time of Pauli, Landau, Feynman, Schwinger Jost, Källen and many others prevented a long term solidification of incorrect ideas.

Sagredo: Thank you my dear friend for your enlightening comments.//

5. The E–J conundrum, Jordan's model

With the *locally restricted vacuum* representing a highly impure state with respect to *all* modular Hamiltonians $H_{mod}(\check{\mathcal{O}})$, $\check{\mathcal{O}} \supseteq \mathcal{O}$ on local observables $A \in \mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$, a fundamental conceptual difference between QFT and QM has been identified. QM (type I_∞ factors) is the conceptual home of *quantum information theory*,[24] whereas in case of localized subalgebras of QFT a direct assignment of entropy and information content to a monad, if possible at all, can only be done in a limiting sense. The present work shows that although QFT started with this conceptual antagonism in the E–J conundrum, its foundational understanding only began more than half a century later and is still far from its closure.

For this reason it is more than a historical retrospection to re-analyze the E–J conundrum from a contemporary viewpoint. In a modern setting Jordan's two-dimensional photon[25] model is a chiral current model. As a two-dimensional zero mass field which solves the wave equation it can be decomposed into its two u,v lightray components

$$\partial_\mu \partial^\mu \Phi(t,x) = 0, \quad \Phi(t,x) = V(u) + V(v), \quad u = t + x, \quad v = t - x$$

$$j(u) = \partial_u V(u), \quad j(v) = \partial_v V(v), \quad \langle j(u), j(u') \rangle \sim \frac{1}{(u - u' + i\varepsilon)^2}$$
$$T(u) = :j^2(u):, \quad T(v) = :j^2(v):, \quad [j(u), j(v)] = 0 \tag{13}$$

The scale dimension of the chiral current is $d(j) = 1$, whereas the energy–momentum tensor (the Wick-square of j) has $d(T) = 2$; the u and v worlds are completely independent and it suffices to consider the fluctuation problem for one chiral component. The logarithmic infrared divergence problems of zero-dimensional chiral $d(V) = 0$ fields arise from the fact that the zero mass field V, different from what happens in higher dimensions.[26] are really stringlike instead of pointlike localized. In fact the V is best pictured as a semi-infinite line integral (a string) over the current (Schroer, 2011c); this underlines that the connection between infrared behavior and string-localized quantum matter also holds for chiral models on the lightray. It contrasts with QM where the infrared aspects are not related to the infinite extension of quantum matter but rather with the *range of forces* between particles. Exponentials of string-localized quantum fields involving integration over zero mass string localized $d = 1+3$ vectorpotentials share with the exponentials of integrals over $d = 1+1$ currents $\exp i\alpha V$ the property that their infrared behavior requires a representation which is inequivalent to the vacuum representation of the field strength or currents; the emergence of superselection rules (Maxwell charges) is one of the more radical consequences of string-localization.

The E–J fluctuation problem can be formulated in terms of j (charge fluctuations) or T (energy fluctuations). It is useful to recall that vacuum expectations of chiral operators are invariant under the fractionally acting 3-parametric acting Möbius group (x stands for u, v)

$$U(a) j(x) U(a)^* = j(x+a), U(\lambda) j(x) U(\lambda)^* = \lambda j(\lambda x) \text{ dilation}$$
$$U(\alpha) j(x) U(\alpha)^* = \frac{1}{(-\sin \pi\alpha + \cos \pi\alpha)^2} j\left(\frac{\cos \pi\alpha x + \sin \alpha}{-\sin \pi\alpha x + \cos \pi\alpha}\right) \text{ rotation} \tag{14}$$

The next step consists in identifying the KMS property of the locally restricted vacuum with that of a global system in a thermodynamic limit state. For evident reasons it is referred to as the *inverse Unruh effect*, i.e. finding a localization-caused thermal system which corresponds (after adjusting parameters) to a heat bath thermal system. In the strong form of an isomorphism this is only possible under special circumstances which are met in the Einstein–Jordan conundrum, but not in the actual Unruh Gedankenexperiment for which the localization region is the Rindler wedge.

Theorem 1 (*Schroer (2013)*). *The global chiral operator algebra $\mathcal{A}(\mathbb{R})$ associated with the heat bath representation at temperature $\beta = 2\pi$ is isomorphic to the vacuum representation restricted to the half-line chiral algebra such that*

$$(\mathcal{A}(\mathbb{R}), \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_+), \Omega_{vac})$$
$$(\mathcal{A}(\mathbb{R})', \Omega_{2\pi}) \cong (\mathcal{A}(\mathbb{R}_-), \Omega_{vac}) \tag{15}$$

The isomorphism intertwines the translations of $\mathbb{R}$ with the dilations of $\mathbb{R}_+$, such that the isomorphism extends to the local algebras

$$(\mathcal{A}((a,b)), \Omega_{2\pi}) \cong (\mathcal{A}((e^a, e^b)), \Omega_{vac}) \tag{16}$$

This can be shown by the modular theory. The following proof extends prior work by Borchers and Yngvason (1999). Let $\mathcal{A}$

[24] Another subject which would have taken different turn with a better appreciation of the problems in transferring notions of quantum information theory to QFT is the decades lasting conflict about the problem of "black hole information loss".

[25] This terminology was quite common in the early days of field quantization before it was understood that that in contrast to QM the physical properties depend in an essential way on the spacetime dimension. Jordan's two-dimensional photons and his later neutrinos (in his "neutrino theory of light" Schroer, 2011c) bear no relation to objects in the real world.

[26] The V are semi-infinite integrals over the pointlike j's, just as the stringlike vectorpotentials in QED are semi-infinite integrals over pointlike field strength (Schroer, 2011a).

denote the C^* algebra associated to the chiral current j.[27] Consider a thermal state ω at the (for convenience) modular temperature 2π associated with the translation on the line. Let $\mathcal{M}$ be the operator algebra obtained by the GNS representation and $\Omega_{2\pi}$ the state vector associated to ω. We denote by N the half-space algebra of $\mathcal{M}$ and by $\mathcal{N}' \cap \mathcal{M}$ the relative commutant of $\mathcal{N} in \mathcal{M}$. The main point is now that one can show that the modular groups $\mathcal{M}, \mathcal{N}$ and $\mathcal{N}' \cap \mathcal{M}\}$ generate a "hidden" positive energy representation of the Möbius group $SL(2,R)/Z_2$ where hidden means that the actions have no geometric interpretation on the thermal net. The positive energy representation acts on a hidden vacuum representation for which the thermal state is now the vacuum state Ω. The relation of the previous three thermal algebras to their vacuum counterpart is as follows:

$$\mathcal{N} = \mathcal{A}(1,\infty), \quad \mathcal{N}' \cap \mathcal{M} = \mathcal{A}(0,1), \quad \mathcal{M} = \mathcal{A}(0,\infty) \tag{17}$$

$$\mathcal{M}' = \mathcal{A}(-\infty,0), \quad \mathcal{A}(-\infty,\infty) = \mathcal{M} \vee \mathcal{M}' \\ \mathcal{M}(a,b) = \mathcal{A}(e^{2\pi a}, e^{2\pi b}) \tag{18}$$

Here $\mathcal{M}'$ is the "thermal shadow world" which is hidden in the standard Gibbs state formalism but makes its explicit appearance in the so-called *thermo-field* setting i.e. the result of the GNS description in which Gibbs states described by density matrices or the KMS stated resulting from their thermodynamic limits are described in a vector formalism. The last line expresses that the interval algebras are exponentially related.

In the theorem we used the more explicit notation

$$\mathcal{M}(a,b) = (\mathcal{A}(a,b), \Omega_{th}) = (\mathcal{A}(e^{2\pi a}, e^{2\pi b}), \Omega_{vac})$$

Moreover we see that there is a natural space–time structure also on the shadow world i.e. on the thermal commutant to the quasilocal algebra on which this hidden symmetry naturally acts. Expressing this observation a more vernacular way: the thermal shadow world is converted into virgin living space.[28] In conclusion, we have encountered a rich hidden symmetry lying behind the tip of an iceberg, of which the tip was first seen by Borchers and Yngvason.

Although we have assumed the temperature to have the Hawking value $\beta = 2\pi$, the reader convinces himself that the derivation may easily be generalized to arbitrary positive β as in the cited Borchers–Yngvason work. A more detailed exposition of these arguments is contained in a paper *Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models* (Schroer & Wiesbrock, 2000).

In this way the semi-infinite interval $(-\infty, -L)$ of a thermal system in a one-dimensional interval $(-L, L)$ of length $2L$ (one-dimensional "box") passes to the split interval ε (the size of Heisenberg's vacuum polarization cloud) $\varepsilon \sim e^{-2\pi L}$. As a result the thermodynamic $L \to \infty$ corresponds to the limit of sharp localization $(e^{-2\pi L}, e^{2\pi L}) \overset{L\to\infty}{\to} (0,\infty)$ on the vacuum side. From this one draws the conclusion that the thermal heat bath entropy for large L passes to the localization-entropy in the vacuum state for small split distance ε

$$En_{kT=2\pi} \simeq L = -\frac{1}{2\pi} \ln \varepsilon \simeq En_{loc} \tag{19}$$

Though it is unlikely that a localization-caused thermal system is isomorphic to a heat bath thermal situation in higher dimensions, there may exist a "weak" inverse Unruh situation in which the volume factor corresponds to a logarithmically modified dimensionless area law i.e. $(R/\Delta R)^{n-2} \ln(R/\Delta R)$ where R is the radius of a double cone and $R/\Delta R = \varepsilon$ its dimensionless surface roughness; the volume in this case is that of a box with two transverse- and one lightlike-directions is the counterpart of the spatial box so that the volume factor V corresponds to a box where one direction, the one responsible for the logarithmic factor, is lightlike. But the analogy with the area proportionality of vacuum fluctuations in Heisenberg's partial charges $Q(R, \Delta R)$ favors the area law which also agrees with the result from 't Hooft's proposal of a brickwall picture ('tHooft, 1996).

Although the thermal aspect of a restricted vacuum in QFT is a structural consequence of causal localization, the general identification of the dimensionless modular temperature with an actual temperature of a heat bath system, or, which is equivalent, the modular "time" with the physical time is not correct; the modular Hamiltonian does not describe the inertial time for which the local temperature defined in terms of the zeroth thermodynamic law agrees with the "Carnot temperature" of the second law (Buchholz & Solveen, 2013).

Properties of states in QFT depend on the nature of the algebra: a monad does not have pure states or density matrices, but only admits rather singular impure states as singular (non Gibbs) KMS states. The identification of states with vectors in a Hilbert space up to phase factors becomes highly ambiguous and physically impractical outside of QM. The state in the form of a linear expectation functional on an algebra and the unique vector (always modulo a phase factor) obtained by the intrinsic GNS construction (Haag, 1996) leads always to a vector representation, but this depends on the particular state used for the GNS construction. In QM the algebras are always of the $B(H)$ type where this distinction between vector states and state vectors is not necessary.

6. Particle crossing, on-shell constructions from modular setting

An important new insight into "particles and fields" comes from a derivation of the *crossing property* of particle physics from the modular properties of wedge-localization. The formfactor crossing states that the n-particle-to-vacuum matrixelement of a local operator B is analytically related to the *connected part* of the formfactors of B between k incoming and $n-k$ outgoing particles in terms of the following identity:

$$\langle 0|B|p_1,..p_n\rangle^{in} = {}^{out}\langle -\overline{p}_{k+1}.., -\overline{p}_n|B|p_1,..p_k\rangle^{in}_{con} \\ B \in \mathcal{A}(\mathcal{O}), \quad \mathcal{O} \subseteq W, \quad \overline{p} = \text{antiparticle of } p \tag{20}$$

Here the momenta $-\overline{p}$ on the backward mass–shell refer to the anti-particles of the $n-k$ crossed particles of the original n-particle state where the transition to the negative momenta involves an analytic continuation within the complex mass–shell. The analyticity following from principle of modular wedge-localization is however not in the s,t,u Mandelstam invariants associated to the momenta, but rather in the rapidity θ variables. It turns out that the better-known crossing properties of the S-matrix do not have to be considered separately, they can be related to those of formfactors by the use of the LSZ reduction formalism. The nontrivial aspect is the possibility to relate a scattering amplitude to that of its crossed form by an analytic continuation which remains on the complex mass shell.

The physical content of formfactor crossing is that the different k to $n-k$ formfactors are analytically related to one master formfactor which may be taken to be the n-particle to vacuum formfactor. The only known non-perturbative general derivation of

[27] One can either obtain the bounded operator algebras from the spectral decomposition of the smeared free fields $j(f)$ or from a Weyl algebra construction.

[28] In Haag (1996) it is shown how to extract the shadow world description from the density matrix (Gibbs states) formalism with the help of the canonical GNS construction.

formfactor crossing uses modular theory,[29] to be more precise the modular theory of a wedge-localized subalgebra. Before a sketch of its derivation will be given, some remarks about its conceptual relation to other consequences of modular localization of wedge regions may be helpful. The conceptual proximity of the particle crossing property to the Unruh (1976) effect through the shared wedge localization is somewhat unexpected. Whereas the latter together with the Einstein–Jordan subvolume fluctuations will probably remain a "Gedankenexperiment" which illustrates consequences of vacuum entanglement, the particle crossing has observational consequences (Epstein, Glaser, & Martin, 1969) and constitutes an important concept of high energy particle physics. This changes the conceptual setting of crossing from that attributed to it in the dual model and ST, a topic which will be taken up in Section 7.

The modern conceptual understanding of crossing came from the recognition that in models of QFT with a mass gap and a complete particle interpretation the S-matrix is more than a global operator of scattering theory, it also possesses a less conspicuous local property namely it is a *relative modular invariant* which intertwines between the interacting wedge algebra $\mathcal{A}(W)$ and its interaction-free incoming counterpart constructed from the incoming free fields $\mathcal{A}(W)_{in}$. Namely the two modular reflections are related through (Schroer, 1999, 2013)

$$J_W = J_{W,in} S_{scat} \quad \text{or} \quad S_W = S_{W,in} S_{scat} \quad \text{using } S = J\Delta^{1/2} \tag{21}$$

a relation which can be traced back to Jost's proof (Jost, 1963) of the TCP theorem and the fact that J_W is only different from Jost's TCP by a π-rotation within the edge of the wedge (which commutes with the Poincaré invariant S_{scat}).

Another idea from modular wedge-localization which is used in the derivation of formfactor crossing is *emulation* of interacting wedge-localized states (state vectors obtained by applying interacting smeared fields $B(f)$ with supp$f \subset W$ to the vacuum Ω) in terms of interaction-free wedge-localized states obtained by applying operators $A_{in}(f)$ to the vacuum (Schroer, 2013, 2012). Emulation involves different algebras acting in the same Hilbert space and sharing the same $\mathcal{P}-$goup representation.

To get some technicalities out of the way, let us first formulate the *free field KMS* relation in the way we need it for later purpose. With B a W-smeared composite of a free field, the modular KMS relation for wedge-localized free fields reads

$$\left\langle BA^{(1)}A^{(2)}\right\rangle = \left\langle A^{(2)}\Delta BA^{(1)}\right\rangle, \quad \Delta^{it} = U(\Lambda(-2\pi t))$$
$$A^{(1)} =: A(f_1)..A(f_k):, \quad A^{(2)} =: A(f_{k+1})..A(f_n):$$
$$\Delta A^{(2)*}|0\rangle = \Delta SA^{(2)}|0\rangle = \Delta^{1/2}JA^{(2)}|0\rangle \tag{22}$$

A smeared free field can be written in terms of creation/annihilation operators integrated with wavefunctions which are the mass–shell restriction of the Fourier transforms of W-supported test functions (for economy of notation f will also be used for the Fourier transform)

$$A(f) = \int (f(p)a^*(p) + \overline{f}_a(p)b(p))\frac{d^3p}{2p_0}, \quad p \in H_m$$
$$A(f)^* = \int (f_a(p)b^*(p) + \overline{f}(p)a(p))\frac{d^3p}{2p_0} \tag{23}$$

where f_a is the wavefunction of the antiparticle. We take the wedge W in the 0–3 directions, so that it is left invariant by Λ_{0-3} Lorentz boosts, and parametrize the mass–shell momenta in terms of W-affiliated rapidities. It is well known that the Fourier transforms of W-supported testfunctions lead to wavefunctions $f(p)$ which are boundary values of functions $f(p(z))$, holomorphic in the rapidity strip in such a way that the analytic continuation of the particle wave function to the other side of the strip is equal to the complex conjugate of the antiparticle wavefunction

$$p(z) = (m\operatorname{sh}z, m\operatorname{ch}z; p_\perp), \quad 0 < \operatorname{Im}z < \pi$$
$$f(p(\theta + i\pi)) = \overline{f}_a(p(\theta)) \tag{24}$$

Rewriting the KMS relation (22) in terms of particle states we obtain

$$\int .. \int \langle 0|B|p_1,..p_n\rangle f_1(p_1)..f_n(p_n)\frac{d^3p_1}{2p_{0,1}}..\frac{d^3p_n}{2p_{0,n}} + \text{contr.}$$
$$= \int .. \int (\Delta^{1/2}J|\overline{p}_{k+1}..\overline{p}_n\rangle, B|p_1,..p_k\rangle)f(p_1)..f(p_n)\frac{d^3p_1}{2p_{0,1}}..\frac{d^3p_n}{2p_{0,n}} + \text{contr.} \tag{25}$$

where the round bracket in the second line denotes the scalar product between the bra and ket vectors and contr. stands for the contraction terms between two Wick-products. They contain a lower number of particles and hence do not contribute to the n-particle terms. The third line in (22) was used inside the inner product in order to rewrite the right hand side of the KMS relation as a matrix element of B between particle states.

To pass to the crossing relation (20) we must show that one can omit the integration with the dense set of strip-analytic wavefunction. Since formfactors in interacting models are generally distributions, this is not possible without knowing that the formfactors are locally square integrable; in this case the relation on a dense set of wave functions implies its validity on all locally L^2 integrable functions and hence (20) follows. Here B is any composite of a free field.

In the presence of interactions the extraction of the particle crossing from the KMS relation is more demanding. Particles are related to (incoming/outgoing) free fields, whereas the fields in the KMS relation are interacting. The crossing relation (20) which we want to derive contains in and outgoing particles which are associated with in/out free fields. We need to know a relation between incoming and interacting wedge localized states. Using the notation: $\mathcal{A}(W), \mathcal{A}_{in}(W)$ for the interacting and incoming free field wedge-local algebra and recalling that both algebras share the same representation of the Poincaré group, one obtains from the equality of the W-preserving Lorentz boosts the equality of the domains of their Tomita operators $domS_{\mathcal{A}(W)} = domS_{\mathcal{A}_{in}(W)}$. This means that for a vector state created by applying a wedge-local operator from $\mathcal{A}_{in}(W)$ to the vacuum there will be a corresponding uniquely defined operator in $\mathcal{A}(W)$ operator which, applied to the vacuum, creates the same vector. Existence and uniqueness is secured by the modular theory applied to the wedge region (Borchers, Buchholz, & Schroer, 2001). We refer to this bijection between wedge local operators as *emulation of wedge localized free fields within the interacting wedge algebra* (Schroer, 2012, 2013) and denote the emulated image by a subscript $\mathcal{A}(W)$

$$|f_1, ..., f_k\rangle = A_{in}(f_1)...A_{in}(f_k) : |0\rangle$$
$$= (: A_{in}(f_1)...A_{in}(f_k) :)_{\mathcal{A}(W)}|0\rangle, \quad \text{supp}f \subset W \tag{26}$$

where, as before, the f inside the bracket state vectors are the wave functions associated with the W-supported testfunctions.

The KMS relation for interacting fields, from which the particle crossing is to be derived, reads now (Schroer, 2010a)

$$\left\langle B\left(A_{in}^{(1)}\right)_{\mathcal{A}(W)}\left(A_{in}^{(2)}\right)_{\mathcal{A}(W)}\right\rangle = \left\langle \left(A_{in}^2\right)_{\mathcal{A}(W)}\Delta B\left(A_{in}^1\right)_{\mathcal{A}(W)}\right\rangle$$
$$\Delta\left(A_{in}^{(2)}\right)^*_{\mathcal{A}(W)}|0\rangle = \Delta^{1/2}JA_{out}^{(2)}|0\rangle, \quad J = S_{scat}J_{in} \tag{27}$$

The identification of the right hand side with a (analytically continued) particle formfactors is similar to the free case; the

[29] For a special case (elastic scattering) Bros, Epstein, & Glaser (1965) derived crossing of the S-matrix within the rather involved setting of functions of several analytic variables.

difference is the presence of the scattering matrix which converts an incoming bra-state into an outgoing state

$$\left\langle BA_{in}^{(1)}(p_1,\dots p_k)_{\mathcal{A}(W)}|p_{k+1},\dots p_n\right\rangle^{in} \simeq {}^{out}\left\langle -\overline{p}_{k+1},\dots -\overline{p}_n|B|p_1,\dots p_k\right\rangle^{in} \tag{28}$$

The equivalence sign expresses the fact that the equality according to (27) only holds after integration with wavefunctions from a dense set of W-localized wave functions, and the Ψ stands for a state obtained by applying an emulated k-particle operator to an $n-k$ incoming state. It depends on n on-shell particle momenta but is not an incoming n-particle state (+ contributions from contractions)[30]; the product of emulations of free field states is not the emulation of the product of the latter. In order to relate the action of an "k emulat" on a $n-k$ particle state one needs an additional idea.

There exists a concept which achieves this: *the analytic on-shell order change.* It arose in the setting of integrable models (Babujian, Fring, Karowski, & Zapletal, 1999) and consists in an analytic interchange of particle momenta within formfactors which, in the presence of interactions, is different from the kinematical interchange in terms of statistics. For simplicity of notation we restrict to $d=1+1$ in which case on-shell formfactors are fully described by rapidities θ. We define a new object (denoted by a superscript an) in a special configuration

$$\langle B|\theta_1\dots\theta_n\rangle^{an} \equiv \langle B|\theta_1,\dots,\theta_n\rangle_{in} \quad \text{for } \theta_1 > \dots > \theta_n \tag{29}$$

Using (bosonic) particle statistics, formfactors can always be written in this naturally ordered form. An analytic ordering change along a certain path leads from the natural order to a different formfactor function which depends not only on the new order but also on the path of the analytic continuation which was used to achieve it. The resulting object is still on-shell, but one generally does not know its interpretations (or representation) in terms of particle states.

Fortunately for the derivation of the momentum space crossing one does not have to know the particle content after the analytic changes. If the formfactors are locally square integrable one can, by using wave functions with ordered θ-supports, always "filter out" the natural order. This is achieved by passing from wedge-local wave functions, which are spread (27) over θ-line, to wave functions supported in naturally ordered θ-intervals. In other words the on-shell analytic ordering property permits to reduce the derivation of the crossing property in the presence of interactions to that of the interaction-free case; the presence of interactions would only show up in the unknown contributions from different orders. Before we attempt to algebraize the analytic ordering idea it is helpful to take a look at the simpler case of integrable models.

Integrable models permit an explicit illustration of the previous arguments, including an operator-encoding of analytic ordering changes into a representation of the permutation group (with the analytic transpositions being defined in terms of the 2-particle elastic scattering matrix). In fact the emulated free fields[31] turn out to be identical to the Fourier transforms of the Zamolodchikov operators which obey the Zamolodchikov–Faddeev algebra (see (30) below).

This simplicity has its mathematical origin in restrictive domain properties of emulats which characterize integrability (Borchers et al., 2001). Emulats in general QFT only inherit the invariance property of their domains under the wedge-preserving subgroup. The requirement that the domain is also invariant under translations turns out to be extremely restrictive (Borchers et al., 2001). In $d>1+1$ the definition of integrability in terms of domain properties of PFG's forces the S-matrix to be trivial $S_{scat}=1$, whereas in $d=1+1$ it allows nontrivial S-matrices which are suitable combinatorial products of elastic 2-particle S-matrices which fulfill the bootstrap properties (matrix-valued scattering functions).[32] In other words the *connected* higher particle scattering contributions vanish, which is the standard definition of integrability in terms of properties of the S-matrix (the infinite number of conservation laws is a consequence). The elastic S-matrices are given in terms of (possibly matrix-valued) scattering functions which have to obey certain analytic properties in order to come from a field theory; these scattering functions permit a classification.

Using these scattering functions as structure functions in a Zamolodchikov–Faddeev algebra (Zamolodchikov & Zamolodchikov, 1979) one obtains the creation/annihilation components of wedge-localized temperate PFGs. At this point one realizes that the above abstract definition in terms of domain properties of PFGs coalesces with the standard definition of $d=1+1$ integrability. Such models are susceptible to solutions in closed form and are therefore called "integrable". Compared with the classical integrability which requires to find a complete set of "conservation laws in involution" (and where integrable systems exist in every dimension), integrability in QFT is limited to $d=1+1$.

The so-called "bootstrap-formfactor construction program" relates the scattering functions to explicitly computed formfactors (Babujian et al., 1999). The last step consists in showing that these formfactors really belong to an existing model of LQP. In order to achieve this one has to establish the nontriviality of double cone localized intersections of wedge-local algebras. This is a very nontrivial step which has been accomplished in terms of the use of modular nuclearity in the work of Lechner (2008). The same author also showed how (in the absence of bound-states) one can construct the wedge-algebra generating PFG's in terms of deformations of free fields (Lechner, 2000). The existence proof for some integrable models is considered to represent a progress compared to the old existence proofs which were limited to unrealistic short distance restrictions in the form of superrenormalizability (Glimm & Jaffe, 1972).

This simplicity of integrable S-matrices (the absence of connected parts for $n>2$) keep integrable models in the proximity of interaction-free models. Hence it is not so surprising that their wedge-generators (the Zamolodchikov–Faddeev algebra generators) can be obtained by *deformations* of free fields instead of the more complicated emulation (Lechner, 2000).

For the convenience of the reader and for later use we add some details on the algebraic structure of emulated free fields for integrable models

$$(A_{in}(f))_{\mathcal{A}(W)} = \int_C f(\theta)Z^*(\theta)\, d\theta, \quad C=\partial\text{strip}, \quad p=m(ch\theta, sh\theta)$$

$$\text{strip} = \{z|0<\text{Im}z<\pi\}, \qquad Z(\theta)\equiv Z^*(\theta+{}'\imath\pi)$$

$$Z^*(z_1)Z^*(z_2) = S(z_1-z_2)Z^*(z_2)Z^*(z_1), \quad z\in C \tag{30}$$

Since integrable models preserve the particle number in scattering processes, the n-fold application of the creation parts $Z^*(\theta)$ to the vacuum are n-particle states. Identifying the velocity-ordered particle

[30] A outgoing free creation operator applied on a $n-1$ incoming state is not an n-particle state. Similarly the action of emulated incoming fields on an incoming state is an infinite superposition of incoming particle states even though the emulated momenta are on-shell.

[31] In earlier publications the special case of an emulated incoming field was referred to as a vacuum-polarization-free-generators (PFG) (Borchers et al., 2001).

[32] In $d=1+1$ the cluster factorization does not distinguish a nontrivial elastic scattering amplitude from $S_{scat}=1$.

state with the incoming states

$$Z^*(\theta_1)Z^*(\theta_2)...Z^*(\theta_n)|0\rangle = |\theta_1,\theta_2...\theta_n\rangle_{in}, \quad \theta_1>\theta_2>\cdots>\theta_n$$
$$\text{anal. transpos. } \langle 0|B|\theta_2,\theta_1,...\theta_n\rangle_{in} = S(\theta_1-\theta_2)\langle 0|B|\theta_1,\theta_2,...\theta_n\rangle_{in} \tag{31}$$

the old degenerate representation related to (bosonic) statistics has been encoded into the natural order while the other orders describe analytic changes inside formfactors. For integrable models the transposition of two adjacent θ uses the two-particle S-matrix.

It follows from repeated application of (31) that the analytic change of a θ through a k-cluster of θ on its right hand side will be a *product* of scattering functions which rewritten *in terms of the full* $k+1$ *S-matrix* corresponds to a *grazing shot S-matrix* defined as (Schroer, 2014)

$$S_{g.s.}(\theta;\theta_1,...\theta_k) = S^k(\theta_1,...\theta_k)^{-1}S^{k+1}(\theta,\theta_1,...\theta_k) \tag{32}$$

This grazing shot concept has been used to generalize the properties of integrable emulations to the generic situation (Schroer, 2012, 2013) by converting the idea of analytic changes of ordering into an algebraic structure; in this sense this is an attempt to generalize the structure of the Zamolodchikov–Faddeev algebra.

The first attempt of an on-shell construction of particle theory after the failure of the S-matrix bootstrap was that by Mandelstam. It ignored the subtlety of analytic on-shell properties by trying to guess their structure based on a postulated double spectral representation for elastic scattering amplitudes (the Mandelstam representation) instead of a derivation from the foundational causal locality principles of QFT. It finally came to an end when Mandelstam supported the incorrect idea of identifying the meromorphic function of Veneziano's dual model with the particle crossing of scattering amplitudes (more in Section 7).

The idea of the present work is suggested from properties of the modular wedge localization and consists in relating on-shell analytic order changes to the action of emulats. For two relatively naturally ordered clusters, the analytic ordering idea for the left hand side in (28) reads

$$\left\langle BA_{in}^{(1)}(\theta_1,...\theta_k)_{\mathcal{A}(W)}|\theta_{k+1},...\theta_n\right\rangle^{in} = \langle B\theta_1,\theta_2,...\theta_n\rangle^{in} + \text{contr.}$$
$$\text{for } (\theta_1,...\theta_k) > (\theta_{k+1},...\theta_n) \tag{33}$$

i.e. all θ in left hand cluster are larger than the those on the right hand side. The contractions result from the incoming Wick product $A_{in}^{(1)}(\theta_1,...\theta_k)$ acting on the n–k particle state; they are delta function contact terms in rapidity space and hence do not contribute if all θ are different. Fortunately other orders are not needed for the crossing relation, but they contain the dynamic information and enter which is important for the full understanding of crossing and enter any constructive approach which tries to generalize what has been learned from integrable models.

That the ordering prescription is crucial for the derivation of the standard form of the LSZ property is corroborated by the derivation of the time-dependent LSZ reduction formula from the foundational properties of QFT (Buchholz & Summers, 2005). In that derivation overlapping wave functions have to be avoided because through such overlaps threshold singularities enter into the problem. This result supports the picture of analytic changes of moving through new threshold singularities at points of coalescence of two θ. It is an indication that ordering changes of two θ lead to nontrivial changes which affect the derivation of the LSZ formula and the crossing relation. The present arguments suggest that both these changes should have their explanation in a better understanding of the consequences of modular localization for wedge-local algebras in QFT. From a modern viewpoint it is clear that the conceptual tools for its solution were simply not available at the time of its proposal.

The ideas about PFGs and of wedge-localized particle states in terms of emulated fields can (and in my opinion should) be viewed as an extension of Wigner's representation-theoretical approach for noninteracting particles and its functorial relation ("second" quantization) with quantum fields but now in the presence of interactions. The conceptual distance between the functorial particle-free field relation and emulation in the presence of interactions is immense. Modular localization, as a mathematical precise formulation of the causal locality principle of LQP, is the only intrinsic property which has the necessary conceptual pugnancy to eventually solve this field–particle problem in the presence of interactions.

7. Impact of modular localization on gauge theories

It is well-known that the *Hilbert space formulation* for renormalizable couplings of *pointlike* fields is limited to spin $s<1$. For $s=1$ vectorpotentials, one is forced to use a Krein space formulation, either in the form of the Gupta–Bleuler formalism, or for massive gauge theories in terms of the ghost field formalism of the Becchi–Rouet–Stora–Tyutin (BRST) operator gauge setting. Usually textbooks on QED do not explain that this deviation from the Hilbert space setting of quantum theory comes from an incompatibility of pointlike zero mass vectorpotentials with the positivity of Hilbert space (closely related to quantum probability) which leads to limitations of viewing models of QFT as obtained by quantizing classical gauge theories. In fact this problem arises for all massless $s\geq 1$ tensor potentials; only their associated pointlike field strengths are fields acting in a Hilbert space. This problem of loss of the Hilbert space setting for interactions which use pointlike vectorpotentials is the origin of gauge theory.

For our aim to present a new formulation which permits to describe the full theory in Hilbert space, it is helpful to recall first some facts about the BRST gauge setting. We will use the so-called BRST operator formalism as it can be found in Scharf's book "Gauge theory, a true ghost story" (Scharf, 2001), but present it in a form which highlights analogies between the nilpotent cohomological BRST s-formalism in Krein space and the d-operation on differential forms of the new Hilbert space setting which requires the use of stringlocal field (the SLF formalism).

The BRST description of massive vectormesons relates the physical Proca field A_μ^P with $\partial^\mu A_\mu^P=0$ with short distance dimension $d_P=2$ to an unphysical field in Krein space A_μ^K with $d_K=1$ and a negative metric scalar Stückelberg field ϕ^K with $d_{sd}=1$. The idea is to compensate the highest short distance singularity in terms of the d=2 derivative $\partial_\mu\phi^K$; for this compensation one needs the opposite sign of the ϕ^K two-point-function; this just uses the well-known (already before gauge theory) short-distance softening effect of indefinite metric which is of cause inconsistent with quantum physics but helpful for renormalization. The idea of gauge-invariance is to formulate a restriction which permits at least to return to a "smaller" physical setting. The relation which intuitively achieves this is of the form

$$A_\mu^K(x) \simeq A_\mu^P(x) + \partial_\mu\phi^K, \curvearrowright \partial^\mu A_\mu^K(x) + m^2\phi^K \simeq 0 \tag{34}$$

The equivalence sign is meant to indicate that relation between the Krein space vectorpotential and its physical Proca counterpart is not yet an operator equality but rather a relation which requires a cohomological interpretation. The reader recognizes in the second relation the Lorentz condition; by relating physical states with suitably defined equivalence classes of Krein states these relations represent cohomological equivalences. In the following we restrict the formalism to massive vectormesons; in this case the space is a Fock–Krein particle space and the BRST formalism can be formulated in terms of indefinite metric free fields.

The BRST formalism extends the Krein space setting by additional indefinite metric free fields: the ghost and anti-ghost fields $u, \tilde{u}$ fields; this permits the reformulation of the content of (34) as operator equalities in terms of a nilpotent ghost charge Q which in turn allows the definition of a nilpotent s-operation

$$sA_\mu^K = \partial_\mu u, \quad s\phi^K = u, \quad s\tilde{u} = -(\partial A^K + m^2\phi^K)$$
$$sB^K = \left[Q, B^K\right]_{\text{grad}}, \quad Q \text{ ghost charge}, \quad s^2 = 0 \tag{35}$$

where the graded commutator is an anti-commutator if B contains an odd number of ghost fields $u, \tilde{u}$. The first line leads to $s(A_\mu^K - \partial_\mu\phi^K) = 0$ which according to (34) is consistent with the physical nature of the Proca field. As shown in Scharf (2001), Aste, Scharf, & Duetsch (1997), and Duetsch, Gracia-Bondia, Scheck, & Varilly (2010) this leads to an operator formulation of *renormalizable gauge theories for massive*[33] *vectormesons* coupled to charge-carrying- or neutral matter fields. The S-matrix in such a setting is characterized by $sS = 0$; for details we refer to the cited papers.

The action of the s via the graded commutator with the ghost charge defines the quantum gauge symmetry transformation so that gauge-invariant operators are annihilated by the action of s. As mentioned the limitation of the operator gauge formalism shows up in the attempt to construct *physical* matter fields[34] which couple to vectormesons. The well-known nonrenormalizability of pointlike massive vectormeson interactions in a *Hilbert space* indicates that pointlike physical fields are more singular than Wightman fields (operator-valued tempered distributions). The literature on the BRST formalism contains no informations about their construction. An illustration can be found in Fröhlich, Morchio, & Strocchi (1979a) where its was shown that the use of unphysical matter fields leads to the wrong result that the Maxwell charge (associated to the identically conserved Maxwell current $j_\mu = \partial^\nu F_{\mu\nu}$) of the gauge-variant matter field vanishes which contradicts rigorous results about states created by physical matter fields acting on the vacuum. Buchholz (1982) has used an appropriated formulation of the quantum Gauss law in order to prove that physical charge-carrying operators cannot be better localized than in arbitrarily narrow spacelike cones whose cores are semi-infinite spacelike strings.

Some more comments on the BRST operator gauge formalism and its relation to classical gauge theory may be helpful. The terminology gauge "principle" is sometimes misunderstood as a special *physical* property of $s = 1$ fields. Its role is however of a pure *technical* kind; working with a formulation in a Krein space, one needs to extract from such an unphysical description physical data referring to objects which act in a Hilbert space; in the past there has been simply no renormalizable formalism in terms of pointlike fields in a Hilbert space setting.

The BRST gauge formalism in Krein space achieves its limited validity in the vacuum sector (generated by the local gauge-invariant fields) by constructing a "symmetry" which involves in addition to the Krein space counterparts of matter fields also "ghost" and anti-ghost operators (35). This formal symmetry (sometimes referred to as a local gauge symmetry) *is by itself not a physical symmetry* in the usual sense; even though its formal invariants are the physical *local observables* whose application to the vacuum state generate the Hilbert space of the vacuum sector. Important physical fields, as those which transfer electric charge, remain outside the quantum gauge formalism. Neither does one know a physically useful generalization of gauge symmetry to higher spin. Indefinite metric spaces entered QFT through quantization of QED (the Gupta–Bleuler formalism), and the BRST setting resulted from generalizing the gauge formalism to interactions involving *massive* vectormesons.

Before describing some of the conceptual-mathematical details of the new setting it is helpful to recall how physical stringlocal charge-carrying matter fields have been formally envisaged in the BRST gauge setting. The formal expressions in the Krein space setting are well known

$$\varphi(x, e) = \varphi^K(x) \operatorname{expig} \int_x^\infty A_\mu^K(x + \lambda e)e^\mu \, d\lambda, \quad e^\mu e_\mu = -1, \tag{36}$$

they already appeared in publications of Jordan and Dirac during the 1930s. But anybody who (besides playing formal games) tried to obtain a perturbative computational control on the basis of such nonlocal composite formal expressions within renormalized perturbation theory knows that this is an impossible task.

The new SLF formalism solves this problem by converting it from its head to its feet; instead of trying to represent physical charge-carrying fields in terms of pointlike gauge-variant fields in a Krein space setting, it bases renormalized perturbation theory direct on stringlocal fields in the Hilbert space. In this way the stringlocal physical fields become the basic fields in terms of which renormalized perturbation theory is formulated (Schroer, 2011a; Schroer-b). For free massive pointlike potentials (Proca potentials) the short distance dimension $d_{\text{Proca}} = 2$ poses no problems. The problems start if such fields interact, since it is impossible to define an interaction density which stays within the power-counting limit $d_{int} = 4$; all interactions of Proca fields are nonrenormalizable.

In classical field theories Hilbert space positivity plays no role; the vectorpotential is a perfectly legitimate and useful classical object; the fact that many different vectorpotentials correspond to the same field strength and the formalization of this observation in terms of the introduction of a classical gauge group does not change this. However the quantum Hilbert space structure and in particular its *positivity property* (related to the quantum probability) has no classic analog from which it could arise via the quantization parallelism, This changes the whole game; for zero mass quantum vectorpotentials there is a *clash between covariant pointlike zero mass vectorpotentials and the Hilbert space positivity;* this clash extends to $s > 1$ tensorpotentials,[35] whereas the associated observable pointlike quantum field strengths remain pointlike fields in the Hilbert space. In fact stringlike potentials $A_\mu(x, e)$ arise by integrating pointlike field strengths over semi-infinite spacelike lines in the direction e (see below) starting at the spacetime point x. This process can be extended to higher spin field strengths (for $s = 2$ the linearized Riemann tensor); each time the short distance dimension improves by one unit until the process ends at a covariant (e is a Lorentz vector) stringlocal sibling of dimension $d_{string} = 1$ of a pointlocal tensor potential which has $d_{pont} = s + 1$; some details will be presented later on.

Whereas the clash in the zero mass case already occurs for *free* tensorpotentials[36] and hence is of a kinematic nature, the weakening of pointlike localization in the *massive* case is a dynamic phenomenon which manifests itself in a subtle *connection between renormalization and locality* in the sense that nonrenormalizability of certain fields implies that they do not exist as pointlike

[33] The free field transformation rules (35) refer to the incoming free fields of scattering theory. In massless gauge theories as QED the ghost charges depend on the coupling (Duetsch & Scharf, 1999).

[34] The gauge-variant matter fields have no physical content and it is also not possible to extract physical matter fields in a perturbative setting.

[35] Example: for $s = 2$ the tensorpotential is the $g_{\mu\nu}$ and the associated field strength is a tensor field with four indices (the linearized Riemann tensor).

[36] The counterpart of vectorpotentials to higher spin. E.g. for $s = 2$ the field-strength is a 4-index tensor (the linearized Riemann tensor) and the associated tensorpotential is the 2-index $g_{\mu\nu}$ tensor.

Wightman fields but that their perturbative interactions becomes renormalizable if formulated in terms of *stringlike* Wightman fields.

The indefinite metric gauge formalism for pointlike massless tensorpotentials can be related via quantization to the classical pointlike formalism, but it is not immediately clear what is the tightest localization which is consistent with the Hilbert space positivity. The before mentioned structural theorem for localization in QED suggests that it should be semi-infinite stringlike. There is a powerful general theorem which states that in theories with mass gaps and pointlike observable algebras the generating fields (fields act always act in Hilbert space unless otherwise stated), which carry superselection charges, are generically stringlocal (Buchholz & Fredenhagen, 1982); in other words, in order to generate the operator algebras of QFT, one does not need generating fields which live e.g. on spacelike hypersurfaces. Stringlike $\Psi(x,e)$ fields are covariantly localized on a semi-infinite spacelike string: $x+\mathbb{R}_+e$, $e\cdot e=-1$. In this new setting pointlike fields $\Psi(x)$ are considered as special e-independent cases. By definition local observables are always pointlike generated (currents, field strength, etc.).

It is the main point of this section to abandon the gauge description in favor of a Hilbert space formulation; for $s\geq 1$ this requires to replace pointlocal vectormesons by their stringlike counterpart. The Krein space gauge setting and the SLF Hilbert space formulation meet on the level of local observables where the property of gauge invariance corresponds to e-independence, whereas the Hilbert space setting provides the missing higher sectors beyond the vacuum sector which cannot be generated by local observables but need stringlike generating operators (e.g. the physical electron fields). The gauge setting arises naturally from the Lagrangian quantization of the classical electromagnetism, whereas stringlocal vectorpotential have no Euler–Lagrange description.

Fortunately perturbative QFT does not depend on an Euler–Lagrange description. The Epstein–Glaser (E–G) formulation (Epstein & Glaser, 1973) of perturbation theory (causal perturbation theory) accepts Lorentz-invariant interaction densities in terms of covariant fields independent of whether these fields are of Lagrangian origin or results of representation-theoretic (Wigner) local quantum physical constructions. However the use of covariant stringlocal fields requires a nontrivial extension of the E–G inductive construction from pointlike to stringlike crossings; such an extension has been achieved in recent but yet unpublished work by Mund (2014a).

The Hilbert space positivity restricts the existing pointlike formulation to $s<1$. According to the aforementioned theorem (Buchholz & Fredenhagen, 1982) generating fields of LQP in theories with a mass gap are at most string-localized. In the following it will be shown that nonrenormalizable couplings involving massive pointlike $s\geq 1$ fields can be rewritten in terms of stringlocal Wightman fields. We believe that perturbative nonrenormalizable pointlike couplings which cannot be converted into stringlike renormalizable couplings (see next section) do not define models consistent with principles of QFT.

Our SLF setting requires to describe interactions of zero mass vectormesons (QED, QCD) as limiting cases of massive interactions; in the limit only the stringlocal Wightman fields survive; their presence is essential for the understanding of physical consequences of infrared divergencies and they are the only physical fields which carry a nontrivial Maxwell charge. As already mentioned a nonperturbative proof of string-localization as the tightest possible localization of charged fields which carry a nontrivial Maxwell charge is based on the quantum Gauss law (Buchholz, 1982). In contradistinction to massive strings in different directions which are unitarily equivalent, charged QED strings are "rigid"; in particular they lead to a spontaneous breaking of Lorentz covariance (Fröhlich, Morchio, & Strocchi, 1979b).

The new SLF setting bases renormalized perturbation theory direct on stringlocal physical fields (Schroer, 2011a; Schroer-b). For free massive pointlike potentials (Proca potentials) the short distance dimension $d_{Proca}=2$ poses no problems. They start if such fields interact since it is impossible to define an interaction density which stays within the power-counting limit $d_{int}=4$, i.e. all interactions of Proca fields are nonrenormalizable.

The first hint into which direction to look comes from the observation that there are two other fields which belong to the localization class of the Proca field and have the short distance dimension $d=1$ instead of (2). They are constructed from the Proca potential in terms of the following definitions:

$$F_{\mu\nu}(x):=\partial_\mu A_\nu^P(x)-\partial_\nu A_\mu^P(x),\quad A_\mu(x,e):=\int_0^\infty F_{\mu\nu}(x+\lambda e)e^\mu\, d\lambda$$
$$\phi(x,e):=\int_0^\infty A_\mu^P(x+\lambda e)e^\mu\, d\lambda,\quad e^2=-1 \tag{37}$$

All three covariant free fields are written in terms of the same basic Wigner $s=1$ creation/annihilation operators $a^\#(p,s_3)$, $s_3=-1,0,1$; unlike in the BRST setting no additional Stückelberg degrees of freedom are introduced, so that the Hilbert space remains identical to that which the Proca field generates from the vacuum.[37] In the presence of interactions the stringlocal scalar ϕ may potentially interpolate particles of any integer spin (Mund et al., 2006), including $s=0$

$$\langle p,s_3|\phi|0\rangle\neq 0,\quad -s\leq s_3\leq s \tag{38}$$

Which boundstate particles actually appear in addition to the "elementary" $s=1$ vectormeson and the matter field with which it interacts depending on the interactions of massive vectormesons with other matter or among themselves.

The important point here is that the covariant string-local nature of ϕ permits a *linear* interpolation, whereas covariant pointlike fields achieve this only by forming (nonlinear) composite fields (Mund et al., 2006). Solely *zero mass stringlocal* fields maintain the standard connection between spinorial indices and physical spin[38] $s=|h|$ which is the same as that for pointlike *massive* fields; in particular they have no ϕ "escorts". The semi-infinite line integrals in (37) lowers the dimension by one unit, so that the stringlocal potential and its stringlocal escort field permit to define formal interaction polynomials within the power-counting restriction. The string-localization shows up in the commutation relation; bosonic strings commute if and only if the entire strings $x+\mathbb{R}_+e$ are spacelike relative to each other.

Between the pointlocal Proca field and its stringlike relatives there exists a (easy verified) linear relation

$$A_\mu(x,e)=A_\mu^P(x)+\partial_\mu\phi(x,e),\quad d_{sd}(A_\mu)=1,\quad d_{sd}(\phi)=1,\quad d_{sd}(A_\mu^P)=2 \tag{39}$$

In contrast to the equivalence relations (34) in the Krein space, these relations are bona fide operator equations in Hilbert space which (in case of free fields) are direct consequences of the above definitions. The ϕ are similar to the Stückelberg fields in the BRST gauge setting (34) but in contrast to the letter they are physical i.e. they interpolate physical states. In the free field case these escort fields (of stringlocal vectorpotentials) ϕ generates the same $s=1$ Wigner particle as A_μ, but in the presence of interactions they may potentially interpolate any integer spin particle, including a scalar

[37] This renders the SLF setting more similar to the Ginsberg–Landau phenomenological theory of superconductivity than the relation of the latter to the Higgs mechanism for which the "fattened" vectormeson need the presence of the Higgs particle.

[38] For massless pointlike fields the relation excludes tensorpotentials.

bound state. The compensation of the most singular part (in the present case of the Proca field) by the derivative of a lower dimensional field (39) is the mechanism by which later on the singular nonrenormalizable pointlike interaction density will be converted into its less singular renormalizable stringlocal counterpart.

In contrast to the role of the scalar Higgs field, which must be added to the zero-order field content, the Hermitian stringlocal scalar $\phi's$ are inexorable companions (intrinsic escorts) of renormalizable massive vectormesons. Together with the Proca field they disappear in the massless limit in which the relation (39) breaks down and only stringlocal vectorpotentials remain.

Before presenting illustrative second-order perturbative model calculations in the new SLF Hilbert space formulation, it is helpful to know how the local equivalence class relation between point- and string-local fields can be extended to the matter fields. Looking at the "gauge theoretic appearance"[39] of (39) it is not surprising that this relation takes the form of a gauge transformation

$$\psi(x) = e^{-ig\phi(x,e)}\psi(x,e) \tag{40}$$

The g-dependent exponential dependence on the physical ϕ field changes the renormalizable stringlocal matter field; the result is a very *singular pointlike field* with unbounded short distance dimensions (non-polynomial increase in momentum space). Such fields have been introduced in Jaffe (1967); they are more singular[40] than operator-valued Schwartz distributions (Wightman fields) and indicate their presence in terms of a breakdown of renormalizability. Any attempt to calculate them directly (i.e. without using the relation to their stringlocal renormalizable siblings) will lead to the well-known problems of nonrenormalizable perturbation theory with infinitely many counterterm parameters, whereas their calculations as objects *within* the renormalizable stringlocal perturbation theory will maintain the same number of parameter as those appearing in the stringlike formulation. In fact they provide a very singular "coordinatization" of the same physical situation. In particular they do not allow the construction of localized operator algebras by smearing with arbitrary compact spacetime supported smooth testfunctions.

The intrinsic nature of the stringlocal physical ϕ fields strengthens the analogy with the massive gauge fields in the Ginsberg–Landau theory of superconductivity. In contradistinction to the Higgs mechanism, which adds additional degrees of freedom (namely the extrinsic Higgs fields) in the belief that vectormesons need them in order to be massive, the SLF setting describes massive vectorpotentials coupled to charged matter *without* adding degrees of freedom, just as the quantum mechanical theory of superconductivity describes short range vectorpotential without requiring additional degrees of freedom. What is not clear at this point but will become evident in the following subsections, is that these scalar stringlocal fields, which together with the other two fields (39) are members of the same relative localization class, play a crucial role in the interaction of massive vectormesons with matter.

It is interesting to note that the local equivalence class picture permits a generalization in which the linear relation between $s=1$ free fields is a special case a more general relation for integer spin $s>1$ fields

$$A_{\mu_1\ldots\mu_n}(x,e) = A^P_{\mu_1\ldots\mu_n}(x) + \partial_{\mu_1}\phi_{\mu_2\ldots\mu_n} + \partial_{\mu_1}\partial_{\mu_2}\phi_{\mu_3\ldots\mu_n} + \cdots + \partial_{\mu_1}\ldots\partial_{\mu_{n_n}}\phi$$

The left hand side represents a stringlocal spin $s=n$ tensor potential associated to a pointlike tensor potential with the same spin. The $\phi's$ $s=n-i, i=1,\ldots,n$ are tensorial stringlocal fields of dimension $d=n-i+1$. Each ϕ "peels off" a unit of dimension so that at the end one is left with the desired spin s stringlocal $d=1$ counterpart of the tensor analog of the Proca field. The main problem of using such generalizations is to identify those couplings which guaranty the existence of sufficiently many (generally composite) local observables generated by pointlike Wightman fields (operator-valued Schwartz distributions). This may be important in attempts to generalize the idea of gauge theories in terms of SLF couplings involving massive $s>1$ fields.

The two-point functions of the above $s=1$ stringlocal fields are e-dependent

$$\langle \Phi_1(x,e)\Phi_2(x',e')\rangle = \frac{1}{(2\pi)^{3/2}}\int e^{-ip(x-x')}M_{\Phi_1,\Phi_2}(p;e,e')\frac{d^3p}{2p_0}$$
$$M_{A^P_\mu,A^P_\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}, \quad M_{\phi,\phi} = \frac{1}{m^2} - \frac{ee'}{(pe-i\varepsilon)(pe'+i\varepsilon)}$$
$$M_{A_\mu,A_\nu} = -g_{\mu\nu} - \frac{p_\mu p_\nu}{(pe-i\varepsilon)(pe'+i\varepsilon)} + \frac{p_\mu e_\nu}{pe-i\varepsilon} + \frac{p_\nu e'_\mu}{pe'+i\varepsilon} \tag{41}$$

Besides these three diagonal expectations there are also mixed e-dependent two-point functions of which only

$$M_{A_\mu,\phi} = -i\left(\frac{e'_\mu}{pe'+i\varepsilon} - \frac{p_\mu ee'}{(pe-i\varepsilon)(pe'+i\varepsilon)}\right) \tag{42}$$

will be needed later on. The ε-prescription defines the distributions as boundary values of analytic functions. A systematic derivation of such relations in the context of the intertwiner formalism for stringlike fields (Mund et al., 2006) will appear in Mund & Schroer (2014a). The appearance of e-dependent time-ordered correlations complicates analytic perturbative calculations as compared to the BRST setting.

But the extra computational effort is unavoidable, because it is the only possibility to construct correlation function involving *physical* zero mass matter fields since the latter *exclusively exist as stringlocal objects*[41] and the massive vectormeson theories offer a natural covariant way (without ad hoc cutoffs) to analyze the infrared behavior. Such constructions are necessary if one wants to show that confinement is a property of zero mass gluon-matter interactions. In fact the expected result is that $m\to 0$ limiting correlations vanish if besides pointlike observable (composite) fields they also contain stringlocal gluons/quarks; the only expected exception is quark–antiquark pairs with an e which matches the direction of the spacelike separation between the pair (a stringlike bridge). One knows from infrared problems in QED that the leading logarithmically divergent contributions must be re-summed before one takes zero mass limits (Yenni, Frautschi, & Suura, 1961).

One should also note that the apparent simplicity of the pointlike BRST perturbation theory as compared to the Hilbert space setting is deceiving; the difficult part in the gauge setting is not the perturbation theory itself, but rather the extraction of the physical results. Physical operators, as the S-matrix, inevitably contain unphysical fields, and to compute their matrixelements between physical particle states is a nontrivial and even ill-defined task since the physical space is not simply a subspace but rather results from a cohomological construction. The s-invariant BRST S-operator in the Bogoliubov formulation depends not only on the physical matter operators but also on the unphysical A^K_μ and ϕ^K free fields and even if one finds a way to compute scattering amplitudes by "sandwiching" S between physical Wigner particle

[39] This is not a gauge transformations between fields of the same kind, but rather an equation which connects string-and point-like fields which are members of the same localization class.

[40] In fact they only allow smearing with a dense class of localized testfunctions.

[41] Even the singular pointlike fields of the massive case disappear in the massless limit.

states it is not clear whether it would agree with the scattering amplitudes which are calculated by doing the same with S_{phys} obtained from the Hilbert space formulation where such problems do not occur. The only secure result of the gauge approach is the physical nature of the gauge-invariant local observables, but from those alone it is not possible to derive the S-matrix.

7.1. SLF perturbation theory involving massive vectormesons

For the perturbative study of interactions of massive vectorpotentials with charged matter, one needs to establish the validity of relations as (39) and (40) in every order of perturbation theory. The zero-order matter fields are pointlike but, as a result of their interaction with the stringlocal vectorpotential, they become stringlike in higher orders, in fact they turn out to be even "more stringy" than the vectorpotentials which mediate the interactions. The important idea which permits to establish these relations in every order within the Stückelberg–Bogoliubov–Epstein–Glaser (SBEG) setting of renormalized perturbation theory will be referred to as "adiabatic equivalence" (AE) since it involves the adiabatic limit in which the spacetime-dependent compact supported coupling $g(x)$ of the SBEG functional formalism approaches the spacetime-independent everywhere constant physical coupling strength g; this will be explained in the sequel.

Before we turn to concrete model illustrations of perturbation theory in terms of stringlike fields, a historical remark about the origin of these ideas may be appropriate. It had been known for a long time that Wigner's infinite spin representations of the Poincaré group cannot be generated by pointlike wave functions (Yngvason, 1970). Further progress had to await the concept of modular localization, which first appeared in the context of integrable models (Schroer, 1999). Of significant importance was the systematic application of modular localization to positive energy Wigner representations in Brunetti et al. (2002). In that paper it was shown that all such representations permit a causal localization in (arbitrary narrow) spacelike cones. Since the core of such a conic region is a semi-infinite spacelike string, the only remaining computational problem was the construction of covariant fields $\Psi(x,e)$ which are causally localized on $x+\mathbb{R}_+e$, $e^2=-1$ and generate operator localized in (arbitrary narrow) spacelike cones (Mund et al., 2006). This finally led to a solution of the age-old problem concerning the field content of Wigner's "infinite spin" representation class.

It then turned out that the construction of stringlocal fields is also useful for the pointlike localizable representations since it resolves the *clash between pointlike localization and the Hilbert space positivity for zero mass* $s \geq 1$ *fields* which one encounters in passing from pointlike field strength to their associated potentials.[42]

The use of stringlike potentials also lowers the short distance dimension; instead of $d_{sd}=s+1$ for pointlike spin s fields, one can always construct a free stringlike field with $d_{sd}=1$ for all s. This allows us to convert interactions between massive nonrenormalizable pointlike fields into renormalizable interaction involving their stringlocal analog. It also shows that this conversion can be used in the opposite direction; the stringlike renormalization theory permits to construct well-defined (but more singular) higher order pointlike interaction densities via the detor of renormalizable stringlike Wightman fields; this roundabout way cannot prevent the singular (non-Wightman) nature known from direct use of pointlike perturbation theory with first-order interaction densities beyond the power-counting limit $d_{int} \leq 4$, but at least the number of parameters stays the same as in its stringlike counterpart.[43]

Although *modular localization* was important for the understanding of the role of stringlocal fields and their role in the reformulation of gauge theory, their renormalization theory can nowadays be carried out without direct use of modular methods. The latter remain present in the background; they furnish the conceptual-mathematical fundament for the ongoing changes in QFT. They shows in particular that the perturbative use of SLF in the Hilbert space is more than a computational substitute of the BRST gauge formulation; in fact, it is the only perturbative formulation in which the full field content (and not just the local observables of the gauge-invariant vacuum representation) complies with the physical principle of causal localization in a Hilbert space.

After having explained the philosophy behind SLF, we will now illustrate these ideas in three different models. As a preparatory step the reader is first reminded of the SBEG setting of perturbation theory. Its central object is Bogoliubov's perturbative operator-S-functional which generates the *time-ordered products* associated with the scalar interaction density $L(\chi)$. The scattering matrix S_{scat} and the quantum fields are then defined in terms of the adiabatic limit of the following definitions

$$S(gL) \equiv \sum_n \frac{i^n}{n!} T_n(L,\ldots,L)(g,\ldots,g) =: Te^{i\int L(x)g(x)}, \quad S_{scat} = \lim_{g(x)\to g} S(gL)$$
$$\psi_g(f) := S(gL)^{-1} \sum_n \frac{i^n}{n!} T_{n+1}(L,\ldots,L,\psi)(g,\ldots,g,f), \quad \psi(f) = \lim_{g(x)\to g} \psi_g(f) \tag{43}$$

Here $g(x)\to g$ is the adiabatic limit in which the spacetime-dependent coupling approaches the coupling constant and the S-matrix and the fields become covariant. A sufficient condition is the existence of mass-gaps, which is satisfied if all fields in the Lorentz-invariant interaction density are massive (Haag, 1996). Since quantum fields are not operator-valued functions but rather operator-valued distributions, the definitions of the S-matrix and quantum fields must be subjected to renormalization which has to be carried out order by order.

In the case of massive scalar QED (Mund, 2014b; Schroer-b) we have two $L's$, a pointlike interaction L^P and its stringlike counterpart L

$$L^P(x) = j^\mu(x)A_\mu^P(x) = L(x,e) - \partial^\mu V_\mu$$
$$L(x,e) = j^\mu(x)A_\mu^S(x,e), \quad V_\mu = j^\mu(x)\phi(x,e), \quad j_\mu(x) =: \varphi^*(x) i \overleftrightarrow{\partial}_\mu \varphi(x):$$
$$S(gL^P + f\psi) \simeq S(gL + f\psi^S)$$
$$A_\mu^P(x) = A_\mu^S(x,e) - \partial_\mu\phi(x,e), \quad \psi^P(x) = e^{-ig(x)\phi(x,e)}\psi^S(x,e) \tag{44}$$

The L^P is the singular pointlike Proca interaction, whereas L is the new stringlike interaction which, as a result of $d_{sd}(A_\mu^S)=1$, stays within the power-counting limit of renormalizable couplings; both $L's$ act in the Hilbert of the free fields which were used in the definition of L^P. The vector V_μ contains the previously introduced intrinsic escort field ϕ of A^S, and $\partial^\mu V_\mu$ with $d_{sd}(\partial^\mu V_\mu)=5$ plays a similar role with respect to L^P as $\partial_\mu \phi$ in (39) with respect to A_μ^P, namely it "peels off" the highest short distance dimension from L^P and converts it into the renormalizable $d_{sd}=4$ interaction density L.[44] The highest divergence is now carried by the derivative $\partial^\mu V_\mu$ term which, integrated with $g(x)$, becomes a boundary term and hence vanishes (in massive theories) in the adiabatic limit $g(x)\to g$. In this way one arrives at the equality (up to problems of

[42] A corresponding result holds for massless higher halfinteger integer spin fields with $s \geq 3/2$.

[43] The growth of the number of independent counterterms parameters with the perturbative order in the direct pointlike setting renders nonrenormaliable interactions rather useless.

[44] For convenience of notation we omit the superscript S for stringlocal objects.

normalization) of the first-order pointlike scattering matrix with its string counterpart

$$\int L^P d^4x = \int L d^4x \quad \text{or} \quad L^P \overset{AE}{\simeq} L \tag{45}$$

which defines the concept of "adiabatic equivalence" of the two interactions.

For notational conveniences, and also in order to maintain formal analogy to the BRST formalism, one views $A_\mu(x,e)$ and $\phi(x,e)$ as zero forms in e, with d_e denoting the differential operator which maps n-forms into $n+1$ forms so that $d_e^2 = 0$. Then the basic relation of string-independence (39) reads

$$d_e(A_\mu(x,e) - \partial_\mu \phi(x,e)) = 0, \quad u := d_e \phi$$
$$d_e(L(\chi, e) - \partial_\mu V^\mu(x,e)) = 0, \quad Q_\mu = d_e V_\mu \tag{46}$$

and the last relation, in which the d_e acts on composites, is a consequence of the d_e action on the basic free fields. For all interactions of massive vectormesons with matter such pairs L, V_μ exist. The content of the bracket in the second line is simply the lowest order nonrenormalizable pointlike interaction; for massive QED see (44).

The differential form calculus is *formally* similar to the nilpotent s-operation of the cohomological BRST gauge formalism (see below). Its conceptual role remains however quite different; in the case at hand the differential formalism separates pointlocal observables from stringlocal fields in the Hilbert space, whereas the main purpose of the BRST s-operation is to allow the return from an unphysical Krein space to a quantum theoretical Hilbert space in which (only) gauge invariant observables act. Operators as (36), which in the BRST terminology may be called "gauge invariant nonlocal matter fields", are outside the range of the perturbative gauge formalism, whereas in the SLF setting they define the basic renormalizable matter fields of perturbation theory. In contrast to the nilpotent s-operation, which is needed for the construction of a Hilbert space, the d_e acting on classical differential zero forms is directly related to the physical localization properties.

If the T-products would not involve distributions with singularities at coinciding points as well at string crossings which impede to pull the ∂_μ through the T, higher order string independence relations as

$$(d_e + d_{e'})(TLL' - \partial_\mu T\, V^\mu L' - \partial'_\nu TL\, V^{\nu'} + \partial_\mu \partial'_\nu T V^\mu V^{\nu'}) = 0 \tag{47}$$

would be an automatic consequence. This relation may be somewhat simplified by splitting it (using the symmetry in $x, e \leftrightarrow x', e'$) into

$$d_e(TLX' - \partial_\mu T V^\mu X') = 0, \quad X' = L', V^{\mu'} \tag{48}$$

The ambiguities of time-ordering at point or string-crossings make the fulfillment of these relations a nontrivial renormalization problem. Their validity as distributional relations, including coalescent $x's$ and string crossings, would imply the string-independence of the second-order scattering matrix, since all derivative terms lead to vanishing boundary terms in the AE limit.

The vanishing of the bracket in (47) also provides a second-order definition of a T-product of singular "pointlike"[45] interactions $TL^P(x)L^P(x')$, which in the standard pointlike setting would be outside the range of renormalization theory

$$TL^P L^{P'} \overset{AE}{\simeq} TLL', \; TL^P L^{P'} \equiv TLL' - \partial_\mu T\, V^\mu L' - \partial'_\nu TL\, V^{\nu'} + \partial_\mu \partial'_\nu T V^\mu V^{\nu'} \tag{49}$$

The derivative terms, which in massive theories lead to vanishing surface contributions after integration over spacetime, account for the fact that this e, e' independent definition of a second-order pointlike interaction leads to the same scattering matrix as its stringlike counterpart. Renormalization means the construction of a time-ordering which fulfills e-independence in the sense of (49).

This is conveniently done by decomposing the time-ordered products in terms of Wick-ordered products. The resulting operator contributions can be ordered according the number of contractions. The term with no contraction obviously fulfills the above identity. The so-called tree-contribution contains one contraction; for contractions containing the time-ordering of derivative of fields this leads to a renormalization problem. The only massive vectormeson coupling in which this problem is absent is massive spinor QED (Mund, 2014b). Loop contributions are as usual absorbed in mass- and coupling-renormalization.

The interesting new phenomena of SLF in the Hilbert space happen in the "tree"-component. In the following this problem and its solution will be sketched for three models: scalar massive QED, its chargeless counterpart (coupling to a Hermitian field H) and some comments on the massive Yang–Mills coupling (interacting massive gluons). In the following three subsections we will be content with the calculation of the second-order S-matrix. The calculation of off-shell correlation of quantum fields and the relation between singular pointlike and renormalizable stringlike matter fields (40) will be left to a separate publication.

For new interesting problems of mathematical physics arising from stringlocal perturbation theory, in particular problems related to the extension of Epstein–Glaser causal renormalization theory to string-crossings, we refer to forthcoming work by Mund (2014a).

7.2. Scalar massive QED

According to the traditional view, massless scalar QED is a pointlike model with two coupling parameters[46]; it is known to be renormalizable in the unphysical pointlike BRST Krein space setting. Unlike its classical counterpart, this quantum gauge description is severely restricted; the positivity requirements of the Hilbert space clash with the pointlike localization and quantum gauge theory is the result of a compromise; the description is limited to local observables which constitute the gauge invariant part, physical matter fields remain outside.

As a consequence, quantum gauge theory is not capable to provide a spacetime description of collisions between electrically charged particles; however there exist calculational successful infrared regularized momentum space recipes for photon-inclusive cross sections. There is presently no spacetime understanding of collision theory analogous to that provided by the LSZ scattering theory[47] in case of models with mass gaps. The traditional point of view is that zero mass interactions are simpler than their massive counterparts; but this refers to purely formal aspects of renormalization theory and ignores the physical-conceptual problems. The latter point into the opposite direction.

The problems of infraparticles in QED (Buchholz, 1986) and confinement in QCD still belong to the conceptual demanding unsolved problems of particle theory, whereas the incorporation of renormalization problems of its massive counterparts can be achieved by extension of the renormalization theory to the new SLF setting in the Hilbert space. Apart from some remarks at the end of next section, the construction of massless limits and new ideas to tackle infrared problems will be left to a separate publication.

[45] The $TL^P L^{P'}$ is generally not pointlike as an interaction density, since there remain e-dependent contact terms which only vanish after integration (i.e. in the AE limit).

[46] The electromagnetic coupling and a parameter related to a quadrilinear scalar field coupling.

[47] The large-time LSZ limits vanish for infraparticle fields (Schroer-b).

The defining first-order stringlocal interaction density of massive scalar QED

$$L(x,e) = gA_\mu(x,e)j^\mu(x) = L^P + \partial^\mu V_\mu$$
$$j^\mu = \varphi^* \overleftrightarrow{\partial^\mu}\varphi, \quad V_\mu = \phi j_\mu \tag{50}$$

is according to (47) d_e-equivalent to its pointlocal counterpart L^P. This secures the e-independence of the first-order S-matrix in the AE limit. In these equivalences the stringlocal intrinsic escort fields ϕ which appears explicitly in V_μ play an essential role. Whereas the first-order relation is a result of the definition of a "stringlocal" interaction, the second-order relation (47) is a nontrivial restriction on the renormalization.

One defines a reference time-ordering T_0 of two-pointfunctions of derivatives of the complex scalar field φ by taking the derivatives outside the two-point function e.g.

$$\langle T_0\partial_\mu\varphi^*(x)\partial'_\nu\varphi(x')\rangle = i\frac{\partial_\mu\partial'_\nu}{(2\pi)^4}\int d^4pe^{-ipx}\frac{1}{p^2-m^2+i\varepsilon}$$

On the other hand the time ordering in Epstein and Glaser's renormalization approach permits delta function counterterms of the same scaling degree as the integrand, for the present case

$$\langle T\partial_\mu\varphi^*(x)\partial'_\nu\varphi(x')\rangle = \langle T_0\partial_\mu\varphi^*(x)\partial'_\nu\varphi(x')\rangle - aig_{\mu\nu}\delta(x-x') \tag{51}$$

where a is a free parameter.

If we were to treat the defining first-order interaction $A_\mu j^\mu$ as involving a pointlike A_μ field in the Krein space of pointlike massless vectorpotentials, the interaction is renormalizable in the perturbative inductive Epstein–Glaser renormalization setting where it leads to two counterterms. The first counterterm (51) appears in the second-order tree approximation and amounts to a modification of the interaction through a second-order contact term (all operator products are meant to be Wick-ordered)

$$aA_\mu(x)A^\mu(x)\varphi^*(x)\varphi(x) \tag{52}$$

with an *independent* coupling parameter a. There is an additional quadrilinear counterterm with a coupling parameter of the form

$$b(\varphi^*(x)\varphi(x))^2 \tag{53}$$

which appears for the first time in fourth order; these two counterterms exhaust the possibilities of counterterm structures (primitively divergent contributions in the Feynman graph setting), which means that the renormalized theory is 3-parametric.

To recuperate local observables acting in a Hilbert space (at the expense of charge-carrying matter fields which remain unphysical fields in the Krein space) one has to *extend the Krein space formulation by ghost operators* as explained in the previous section; in this way one arrives at the *BRST gauge formulation which fixes the parameter a in* (52) *to a numerical value* $a=1$ according to the rules of a formal "gauge symmetry". By itself this term has no direct physical interpretation apart from its role in the extraction of local observables from an unphysical description. For the formal description and the perturbative calculations of the two-parametric massive scalar QED one needs the full BRST "ghost program", even though the physics is only contained in the small subalgebra generated by "gauge invariant" local observables. The gauge symmetry is a technical trick and not a physical symmetry; in particular its spontaneous breaking is physically meaningless.

In the SLF Hilbert space setting on the other hand, the second order with the correct value of a is "induced" from the model-defining first-order $A\cdot j$ interaction; it is simply the result of the implementation of locality in Hilbert space setting. No additional principle as gauge symmetry has to be invoked in order to fix a to its correct numerical value; models QFT are realizations of the foundational causal localization principle. The difficult task is to trace the richness of models back to different physical manifestations of this principle. The *induction mechanism* exists only for higher spins $s\geq 1$, for lower spins the renormalization theory is the well-known counterterm formalism of pointlike interactions.

For the case at hand this is done as follows. From the results in the previous section we know that the second-order locality requirement for the S-matrix in the presence of stringlike fields amounts to the vanishing of the d_e operation on the renormalized tree component

$$d_e(TA\cdot jA'\cdot j' - \partial^\mu T\phi j_\mu A'\cdot j')_{1-con} = 0$$
$$-A_e := d_e(T_0A\cdot jA'\cdot j' - \partial^\mu T_0\phi j_\mu A'\cdot j')_{1-con} = N_e + \partial^\mu N_{e,\mu}, A = A_e + A_{e'} \tag{54}$$

and a similar expression in which the unprimed and primed x,e are interchanged; the total anomaly A from the one-contraction terms is simply the sum of the two contributions. They originate from the divergence of propagators

$$\partial^\mu\langle T_0\partial_\mu\varphi^*(x)\partial'_\nu\varphi(x')\rangle = -i\partial'_\nu\delta(x-x') + \mathrm{reg}$$
$$\partial^\mu\langle T_0\partial_\mu\varphi^*(x)\varphi(x')\rangle = -i\delta(x-x') + \mathrm{reg} \tag{55}$$

where reg stands for the regular contributions which results from applying the wave operator to the free field $\varphi^*(x)$ inside the time ordering. The anomaly contribution is not the only delta contribution, the T_0LL' also contributes since according to the rules of minimal scaling we are required to introduce a counterterm (51) with an undetermined parameter a. For two derivatives the T_0 passes to a T which contains a free renormalization parameter a (51) whereas we keep $T=T_0$ for φ propagators with a lower number of derivatives. The T propagators also appear in the φ-contractions of the tree contribution $TLL'|_{1-con}$. Instead of presenting the lengthy but straightforward calculation of the $N's$ we only present the result

$$N = 2\delta\varphi^*\varphi A\cdot A', N_\mu = \delta\varphi^*\varphi(\phi A'_\mu + \phi' A_\mu) \tag{56}$$

where the δ stands for $\delta(x-x')$.

By inspection one now realizes that the choice $a=1$ in (51) leads to a compensation of the N-anomaly with the normalization term from TLL'. The N_μ contributes to the renormalization of the $TV_\mu L'$ operator but does not contribute to the renormalization of the second-order S-matrix. As a consequence of the identity $d_e\partial^\mu\phi = d_eA^\mu$ there are no delta anomaly- contributions from $\phi - A_\nu$ contractions. One obtains the expected second-order quadratic in A_ν contributions which in the gauge formalism results from imposing gauge invariance

$$TLL' = T_0LL' + 2i\delta(x-x')L_2, \quad L_2 = 2\varphi^*(x)\varphi(x)A\cdot A'$$
$$S = \int\left(igL - \frac{1}{2}g^2L_2\right) - g^2\frac{1}{2}\iint T_0LL' + \mathrm{higher}$$
$$= ig\int L - \iint\frac{g^2}{2}TLL' + \mathrm{higher} \tag{57}$$

The separate e,e' dependence is a consequence of the independent directional fluctuations i.e. reminder that e is not the gauge parameter of the noncovariant axial gauge but rather the fluctuating string variable of a covariant stringlocal potential. Since the anomaly contributions are Wick-ordered quadrilinear terms there is no problem with setting $e=e'$; the only problematic aspect is to identify the $e's$ in propagators. In momentum space scattering amplitudes one can always avoid the dangerous e-directions by choosing the $e=e'$ such that the denominator in the propagator does not vanish. Then the formalism guaranties that each contribution to a 2-particle scattering amplitude is well-defined and their sum (the scattering amplitude) is the e-independent sum of these contributions is guarantied by the formalism (the string-independence of the S-matrix).

In (57) the last equation has absorbed the L_2 contribution into a redefinition of the T-product. This is a notational simplification for tree contributions of arbitrary high order which the gauge description does not suggest. As mentioned in (49) the SLF setting permits a "backdoor" construction of pointlike interaction densities; their momentum space behavior corresponds to what one expects from the pointlike counterterm formalism but, different from the latter it introduces no new undetermined coupling parameters. For such computations it is necessary to use the N_μ for the renormalization of the $TV_\mu L'$ derivative terms. This observation is restricted to pointlike interaction densities of arbitrary order but does not yet extend to field correlations; for the latter one has to extend the Bogoliubov S-matrix formalism, a step which is well known for pointlike fields, but still needs to be elaborated for stringlocal fields.

The structure of the definition (49) shares with the naive expression obtained from second-order pointlike perturbation theory the large momentum increase, but the mass–shell restriction of the former leads to the cancelation of leading high momentum contributions which is the momentum space counterpart of the on-shell "peeling property" in x-space. This difference between off-shell correlations and the on-shell lowering of the p-increase has no counterpart in the pointlike Feynman formalism. As everything which is different from the standard pointlike formalism, its origin is the powerful Hilbert space positivity which starts to assert itself in massive $s \geq 1$ interaction; as such it is a completely new phenomenon with no counterpart in the Krein space gauge setting.

7.3. Couplings to Hermitian fields and the Higgs model

Although having no counterpart in classical theory, one may ask whether it is possible to couple a massive vectormesons to a Hermitian scalar fields H as kind of "charge-neutral" counterpart of massive scalar QED. A second-order BRST operator gauge treatment of such a situation which is suitable for a comparison with our SLF setting has been given by the University of Zürich group (Scharf, 2001 and references therein) and more recently in Duetsch et al. (2010). It is appropriate to briefly recall their results before presenting the solution in the SLF Hilbert space setting. For comparison it is helpful to reformulate their derivation in analogy to our Hilbert space formulation (Mund & Schroer, 2014a).

The first-order pair L, V_μ which corresponds to the lowest pointlike interaction with a Hermitian field H is[48] ($\phi_{Scharf} \sim m\phi$)

$$L^P = m(A^P \cdot A^P H + cH^3) = L - \partial_\mu V^\mu \text{ with:}$$
$$L = m\left(A \cdot AH + \frac{1}{2}A \cdot (\phi \overleftrightarrow{\partial} H) - \frac{m_H^2}{2}\phi^2 H + cH^3 + u\tilde{u}H\right)$$
$$V_\mu = m\left(A_\mu \phi H + \frac{1}{2}\phi^2 \overleftrightarrow{\partial}_\mu H\right) \tag{58}$$

where the superscripts K on A_μ, ϕ, L and V_μ have been omitted for notational convenience (for the notation see (35)). The mass factor m (the vectormeson mass) has been introduced in order to keep track of the overall "engineering dimension" $d_{en} = 4$. Even though the conceptual content of the Hilbert space approach is quite different from the gauge theoretical approach, there are close formal correspondences between the differential form d_e formalism with the BRST nilpotent s-operation. Instead of starting with pointlike trilinear interaction L^P and converting it into an L and a divergence of a V_μ, there is the more general looking possibility to start with a trilinear Ansatz for a stringlike $\hat{L}$ within the power-counting restriction and find the correct L and V which fulfills the string-independence string-independence $d_e(L - \partial V) = 0$ in a unique way (up to a contribution in V whose divergence vanishes). We may call this the first-order "induction".

We now pass to the second-order implementation of the BRST s-symmetry (the operator form of gauge invariance in the Krein space). The appearance of a $u\tilde{u}H$ term, which only vanishes on *Kers/Ims*, has no counterpart in the SLF setting; it simply does not occur in the Hilbert space setting. Again one computes the anomalies of the one-contraction $(1-c)$ contributions of the s operation according to the rules (35) and compensates them with corresponding normalization terms by choosing the free normalization parameter in TLL' in such a way that, in analogy to massive QED they match the well-defined anomaly A. The induced counterterms which together with the T_0-product define the renormalized T-product ($Q_\mu := sV_\mu$)

$$A = sT_0 LL' - (\partial^\mu T_0 Q_\mu L' + (x \longleftrightarrow x')) = sN + N_\mu$$
$$\text{with} \quad TLL' = T_0 LL' + C, \quad TQ_\mu L' = T_0 V_\mu L' + C_\mu$$

For the calculation of the renormalized S-matrix we only need the one-contraction (tree) component of the anomaly. As has been shown in (Scharf, 2001, p. 147) this leads to four induced delta function anomaly terms $N = \delta(x - x')L_2$ with

$$L_2 = AAH^2 + AA\phi^2 - \frac{1}{4}m^2 m_H^2 \phi^4 - \frac{1}{2}m_H^2 \phi^2 H^2 + cH^3 + c'H^4 \tag{59}$$

Here the c' is an additional coupling which, although still free in second order, is needed for the compensation of anomalies in third order which leads to the value $c' = -\frac{1}{4}(m_H^2/m^2)$. In other words the Mexican hat potential is fully induced by the gauge s-invariance of the S-matrix. There is no place for symmetry breaking shifts in field space.

Again the sum of the local second-order term $(g^2/2)L_2$ is not physical by itself, but the sum

$$\frac{g^2}{2}\left(\int L_2(x)d^4x + \frac{1}{2}\iint T_0 L(x)L(x')d^4xd^4x'\right) \tag{60}$$

is the second-order contribution to the gauge-invariant S-matrix. As in (57) the form of the induced interaction L_2 depends again on the definition of the T_0 with which the anomalies were computed; and as in the previous case of scalar massive QED one can absorb the quadratic terms in A in (59) into a change $T_0 \to T$. What remains is the quadrilinear H-ϕ potential which together with the A-independent terms from L_1 can brought into the form of a Mexican hat potential as shown in Scharf (2001). But here this is a result of a *second-order gauge induction and not of a symmetry-breaking interaction*; the numerical coefficients of the induced potential are ratios of the masses and do not depend on a symmetry-breaking field shift. Together with the first-order $H - \phi$ contributions they can be written in the form of a Mexican hat potential.

Contrary to the destruction of gauge invariance by a numerical field shift in the gauge-dependent field of scalar QED and subsequent adjustments in terms of special gauges, *the induced Mexican hat potential results from the preservation of gauge invariance for the coupling of a Hermitian field to a massive vectormeson using the BRST gauge invariance of the S-matrix through the relation* $sS = 0$. Since all operators are massive there are no infrared problems. As a result the second-order inductions of a Mexican hat potential from the implementation of the BRST s gauge invariance is totally different from the introduction by hand of a Mexican potential whose purpose is the (impossible task) to break the gauge symmetry in order to generate a mass. The work of the university of Zürich group (Scharf, 2001; Aste et al., 1997) should have caused the ringing of bells with respect to the Higgs issue, but it was ignored.

[48] A term $A^P \partial H^2$ turns out to be a total derivative since $\partial A^P = 0$.

In the SLF setting the calculation proceeds in a similar fashion. The e-independence first-order argument results in

$$L = m\left(A\cdot(A^P H + \phi\partial H) - \frac{m_H^2}{2}\phi^2 H + cH^3\right)$$
$$V_\mu = m\left(A_\mu^P \phi H + \frac{1}{2}\phi^2 \overleftrightarrow{\partial}_n H\right) \tag{61}$$

Again the m factors keep track of the engineering dimension. Different from the previous case there are off-diagonal propagators between A, A^P and ϕ. It turns out that the best way to handle this problem is to use one A^P instead of only $A's$; this can be done as long as the power-counting restriction $d_{int} \leq 4$ is obeyed. Apart from the absence of the $u\tilde{u}H$ term and the difference in normalization between the negative metric ϕ^K and the physical escort field ϕ, the algebraic steps of the implementation of second-order string independence in the spacetime d_e differential form calculus are following the same steps as those of the nilpotent s calculus. Therefore it is not surprising that also the results are accordant. One expected difference is the appearance of both e and e' in the Mexican hat potential; this is similar to the second-order expression of the previous massive scalar QED model. The appearance of in e, e' asymmetric term in addition to the symmetric Mexican hat contribution is however unexpected. This term vanishes on the diagonal $e = e'$. There is no problem to let the directions coalesce in the Wick-ordered Mexican hat potential, the problem is the propagator of the tree-component. For each momentum space region this is possible by choosing $e = e'$ such that the denominator does not vanish. This suffices to insure that each contribution to the second-order tree approximation is well defined and by construction the result of adding up the various contributions is independent of any e. The full second-order contribution will be presented in a joint paper with Mund (Mund & Schroer, 2014a).

The calculation in the stringlocal Hilbert space setting confirms the results of the BRST gauge setting. This confirmation is important because the physical content of the gauge formalism is restricted to the gauge-invariant local observables but the S-matrix is a global object.

As mentioned before, it is not necessary to go through detailed calculation if one only wants to see the inconsistency of the Higgs-mechanism with the principles of QFT. From a conceptual viewpoint the fast way is to argue that couplings of massive vectormesons to any matter cannot produce conserved currents with diverging charges (the spontaneous broken symmetry condition). Their "Maxwell charge" is always screened and in case of only Hermitian matter, *the identically conserved Maxwell current is the only current*. In zero order i.e. for a free massive vectormeson one has

$$\partial^\mu F_{\mu\nu} = j_\nu^{Maxwell} \sim m^2 A_\mu^P \tag{62}$$

and higher order corrections can be computet by using the SBEG renormalization theory for fields. It is somewhat strange that the followers of the Higgs mechanism did not at least check the zero-order Maxwell current of a massive vectormeson and verify that its charge is screened and not spontaneously broken. In the following table all possible situations related to conserved currents have been collected

$$\text{screening}: \ Q = \int j_0(x)d^3x = 0, \quad \partial^\mu j_\mu = 0$$
$$\text{spont.symm.-breaking}: \ \int j_0(x)d^3x = \infty$$
$$\text{symmetry}: \ \int j_0(x)d^3x = \text{finite} \neq 0 \tag{63}$$

In order to avoid any misunderstanding, the present critique is not directed against discoveries made by metaphoric arguments; many discoveries, including Dirac's important idea of antiparticles, were based on incorrect models or theories (the hole theory). Metaphoric observations are valuable placeholders but start to be harmful if in due time they are not replaced by arguments compatible with the foundational principles of QFT. The idea that QFT can say something about the masses of elementary particles (masses of the interaction-defining fields) is incorrect; its causality principles are expected to determine masses of bound states which are interpolated by composite fields but for there description one still has to rely on methods of lattice approximations.

The case of Goldstone's spontaneous symmetry breaking is no exception. The definition of a spontaneous symmetry breaking is the existence of a conserved current whose charge (the would be generator of a symmetry) cannot generate a symmetry because the integral over the zero component of the current diverges. The Goldstone theorem says that this can only happen if the energy momentum spectrum starts at zero; for self-interacting bosons its content is more specific in that there must be a zero mass Goldstone boson which couples to the conserved current and prevents the convergence of the integral over the zero component of the current for large distances. The shift in field space is not the definition but only a mnemonic trick (a "pons asini") to find a model of self-interacting scalar fields whose first-order interaction leads to such a current; the intrinsic observable properties of such a situation (the correlation functions) do not contain a field shift but only scalar fields and their physical masses. Nevertheless it is within the range of metaphorical tolerance to say that "the shift breaks the symmetry". The problem with the Higgs mechanism is that this metaphor created the idea if a spontaneous mass creation which is not compatible with the structure of QFT.

The situation of coupling of massive vectormesons to matter is totally different. In that case there is the Schwinger–Swieca theorem which says that the charge of the Maxwell current is screened and in the H-model this is the only current. This case involves $s \geq 1$ for which is known that the coupling of massive vectormesons in the Hilbert space leads to nonrenormalizable interactions as a result of violation of the power-counting bound for interacting $d=2$ Proca potentials. The trick for $s=1$ is to use the BRST Krein space gauge setting but this severely limits the confiable range of validity to the small subset of gauge invariant observables (the vacuum sector). The new Hilbert space formulation requires to replace the pointlike by stringlocal vectorpotentials and in this way secures the physicality of all operators; the important field operators are stringlike. But it makes good sense to use stringlike operators to generate particle states because the difference between point-and stringlocal disappears on the level of particle states; there simply are no point- and stringlike particles. Different from the Goldstone situation where the issue is the construction of a first-order interaction (in terms of free fields which already have the masses of the physical particles) which leads to a lowest order current with the Goldstone properties, the lowest renormalizable order in the s=1 situation must be constructed according to the above $L - \partial V$ requirement (the s respectively the d_e invariance) whose validity must be insured in higher orders. In both cases there appears a second-order induced Mexican hat potential, but in contrast to the Goldstone case this has no relation with a symmetry-breaking field shift.

To discover something important *together with the correct and final theoretical explanation* is an unreasonable requirement on the discoverer (in this case Peter Higgs), this is rather the responsibility of the particle theoreticians who use the observation. Critique is the live-blood of any highly speculative theoretical research especially if it takes place on the frontiers of particle physics. Interestingly a critical view was already around at the time of Higgs' discovery: namely Schwinger's suggestion that currents of massive vectormesons lead to "screened charges" and Swieca's subsequent proof which led him to the terminology "Schwinger–Higgs screening" (Swieca, 1976), see also Buchholz & Fredenhagen (1979). Unfortunately these early attempts were ignored and vanished in the maelstrom of time. Schwinger did not mention that in massive gauge theories there are two currents: the Maxwell current and the particle-antiparticle counting current which only coalesce in the massless limit. For H-couplings there is no counting current and the coupling disappears (decomposes into free fields) in the massless limit.

The arguments against spontaneous mass creation (the "Higgs mechanism") do in no way rule out that a H-field may be necessary for other foundational reasons. Whereas the calculations in this section show that this is not the case in massive abelian gauge theory, the gauge invariance of the second order S-matrix for selfinteracting massive vectormesons requires the presence of a H-field. In other words in order to obtain a consistent theory of electroweak interactions it is not sufficient to pass from the Fermi theory to one in which the interaction is mediated by massive vectormesons and the photon. Consistency as a nonabelian gauge theory reqiuires also the presence of a H-field as a kind of "dynamical escort" of massive selfinteracting vectormesons. The new stringlocal Hilbert space setting, in which all fields are physical, is expected to reveal more about the connection with the foundational causality principles of QFT.

7.4. Self-interacting massive gluons and remarks on confinement

For abelian massive gauge theories in the SLF Hilbert space formulation there are no structural theoretical reasons for enlarging the field content beyond the matter fields to which one wants to couple the massive vectormesons since its escort fields do not create any additional degrees of freedom. This is less clear in case of self-interacting massive gluons. Although the arguments against the consistency of the Higgs mechanism are generic (independent of the kind of vectormeson interactions), there could be other consistency requirements coming from the foundational modular localization properties in THE Hilbert space which make it necessary to introduce additional degrees of freedom. Present calculational attempts indicate that this is not the case; to exclude such situations higher order calculations are necessary (Mund & Schroer, 2014b).

The escort fields ϕ fields do not count in this balance since they are part of the Hilbert space formalism for all higher spin interactions; they are already present in the interaction-free case where they enter the relation between the pointlike Proca potential and its stringlike sibling. The masses of self-coupled massive vectormesons are totally independent and the mass of each escort is equal to that of the stringlocal vectormeson which it escorts. On the other hand the masses of coupled H-fields are independent and (as all masses, except for that of the A-dependent escorts which must be identical to the A masses) are part of the interaction-defining free field content.

In the remainder of this subsection we will present the first-order stringlocal Y–M interactions which are obtained from the $d_e(L-\partial V)=0$ argument which also determines the first-order pointlike interaction density L^P with its $d_{sd}>4$ scaling degree. For simplicity we take the equal mass $O(3)$ Y–M model. The starting point is the reduction of the power-counting violating $d=5$ dimension pointlike interaction L^P by peeling off the highest dimension 5 and in this way obtaining a $d=4$ stringlike interaction density L

$$L^P=\sum \varepsilon_{abc}F_a^{\mu\nu}A_{b,\mu}^PA_{c,\nu}^P=L-\partial^\mu V_\mu \quad \text{or} \quad d_\varepsilon(L-\partial^\mu V_\mu)=0 \tag{64}$$

$$L=\sum_1^3 \varepsilon_{abc}\left\{F_a^{\mu\nu}A_{b,\mu}A_{c,\nu}+m^2A_a^{P\mu}A_\mu^b\phi_c\right\}, \quad V_\mu=\sum \varepsilon_{abc}F_a^{\mu\nu}(A_{b,\nu}+A_{b,\nu}^P)\phi^c \tag{65}$$

Actually we could have started with the most general trilinear Ansatz for $\hat{L}$ in terms of A and ϕ. Since there are four such terms, this Ansatz would contain four different types of yet undetermined $f_{abc}^i, i=1,\ldots,4$. Then asked the question would be whether within this general Ansatz for $\hat{L}$ there exists a $\hat{V}_\mu$ such that

$$d_e(\hat{L}-\partial^\mu\hat{V}_\mu)=0 \tag{66}$$

The only solution (up to additional divergence free contributions to V_μ) of this requirement in the case of equal masses turns out to be the first line of (64). Defining the content of the bracket as L^P we realize that the first-order stringlocal S-matrix is equal to the first-order pointlike counterpart since the two different first-order interaction densities are adiabatically equivalent (the boundary term from the divergence of V_μ vanishes in the adiabatic limit)

$$\int L^P=\int L, \quad L^P \overset{AE}{\simeq} L \tag{67}$$

This is the beginning of an extremely restrictive *induction mechanism* which has no counterpart in the nonrenormalizable pointlike $s\geq 1$ setting. For the full Lie-algebra structure (64) one has to proceed to the induced second order which will be done in Mund & Schroer (2014b).

These observations generalize those which were already made in the abelian case in Section 6.2; the locality principle together with Hilbert space positivity leads to restrictions between couplings which are analogous to those of classical gauge theory (the geometry of fibre bundles). Here they are simply the result of the Hilbert space positivity which for interactions which couple $s\geq 1$ fields requires the use of string-localization. There is absolutely no need for any support from the fibre-bundle setting of classical gauge theory; QFT does not need any "crutches" from classical field theory such as those which are provided by the classical-quantal parallelism of quantization. Any quantum fields obtained from covariantizing Wigner's classification of positive energy representation of $\mathcal{P}$ can be coupled to a scalar density which defines the first-order interaction density of a QFT and in case its short distance dimension falls within the power-counting range $d_{sd}\leq 4$ the interaction density is on the best way to define a renormalizable model of QFT. The above "self-induction" mechanism also works for unequal masses; in this case the $f's$ depend also on mass-ratios.

The potentially most important consequence of the Hilbert space SLF formulation is a profound insight into hitherto incompletely or not understood infrared phenomena as "infraparticles" and confinement. Concerning the latter, the remarks that one finds in the literature do not go beyond the statement that the perturbative expressions for the massless gauge-variant correlations of gluon- or quark-fields are infrared divergent and that this indicates the breakdown of perturbation theory. But the behavior of pointlike gauge-variant matter fields in a BRST gauge setting is physically irrelevant; what one needs is an understanding of *the infrared property of massless limits of massive stringlocal gluon correlations* and the only way to do this is offered by the SLF formalism in Hilbert space, a task which is outside the physical range of gauge theory. One expects that all correlations vanish which contain besides pointlocal composites also gluon/quark fields; in fact this seems to be the only way in which the localization principles of QFT can realize confinement. Pointlike physical fields never lead to confinement.

The infraparticle situation is slightly more accessible . The Yennie–Frautschi–Suura (YSF) proposal (generalizing previous model calculations by Bloch and Nordsiek) introduces an ad hoc infrared regularization ε in terms of which the scattering amplitudes involving charged particles are logarithmically divergent for $\varepsilon\to 0$. The leading logarithmic divergencies are then summed up to a coupling-dependent power behavior containing factors $\lambda^{f(g)}$ which vanishes for $\lambda\to 0$. The vanishing of the scattering amplitude shows that the LSZ scattering theory is not the correct concept for obtaining nontrivial scattering information for "infraparticles"; in fact the presence of milder cut-type singularities which replace the one-particle mass shell poles confirm that such milder singularities cannot counteract the large-time dissipation of wave packets in the LSZ time-dependent scattering theory so that one obtains zero for $t \to \infty$. Low order perturbative calculations also show that the vanishing can be prevented by passing from scattering amplitudes to photon inclusive cross sections before letting $\lambda \to 0$. This limit constructions should be viewed as a perturbative analog of recent more abstract representational proposals to describe charged states (Buchholz, 2013; Buchholz & Roberts).

Although both QED and Y–M gluons couplings lead to string-local fields without singular pointlike counterparts, their mathematical structure and physical manifestations are very different. *Interacting vectorpotentials in QED are integrals over pointlike observable zero mass field strength whereas this property is lost in massless Y–M interactions*. This implies in particular that massless gluon strings cannot be approximated by local observables. Such objects are inherently nonlocal in an irreducible sense. This severe nonlocality cannot occur in $s < 1$ models, even global objects as charges (integrals over pointlike currents) can always be approximated by compact localized matter. The emergence of Inherently noncompact fields from collisions of ordinary matter would create havoc with causality; this only can be avoided if they remain virtual objects whose use is necessary in order to formulate the interaction density but disappear in the correlation functions. Confinement in the sense of vanishing correlation functions which contain in addition to pointlike composites also irreducible stringlike gluons solves this problem in a radical way.[49]

The definition of interacting zero mass vectormesons as limits of their much simpler massive counterparts in terms of their correlation functions (from which one may reconstruct the operator formulation) accounts for the fact that the limit represents an inequivalent representation in which the Wigner–Fock structure of the Hilbert space is lost. Structures which are expected to be independent of the mass, as the Callen–Symanzik beta-function $\beta(g)$, should be computed in the massive case since a direct perturbative derivation of the Callen–Symanzik is not possible due to the presence of infrared divergencies. A derivation of the C–S relations for stringlocal and hence renormalizable massive vectormesons should be possible and provide a proof (and not only a consistency argument based on additional assumptions outside mathematical control[50]) of asymptotic freedom and in this way close that old but unfinished subject.

The above confinement scenario presents an interesting contrast to another kind of stringlocal matter: the QFT of Wigner's zero mass "infinite spin" positive energy representation class. Actually the understanding of the importance of string-localization for the conceptual progress of QFT started with a paper (Mund et al., 2006) in which the main point was the presentation of the QFT behind this mysterious 1939 Wigner representation. As a positive energy representation it shares properties as the stability of matter and coupling to the gravitational field with the massive and massless finite helicity representations. It turns out that the infinite spin Wigner representations contain no pointlike covariant wave functions at all and there are convincing arguments that the associated net of local algebras admits no compact localized subalgebras generated by composite pointlike fields; such representations describe noncompact matter par excellence.

Whereas gluon or quark matter cannot emerge from collisions of normal matter (which interacts in a compact region), Wigner's noncompact free infinite spin matter, once it got inside our universe, cannot be registered in earthly particle counters. In fact it is totally inert apart from gravitational manifestations (Schroer-a). This means that the presence of such inherent noncompact matter would change the gravitational balance of normal matter in a galaxy. When Weinberg wrote his book on QFT he rejected the infinite spin matter because "nature does not make use of it"; at that time its strange noncompact localization properties were not yet known, apart from the fact that all attempts to describe this matter in terms of pointlike covariant fields had failed. Although its property of eluding registration in particle counters would still cause stomachaches with high energy physicists, it seems that astrophysicists should like such inert matter whose only arena of action are galaxies.

It may be helpful for the reader to use again Galileo's method of codification in terms of a dialog between Sagredo and Simplicio.

Sagredo: Dear friend Simplicio, are you still claiming that the abelian Higgs mechanism is only a metaphor for the coupling of Hermitian (chargeless) scalar fields to massive vectorpotentials i.e. the neutral analog of the massive scalar QED? And would this mean that the mass of the massive vectormeson and the Hermitian Higgs field in the simplest (abelian) coupling does not originate from a spontaneous symmetry breaking of the scalar two-parametric QED[51] in terms of a "field shift" (the "gauge-breaking" defining Mexican hat potential)? Is the picture of a distinguished particle whose interaction does not only create the mass of the vectormeson but also its own mass (often referred to as the self-creating "God particle") inconsistent with the principles of QFT?

Simplicio: This is more or less my point of view, but I would suggest to look at the present situation in a historical context. History is more lenient, in particular it explains how the protagonists of the "Higgs mechanism" were led to their ideas within the prevalent Zeitgeist which dominated the post QED particle theory. For a long time (and to a certain extend even nowadays) its was believed that interactions of zero mass vectormesons are simpler than those involving their massive counterpart (spinor or scalar QED) and that therefore one should try to understand the massive interaction by starting from massless models and think about ideas of how to generate masses.

Sagredo: But isn't this true, are massless propagators and their use in Feynman loop integrations not much simpler than integration involving massive propagators; and above all isn't "massive QED" nonrenormalizable because any coupling of a massive vectorpotential (a Proca field) would lead to power-counting violating interactions of short distance scale dimensions $d_{int} > 4$?

Simplicio: Not quite, at least if you recall what QFT is about, namely to understand particle theory in terms of the foundational localization principle of QFT. The most basic structure of quantum physics is the Hilbert space positivity and this is violated in both cases. In QED the violation enters explicitly through the use of pointlike massless vectorpotentials which only exist in indefinite metric Krein spaces, and its massive counterpart is nonrenormalizable as a result of the short distance dimension d=2 of the Proca potential and only becomes formally renormalizable by the use of a $d=1$ potential (together with a negative metric scalar Stückelberg field) in Krein space. The required formalism is the BRST gauge setting which is somewhat more elaborate than the QED Gupta Bleuler formalism. In both cases the physics is reduced to the vacuum sector which the gauge-invariant observables generate by acting on the vacuum state. This restrictive nature of the quantum gauge formalism (i.e. its limitation to the vacuum sector) is shared between the massive and massless case, but the infrared problems from massless vectormesons come on top of these limitations.

Sagredo: Yet people compute scattering amplitudes, which certainly cannot be obtained within the vacuum sector of gauge theories. How can one understand this?

[49] Only quark–antiquark pairs separated by a finite string can avoid confinement since their compact nature avoids the problem cause by noncompact matter.

[50] The assumption that in deriving the Callan–Symanzik equation one can separate low from high momenta.

[51] Different from spinor QED which only has one coupling parameter, the application of the standard pointlike renormalization formalism to a scalar gauge coupling leads to an additional quadrilinear self-coupling of the matter field.

Simplicio: This is indeed a sore point of quantum gauge theory which has no analog in classical gauge theory. Strictly speaking the S-matrix should be computed from the LSZ limit of fields, but there is a formalism which goes back to Bogoliubov which represents the nth order S-matrix in terms of formal spacetime integrals over time-ordered products of the (first-order) interaction density. The Krein space gauge setting uses this formalism within the BRST operator formulation and claims that the BRST condition $sS=0$ in terms of the nilpotent BRST s-operation insures that the resulting S lives in the Wigner–Fock space of physical particles. But this requirement cannot be formulated within the vacuum sector of the local observables, so the conceptual clarity remains less than perfect.

Sagredo: All these problems arise because one tried to resolve the conceptual clash between $s \geq 1$ pointlike interactions and Hilbert space positivity at the expense of the Hilbert space in favor of keeping the pointlike field formalism for tensor potentials. Can one not take the other direction by letting the Hilbert space positivity decide which is the tightest covariant localization consistent with it?

Simplicio: Yes one can, provided one is prepared to make a new conceptual investment of the same caliber as that which led from the old (Wentzel, Heitler) noncovariant perturbation theory to that of post WWII covariant QED which included vacuum polarization (loop contributions). It turns out that the clear answer to your question is to use covariant stringlocal fields localized on spacelike lines $x+\mathbb{R}_+e$. But this is much easier said than done, it amounts to a nearly revolutionary change of QFT of almost all of its perturbative aspects except of its causal localization principle which becomes strengthened. This is not surprising because people did not opt for gauge theory because they were unaware of the physical importance of the Hilbert space positivity but rather as a result of lack of apparent alternatives. There were hints in what direction to look at by Mandelstam and DeWitt but they consisted in the restriction of the formalism to field and missed the short distance improving stringlocal potentials. Others observed that the axial gauge is, together with the noncovariant Coulomb gauge, consistent with Hilbert space positivity, but failed to treat the e-variable as a fluctuating spacetime variable by assigning to it the role of a gauge parameter which is the same in all fields. This misunderstanding caused serious renormalization problems of short-mixed with long-distances which finally led to the abandonment of this gauge.

The correct understanding came in a roundabout way from the solution of the localization problem related to the infinite spin Wigner representation by methods of modular localization (Brunetti et al., 2002; Mund et al., 2006). The related free field theory turned out to describe noncompact localized "stuff": not only potentials but all covariant fields are stringlocal. From here arose the idea that all massless $s \geq 1$ free potentials are covariant relatives in the Hilbert space of the noncovariant Coulomb (radiation) representations. Though the price to pay in terms of localization is surprisingly little since the smallest causally closed noncompact localization region is an arbitrarily narrow spacelike cone whose core is a semi-infinite string, to deal with the renormalization theory in the Hilbert space of stringlocal fields with independent directional fluctuations is a quite unaccustomed new problem.

Physicists of the older generation as the principle protagonist of the BRST formulation Raymond Stora conisdered that gauge theory as a (surprisingly successful) placeholder for a future Hilbert space formulation. One expects that the restrictive Hilbert space positivity leads to new insights outside the range of gauge theory (a different view of the "Higgs mechanism", the ability of Y–M interactions to exist without the classical fibre-bundle "crutches", a deeper and more specific understanding of what hides behind infrared divergencies as the confinement problem).

Sagredo: Are their any new physical concepts which have no counterpart in the pointlike setting?

Simplicio: Yes there are several. One remarkable new aspect is the appearance of "escorts" of stringlocal massive vectorpotentials; these are stringlocal scalar Hermitian fields ϕ (one for each massive vectormeson). Its name refers to the fact that (unlike a H-field) it has the same mass and the same coupling strength as the vectormeson, but (also unlike the H) it does not add new degrees of freedom to those which are already contained in the stringlocal vectormeson i.e. it is a kind of "massive gluonium field". In the abelian Higgs model, whose physical content in the Hilbert space SLF description is just the unique renormalizable $A \cdot AH$ coupling, this first-order interaction density induces a second-order Mexican hat like potential in H and ϕ. Naturally the numerical coefficients depend only on the ratio of the two masses of the massive A and the H. The new Hilbert space SLF setting turns the Higgs mechanism from its hat to its feet: instead of spontaneously creating masses with the help of a symmetry-breaking Mexican hat potential the renormalizable interaction between massive A and H fields associated with a string-independent S-matrix *induces* a second-order Mexican hat potential. Its form depends on the masses of the defining free fields and the requirement that the Hilbert space S-matrix should be independent of the $e's$ (i.e. it should be a kind of global counterpart of the local observables).

Whereas the shift in field space in Goldstone's model is a quasiclassical device whose physical aim is to prepare a situation in which a conserved current is prevented from leading to a global charge, its application to the gauge-dependent scalar field of QED is not supported by any physical idea. Quantum gauge "symmetry" (misleadingly also called "local symmetry) is, contrary to classical theory in which Hilbert space positivity is not an issue, not a physical quantum symmetry but rather a prescription how to extract a vacuum representation of local observables from an unphysical setting in Krein space. Renormalized perturbative QFT is also not able to incorporate metaphors about the origin of masses. The hope that by formally breaking a symmetry one may save parameters (as compared to using the physical masses in the definition of the model-defining first-order interaction) only lead to frustration upon realizing that such formal devices do not reduce the number of parameters of electroweak interactions; all ways of breaking correspond to all possibilities of choosing masses. One avoids all physically meaningless manipulations by studying directly renormalizable couplings of Hermitian H fields to massive vectormesons in the BRST gauge setting instead of breaking the latter for couplings of complex scalar to massless vectormesons. In this way one realizes that its characteristic intrinsic physical property is the Schwinger–Swieca–Higgs charge screening and not the ssb Higgs mechanism.

Sagredo: If it is that simple as you presented it, namely a kind of neutral counterpart of the Maxwell theory of charged matter, why was such a coupling not studied before Higgs?

Simplicio: Thinking in terms of quantizing Maxwell fields coupled to charge-carrying quantum-matter the generalization to massive vectormesons appears natural; but the idea of coupling neutral (Hermitian) H-fields is quite removed from Lagrangian quantization of classical fields,[52] in particular since such "chargeless" interactions are only possible with massive vectormesons and disappear for $m \to 0$ i.e. have no counterparts in classical electromagnetism. The first indications of what may be different with massive vectormesons came from Schwinger (1963) who suggested that in such a case the charge is "screened" (vanishes); as a model which only exists in the screening phase he proposed the $d=1+1$ rigorously solvable "Schwinger model" (Schroer & Jorge, 2010). In a subsequent structural (nonperturbative) proof of

[52] More precisely of classical fields obtained by reading quantum fields (Dirac spinor, …) and their quantum symmetries back into the classical realm.

charge-screening by Swieca (1976) it became clear that in couplings of massive vectormesons to complex matter there are two conserved currents namely the identically conserved Maxwell current from the divergence of the massive field strength $j_\mu = \partial^\nu F_{\mu\nu}$ and the particle–antiparticle counting current of complex fields; they only coalesce in the massless limit. Swieca emphasized that in case of a self-conjugate H-field the Maxwell charge (the only conserved charged in the abelian Higgs model) is screened and not spontaneously broken; for this reason he used the terminology Schwinger–Higgs mechanism in his publications.

Unfortunately the Higgs mechanism of spontaneous mass generation was proposed by several authors at the same time with identical computational recipes involving field shifts in the gauge-variant complex field of scalar QED so that the shared conceptual error was protected by the "many people cannot err" dictum. Swieca's scientifically successful but sociologically futile attempts may serve as an illustration that there was a well-founded early scientific criticism of these ideas, but the beginnings of a correct understanding were finally lost in the maelstrom of time. After Glashow, Weinberg and Salam supported the spontaneous symmetry breaking the Higgs issue became sealed and the chance for a correct understanding within QFT evaporated.

Sagredo: Even if the "Higgs symmetry breaking" is only a metaphor for a coupling of a Hermitian field to a massive vectormeson as you claim, couldn't it survive as a mnemonic trick which at the end more or less describes what you want? If a metaphoric idea leads to results which later on finds a derivation for which every step is consistent with the principles of QFT, *isn't it justified to credit the discoveries*? After all we attribute the discovery of antiparticles to Dirac even though his hole theory was later recognized as being incorrect.

Simplicio: This is an important point, and yes they do deserve recognition as in many other cases besides Dirac. In a science about foundational properties of matter, the frontiers are often in a very speculative state and discoveries via metaphors are helpful as placeholders for a later understanding. But at the times of Pauli, Feynman, Lehmann, Landau, Kallen, Schwinger, 't Hooft, Veltman, Jost, … none of the many less than correct proposals which resulted from conceptual misunderstandings had the chance to survive for more than a decade (SU(6), peratization, …). The valuable discoveries, as those of renormalized perturbation of nonabelian gauge theories, went through many refinements; starting with 't Hooft and Veltman, passing through Faddeev–Poppov, Slavnov and reaching the level of technical maturity in the BRST formalism before the recent proposal of the use of stringlocal potentials in a Hilbert space setting (which is still very much in its infancy) again returned to it.

There are only two exceptions to the continuous unfolding of a discovery: the five decades old discovery of String theory (which has neither observational nor theoretical credentials) and the Higgs mechanism which is the only experimentally successful discovery which managed to survive for more than 4 decades without any theoretical modification . The (often well-founded) early critique was unable to penetrate the thick protective sociological layer of approval and finally vanished in the maelstrom of time. Present Big Science and a Nobel prize guaranty that the issue will remain protected against scientific critique.

To be more concrete, QFT is a foundational theory based on the quantum adaptation of causal localization. Its perturbative implementation in the most accepted (Bogoiubov) formulation is based on interaction-defining free fields and a first-order interaction density. Hence the definition of a model includes the masses of these fields; renormalization theory insures that these masses are identical to the masses of the observed particles which are considered elementary within that model. QFT is *not a theory which can say anything about masses of the defining "elementary" fields*. Renormalized perturbation theory in its present stage is not able to say something about bound states, associated to composite fields; for this task one presently uses lattice approximations. In case of the massive vectormeson-H coupling one only needs to specify the first-order AAH coupling, the rest (which include the second-order "Mexican hat" shaped H-self-interactions) do not belong to the defining properties of the model but are *induced* by the powerful BRST gauge conditions (the s-invariance of the S-matrix) or by the even more powerful Hilbert space positivity condition in the new SLF setting (the differential calculus implementing string independence).

All the numerical aspects of the induced second-order potential are fixed in terms of the masses of the model-defining free fields; there is no place for gauge symmetry breaking field shifts. By ignoring the gauge aspects of scalar QED one can of course envisage a quasiclassical picture of how to attain such a H-coupling, but when setting up the renormalized perturbation theory involving a massive vectormeson one has to liberate oneself from such pictures and follow the BRST rules of gauge theory starting with the $gAAH$ coupling of massive vectormesons to Hermitian scalar fields and let the BRST operator formalism do its job (the implementation of $sS=0$) (Scharf, 2001). Otherwise one may overlook the fact that the Mexican hat potential is induced in order g^2 of the gauge-invariant S-matrix and depends on the two masses. Of course one may re-construct from these two data the strength of a quadrilinear self-coupling of an imagined scalar QED and the gauge breaking shift parameter in field space, but why does anybody want to construct something, which is it best a quasiclassical image, if the first order which contains the mass of the vectorpotential as well as the H-mass is already prepared for renormalized perturbation theory in the BRST setting.

This is totally different from setting up perturbation theory of a Goldstone spontaneous symmetry breaking. Here the starting point is the intrinsic definition of spontaneous symmetry breaking. Contrary to popular opinion it is not the shift in field space but the physical (observable) attribute of a conserved current whose charge (the would-be symmetry generator) diverges. A structural theorem (Ezawa & Swieca, 1967) says that this can only occur in the presence of a zero mass Goldstone particle which destroys the convergence of the charge for large distances.[53]

The task is therefore to find renormalizable couplings in which zero mass and massive fields interact in a way which leads to a conserved "Goldstone current". A convenient way to get there is to use the quasiclassical device of a field shift which itself is not a physical parameter. This current is very different from that associated to a massive vectormesons which is the only current of the abelian H-model. The Maxwell current of a massive vectormeson (the only current of the H-model) leads to "screened charge" $Q=0$, whereas a spontaneously broken symmetry manifests itself in terms of a diverging charge $Q=\infty$. Nothing could be more different than that! The unification of both phenomena under the roof of spontaneous symmetry breaking is a conceptual misunderstanding. For the construction of the formel Goldstone current the shift in field space is a useful trick (although it does not help in the construction of the renormalized Goldstone current) whereas the Higgs breaking mechanism has no concrete aim since the logic is that of the BRST formalism applied to a AAH coupling.

Interacting massive gauge theories exist with arbitrary masses and in the Hilbert space setting each massive vectorpotential is accompanied by its scalar stringlocal escort ϕ which carries the same mass. For equal vectormeson masses (in particular for zero masses) the induction mechanism imposes a Lie-structure on the self-couplings of vectormesons; solutions with completely independent couplings but

[53] His Cargese lecture notes (Swieca, 1970) on this topic are highly recommended since they reveal the clarity and depth on which these issues were once understood. His profound knowledge about spontaneous symmetry-breaking led him some years later to the "Schwinger–Higgs charge screening mechanism" (Swieca, 1976).

equal masses would violate the Hilbert space positivity. *H*-fields may be coupled in addition, but their presence is not necessary for "fattening" massless gluons and *H*-fields. If, as it seems to be the case, H-fields are necessary for the consistency of the gauge theory (or better: to uphold causality in a Hilbert space setting) then the problem to understand such a situation points into a quite different direction than the metaphor of a spontaneous symmetry breaking.

Interacting zero mass vectormesons (QED, QCD) are outside the range of the standard field-particle setting in a Wigner–Fock Hilbert space. In that case one needs to go the round-about way of computing appropriately infrared-renormalized correlation functions of string-local vectormesons in the massless limit and then reconstruct the operators using Wightman's reconstruction theorem. One expects that the characterizing property of confinement will be the vanishing of correlation functions containing in addition to local observables also self-interacting massless stringlocal gluon and quark fields.

Sagredo: In your new setting the stringlocal fields $\Psi(x,e)$ are renormalizable in the standard sense of the power-counting criterion, which in particular means that they are localizable in the sense of Wightman's testfunction smearing with the Schwartz $\mathfrak{D}$ functions in the (x,e) variables. On the other hand you claim that your new stringlocal renormalization theory also allows us to construct associated singular pointlike fields whose short distance scaling degree is unbounded (increasing with perturbative order) which explains their pointlike nonrenormalizability in terms of their worsened localizations. Do these singular pointlike fields play any useful role?

Simplicio: Most of the intuition which comes with the definition of the model in terms of a pointlike massive field interaction is preserved; in fact the associated renormalizable stringlike interaction is obtained by "peeling-off surface terms" which carry the leading short distance singularities and hence do not contribute in the adiabatic S-matrix limit. This has the interesting consequence that the high-energy behavior of scattering amplitudes is better than that which one naively reads off in momentum space by going in a simple-minded way to the mass shell from the Fourier transformed singular pointlike correlation functions. Hence phenomenological arguments in favor of the presence of Higgs particles based on the use of Feynman diagrams must be taken with a grain of salt; the perturbative content of $s \geq 1$ stringlocal interactions cannot simply be encoded into Feynman diagrams (including contributions from counterterms). The intuitive appeal of pointlike couplings is however completely lost in the massless limit; in that limit the singular pointlike fields disappear and the stringlike localization in QED becomes more stiff since different *e*-directions of the localization lines along which infrared photons "hover" cease to be unitarily equivalent (spontaneous breakdown of Lorentz covariance in electrically charged sectors); this is the regime in which the standard field-particle relation is lost.

QFT is presently undergoing significant changes. There are several forthcoming papers which promise to clarify the mathematical problems coming from causal string crossings in addition to the already existing Epstein–Glaser renormalization theory for point-crossings. The development of these new ideas will be slow because foundational knowledge about QFT has been lost seems to be limited to a minority of experts.

Sagredo: I thank you dear friend for sharing your thoughts, and I hope that your pessimistic assessment about foundational particle theory in the shadow of Big Science remains a warning and does not become a prediction about its future. It will take some time to fully comprehend what you told me; lots of important issues to think about lie before me before I will meet you again.//

8. The dual model, misunderstandings about particle crossing

The idea to avoid the use of singular fields, which led to the problem of ultraviolet divergencies, and instead formulate particle physics in terms of the S-matrix goes back to Heisenberg. It was abandoned soon afterwards when the success of renormalized perturbation theory in QED left no doubts that the conclusion of inconsistency of QFT based on those divergencies was premature. The problem which perturbative methods had with strong interactions led to adaptation of the Kramers–Kronig dispersion relations to particle physics. It was modest in scope[54] but after a decade it came to closure by achieving all its objectives (the only project in particle theory which came to a successful closure) which included the support of the validity of the locality principle at that time new high energy region.

This success encouraged several theoreticians to formulate a new constructive S-matrix setting in which the perturbative analytic particle crossing property for the S-matrix (and later formfactors) played the important role. Together with unitarity and Poincaré invariance it became known as the "S-matrix bootstrap" but it was soon abandoned as a result of the unmanageable nonlinear problems arising from simultaneously implementing these three properties "by hand". Without any demonstrable success it nevertheless enjoyed a lot of support even by people who on different topics had been quite critical as e.g. Freeman Dyson. A related problem was the insufficient understanding of the conceptual origin of particle crossing; its derivation from the locality principle for some very special scattering amplitudes did not lead to sufficient insights, and the prohibitively difficult method of analytic functions (Bros et al., 1965) of several complex variables led to an early end of these attempts.

Another attempt to obtain a constructive computational access to particle theory in terms of an on-shell project based on S-matrix properties was formulated by Mandelstam (1968). In analogy to the successful use of the Jost–Lehmann–Dyson spectral representation which led to a rigorous proof of dispersion relation, Mandelstam postulated the validity of a double spectral representation for the elastic scattering amplitude as a starting point for getting access to analytic on-shell properties, including the crossing property.

The era of genuine misunderstanding of particle crossing started with Veneziano's (Di Vecchia, 2008) construction (based on properties Euler's beta function) of a meromorphic function of two variables which had an infinity of first-order poles in the two variables which were related by an analytic crossing relation. Although his presentation did not contain any physical argument why this mathematically constructed function which is meromorphic in variables which he identified with the invariant *s*, *t*, *u* variables (the "Mandelstam variables") should be related with the elastic part of a scattering amplitude, his construction created a lot of excitement within which a critical attitude had little chance. Apparently the results on integrable models, which could have revealed that although scattering amplitudes can be meromorphic in the rapidity variables but not in the Mandelstam variables, were not known to the dual model community.

Instead of speculating about what went on the mind of peoples who excepted Veneziano's use of the dual model meromorphic function as an approximation of an elastic scattering amplitude (to be improved by "unitarization"), it is much easier to understand what kind of quantum field theoretic idea leads precisely to such dual model function. This clarification is due to Mack, and his construction is here referred to as the "Mack-machine"; this name is chosen because it can not only produce Veneziano's dual model and similar dual models constructed later, but in a certain sense it also contains all dual models (all crossing symmetric *meromorphic* functions[55] in s,t,u).

[54] Its main aim was to make sure that the causal locality principle of QFT continues to be valid at the energies of the newly emerging High Energy Physics.

[55] Even in the simplest context of integrable models elastic crossing symmetric scattering amplitudes are not meromorphic in the Mandelstam variables (but rather in the exponential "rapidity variables")

The construction uses conformal global operator expansions for pairs of operators which, in contrast to the Wilson–Zimmermann short distance expansions, are known to converge

$$A(x)B(y)\Omega = \sum_k \int d^4z \Delta_{A,B,C_k}(x,y,z)C_k(z)\Omega \tag{68}$$

$$\langle A_1(x_1)A_2(x_2)A_3(x_3)A_4(x_4)\rangle \rightarrow 3 \text{ different expansions} \tag{69}$$

and applies them to all pairings inside the 4-point function (second line). Each pair of operators has a converging expansion on the vacuum in which the resulting operators C_k stand for a list of composites which can be connected with the given pair through nonvanishing conformal 3-point functions Δ. Used inside the 4-point function, this leads to three different ways of decomposing the 4-point function into a sum over two three-point functions multiplicatively connected by an integration over the z-variables. Mack showed that the Mellin transform of this infinite sum over operators C leads precisely to the pole representation of the meromorphic functions which define dual models; the position of the first-order poles is given in terms of the spectrum of scale dimensions of those $C's$ which couple to the A,B pairs. Veneziano's model corresponds to a certain chiral conformal model, but any conformal 4-point function in any spacetime dimension upon expansion of its 4-point function and Mellin transforming the resulting series always leads to a dual model in the sense of defining a meromorphic function with first-order poles which fulfills a crossing relation. The set of contributing poles is (up to a shared factor) a subset of the anomalous dimension spectrum of the conformal theory. What initially looked magic and unique[56] in Veneziano's is now "mass-produced" by the Mack-machine; demystified in this way it makes no sense to identify the dual model with scattering amplitudes.

A scattering function cannot be meromorphic in the Mandelstam variables but, under special circumstances (integrability) it is meromorphic in the rapidity variables. Conformal theories are interesting quantum field theories from which one can learn a lot about the inner workings of the modular localization properties, but they certainly contain no information about scattering of particles; in fact *interacting conformal models contain no particles at all*, they are rather theories of anomalous scale dimensions which live on a covering of the compactified Minkowski space. Mellin transforms of their 4-point functions may be called dual models, but this has no bearing on interactions between particles. It does not make sense to apply ideas of unitarization to them as if they would define a kind of nonunitary approximation of an S-matrix.

This could have been the end of a misunderstanding and led to the closure of this unfortunate chapter of misguided particle physics. In fact it probably would have been the end if not an even stranger twist would have greatly increased the mysterious aspects and with it the attractiveness of ST. This consisted in the observation that the oscillator algebra resulting from the Fourier decomposition of a certain chiral 10-component conformal current algebra formally related to supersymmetric version of the Polyakov action:

$$\int d\sigma\, d\tau \sum_{\xi=\sigma,\tau} \partial_\xi X_\mu(\sigma,\tau) g^{\mu\nu} \partial^\xi X_\mu(\sigma,\tau), \quad \sigma,\tau = t \pm x$$
$$X = \text{potential of conformal current } j \tag{70}$$

permits a positive energy representation of the Poincaré group which decomposes into a discrete infinite sum of irreducible representation (an infinite (m,s) "tower"). This action is conveniently formulated on the oscillator variables obtained by Fourier transformations in the standard circular compactification of conformal theories.

The construction of such a tower (an infinite component field fields) from an *irreducible algebraic structure* was Majorana's project which he formulated in 1932 with the idea to achieve something similar to what the $O(4,2)$ group representation theory does for the hydrogen atom spectrum in QM. This project was revived in the 1960s when it acquired some popularity under the name "dynamic infinite group representation project" (Fronsdal, Barut, Kleinert, …Bogoliubov, Logunov, Oksak, & Todorov, 1989). Majorana's project as well as its later revival restricted this search to irreducible representations of extensions of the Lorentz group. The only known solution up to date is the representation *on the irreducible oscillator algebra of the supersymmetric 10 component current algebra,* the so-called superstring representation of the Poincaré group. This is a group theoretic fact which, although discovered by string theorists, has no relation to Mandelstam S-matrix based on-shell project.

To understand in a more generic way the prerequisites one need to encounter the representation of a noncompact group as a kind of internal symmetry group on the component space of a multicomponent chiral conformal algebra, it is helpful to be reminded of same basic fact of LQP in which inner symmetries arise from the local net of observable algebras in the vacuum representation. The inequivalent local representation classes (superselection sectors) can in typical cases be combined with the vacuum representation within a larger *field algebra net* (Haag, 1996). There are convincing arguments why a continuous set of superselection sectors (in the presence of zero mass particles as QED one must pass to charge-classes Buchholz, 2013) and noncompact internal symmetries of the field algebras cannot occur in higher than two dimensions. The superselection analysis is very different in $d=1+1$ dimensions and such cases can occur; in fact the abelian chiral current models are examples.

As an illustration let us look at a n-component current algebra

$$\Phi_k(x) = \int_{-\infty}^{x} j_k(x), \quad \langle j_k(x) j_L(x')\rangle \sim \delta_{k,L}(x-x'-i\varepsilon)^{-2}$$
$$\Psi(x,\vec{q}) = :e^{i\vec{q}\,\vec{\Phi}(x)}: \text{ carries scale dimens. } d(q) \sim q^2$$
$$\text{suggests analogy } q \sim p, \quad \vec{q}\cdot\vec{q} \sim p_\mu p^\mu, \quad d_{sd}, \sim m^2 \tag{71}$$

Here we have substituted the somewhat confusing letter X (70) in favor of Φ for the multicomponent current potential because we want to avoid a notation which may suggest the wrong idea of an operator which embeds a chiral conformal theory on a lightray (or on its compactified circle) into a n-dimensional Minkowski spacetime so that at its development in time it looks like a 2-dimensional surface (a tube, in case of a chiral theory on a circle). This picture of a covariant string generating a spacetime tube-like world-sheet is incorrect inasmuch as it is incorrect to think that the classical covariant particle Lagrangian $\sqrt{ds^2}$ leads to a covariant quantum embedding described in terms of a covariant operator $x_\mu^{op}(\tau)$. In fact Lagrangian quantization is the wrong guide; there simply exists *no covariant position operator* whose spectral projectors fulfill the requirements of covariant localization. Wigner was well aware of this limitation when he constructed relativistic particles by representation theory and not by quantization.

In the book on string theory by Polchinski he used this classical relativistic particle Lagrangian as a "trailer" for presenting a relativistic quantum theory of strings based on the Nambu–Goto action which replaces the ds^2 under the square root by the corresponding covariant surface differential. Hence instead of

[56] The uniqueness, which was already expected to be follow from the bootstrap principles, was a precursor of the reductionist idea of a theory of everything (TOE) which originated in connection with ST.

being helpful, this analogy turns out to be a squid load. The quantization of the Nambu–Goto Lagrangian according to the correct rules for quantization in the presence of reparametrization invariance resembles that of quantizing the Einstein–Hilbert action; It is certainly non-renormalizable and has no natural relation to the Poincaré group which acts on the embedding Minkowski spacetime (Bahns, Rejzner, & Zahn, 2014). There is another approach to the square root N–G Lagrangian which is due to Pohlmeyer (Bahns (2004)); it is based on the observation that the classical system is integrable. So instead of confronting the problem of quantization of reparametrization-invariant actions which inevitably leads to renormalization problems, he proposes to quantize the Poisson relations between the infinitely many conserved "charges". The problem with this quantization is that one loses the connection with localization in spacetime and Poincaré covariance.

On the other hand the Polyakov Lagrangian has a direct relation to chiral conformal QFT, so one believes to be on conceptually safe grounds. Here the problem is that the representation of the irreducible oscillator algebra behind the operator formalism (71) which serves for the representation of the Poincaré group (and the ensuing intrinsic localization concept which comes with positive energy representation of the Poincaré group Brunetti et al., 2002) is not the same as the one which localizes the chiral model on the lightray. In other words the Hilbert space representations of the oscillator algebra are not equivalent. The charge spectrum of the chiral theory is the whole $\mathbb{R}^n$ and the sigma-model fields Ψ in (71) are the charge carriers. On the other hand the spectrum of the representation of the Poincaré group is contained in the forward light cone and has mass gaps. The the spectrum of the zero mode multicomponent charge operator covers the full spectrum of the charge superselection structure. The treacherous nature of the analogy between the mass spectrum and the conformal dimensional spectrum in (71) has been overlooked by string theorists.

These analogies become even more seductive if one realizes that a particular discrete particle representation of the Poincaré group (the superstring representation) does appear on the oscillator algebra of a 10 component supersymmetric current model (unique up to a finite discrete "*M*-theoretic" variation). But what has this group theoretic coincidence between a spectrum of a discrete Poincaré group representation on the oscillator algebra of a supersymmetric 10-component abelian current to do with Mandelstam's S-matrix project? The answer is nothing beyond the appearance of crossing symmetric analytic functions. Nevertheless the group theoretic content of this relation is interesting from a historical viewpoint because it is the only known solution of the 1932 Majorana project to find an irreducible algebra which carries a purely discrete representation of the Poincaré group.

In distinction to the string-localization of matter fields interacting with vectorpotentials in previous section, the representations occurring in the superstring representation are pointlike generated. This was precisely what the calculations of the (graded) spacelike commutator of the putative string-fields by string-theorists in the 1990s showed (Martinec, 1993; Lowe, 1994). The situation is somewhat confusing as a result of the fact that the distribution representing the infinite component quantum field is extremely singular since the localization points of all pointlike components fall on top of each other. It is an interesting historical question why the string community agreed with the authors that the localization is stringlike (a point on an invisible string?). Looking back with some hindsight, the dual model and the string theory are certainly the most curious results from an epoch in which conceptually unguided calculations combined with sophisticated mathematics were expected to lead to a unified theory of everything (a TOE). Historians of science will have a lot of problems to understand the related Zeitgeist, but the almost 50 years lasting popularity (longer than the phlogiston theory) will leave them no choice but to try to explain to a curious public what really went on in the minds of people.

9. Localization and phase-space degrees of freedom

In a course on QM one learns that the number of "degrees of freedom" (quantum states) per unit cell of phase space is finite. Already in the beginning of the 1960s it became clear that this is not compatible with the causal localization in QFT. The first computation revealed that the infinity is not worse than that of a compact set (Haag & Swieca, 1965) which in later work of Buchholz and Wichmann became sharpened to the cardinality of a *nuclear set* (Haag, 1996); together with modular localization theory it led to the important concept of modular nuclearity (Haag, 1996).

The physical motivation of these investigations is the desire to understand the connection between field localization and the presence of particles; in particular the circumstances under which the causal localization properties of quantum fields lead to particles with discrete masses including the important property of *asymptotic completeness*.[57] One remarkable result in more than eight decades lasting attempts to prove the existence of models of QFT with interactions and to obtain mathematically controlled approximations is the before mentioned existence proof for certain strictly renormalizable integrable models. Such models are characterized in terms of its factorizing S-matrices which permit a classification in terms of matrix-valued 2-particle scattering functions (Section 6). In that case one knows the particle structure and one would like to find the net of local algebras and the their generating quantum fields whose collision theory reproduces the known particle content. The S-matrix determines the structure of the wedge algeℓbras. In order to obtain a nontrivial net of compact localized double cone algebras one can use the aforementioned modular nuclearity property of phase space degrees of freedom which follows from the analytic properties of the scattering functions.

On the positive side these models have a realistic short distance behavior as one expects it from renormalizabity, i.e. they are not superrenormalizable as polynomial self-interactions between scalar $d_{sd}=0$ (logarithmic divergent short distance behavior) fields in two dimensions.[58] The fact that integrability in QFT can only be achieved in $d=1+1$ did not affect their usefulness as a "theoretical laboratory" of QFT. The existence of these models can be controlled with the help of "modular nuclearity" (Lechner, 2008).

Another important use of these ideas consists in the *exclusion* of models with unphysical causality properties. Lagrangian quantization seems to lead inevitably to divergent renormalized perturbative series, and hence it is not suited for addressing problems of existence of models. It is however important to maintain the formal causality properties of Lagrangian quantization in the better mathematically controlled LQP setting of QFT. Whereas the spacelike Einstein causality property is easily taken care of, the relevance of the causal completion (causal shadow) property is sometimes overlooked. One reason is that this quantum counterpart of causal propagation cannot be formulated in terms of individual fields; its precise formulation needs the algebraic setting as in Section 4.

It is easy to write down generalized free fields which fulfill Einstein causality but violate the causal completeness property

[57] The equality of the Hilbert space with a Wigner–Fock particle space.

[58] The $d=1+1$ superrenormalizable theory can still be treated within a measure-theoretic functional quantization setting (Glimm & Jaffe, 1972), no use of modular localization properties is needed.

(the local version of the old time-slice property, Haag & Schroer, 1962). A recent illustration of a violation of this important physical property is the conformal covariant generalized free field which results from a normal free field on a AdS spacetime through the mathematical AdS_{n+1}–CFT_n isomorphism (Duetsch & Rehren, 2002). The physical defect of fields which violate the causal completeness property is that they lead to a "poltergeist effect" in the causal shadow region; as one "moves up" from the spacetime region $\mathcal{O}$ into its causal completions $\mathcal{O}''$ there are causality violating degrees of freedom apparently coming from nowhere.

The LQP setting reveals that this physical defect is of a general nature and may be viewed as a manifestation of the holistic nature of spacetime localization. As the holistic nature of life needs the right amount of chemicals, the holistic nature of causal localization in spacetime needs the right cardinality of degrees of freedom which is appropriate for causal localization. Starting from a physical AdS theory, one obtains an "overpopulated" CFT model which leads to the mentioned poltergeist phenomenon. In the opposite direction a "physical healthy" CFT passes to an "anemic" AdS theory which does not have enough degrees of freedom which are needed for a nontrivial realization of causality; in the case at hand one has to go to noncompact spacetime regions in order to find at all degrees of freedom (Rehren, 2000).

It is interesting to note that this pathology is absent in holographic projections onto null-surfaces; unlike in isomorphic correspondences, holographic projections dilute (loss of imbedding information) degree of freedom by the right amount which fits the lower dimensional surface.

It is interesting to take a closer look at a special misinterpretation which played an important role in ST. As mentioned before, the irreducible oscillator algebra of the 10 component chiral current admits two inequivalent representations, one which is important for the invariance under the conformal Möbius group and the pointlike localized fields on a lightray, and the other which carries the mentioned 10 dimensional superstring positive energy representation of the Poincaré group. Both representations are pointlike generated; this is a property shared by all positive energy representations with the exception of Wigner's infinite spin representations. But there is a huge difference in the cardinality of freedom; the oscillator representation carries the superstring Poincaré group representation, but certainly not the superstring field representation which is canonically associated with it and hence it is not possible to view the one as embedded into the other. The misplaced terminology "ST" which refers to a stringlocal object in a target spacetime is the result of an incorrect picture.

At best this terminology could refer to an internal oscillator chain (after taking out the zero mode degree of freedom) "over" a spacetime localization point which carries the (m,s) representation as well as additional operators which are not needed for the representation of the Poincaré group, but interlink the different levels of the (m,s) tower and in this way secure the embedding of the reducible superstring representation of the Poincaré group into an irreducible algebra. Such a tower of free fields piling up over one point leads to pointlike singularities which are beyond those of ordinary (Wightman) QFT even though each individual component is an ordinary free field. Perhaps this could have been the reason why, despite their correct calculation, the authors in Lowe (1994) and Martinec (1993) presented their result as a confirmation of stringlike spacetime localization by declaring the localization point to be the center point on an imagined spacetime string.

As previously mentioned the embedding of lower dimensional QFTs into higher dimensional ones and its Kaluza–Klein inverse are also inconsistent with the holistic localization principle. Arguments based on quasiclassical approximations or on "massaging" Lagrangians do not help on issues which directly relate the cardinality of degrees of freedom with quantum causal localization. Different from quantum mechanical matter the spacetime dimension is an inseparable part of what constitutes causally localized quantum matter. The only known exceptions are holographic projections onto null-surfaces; in this case the cardinality of degrees of freedom is thinned out in the right way (Schroer, 2011b).

These insights into the connection between the cardinality of degrees of freedom and localization immediately disproves the Maldacena conjecture which claims that both sides of the AdS_5–CFT_4 represent *physical* theories. As a coauthors of a 1962 paper (Haag & Schroer, 1962) which led to the concept of the causal completion property it is somewhat distressing to look at the present situation in which globalized communities of particle theorists have fallen behind previously attained levels of knowledge about important concepts.

Sagredo: Dear Simplicio, some of our friends tell me that you claim the dual model and ST led to a derailment of an important part of particle theory?

Simplicio: Although my attitude with respect to those attempts concerning a "theory of everything" has been indeed very critical, I have good reasons to avoid expressing my critique in this way. What prevents me is the fact that I share the goals of an S-matrix-based alternative to the quantization approach. Hence criticizing a certain unfortunate direction which this has taken in the form of string theory should not be misunderstood as a dismissal of the aims of the project.

After the successful closure of the dispersion relation project it seemed natural to look for a setting in which the analytic properties derived from the relativistic causality of QFT can be extended in such a way that they may be used for dynamical calculations in particle physics. So instead of starting with quantized fields and deriving properties of interacting particles (scattering amplitudes, formfactors), why not start directly with objects referring to particles and address the problem of whether these results can be backed up by a more foundational QFT to a later stage. It is customary to refer to such a particle-based construction as an "on-shell" projects and to quantum field based approach as "off-shell" since scattering amplitudes and formfactors are formally related to mass shell restrictions of Fourier transforms of field correlations. Different from the off-shell project of QFT for which one will know the physical content of the model-defining field theoretic interaction only at the end of the calculation, the on-shell particle-based project is a "top-to-bottom" setting in which the physical properties are laid out before one starts to work one's way down to the field theoretic description.

The problem is of course that our conceptual/mathematical understanding works best on the level of the foundational causal localization principles of quantum fields, whereas it is difficult to directly convert attributes of particles with the help of analytic properties of scattering amplitudes into concrete predictions. Whereas the foundational properties of fields lead to analytic properties of off-shell field correlations, it is extremely hard to extract from them on-shell analytic properties. Even in perturbation theory where the graphical aspects of crossing properties are obvious, the proof that there is an analytic on-shell path which relates a scattering amplitude to its crossed counterpart is anything but simple. Stanley Mandelstam, one of the protagonists of an on-shell project, knew that on-shell analytic properties beyond those which were needed for the derivation of the particle analog of the Kramers–Kronig dispersion relations are hard to get at. His proposal of the Mandelstam double spectral representation for the elastic scattering amplitude was a guess and not a derivation from the causality principles. It was Venezianos guess of a dual model and its later conversion into string theory which led to the derailment of Mandelstam's project.

Looking back at that epoch with today's hindsight it is clear that there was no chance for such a project to succeed at that time. An important aspect of the S-matrix which tightens its link with the causality principle of local quantum physics was still missing namely the fact that the S-matrix, in addition of describing the collision of particles, is also a relative modular invariant of the wedge algebra $\mathcal{A}(W)$. For integrable models of QFT (a property which unfortunately is limited to $d=1+1$ and which forces the S-matrix to be purely elastic) the on-shell project has a unique solution; in this case one can really start from the classification of S-matrices and arrive at a unique integrable QFT which is associated to that integrable S-matrix. Even without integrability there are some ideas, but due to the complexity of the problem there has been no significant progress.

Sagredo: But how was string theory related to Mandelstam's on-shell project and what was its impact?

Simplicio: Mandelstam realized that an on-shell approach to particle theory idea must start with a profound understanding of the analytic crossing property of scattering amplitudes of which the elastic part is the simplest. As a starting point he postulated a two-variable representation which became known under the name "the Mandelstam representation". Unfortunately no crossing symmetric solution of this representation was found.

In order to understand the next step one needs to recall a bit of the spirit of the times. When a seemingly well-defined but nonlinear problem did not admit any solution this was sometimes taken as a hint that if the problem admits any solution at all, this should be rather unique. This was the view about solutions of the nonlinear Schwinger–Dyson equations and this was not different in case of the nonlinear bootstrap project. It may nowadays appear naive, but the idea that Poincaré invariance, unitarity and the crossing property lead to a unique S-matrix (a TOE apart from gravity) had a strong spell on many people even prominent physicists as Freeman Dyson supported it for some time.

When Veneziano, while playing with properties of Euler beta functions, found a meromorphic crossing symmetric functions with an infinite family of first order poles, there was a lot of commotion in the phenomenologically motivated particle theory community. Veneziano's proposal to view it as a model of an approximation (it was not unitary and had no elastic cut) to a crossing symmetric scattering amplitude received widespread acceptance and also Mandelstam's blessing. Nowadays we know that such functions occur in models of conformal QFT and have no relation to scattering amplitudes. When the use of the dual model functions in scattering theory was finally given up, the reason was not the existence of a conceptual flaw but rather the fact that new experimental results removed the phenomenological basis for the interest in such models. This was the end of the Mandelstam on-shell project but not that of the dual model formalism. The new idea was that Veneziano's mathematical dual model observations were anyhow too sophisticated for strong interaction phenomenology and one should find a more foundational application. This was the birth of string theory which pushed the somewhat modified dual model formalism from its application to strong interactions all the way up to the Planck scale; in this way it became the millenniums TOE.

Sagredo: But doesn't this mean that string theory rid itself from the impossible relation to Mandelstam's on-shell project? How does this fit in with your belief that ST a failed theory?

Simplicio: The 10 dimensional free superstring is a second quantized version of the so-called superstring representation. This is a positive energy Wigner representation on the irreducible operator algebra associated with a certain supersymmetric 10-component abelian chiral current algebra. One has all the right to be surprised about the existence of such a representation since it is the only known entirely discrete positive energy representation on an irreducible algebra; representations on field algebras coming from QFT inevitably have the continuous contribution from scattering theory. It is the first and only known solution of Majorana's 1932 problem (Majorana, 1932) to find an irreducible algebra which can support an infinite component *discrete* positive energy Wigner representation (an "infinite component field equation"). This group theoretic problem was solved by the string theorists construction of the "superstring representation" on the algebra of the supersymmetric 10-component abelian chiral current model; this is their achievement.

Sagrado: But what about strings in spacetime?

Simplicio: The terminology "string theory" is misleading since the superstring field creates states which decompose into irreducible pointlike generated irreducible Wigner components. The only positive energy Wigner representations which are genuinely stringlocal are the massless infinite spin representations but they are absent in the superstring representation. Since the relation between states and field operators in case of linear (free) fields is unique, the pointlike nature of states passes immediately to the fields. By projecting states on finite invariant energy subspaces one can explicitly see that the "string" field is the singular limit of pointlike ordinary fields.

There is an important philosophical message which these failures reveal. Independent of how theoretical discoveries are obtained, the aim must always be to understand them as a realization of physical principles. String-localization cannot be based on similarities of an infinite (m,s) tower spectrum with that of a quantum mechanical chain of oscillators; causal localization is a totally intrinsic property of local quantum physics and the concept of modular localization expresses this fact in its conceptual/mathematical most concise form.

Of course what we consider to be a foundational principle is subject to future refinements. The idea of finding a TOE by playing mathematical games is not the way in which the material world reveals itself to us. Such a theory is its own principle whereas all our experience shows that the real interesting part of nature is that it offers a wealth of different realizations of its principles.

The explanation of why the popularity of a TOE reached its peak at the turn of the millennium will be problem for historians of science. As the phlogiston theory, string theory lasted too long in order to be overlooked in the history of physics. Whereas the phlogiston theory was abandoned as a result of contradictions with measurements, the contradictions of string theory with existing principles of particle physics were always present for anybody with a strong conceptual awareness. The final word about its legacy is up to historians of science.

My dear Sagredo, at this late hour I propose to close our dialog.//

10. Resumé and concluding remarks

QFT provides particle theory with an important conceptual structure: the causal localization principle. It results from the amalgamation of the Faraday–Maxwell–Einstein classical causality in Minkowski spacetime with the operator-algebraic formulation of quantum theory in Hilbert space. Its conceptual strength is matched by its concise mathematical formulation: the adaptation of the Tomita–Takesaki theory of operator algebras in the form of modular localization. One reason for submitting the present work to a history/philosophy oriented physics journal is the fact that this new framework of QFT sheds additional light on a famous debate in the history of QFT namely the dispute between Einstein and Jordan which finally led Jordan to the discovery of QFT. Its main message concerning the vacuum-polarization caused statistical mechanics nature of the spacetime-restricted vacuum has sometimes been misinterpreted in terms of the quantum mechanical particle–wave duality (Duncan & Janssen, 2008).

The new modular localization-based formulation removes the alleged spacetime string-localization from string theory and shows that such models are special examples of infinite component pointlike fields. It reveals the conceptual origin of the particle crossing property and explains the solvability of integrable models in terms of the simplicity of generators of modular-localized wedge algebras. It suggests to construct nonperturbative QFTs by starting from the modular structure of wedge algebras and obtain compact localized operator algebras in terms of intersections of wedge algebras.

The enormous conceptual range of modular localization unfolds in numerous applications. In certain cases this led to clashes with existing results and their interpretation. This happened in particular with ideas which originated in string theory as dimensional embeddings and reductions (the use of Kaluza–Klein ideas outside of (quasi)classical approximations) and Maldacena's incorrect claim that the mathematical AdS–CFT isomorphism can be used to relate two causally localized QFTs in different spacetime dimensions.

On the constructive side it led to a deeper conceptual understanding of the limitations of BRST gauge theory and how to overcome them in a new Hilbert space setting of stringlocal fields. This in turn led to a demystification of the Higgs mechanism and its alleged symmetry-breaking Mexican hat potential in terms of massive vectormesons coupled to Hermitian (instead of complex) fields and their induced second-order interactions. The new Hilbert space setting of interacting higher spin fields leads in particular to the new concept of stringlocal Hermitian "escort fields" which in the case of $s=1$ are in many aspects Higgs-like, except that they appear as an inexorable part of the massive vectormesons rather than independent scalar fields to be coupled to massive vectormesons. This new concept has no counterpart in the pointlike gauge setting and therefore cannot be adequately described in the terminology of pointlike fields.

These theoretical results present a new meeting ground of ideas coming from foundational local quantum physics with problems arising from the observation-oriented research on the Standard Model. It does not exclude Higgs couplings but it denies the existence of a Higgs mechanism of mass creation by symmetry breaking.

An unsolved problem of at least comparable importance is the derivation of gluon/quark confinement from the QCD coupling. As explained in the text, the problem amounts to establish the vanishing of all *correlation functions which contain stringlocal gluon or quark operators*[59] in the limit of vanishing gluon mass so that only pointlike correlation functions of composites (hadron and gluonium) survive. The stringlike nature results from the very restrictive Hilbert space positivity, which was not available in the Krein space gauge setting. The analogy to the YFS summation technique of leading logm terms in the limit m–>0 (which leads to vanishing QED scattering amplitudes for collisions of charged particles with a finite number of outgoing photons) should be a valuable guide for proving the vanishing of correlation functions which contain in addion to pointlike composites also gluon or quark operators.

The purpose of this article has been accomplished if it succeeds to draw attention to the enormous unifying power of modular localization for problems of QFT and particle physics.

[59] The exception are $q-\overline{q}$ pairs for which the string-directions have been chosen in a particular manner.

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