Directional Domains of Physical Motion - Einstein's Blunder -

Bernd Schmeikal

Faculty of Social Science, University of Vienna, Alumnus of Kurt Gödel Research Center
E-mail: schmeikalbernd@antony.one

Abstract. Einstein's blunder was to take over Minkowski's World Point and World Line concepts. The early treatises on Relativity and Relativity Principle contained an error concerning rotation of so called rigid bodies and some invalid concept of *world* and *worldline*. The theory showed a clearly recognizable paradox, which had enormous consequences for the mathematical execution of its layout. If we remove those errors, Relativity and Quantum Theory come closer. This required at least to provide Relativity Theory with extended algebraic means, and to define a better concept of the *directional domain* of quantum motion in the algebraic basis. Once this is carried out, we come up with a better understanding of matter and space-time that involves causality amending, the interaction of fermion boosts, warps, enhancement and shrinking of space-time-volumes. Local Four-volumes, - as directors of the Clifford algebra of the Minkowski space-time,- are no longer invariant pseudoscalars like in the old theory, but are converted into time.

Keywords: Time Travel, Quantum Motion, Quantum Information, Isospin, Clifford algebra, Causality amending.
PACS: 03.67.-a, 12.10.Dm, 04.20.Gz, 02.20.Sv, 12.38.Aw, 11.30.Hv

1. PROLOGUE

Future events influence the past and shape it. But the influence is subtle. Information and energy can flow backward in time. In this paper, there is, on a few pages, outlined the basis of a theory, both new and old, which clarifies why this is possible. The strong force of nature influences the inner of nucleons. The impact of future events interacting with the inner partons extends all the deeper into the interior fields of the protons the further the future of these events that are amending the causality. Relativity Theory is not wrong. Quantum Theory not either. But both contain little errors. If we remove those errors, Relativity and Quantum turn out as one unitary theory. In those troublesome days between the world wars, a few mathematicians and physicists helped Einstein to come out with his theory. Minkowski, who died aged 45 already in 1909, contributed his idea of *world* made of points, *worldlines*, and group G_c, and thereby introduced the valuable concept of *line-element*, but a false concepts of direction and orientation. He did not become aware of it. Since then this lack of insight is prevailing. We seem not to go beyond the idea of worldlines. But these are not entirely compatible with the idea of spinors and spin-wavefunctions. Yet, then, concepts of spinors and matrices of geometric algebra, Dirac matrices, were already there! Paul Dirac [1], Gustave Juvet [3], Fritz Sauter [2], Jan Arnoldus Schouten [4], Walther Mayer and even Albert Einstein [5] had already found out that geometric algebra and hypercomplex numbers were needed to understand motion. But the Lorentz boost had to proceed in *"direction x"*-fashion, something that turns out impossible as soon as we enter the domain of isospin and *strange, colored* wave-functions of matter. In this paper we work out the new concept of *directional domain* based on structure. On such a basis Feynman's causality amending quantum electro- and chromodynamics can be understood. The interaction of remote events with the inner of partons is explained. We arrive at that general motion group G_c that Minkowski had searched for, which describes the unity of material motion and space-time. We explain the boost together with warp and recycling of time and space-time-volume. For the Clifford director J, the basic 4-volume in Clifford space-time, is not preserved, but it can be converted into what we call time. Warp-driven space-travel means time-travel. May this paper provide some of the terrestrial mathematical basis for extraterrestrial time-travel technology.

2. HISTORIC DELAY

Paul Dirac's book [1] on Principles of Quantum Mechanics and some earlier works by Gustave Juvet [3] , Fritz Sauter [2], Jan Arnoldus Schouten [4] , Walther Mayer and even Albert Einstein [5] himself, hint at the fact that the mere Minkowski space $\mathbb{R}^{3,1}$ provides only a somewhat depleted concept of space-time, and that some more complete idea should rather include spinors and hypercomplex elements of the Clifford algebra which it creates. In particular we must emphasize the 1930 publication of Gustave Juvet who wrote: *"The hypercomplex numbers that we are concerned with here are the numbers that were considered quite recently by several authors, the first of which in time was Clifford."* Juvet had designed the general hypercomplex Clifford-number in a 16-dimensional basis of that Clifford algebra generated by Euclidean 4-space, that is, in nowadays denotation the algebra $Cl_{4,0}$.

3. CORRECTION OF THE CONVENTIONAL SPACE-TIME CONCEPT

3.1. The mainstream concept

To limit the plot, let me sketch the beginning and the end of the drama. The story is always the same. In his 1905 publication [6],[7], Einstein investigates the *"Transformation of Co-ordinates and Times from a Stationary System denoted to another System in Uniform Motion of Translation Relatively to the Former"* (§3 in [7])[1] We are very familiar with the results. He submitted the calculation as follows, *"so that the transformation equations which have been found become:"*

$$\begin{aligned} \tau &= \beta\left(t - \frac{v\,x}{c^2}\right) \\ \xi &= \beta(x - v\,t) \\ \eta &= y \\ \zeta &= z \end{aligned} \tag{3.1}$$

where $\beta = 1/\sqrt{(1-\frac{v^2}{c^2})}$ (sic!)

- note that the x marks a, preferred or not preferred, direction of motion alongside some base unit e_1 in the coordinate system K,- and since then, this peculiar, reduced design has been pasted over and over again. Obviously, most of us thought this had no serious consequences for the theory and its application. But it had, and it still has some. The last act in the drama is called "Lorentz transformation" in Wikipedia, the free encyclopedia. Therein we find subsection "Coordinate transformation" of the most famous act "Physical formulation of Lorentz boosts". The transformation equations now bear the title "Lorentz boost (x direction)":

$$\begin{aligned} t' &= \gamma(t - \frac{vx}{c^2}) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned} \tag{3.2}$$

where $\gamma = 1/\sqrt{(1-\frac{v^2}{c^2})}$ (sic!)
As so often in our trailblazing writings, the self-evidentness is well hidden, the self-evident well cam-

[1] `http://hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_1905_relativity.pdf` : "This edition of Einstein's *On the Electrodynamics of Moving Bodies* is based on the English translation of his original 1905 German-language paper (published as *Zur Elektrodynamik bewegter Körper*, in Annalen der Physik. 17:891, 1905) which appeared in the book *The Principle of Relativity*, published in 1923 by Methuen and Company, Ltd. of London. Most of the papers in that collection are English translations from the German *"Das Relativitätsprinzip"*, 4th ed., published by in 1922 by Tuebner. All of these sources are now in the public domain; this document, derived from them, remains in the public domain and may be reproduced in any manner or medium without permission, restriction, attribution, or compensation. Numbered footnotes are as they appeared in the 1923 edition; editor´s notes are marked by a dagger (†), and appear in sans serif type. The 1923 English translation modified the notation used in Einstein´s 1905 paper to conform to that in use by the 1920´s; for example, c denotes the speed of light, as opposed the V used by Einstein in 1905. This edition was prepared by John Walker. The current version of this document is available in a variety of formats from the editor´s Web site: `http://www.fourmilab.ch/`"

ouflaged. It appears in the first and last act, and is best expressed by the bracket term nicely guiding the Lorentz boost: ”(x direction)”. The first act means Einstein 1905, the last Wikipedia today. Note the replacement of β by γ, and altered use of the fraction β in later years, and of course the early exchange of Einstein’s big V by a small c. Let us further consider Hermann Minkowski´s contribution to *Das Relativitätsprinzip - Eine Sammlung von Abhandlungen* [8] as, say, second act in the drama. This illuminating simplification takes the form: ”[..] A full understanding of the rest of those transformations can be obtained by considering such among them for which y and z remain unchanged”, [9] (p. 41).[2] What was and still is the purpose of this simplification? Was it to make the mathematics more easy? Obviously, the hyperbolic squeeze maps represented by those small terms of $c^2t^2 - x^2 = 1$ and $c'^2t'^2 - x'^2 = 1$, [9] (p. 41), adopt a manageable appealing form. But is that all? Not at all! For, reducing the phenomenon of motion to ”(x direction)” expresses the fact that boosting motion of a ”substantial point”, [9] (p. 47), along a line, or respectively, boosting the line of a ”point mass”, [9] (p. 50)[3], would be enough to describe motion of ”substantial points” in what he called ”the world”.

3.2. The big lack in the concept

Minkowski had in mind to make up the world by worldlines, and lines have a direction. They do have direction easily determined by calculus or, may be, some more sophisticated considerations. At any case, in our time, it would not be that easy to describe motion of ”point mass” or even ”point energy” moving on worldlines in simple grade-one directions. Quite clearly, Minkowski did not describe, in his lecture, or in any other lecture, motion of energy in directed space-time-volumes or even space-time-areas. He saw things exactly in the way he constructed them. And how did he see things? ”I will call a point in space at a given time, i. e. a system of values x, y, z, t a worldpoint. The manifold of all possible systems of values x, y, z, t will be called the world. [..] We then get an image, so to say, of the eternal course of life of the substantial point, a curve in the world, a worldline, whose points can be clearly related to the parameter t from $-\infty$ to $+\infty$. The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.”

It seems Arnold Sommerfeld was aware of some specific weakness of Minkowski’s and Einstein’s conceptions when he added his supplementary notes to the fifth edition (1923) of *The principle of relativity*, the papers by Hendrik Lorentz, Albert Einstein and Hermann Minkowski. He brings in Max Born’s work on inert masses and relativity [10] where each volume-element could suffer Lorentz contraction. Consider a uniformly rotating rigid disk. Then, Ehrenfest pointed out, such a disk could not be set in rotation (Ehrenfest´s paradox).[4] Sommerfeld directed attention to the articles by Herglotz [11] and Nöther [12] , showing that such a relativistic rigid body had only three degrees of freedom of motion. This was an incomprehensible limitation.

3.3. Why SR has no time travel

Then, Minkowski denoted the Lorentz group by $\mathbf{G}_c$ with ”c the velocity of the propagation of light in empty space”, and he attempted to ”state the existence of the invariance of the laws of nature with respect to the group $\mathbf{G}_c$”. But in perfect correlation with the so called Herglotz-Nöther-theorem, any Lorentz transformation conceived in the above way preserves the oriented space-time-volume. Considering the above shrink version ”Lorentz boost (x direction)”; the Lorentz transformation does not dilate the oriented space-time-area of the boost. In terms of the quadratic Clifford algebra $Cl_{3,1}$, the differential volume element dV is a grade 4 multivector of differential 4-forms given by the exterior

---

[2] English translation of the German lecture *Raum und Zeit* is taken from [9].
[3] In the German lecture [8] , these words (”substantieller Punkt” and ”Massepunkt”) appear on pages 61 and 63.
[4] As Galina Weinstein [13] put it: ”a uniformly rotating rigid disk would be a paradoxical object in special relativity, since on setting it into motion its circumference would undergo a contraction whereas its radius would remain uncontracted.”

product

$$dV = e_{1234}|dx\,dy\,dz\,dt| = e_1 \wedge e_2 \wedge e_3 \wedge e_4\,|dV| \tag{3.3}$$

in the basis e_1, e_2, e_3, e_4 having signature $\{+ + + -\}$. We can learn from the excellent book by Josè G. Vargas how demanding it is to understand the transformation of an n-volume differential form. [14], p. 38. "What we are missing is a yardstick for every direction so that, even along a coordinate line, we shall be able to tell how long something is. [..] we also need to know how much a coordinate line x^1 is tilted with respect to another coordinate line x^j [..] all this is achieved by the introduction of a metric." In such a lucky case where quantum motion seems to assemble quadratic space $\mathbb{R}^{3,1}$ with its quadratic Clifford algebra, we can be sure that the above definition of a directed space-time-four-volume makes sense. So, let us calculate the differential 4-volume dV at rapidity ϕ , that is, $\phi = \operatorname{arctanh}\frac{v}{c}$. Our title "Lorentz boost (x direction)" corresponds with the equations

$$\begin{aligned} e_1' &= (\cosh\phi)\, e_1 - (\sinh\phi)\, e_4 \\ e_2' &= e_2 \\ e_3' &= e_3 \\ e_4' &= -(\sinh\phi)\, e_1 + (\cosh\phi)\, e_4 \end{aligned} \tag{3.4}$$

The transformed 4-volume is scaled by the wedge product $e_1' \wedge e_2' \wedge e_3' \wedge e_4'$ which is essentially determined by the Lorentz boosted directed space-time area e_{14}'. This is equal to

$$\begin{aligned} e_{14}' = e_1' \wedge e_4' &= \\ = \big((\cosh\phi)e_1 - (\sinh\phi)e_4\big) \wedge \big(-(\sinh\phi)e_1 + (\cosh\phi)e_4\big) &= \\ = -(\sinh\phi)(\cosh\phi)Id + (\cosh\phi)^2 e_{14} + (\sinh\phi)^2 e_{41} + (\sinh\phi)(\cosh\phi)Id &= \\ = \big((\cosh\phi)^2 - (\sinh\phi)^2\big)e_{14} &= \\ = e_{14} \quad \text{with identity } Id \in Cl_{3,1} & \end{aligned} \tag{3.5}$$

Altogether we have obtained that a Lorentz boost in direction of the unit vector e_1, "x-direction", preserves the directed 4-volume and we have

$$e_{1234}' = e_{1234} \quad \text{and} \quad dV' = dV \tag{3.6}$$

This invariance is connected with Ehrenfest´s paradox, the Herglotz-Nöther-theorem and last not least with Arnold Sommerfeld´s hint in the later editions of [8] . What has happened? What was and what is missing? Do we have to consider the warp-drive to understand, not only time-travel, but also relativistic quantum motion? Sure, the warp-drive controls and changes the scale of space-time 4-volumes. Time travel is based on the variance of space-time areas ad space-time volumes, not their invariance. So, how shall we go on? Actually, we do not yet need to consider time travel, to see what went wrong. Some minor idea in Minkowski´s great "world-image" is wrong! What is it? Is it perhaps clarified in the last act of the SR-Drama, in the Wikipedia?

3.3.1. Did Wikipedia compile the truth?

In the time between, say, 1939 and today, there have been given numerous representations of the Lorentz groups, [15] , [16], [17] , [18], [19], [20] , also its different components and manifestations, beginning with Eugene Paul Wigner´s investigation of the inhomogeneous Lorentz group and culminating in Mark Aronovich Naimark´s *Linear representations of the Lorentz group*, a book that scholars of Walter Thirring would have had to read. Meanwhile this mathematical problem of representation seems to have been solved. But still we do not know how we could achieve Minkowski´s "substantial points", and how and why should energy move on "worldlines"? Perhaps there are substantial, phenomenal reasons why it does not and cannot move in grade-1 directions!

In section on Proper Transformation, in Wikipedia [22], it is explained that the "composition of any two Lorentz boosts is equivalent to a boost followed or preceded by a rotation on the spatial coordinates, in the form of $R(\rho)B(w)$

or $B(\bar{w})R(\bar{\rho})$. The w and $\bar{w}$ are composite velocities, while ρ and $\bar{\rho}$ are rotation parameters (e.g. axis-angle variables, Euler angles, etc.)." In this very section the free Encyclopedia compiles quite nicely what has conquered the inter-institutional electronic academic space-time during the last six decennia or so, and it endows physicists with the complete boost matrix. Now each "Cartesian direction", as it is called, that is, x,y,z, contribute to the boost. But the former weakness remains, namely the boost direction is conceived as motion alongside a worldline of grade one. The directional domain of motion is just projected out onto the Cartesian directions. So we have again one line, a worldline, and the original easy-travel "Lorentz boost (x direction)" is merely rotated a little, thus bringing forth the boosted unit vectors, while the space-time 4-volume is preserved, as before. But in empirical reality, in *relativistic quantum motion*, $J = e_{1234}$ is not invariant, but is controlled by the nuclear force.

During all those years, from 1905 until today, this *directional pauperization* of Relativity and Quantum Mechanics, and some rather serious conceptual and mathematical errors have been repeated over and over again. When you watch one of those beautiful films on Youtube, such like "*Moving through Spacetime*" [21] , you may become aware of it. You can hear some "DrPhysicsA" saying: "Space-time is a 4-dimensional creature, the normal 3 dimensions of space plus time [..] to make it easy, we start with one dimension of time and one dimension of space which we call the x-axis. So people are free to move along the x-axis, but not in the y- and z-direction. Just keep things simple. And you recall that we said there were two things that you couldn´t do in space-time [21] (youtube video, 26^{th} second). The first thing is, you couldn´t move backwards in time. Time is going upwards, this arrow (note: in some of the diagrams) suggests that you are going backwards in time, and you are not allowed to go backwards in time because there is no known mechanism for doing so. The second thing I said was that when one draws a space-time-chart, we usually organize it such that the speed of light is given by the 45°-line, and I said that you couldn´t, therefore, travel at any speed greater than the speed of light.

Anything in this area here is forbidden because that´s greater than the speed of light. If you stand still, then you don´t travel in the x-direction at all. You simply go up the time-direction." Those rules of time-travel seem to be surprisingly simple,

- you cannot move backwards in time
- moving forward in time, you cannot drop out of the forward-light-cone.

But they are wrong.

3.4. Universal Relativity Theory

Aspiration, faith in progress, competition, sociopolitical givens and other collective habits forced Albert Einstein to publish his theory before it could meet his own demands for perfection. He had to compromise. He had to acknowledge a few concepts with which he and others in the group of high ranking physicists were not fully satisfied. One such concept was Minkowski´s world made of worldlines and substantial points traveling along such lines. In his "*Concise Statement*" Hendrik Antoon Lorentz [23] (p. 45) mentions the original denotation and the expectations that Einstein combined with his new theory of space-time. There he uses the label "*Universal Relativity Theory*". "If we avail ourselves of the simplifying circumstance that the velocities of the heavenly bodies are slight in comparison with that of light, then we can deduce the theory of Newton from the new theory, the "universal" relativity theory, as it is called by Einstein. Thus all the conclusions based upon the Newtonian theory hold good, as must naturally be required. But now we have got further along." The Universal Relativity Theory (URT) naturally has this unity of Relativity and Quantum Mechanics. These two need not even be made compatible. But they are one from the outset, if motion is conceived properly.
Of course Zbigniew Ocziewicz should be mentioned here as he took into account Harald Keres space-of-locations [24] as a congruence of world lines where there is no absolute 3-dimensional space of locations. Since quite a while Zbigniew reconsidered the relative concept of rest and points at the importance of groupoids, as a material body comes up in form of the reference system of a groupfree split. There seem to be only a few who are ready and able to follow Oziewicz's Ansatz [25]. But most probably his approach is closest to the idea of Universal Relativity, as far as I can see now.

3.5. Directional Domain and Spinor Space

3.5.1. Euclidean space and Pauli algebra

The founders spoke about "direction" of particles movements without overmuch analytical effort. They set a simple pattern for directional motion. So, directional localization of particles or quanta was conceived in a simple way,- "the whole world presents itself as resolved into such worldlines',- but the *direction field*, given they had an idea what exactly that could be, did not contain directional components of different grade. When I say so, this is not "ideology", but we are becoming aware of a real lack of insight. In 1905, and even centuries later, we did not yet understand the meaning of motion. We did not understand its directional components. Would we have seen that clearly, we would also have understood the quantum nature of relativistic motion. We would have seen how directional curvature is naturally derived by the structure of the direction field. We would immediately have understood why fields have to be "colored" and "strange", in the sense of HEPhy, would have seen the interplay between local and global events. If I say so, this is not a false promise, but I want to show you why this is actually the case. In the long run we have had to realize quantum motion as a form of directional transposition that transforms the grade of the geometric pattern of what we may call the directional propagation of energy. Macro-, micro-, nano- and femto-scopically, energy does not travel on worldliness. While it propagates alongside a linear worldline-direction, it stretches a directed space-time area, orthogonal to the grade1-propagation, somewhat like an umbrella, and thereby expands the exterior product of the directed line and the directed area, to form a 3-space-time volume. This happens in accord with facts like that quanta jump, photons bridge space, molecules are entangled and all the rest of it.

Let us go back in time to 1905 and take the direction finders at their word, those geniuses who set the pattern. That is, let us find out about the directional components of energy propagation from a traditional micro- and macroscopic platform, trying to somehow preserving the idea of a solid body and Minkowski space, but providing methodological freedom for the associated quantum motion in geometric algebra.

First, consider Pauli algebra and Euclidean $\mathbb{R}^3$. Pauli algebra is the geometric Clifford algebra generated by Euclidean 3-space. We use it to construct non-relativistic wave functions and their associated spinor spaces. So, even in classical, non relativistic settings of Euclidean direction fields, particles have spin. The rotation group is double-covered by the spin group. Now go into details, and find out what exactly that means with respect to "direction".

In the non-relativistic theory of spinning fermions we consider column matrices, the Pauli spinors. These are 2×1 matrices or column vectors with two rows and entries in the double field of complex numbers. But in Clifford algebra, we usually represent them as objects in the matrix algebra $Mat(2, \mathbb{R})$, that is, 2×2 square matrices with one empty column. Thus, we substitute Pauli spinors by square matrix spinors. These square matrix spinors disclose how wave functions of spinning particles relate to the primitive idempotents in the geometric algebra, and why the domain of propagation of a fermion has a structure. We learn from this that the propagational range of a fermion has a directional structure. This holds quite generally, and even more so, in the relativistic case, for Dirac spinors and more general wave functions. In those general phenomenologies, the space-time spinors have a non-trivial composite graded directional structure involving four different dimensions. This seems to be a surprise, as we believed that a particle propagates along a worldline, say, in direction e_1, but even in the Pauli algebra in turns out that the directional structure of the propagation is not given by e_1 or any other single Euclidean vector, but rather by terms $(Id \pm e_1)/2$ or, just as well, $(Id \pm e_3)/2$. These are idempotents primitive in the Pauli algebra. This algebra is identical with the Clifford algebra of Euclidean 3-space, with the basis vectors represented by the Pauli matrices. Now, consider the idempotent

$$f = \frac{1}{2}(Id + e_3) \in Cl_{3,0} \quad \text{identical with } f = \frac{1}{2}(Id + \sigma_3) \tag{3.7}$$

$$f = \frac{1}{2}\left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and we have $ff = f$.
The spinning electron has a $2x2$ matrix-spinor with only the first column being non-zero. This can be articulated by a

formal design

$$\psi \in \mathrm{Mat}(2,\mathbb{C})\, f \tag{3.8}$$

Multiplying such a spinor by any other element $u \in Cl_{3,0}$ we obtain an element of the same type, that is, $u\,\psi = \phi$ is a Pauli spinor too, and such elements with a vanishing second column form a left ideal S of $Cl_{3,0}$. Space S contains no other left ideal than itself and the zero ideal $\{0\}$. It is therefore called minimal in $Cl_{3,0}$. As a real linear space, S has a basis $\{f, f_1, f_2, f_3\}$. This basis has dimension 4 and contains pairs of units with mixed grade only:

$$\begin{aligned} f &= \frac{1}{2}(Id + e_3) \simeq \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ f_1 &= \frac{1}{2}(e_{23} + e_2) \simeq \begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix} \\ f_2 &= \frac{1}{2}(e_{31} - e_1) \simeq \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \\ f_3 &= \frac{1}{2}(e_{12} + e_{123}) \simeq \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{3.9}$$

Be aware, how these four elements $\in Cl_{3,0}$ combine pairs of even and odd base unit elements of the 8-dimensional Pauli algebra. Pertti Lounesto [26] (p. 60f.) investigated this space S with its associated division ring

$$F := f\,Cl_{3,0}\,f \simeq \left\{ \begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix} \,\middle|\, c \in \mathbb{C} \right\} \tag{3.10}$$

For no element in F is invertible in $Cl_{3,0}$; yet, for each non-zero $a \in F$ there is a unique $b \in F$ such that $a\,b = f$. Hence, F is a division ring with unity. Lounesto showed why F must be isomorphic with the complex numbers, as we have

$$f_3^2 = -f \tag{3.11}$$

and provided the minimal left ideal by a natural right F-linear structure, mapping

$$S \times F \longrightarrow S \quad \text{by} \quad (\psi, \lambda) \longrightarrow \psi\,\lambda \tag{3.12}$$

In this way it became legitimate to denote S as a spinor space. This does not only bring mathematical order into the idea of spinor space, but it has relevant consequences for an exact concept of the directional domain of quantum motion. Consider the unitary plane wave component of classical quantum mechanics, pinned down in sloppy manner as stationary component of "plane wave in direction x". When we were students, we often put

$$\psi = e^{\,ikx} \tag{3.13}$$

The reader may compare how Wikipedia tried to correct the sloppy "direction x" sketch of SR by turning over to a unitary vector basis. In case of a plane wave that carries physical impulse "in x direction", we should consider the term

$$\psi = e^{\,ike_1} = (cos\,k)\,Id + i\,(sin\,k)\,e_1 \tag{3.14}$$

Therefore, the directional domain of a plane wave in direction e_1 carrying scalar momentum $\hbar\,k$, as given by the de Broglie relation, is the 2-dimensional space of paravectors spanned by $\{Id, e_1\}$. This space contains that idempotent which is constituting the associated spinor space S.

3.5.2. Minkowski space-time and its directional domains

In accordance with this procedure, if we want to understand the idea of a natural Minkowski spinor and its associate directional domain, we must investigate the minimal left ideals in which such space-time spinors are located, that is, we shall have

$$\mathfrak{S} := \left\{ s \in f\left(\mathbb{C} \otimes Cl_{3,1}\right) \right\} \tag{3.15}$$

So we have to answer, at first, *what is the idempotent primitive in the Clifford algebra* of the Minkowski space $Cl_{3,1}$. Or do we have, perhaps more than one relevant primitive idempotent? Indeed, the quadratic Clifford algebra $Cl_{3,1}$ of the Minkowski space can be decomposed into three parts

$$Cl_{3,1} := Ch \oplus W \oplus G \tag{3.16}$$

where Ch is a 10-dimensional space with positive definite signature. W and G are well known quaternion spaces. The weak component of spin, often used in mathematical theories, is given by the spatial bivectors which squared give minus the identity, $-Id$. These bring upon the space

$$W := \bigwedge^{2} \mathbb{R}^{3,0} = span_{\mathbb{R}}\{e_{12}, e_{13}, e_{23}\} \tag{3.17}$$

The third component is a second quaternion module,- the observer's "time-space" of base unit monomials which squared give $-Id$ too.

$$G := span_{\mathbb{R}}\{e_4, e_{123}, J\} \quad \text{with} \quad J := e_{1234} \tag{3.18}$$

the oriented unit space-time volume; this also contains three quaternionic unit monomials of the Clifford algebra $Cl_{3,1}$ which squared give $-Id$, the negative identity.
The whole subspace with positive definite signature $Ch \subset Cl_{3,1}$ is

$$Ch := \bigoplus_{\chi}^{6} ch_{\chi} = span_{\mathbb{R}}\{Id, e_1, e_2, e_3, e_{14}, e_{24}, e_{34}, e_{124}, e_{134}, e_{234}\} \tag{3.19}$$

Theorem 1 *Directional Domain of quantum motion in Minkowski space is given by six Color Spaces. Space Ch has six abelian components. Those are the chromatic subspaces ch_{χ}, each one given by a maximal abelian subalgebra of some special linear Lie algebra $sl(4,\mathbb{R}) \subset Cl_{3,1}$*

$$\begin{aligned}
ch_1 &= span_{\mathbb{R}}\{Id, e_1, e_{24}, e_{124}\}, \quad & ch_2 &= span_{\mathbb{R}}\{Id, e_1, e_{34}, e_{134}\} \\
ch_3 &= span_{\mathbb{R}}\{Id, e_2, e_{34}, e_{234}\}, \quad & ch_4 &= span_{\mathbb{R}}\{Id, e_2, e_{14}, e_{124}\} \\
ch_5 &= span_{\mathbb{R}}\{Id, e_3, e_{14}, e_{134}\}, \quad & ch_6 &= span_{\mathbb{R}}\{Id, e_3, e_{24}, e_{234}\}
\end{aligned} \tag{3.20}$$

Clearly, these 4-dimensional Cartan subalgebras that form commutative subspaces, carry information about 4-momenta in quantum motion. What we observe in our macroscopic laboratory space-time, discloses to our eyes the Minkowski line elements. But, just as the Pauli algebra, adapted to the non-relativistic problem of spin, has the *directional domain* of its spinor space, so the Minkowski algebra should provide the *directional domains of relativistic fermion spinor spaces*. Again, these are determined by its primitive idempotents. But in the Minkowski algebra there are six lattices generated by six quadruples of primitive idempotents, namely the commutative modules $\{ch_1, ch_2, ch_3, ch_4, ch_5, ch_6\}$ of quantum motion. Such motion involves many more degrees of freedom, namely, in essence, the color and flavor of causality amending strong interaction. Spinors in Minkowski space-time have higher complexity than Dirac spinors, as they have a natural directional domain with a 4-dimensional graded structure. Space-time spinors can be defined neatly, as was carried out in the abstract of report [27]. I repeat that definition here. It is installed in such a way that the idempotency-property of spinors is preserved by the associated motion group, - what Minkowski liked to denote as G_c. Indeed, G_c takes now the appropriate "non standard, non worldline" form.

Sometime in 2016 I had shown Joel Isaacson why we have those six different quarks with their corresponding flavor and color. I had found that the root spaces are connected with six specific commutative graded modules of Clifford numbers which form Salomon's Seal or better the David Star. I wrote Joel *"I badly need some of your hints, namely, that Ne'eman obviously had seen the meaning of the David-Star for the constitution of the SU(3)-model of hadrons. I cannot find this beautiful email of yours. Can you repeat what you said then?"*[..] Joel gave me an immediate reply. *"I am glad to hear from you. Below is the correspondence I have relayed to you before."* Joel found the email he had sent me on February 16, 2016 saying *"Dear Bernd, This is very old email from 2004. Yuval Ne'eman got a copy of my Steganogramic paper and below are some of the immediate questions he raised. Later he proposed that we work together on finding links from my RD to the generators of SU(3). Unfortunately his health was deteriorating and he died in April 2006 before we could do much work together.*

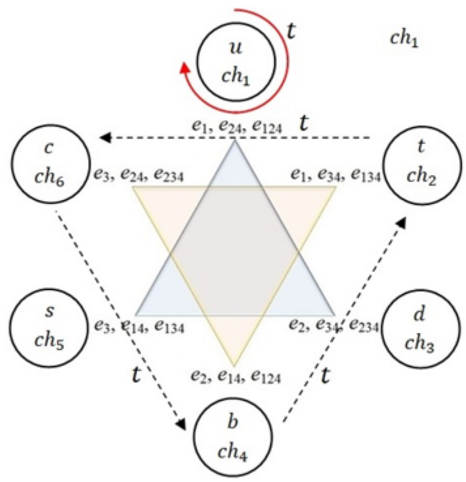

FIGURE 1. David-Star of the Strong Force

Note that he thought that the Star of David (what you call Solomon's seal) may relate to all this... Best, – Joel" Coming up is the David Star in the Clifford algebra of our macroscopic appearance of Minkowski space.

Spinors of fermions are derived as polarized isotropic multivectors in the Clifford algebra of the Minkowski spacetime. It is assumed that each spinor s Clifford multiplied with its gradeinverse $\hat{s}$ constitutes that primitive idempotent $f = s\hat{s} \in Cl_{3,1}$ in the minimal left ideal of which it is located, that is, first of all, we have $f \in f\left(\mathbb{C} \oplus Cl_{3,1}\right)$. Each such Cartan spinor in the Minkowski space-time algebra is decomposable into a product of extension and torsion. The list of fermions, derived on this basis, is complete. The spinors presented, obey the symmetries of the forces of nature. They do not rely on the addition of auxiliary bundles. Relativity Theory, conceived on the basis of a correct concept of "direction", as given by the primitive idempotent structure of its geometric algebra, is a quantum theory. It gives us the correct variant and invariant phenomena of the standard model of particle physics. Hence, before we go on, let us ask: What are the transformative properties of such extended form of *Universal Relativity*. Obviously, that group which was bound to the inappropriate concept of worldline, Minkowski´s G_c, must have some different form now. It must, in some peculiar way, be related to the HEPhy standard model. It also takes care of the old and new properties that we have experienced in nowadays quantum physics and laser optics. I have already anticipated many of the results in [28], [29], but still without making explicit use of the new concept of direction, and without establishing the relation to the old idea of Universal Relativity and the new way of speaking about causality. In the beginning of the last century, partly because of sociological givens, Einstein just could not yet find out about the connection of his most general idea of relativity with Feynman´s investigation of light phenomena, path integrals and particles travelling time-reversed. Relativity is most intimately connected with quantum-fields amending causality and entangled phenomena.

4. PHENOMENA

4.1. Multiple phenomenology of motion

Due to its many degrees of freedom, quantum motion has an incredible complexity. Extended Algebraic Relativity goes far beyond boosts and worldlines. Motion of matter in space-time is quantum motion. Quantum motion, mathematically speaking, is carried out in spaces that require several more degrees of freedom, without breaking the systemic whole of both RT and the model of particle physics. Most of those new degrees of freedom are old and known, but are new in the context of Universal Relativity. They involve spin and three types of isospin, formerly called t-, u- and v-spin, hence strangeness, baryon number, color, hypercharge, therefore charge and color-charge, chirality, helicity, and some more. Every motion in space extends to the inner isospin-spaces of colored and strange phenomena,- using words coined by terrestrial physicists. The main mystery is in the fact that outer space and time are linked with and coupled to the motion of quarks and gluons in the inner space of strongly interacting, strange and colored subnuclear phenomena. To explain the multiple, complex phenomenology of motion, let us first ask, what is a proton?

A proton is for us what it shows us in observation. We expose protons to different kinds of collisions. Thereby momentum is transferred to the inner spaces. Protons appear to us as stochastic samples of more or less dense clusters of quarks and gluons depending on collision energy and resolving power of the incident particle. The number of quarks that become visible within some small interval of collision-time can be fairly large and it occupies many locations and all the above mentioned degrees of freedom of quantum motion. It is because of that diversity and considerable uncertainty, that some of us speak of protons as clusters of partons. The word ”parton” is a placeholder for subnuclear particles. Physicists have not yet realized that phenomena of entanglement are not bound to photons and laser experiments. But these originate in the strange, ephemeral world of partons. Briefly, quarks are entangled.

One of the weirdest things is in the observation that single quarks occupy many locations in the universe. Quarks are not only confined within nucleons, but they are also cosmically entangled. Fermion entanglement can bridge distances as large as the universe. Events from the distant future can affect the inner of a proton. Though we can be sure, these phenomena of causality amending are real facts, we have not installed any suitable experiments for appropriate verification. There are no proper equations of motion and no coupling constants for outer-inner-interactions. The mathematics of basic events is somewhat complicated, and at present, I will not repeat all of it here. But readers can look into my book ”*Decay of Motion - the Anti-Physics of Space-Time*” [29] and find the rigor on ”Universal Gates of Minkowski algebra” in chapter 7, ”Color Braids”.

5. COLOR GROUPS OF ISOSPIN AND FLAVOR

By the time one became aware of six principal motion groups which fixate the corresponding leptons, as they do not partake in strong interaction, and in addition they preserve properties like idempotency, quantum numbers, annihilation relations and so forth. You find those symmetry groups compiled in the Appendix. They are essential space-groups, not auxiliary addons for gauging, but pure geometric features that are capable to restore material properties. They are either symmetric linear or symmetric unitary, depending on whether or not we apply the Weyl-trick to the rotatory elements of $SL(3,\mathbb{R})$, that is to say, in Gell-Mann notation, to the second, fifth and seventh matrix, or, here, rows 2, 5, 7 in all 6 columns of the attached table.

5.1. What leads to the correct Motion group G_c?

First attempts to expose the algebraic theory of special relativity as quantum theory were undertaken in 1996 [30]. Those Lie algebras which turn out relevant for the constitution of both the Standard model of particle physics and the Lorentz group of relativistic motion have been published in [30], [26] and later publications. They have been successfully used since then. Surprisingly, all six commutative subalgebras ch_χ $(\chi = 1,\dots,6)$ are characteristic for some corresponding rank 3 algebra with root-system A_3. How come, that, in the history of High Energy Physics, the root-system A_2 for a Lie group with rank 2, namely the $SU(3,\mathbb{C})$, - and alternatively in non-compact space the $SL(3,\mathbb{R})$), - play a much more important role than $SU(4,\mathbb{R})$? Sure you know, it is an extremely small step to go from $Cl_{3,1}$ to $SU(4,\mathbb{R})$. It is obvious that empirical findings, and theoretical physicists, by the time, promoted a favorable attitude towards $SU(3,\mathbb{C})$ whereas the $SU(4,\mathbb{R})$ was rather considered for the sake of mathematical completeness, e. g. in books like *Lie groups for Pedestrians*, written by Harry Lipkin [31], in the earlier days of HEPhy. What theoreticians usually are doing in such cases of over-determination, is that they remove what is not needed, and eventually add something that may be missing. So some might think, why not remove the grade-3 space-time-volume in each colorspace and be satisfied with the remaining two base-elements, e. g. take as maximal abelian Cartan subalgebra the commuting pair e_1, e_{24}, and proceed from there? One could do so, but some of the most relevant properties of the algebraic model would get lost.

- Using all three base elements of commutative Cartan subspaces ch_χ, we define angular momenta and isospin in a most natural way.
- We obtain phenomena of interlocked torsion [27].

- we explain strangeness.
- We obtain a set of unitary groups that describe interrelated fermions.
- Spinors, representing particle states, have components of the same type as the ’operators’ acting on them. That is, particles come up as both, acting and acted on. Most naturally, they represent both, observers and observed.
- We begin to understand phenomena of boost, warp and time-travel.

Even if we would not be able to see the whole of Universal Relativity, those six points listed offer such a big advantage that it would be silly, not to become aware of the graded isospin-groups with their directional domains. Another advantage which, however, I do not speak about in this paper, is in the logic [32], [33], linguistic, the psychic and neuronal anchorage of such approach. I have worked out many more than one representation of the space-time group, each slightly different from the other. This redundancy had its own value, as it could clarify what group it would be that Minkowski’s G_c would turn into, once we used the appropriate concept of directional domain . I had thusly pinned down at first two representations, one for $sl(3,\mathbb{R})$ and one for $su(3,\mathbb{C})$ in standard notation, where the two Cartan subalgebras of the corresponding groups,- physicists usually denote those by forms like $H_0 = \{T_3, T_8\}$,- are elements that belong to the first color space ch_1. The T are ’traditional’ generators in the fundamental representation, that is, for example, T_3, T_8 are one half of the corresponding λ’s, that is, in representation below. I gave a third representation in [32] which

- reproduces the primitive idempotents as diagonal units,
- reproduces the original Gell-Mann matrices,
- allows for an iterant representation compatible with the observed frequency doubling in phenomena of entanglement.

Consider

5.2. Algebra of the motion group

$sl(3,\mathbb{R})) \subset Cl_{3,1}$ stabilizing first primitive idempotent

$$\begin{aligned}
&\lambda_{1,1} = \frac{1}{2}(-e_{34} + e_{134}); \quad \lambda_{1,2} = \frac{[1]}{2}(-e_{23} + e_{123}); \quad \lambda_{1,3} = \frac{1}{2}(-e_{24} + e_{124});\\
&\lambda_{1,4} = \frac{1}{2}(-e_3 - e_{234}); \quad \lambda_{1,5} = -\frac{[1]}{2}(e_{13} + J);\\
&\lambda_{1,6} = \frac{1}{2}(e_2 + e_{14}); \quad \lambda_{1,7} = \frac{[1]}{2}(e_4 + e_{12});\\
&\lambda_{1,8} = \frac{1}{2}(-e_1 + e_{24});\\
&\Lambda_8 = \frac{1}{\sqrt{3}}(\lambda_3 + 2\lambda_8) = \frac{1}{2\sqrt{3}}(-2e_1 + e_{24} + e_{124});.
\end{aligned} \tag{5.1}$$

Note, the unit denominator in the second, fifth and seventh generators is set in brackets to indicate the possibility to apply the Weyl trick and substitute it by the imaginary unit. From these eight elements we can calculate the familiar shift generators of the group $SL(3,\mathbb{R})$). Most of us know how these things are handled. Some of the basic relations have been shown in the article [34]. The surprising advantage of such ’$su(3)$-app’ is that

- it is based on quantum geometry of matter

but

- leads directly to relativistic phenomenology of motion.

But

- that motion is not in ’diection x’ style, is not in a worldline. But it discloses to us a superposition of boosts in the representation adjoint to $\lambda_{1,3}$.

An observer, a nucleus, in the not dashed frame $\mathsf{F} = e_1, e_2, e_3, e_4$ claims to see events in his frame having coordinates x, y, z, t. The dashed frame $\mathsf{F}' = \{e_1', e_2', e_3', e_4'\}$, an energy carrying field, drives with velocity v relative to frame F. An observer 'moving' in this dashed frame F', - think about positrons or antiprotons irradiated from nucleus - is said, by me, to be 'iso-spinning' the first coordinate. It processes all coordinates with rapidity $\theta = \arctan\frac{v}{c}$ at isospin $\lambda_{1,1}$, but stabilized e_1 that is, in exact terms of language: The first component of isospin

- stabilizes $e_1' = e_1$,
- it warps e_2',
- it boosts e_3',
- it dilates e_4' ,
- it converts $e_{1234}^{'}$

It does so with rapidity $\theta = \arctan\frac{v}{c}$. To understand a little better, the various degrees of freedom in quantum motion, let us list the essential sub-tables for the three components of t-spin. If we would like to pin down a small package version in analogy with the 'direction x' boost map, it would look like this:

6. BOOST AND WARP

The action of t-spin $\lambda_{1,1}$

$$\begin{aligned}
e_1' &= e_1 \\
e_2' &= (\cosh\theta)e_2 - (\sinh\theta)e_{1234} \\
e_3' &= (\cosh\theta)e_3 - (\sinh\theta)e_4 \\
e_4' &= (-\sinh\theta)e_3 + (\cosh\theta)e_4 \qquad [..] \\
e_{34}' &= e_{34} \\
e_{134}' &= e_{134} \qquad [..] \\
e_{1234}' &= -\sinh\theta)e_2 + (\cosh\theta)e_{1234}
\end{aligned} \tag{6.1}$$

The action of t-spin $\lambda_{1,2}$

$$\begin{aligned}
e_1' &= e_1 \\
e_2' &= (\cos\theta)e_2 - (\sin\theta)e_3 \\
e_3' &= (\sin\theta)e_2 - (\cos\theta)e_3 \\
e_4' &= (\cos\theta)e_4 - (\sin\theta)e_{1234} \qquad [..] \\
e_{23}' &= e_{23} \\
e_{123}' &= e_{123} \qquad [..] \\
e_{1234}' &= (\sin\theta)e_4 + (\cos\theta)e_{1234}
\end{aligned} \tag{6.2}$$

The action of t-spin $\lambda_{1,3}$

$$\begin{aligned}
e_1' &= e_1 \\
e_2' &= (\cosh\theta)e_2 - (\sinh\theta)e_4 \\
e_3' &= (\sinh\theta)e_{1234} + (\cosh\theta)e_3 \\
e_4' &= -(\sinh\theta)e_2 + (\cosh\theta)e_4 \qquad [..] \\
e_{24}' &= e_{24} \\
e_{124}' &= e_{124} \qquad [..] \\
e_{1234}' &= (\sinh\theta)e_3 + (\cosh\theta)e_{1234}
\end{aligned} \tag{6.3}$$

6.1. Miniaturized scenario of t-spin

- In all components, t-spin preserves e_1. The moved base unit e_1' is stationary.
- The first component of t-spin boosts e_3 and dilates time.
- The third component of t-spin boosts e_2 and dilates time.
- The first component of t-spin boosts e_2 and enhances space-time-volume
- The third component of t-spin anti-boosts e_3 by anti-enhancing 4-volume
- The second component of t-spin rotates plane $\{e_2, e_3\}$
- The second component of t-spin warps spacetime. It converts space-time-volume into time by elliptic transformation.
- The first component of t-spin preserves the second color space ch_2
- The third component of t-spin preserves the first color space ch_1
- The second component of t-spin stabilizes the commutative module of Euclidean 3-volume

Bottom line: a cylindrical boost of fermionic energy in the plane $\{e_2, e_3\}$ is an isospinning field. t-spinning motion preserves the image of a symmetric stationary $e_1' = e_1$ with stabilized quasi Euclidean volume $e_{123}' = e_{123}$ which appears as independent of time. The two boosts on e_2 and e_3 go together with a planar rotation of $\{e_2, e_3\}$ and a space-time warp that transforms time into space-time volume. Said in linguistic terms, something diachronic is transformed into something synchronic. I denote this as a fundamental thermodynamic feature of time.

There are only a few of us who understand the meaning of the space-time-group, and still fewer who see its phenomenological origin and its mathematics, I wish to mention a conversation with Rolf Dahm, whome I thought I had to explain some of the figures coming up as a consequence of reloading of the symmetric unitary Lie groups then found by Gell-Mann. I did so on the phone. To my surprise he answered, yes, you consider some $su(3, \mathbb{C})$-subgroup for t-spin, one of the three components is always resulting from the commutation relation in the remaining pair. Hence, the second component is elliptic and gives some specific local curvature. Rolf Dahm had read Helgason's books on groups in geometry. I recalled his extraordinary insight that I found in two of his articles [35], [36]. Yes, this simple matter discloses the curvature in nuclear spin. Rolf has a very interesting interpretation of my observations in terms of Plücker's view and some of his own more advanced insights into projective geometry. These physical events touch most essential mysteries of the strong force action. Motion in the inner force field of the partons is hyperbolically balanced by the outer space-time-volume. This is surprise, for sure, but it does not make Relativity Theory much more surprising than it already is. Taking into account all the other isospin generators of these groups, we find out that any motion in strong interacting space must be compensated by thermodynamic transformation of directed 3-volume e_{123} and 4-volume J. What is most surprising is that time, directed volume and space-time-volume experience a trigonal recycling $e_4 \to e_{123} \to e_{1234} \to e_4$ which parallels the color- and flavor-rotations.

7. APPENDIX

	f	l	a	v	o	r
	ch_1	ch_2	ch_3	ch_4	ch_5	ch_6
λ_1	$\frac{1}{2}(-e_{34}+e_{134})$	$\frac{1}{2}(-e_{24}+e_{124})$	$\frac{1}{2}(-e_{14}-e_{124})$	$\frac{1}{2}(-e_{34}+e_{234})$	$\frac{1}{2}(-e_{24}-e_{234})$	$\frac{1}{2}(-e_{14}-e_{134})$
$\lambda_2(\times i)$	$\frac{1}{2}(-e_{23}+e_{123})$	$\frac{1}{2}(e_{23}-e_{123})$	$\frac{1}{2}(e_{13}+e_{123})$	$\frac{1}{2}(-e_{13}-e_{123})$	$\frac{1}{2}(-e_{12}+e_{123})$	$\frac{1}{2}(e_{12}-e_{123})$
λ_3	$\frac{1}{2}(-e_{24}+e_{124})$	$\frac{1}{2}(-e_{34}+e_{134})$	$\frac{1}{2}(-e_{34}+e_{234})$	$\frac{1}{2}(-e_{14}-e_{124})$	$\frac{1}{2}(-e_{14}-e_{134})$	$\frac{1}{2}(-e_{24}-e_{234})$
λ_4	$\frac{1}{2}(-e_3-e_{234})$	$\frac{1}{2}(-e_2+e_{234})$	$\frac{1}{2}(-e_1+e_{134})$	$\frac{1}{2}(-e_3-e_{134})$	$\frac{1}{2}(-e_2-e_{124})$	$\frac{1}{2}(-e_1+e_{124})$
$\lambda_5(\times i)$	$\frac{1}{2}(-e_{13}-J)$	$\frac{1}{2}(-e_{12}+J)$	$\frac{1}{2}(e_{12}-J)$	$\frac{1}{2}(-e_{23}+J)$	$\frac{1}{2}(-e_{23}-J)$	$\frac{1}{2}(e_{13}+J)$
λ_6	$\frac{1}{2}(e_2+e_{14})$	$\frac{1}{2}(e_3+e_{14})$	$\frac{1}{2}(e_3+e_{24})$	$\frac{1}{2}(e_1+e_{24})$	$\frac{1}{2}(e_1+e_{34})$	$\frac{1}{2}(e_2+e_{34})$
$\lambda_7(\times i)$	$\frac{1}{2}(e_4+e_{12})$	$\frac{1}{2}(+e_4+e_{13})$	$\frac{1}{2}(e_4+e_{23})$	$\frac{1}{2}(e_4-e_{12})$	$\frac{1}{2}(e_4-e_{13})$	$\frac{1}{2}(e_4-e_{23})$
λ_8	$\frac{1}{2}(-e_1+e_{24})$	$\frac{1}{2}(-e_1+e_{34})$	$\frac{1}{2}(-e_2+e_{34})$	$\frac{1}{2}(-e_2+e_{14})$	$\frac{1}{2}(-e_3+e_{14})$	$\frac{1}{2}(-e_3+e_{24})$
λ_8^*	$\frac{-2e_1+e_{24}+e_{124}}{2\sqrt{3}}$	$\frac{-2e_1+e_{34}+e_{134}}{2\sqrt{3}}$	$\frac{-2e_2+e_{34}+e_{234}}{2\sqrt{3}}$	$\frac{-2e_2+e_{14}-e_{124}}{2\sqrt{3}}$	$\frac{-2e_3+e_{14}-e_{134}}{2\sqrt{3}}$	$\frac{-2e_3+e_{24}-e_{234}}{2\sqrt{3}}$

Fermion stabilizer groups in Clifford algebra $Cl_{3,1}$ of Minkowski space

FIGURE 2. Six fermion stabilizer groups and Flavor in Minkowski algebra

ACKNOWLEDGMENTS

Thanks to Zbigniew Oriewicz and to José Vargas who were among the first who saw the importance of this topic of a comprehensive approach to some not auxiliary, material space-time group as was coming up in Clifford algebra. Many thanks to Rolf Dahm for our many valuable discussions. I am also recalling the patience of Klaus Gürlebeck, who once had given me the impulse to work out space-time-spinors for one of the Bauhaus conferences. I enjoyed the conversation with his associate Dmitrii Legatiuk in Cafe Museum, Vienna. I am grateful to Wolfgang Sproessig who gave me a book by Catoni, Zampetti and others long after I had presented my findings. Evidently my only way to deal with the authors' insights was to imagine that they were inspired by my writings. Cordial thanks to "little wing", Castalia Francon, to reconsider our thought train on Real Quaternions-And Logic Space Algebra from Nothing.

REFERENCES

1. Paul Adrien Maurice Dirac. *The Principles of Quantum Mechanics*, first published by Oxford University Press in 1930. 4th edition, Oxford University Press: Oxford, 1958.
2. Fritz Sauter. *Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs*. (On the behavior of an electron in a homogeneous electrical field according to the relativistic Theory of Dirac) Z Phys 1931, 69, 742.
3. Gustave Juvet. *Operateurs de Dirac et equations de Maxwell*. Sein Werk aus seiner Sicht. Kindler: München, 1982. By G. Juvet, Lausanne. Translated by D. H. Delphenich. `http://neo-classical-physics.info/uploads/3/0/6/5/3065888/juvet-dirac-operator-and-maxwell.pdf`, (accessed on 3rd May 2017).
4. Jan Arnoldus Schouten. *Zur Generellen Feldtheorie; Raumzeit und Spinraum*. ZS F Phys 81, 1933, 405-417.
5. Albert Einstein, Walther Mayer. *Semivektoren und Spinoren*. Sitzungsber. d. Preuß. Akad. Nr. 32, 1933, 522-550.
6. Albert Einstein. *Zur Elektrodynamik bewegter Körper*. Annal Phys Chem. 17, 1905, 891-921.
7. Albert Einstein. *On the Electrodynamics of Moving Bodies*. June 30, 1905. `http://hermes.ffn.ub.es/luisnavarro/nuevomaletin/Einstein1905relativity.pdf` (accessed on 7th May 2017)
8. Hendrik Antoon Lorentz, Hermann Minkowski, Albert Einstein. *Das Relativitätsprinzip*. - Eine Sammlung von Abhandlungen mit einem Beitrag von Hermann Weyl und Anmerkungen von Arnold Sommerfeld, Vorwort von A. Blumenthal, sechste Auflage, unveränderter Nachdruck der fünften Auflage von 1923. Wissenschaftliche Buchgesellschaft Darmstadt, 1958.
9. Hermann Minkowski. *Space and Time*. Minkowski's Papers on Relativity, Free Version, `http://www.minkowskiinstitute.org/mip/MinkowskiFreemiumMIP2012.pdf`, (accessed on 9th of May 2017).
10. Max Born. *Inert masses and the relativity principle*. Annalen der Physik 29 (3), 1909, 571-584.
11. Herglotz, G. *Über den vom Standpunkt des Relativitätsprinzips aus als 'starr' zu bezeichnenden Körper*. Annalen der Physik, 31, 1910, 393.
12. Fritz Noether. *On the understanding of the fixed bodies in the theory of relativity*. Annalen der Physik, 31 (5), 1910, 919-944.
13. Galina Weinstein. *Einstein's Uniformly Rotating Disk and the Hole Argument*. eprint arXiv: 1504.03989, April 2015.
14. Josè G. Vargas. *Differential Geometry for Physicists and Mathematicians - Moving Frames and Differential Forms: From Euclid Past Riemann*. World Scientific, Singapore, New Jersey et al. 2014
15. Eugene Paul Wigner. *On unitary representations of the inhomogeneous Lorentz group*, Annals of Mathematics, 40 (1), 1939, 149-204.
16. Paul Adrien Maurice Dirac. *Unitary representations of the Lorentz group*. Proc Roy Soc A, 183 (994), 1945, 284-295.
17. Valentine Bargmann. *Irreducible unitary representations of the Lorenz group*. Ann of Math, 48 (3), 1947, 568-640.
18. Israel Moiseevich Gelfand, Mark Aronovich Naimark . *Unitary representations of the Lorentz group* (PDF), Izvestiya Akad. Nauk SSSR. Ser. Mat. (in Russian), 11 (5), 1947, 411-504, retrieved (2014-12-15, Pdf from Math.net.ru).
19. Mark Aronovich Naimark. *Linear representations of the Lorentz group* (translated from the Russian original by Ann Swinfen and O. J. Marstrand), Pergamon Press: Oxford, 1964.
20. Sigurdur Helgason. *Groups and geometric analysis. Integral geometry, invariant differential operators, and spherical functions* (corrected reprint of the 1984 original), Mathematical Surveys and Monographs, 83, American Mathematical Society, 2000.
21. 'DrPhysicsA', *Moving through Spacetime*, `https://www.youtube.com/watch?v=5ODGnbkobNg` , (accessed on 16th May 2017).
22. `https://en.wikipedia.org/wiki/Lorentz_transformation`, (accessed 16th May 2017).
23. Hendrik Antoon Lorentz. *The Einstein theory of relativity; a concise statement*. First published in the Nieuwe Rotterdamsche courant of November 19, 1919. Brentano's, New York, 1920. This book has an editable web page on Open Library. `https://archive.org/details/einsteintheoryr00einsgoog`, (accessed on 20th May 2017).
24. Harald Keres. 1973 *General relativity principle as the principle of universal relativity of mechanical motion*. Methodological Analysis of Theoretical and Experimental Foundations of Gravity Physics, eds A Z Petrov and P S Dyshlevyi. Naukova Dumka, Kiev 1973. p 133.
25. Zbigniew Oziewicz. *Centre-of-mass for the finite speed of light*. 2014 J. Phys.: Conf. Ser. 532 012021. `http://iopscience.iop.org/article/10.1088/1742-6596/532/1/012021/pdf`, (accessed 22nd April 2018).
26. Pertti Lounesto. *Clifford algebras and Spinors*. Cambridge, 2001
27. Bernd Schmeikal. *The Universe of Spacetime Spinors*. Proceedings of the 9th International Conference on Clifford Algebras and their Applications in Mathematical Physics. Digital Proceedings ICCA9, Bauhaus University, Weimar, 2011. `https://www.researchgate.net/publication/260750946The-Universe-of-Spacetime-Spinors`
28. Bernd Schmeikal. *Decay of Motion-The Anti-Physics of Space-time*. Nova Scientific, New York, 2014.
29. Bernd Schmeikal. *Transposition in Clifford Algebra*, in Clifford Algebras, Applications to Mathematics, Physics and Engineering; edited by R. Ablamowicz R., Birkhäuser: Boston 2004, 351-372.
30. Bernd Schmeikal. *The generative process of space-time and strong interaction - quantum numbers of orientation*. In Clifford Algebras with Numeric and Symbolic Computations. Ablamowicz R., Lounesto P, Parra J. M., Eds., Birkhäuser: Boston 1996, 83-100.
31. Harry J. Lipkin. *Lie Groups for Pedestrians*. Dover, New York, 2002.
32. Bernd Schmeikal. *Four Forms Make a Universe* Advances in Applied Clifford Algebra 25, no. 1, 2015. 1-23, doi:10.1007/s00006-015-0551-z.
33. Bernd Schmeikal. *Free Linear Iconic Calculus*, AlgLog Part 1: Adjunction, Disconfirmation and Multiplication Tables. Vienna, 2015. doi:10.13140/RG.2.1.2083.1841.

34. Bernd Schmeikal. *On Motion*. Clifford Analysis, Clifford Algebras and their Application, (CACAA), 2 (1), Cambridge, UK 2013.
35. Rolf Dahm. On A Microscopic Representation of Space-Time III. ICGT 30, Ghent and ICCA 10, Tartu, http://arxiv.org/abs/1508.06872, submitted to AACA, 2015.
36. Rolf Dahm. *On a Microscopic Representation of Space-Time IV*. private communication in Vienna 2016.