In this paper, we study the preventive maintenance (PM) scheduling for a multi-component system and spare parts inventory. The objective of this policy is optimization both the cost and the reliability. We consider three typical preventive maintenance actions: mechanical service, minimal repair and preventive replacement. This policy consists of carrying out, for each preventive maintenance time, the possible maintenance actions for all components of system at once. The preventive maintenance time will be derived from the joint optimization of maintenance cost and spare parts inventory for each component. An optimal planning of the PM actions for the whole system is based on maximizing the maintenance benefit. Numerical example is given to explain the proposed maintenance policy and to show its validity and effectiveness.

Keywords: Minimal repair; spare part; Multi-component; Maintenance cost; Reliability.

1 Introduction

The maintenance efficiency of industrial systems is an important economic and business issue. The main difficulties come from the choice of maintenance actions and preventive maintenance time that minimize cost and maximize the reliability. Therefore, in this work, we examine the planning of the PM actions for whole system along the lifetime. This planning is based on the maximization of the reliability and the minimization of maintenance cost. For that we introduce the maintenance benefit
developed by Tsai and al. [15].
On the other hand, the realization of maintenance activities is dependent on the availability of the maintenance resources. These resources are mainly the human resources, the spare parts, materials handling equipment and the tools.

Nevertheless, most the maintenance models assume that the replacement items are drawn from an infinite stock ([15]; [7]; [9]). However, in some 'real life' situations, the availability of spare parts is an important factor in maintenance efficiency [10]. Therefore, in the last decades, many researchers and studies have proposed the joint optimization of spare parts inventory and maintenance policies ([1]; [3]; [4]; [6]). These studies have proved that the joint strategy of stock provision and preventive maintenance is the effective way to reduce maintenance cost and increase system availability.

In this paper, we consider joint optimization of maintenance and spare part inventory policy for a single-unit system subject to random failure. Each component has two kinds of failures, major and minor ones. The maintenance cost takes into account the costs of preventive replacement, corrective replacement, minimal repair, shortage, and holding; the lead time to receive an ordered spare is constant. For each component of the system, we determine jointly the optimal ordering time ($d^*$) and the optimal preventive replacement time ($T^*$) so as to minimize the expected cost rate.

In order to minimize the number of downtime, we propose a periodical preventive maintenance policy for the system. The preventive maintenance time of system ($T_s$) is the minimum one among the optimal preventive replacement times of all components. In this work, we consider three typical preventive maintenance actions: mechanical service, minimal repair and replacement preventive. The activities of mechanical service include lubricating, cleaning, checking and adjusting, etc. which is set to increase extrinsic reliability. Repair minimal serve to slow down the degradation of components and to increase intrinsic reliability. The selection of action for the components on every preventive maintenance time is based on maximizing maintenance benefit.

2 Joint optimization of spare parts inventory and maintenance policy for single-unit system

2.1 Model description and notation

We consider a system with one component subject to random failure and with one spare in stock. A system has two kinds of failures, major and minor ones. Park and Sun [11] have developed an ordering policy for preventive age replacement with minimal repair.
The system is replaced at \( T \) as soon as a spare is available. An order for a spare is delivered after a lead time \( L \). An order for a spare is placed at a scheduled ordering time \( d \) \( (d \leq T - L) \) or at a major failure, whichever occurs first. The objective is to determine jointly the optimal ordering time \( (d^*) \) and the optimal replacement time \( (T^*) \) so as to minimize the expected cost rate.

### 2.2 Notations and assumptions

- \( f(x) \): probability density function
- \( F(x) \): cumulative distribution function
- \( F(x) \): survivor function of time to failure
- \( g(x) \): probability density function of time to major failure
- \( G(x) \): cumulative distribution function of time to major failure
- \( G(x) \): survivor function of time to major failure
- \( p \): probability that a failure is a minor one
- \( d \): scheduled time for spare ordering
- \( L \): lead time between order and receipt of a spare part
- \( T \): scheduled time for preventive replacement \( (T \geq d + L) \)
- \( T_L \): the expected life of system
- \( c_p \): preventive replacement cost
- \( c_r \): corrective replacement cost \( (c_r \geq c_p) \)
- \( c_h \): holding cost of a spare per unit time
- \( c_s \): downtime cost per unit time due to spare shortage
- \( c_f \): average cost of a minimal repair
- \( c_{MS} \): average cost of a mechanical service
- \( R_i(t) \): reliability of component \( i \) at time \( t \)
- \( R_{ij}(t) \): reliability of component \( i \) in the \( j^{th} \) period at time \( t \)
- \( R_0 \): initial reliability of new component
- \( R_{min} \): minimum level of reliability of component
- \( C(d, T) \): expected cost rate for an infinite time span

### 2.3 Cost model

Since each replacement is a regeneration point, the time between successive replacements can be regarded as one cycle. The expected cost per cycle includes the replacement, repair, holding and
There exist the following four mutually exclusive and exhaustive possibilities in every cycle:

**i.** The operating unit fails before the scheduled ordering time \( d \), the average duration of this cycle is:

\[
D_1(d, T) = \int_0^d (x + L)g(x) \, dx
\]  

(1)

**ii.** The operating unit fails between \( d \) and the arrival of the ordered spare \( d + L \), the average duration of this cycle is:

\[
D_2(d, T) = (d + L) \int_d^{d+L} g(x) \, dx
\]  

(2)

**iii.** The operating unit fails between \( d + L \) and the scheduled preventive replacement time \( T \), the average duration of this cycle is:

\[
D_3(d, T) = \int_{d+L}^T xg(x) \, dx
\]  

(3)

**iv.** The operating unit does not fail before \( T \), the average duration of this cycle is:

\[
D_4(d, T) = T \int_T^\infty g(x) \, dx = T \overline{G}(T)
\]  

(4)

Thus the expected cycle length, \( D(d, T) \), is computed as follows.

\[
D(d, T) = L + \int_0^d \overline{G}(x) \, dx + \int_{d+L}^T \overline{G}(x) \, dx
\]  

(5)

Where the probability density function of time to major failure is given by the following relation [11]:

\[
g(x) = (1-p)f(x)\overline{F}(x)^{-p}
\]  

(6)

Thus the survivor function is \( \overline{G}(x) = \overline{F}^{1-p} \). The expressions of different relevant costs are given below.

1. The expected replacement cost per cycle:

\[
c_r G(T) + c_p \overline{G}(T) = c_p + (c_r - c_p)G(T)
\]  

(7)

2. The expected repair cost per cycle:

\[
\frac{p \, c_f}{(1-p)} G(T)
\]  

(8)
3. The expected holding cost per cycle:

\[ c_h \left[ \int_{d+L}^{T} (x - d - L)g(x) \, dx + (T - d - L)G(T) \right] = c_h \int_{d+L}^{T} \overline{G}(x) \, dx \]  \hspace{1cm} (9)

4. The expected shortage cost per cycle:

\[ c_s \left[ \int_{0}^{d} Lg(x) \, dx + \int_{d}^{d+L} (t + L - x)g(x) \, dx \right] = c_s \int_{d}^{d+L} G(x) \, dx \]  \hspace{1cm} (10)

Since the cost per cycle is the sum of (2), (3), (4) and (5), the expected cost per cycle, \( N(d, T) \), is

\[ N(d, T) = c_p + \left[ (c_r - c_p) + \frac{p}{(1 - p)} \right] G(T) + c_h \int_{d+L}^{T} \overline{G}(x) \, dx + c_s \int_{d}^{d+L} G(x) \, dx \]  \hspace{1cm} (11)

From the renewal reward theorem, the expected cost rate for an infinite time span is the expected cost per cycle divided by the expected cycle length. Hence the expected cost rate, \( C(d, T) \), is

\[ C(d, T) = \frac{N(d, T)}{D(d, T)} = \frac{c_p + A.G(T) + c_h \int_{d+L}^{T} \overline{G}(x) \, dx + c_s \int_{d}^{d+L} G(x) \, dx}{L + \int_{0}^{d} G(x) \, dx + \int_{d+L}^{T} \overline{G}(x) \, dx} \]  \hspace{1cm} (12)

Where,

\[ A = (c_r - c_p) + \frac{p \cdot c_f}{(1 - p)} \]

### 2.4 Optimization Procedure

The problem is to determine jointly the ordering time \( (d^*) \) and the replacement time \( (T^*) \) so as to minimize the expected cost rate \( C(d, T) \). We formulate this problem as constrained nonlinear optimization:

\[
\begin{align*}
\min_{(d,T)} & \quad C(d, T) \\
\text{subject to} & \quad (d_0, T_0) = (0, 0) \\
& \quad T \geq d + L
\end{align*}
\]  \hspace{1cm} (13)

Since \( C(d, T) \) is unimodal and pseudo convex in \( d \) and \( T \), the optimal values of \( d^* \) and \( T^* \), can be obtained by Matlab software using the function fmincon. This function finds a constrained minimum of a scalar function of several variables starting at an initial estimate.

**Numerical Example:**

To illustrate the advantage of this procedure compared to the procedure proposed by Park and Sun [11], we use the example set by the following parameters:

- The lifetime distribution is a Weibull: \( \overline{F}(x) = \exp\left(-\left(\frac{x}{100}\right)^2\right) \)
- The probability of minor failure: \( p = 0.6 \)
- The length of lead time: \( L = 30 \)
- The cost parameters are: \( c_p = $800, c_r = $1200, c_f = $50, c_h = $20, c_s = $200 \).

Using the procedure suggested by Park and Sun, we find the optimal values \( d^* \) and \( T^* \) after four iterations which takes more time. But the function \( \text{fmincon} \) makes it possible to find these values after a simple calculation. Table 1 shows that the function \( \text{fmincon} \) gives results more accurate than the procedure proposed in [11].

<table>
<thead>
<tr>
<th>Procedure</th>
<th>( t^* )</th>
<th>( T^* )</th>
<th>( C(t^<em>,T^</em>) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function ( \text{fmincon} )</td>
<td>29.4886</td>
<td>66.5576</td>
<td>22.5283</td>
</tr>
<tr>
<td>Procedure Park et Sun</td>
<td>29.85679</td>
<td>67.48926</td>
<td>24.04418</td>
</tr>
</tbody>
</table>

Table 1: Results of two procedures

3 Preventive maintenance scheduling for multi-component system

3.1 Maintenance actions and Reliability

We combine three typical PM actions. To scheduling the PM program, the reliability of various PM actions must be identified in advance.

3.1.1 Mechanical service (MS)

This type action can only improves the extrinsic conditions of failures. Several typical activities for this type are, for example, lubricating, adjusting/calibrating the position or load carried to the mating parts, etc.

For this action, the reliability of component \( i \) in the \( j^{th} \) preventive maintenance stage is defined as:

\[
R^i_j(t) = R^i_{0,j}R_i\left(\frac{1}{m_1}(t - (j - 1)T)\right); \quad (j - 1)T \leq t \leq jT
\]

(14)

Where:

- \( m_1 \) is the improvement factor which is set between 0 and 1. It can be evaluated according to the enclosed activities ([13]; [14]).
• $R_{0,j}^i$ is the initial reliability of component $i$ in the $j^{th}$ stage. It can be expressed as:

$$R_{0,j}^i = R_{f,j-1}^i = R_{0,j-1}^i R_i(T) \quad (15)$$

• $R_{0,j-1}^i$ and $R_{f,j-1}^i$ indicate the initial and final reliabilities of this component in the $(j - 1)^{th}$ stage.

### 3.1.2 Minimal repair (MR)

The minimal repair includes the activities of (MS) and repairing/replacing for some simple parts such as springs, seals, belts and bearings, etc. It can rightly recover the intrinsic damage except the extrinsic condition improved.

In this case, the reliability of component $i$ in the $j^{th}$ preventive maintenance stage is defined as:

$$R_j^i(t) = R_{0,j}^i R_i \left( \frac{1}{m_2} (t - (j - 1)T) \right); \quad (j - 1)T \leq t \leq jT \quad (16)$$

Where:

• $R_{0,j}^i$ is the initial reliability of component $i$ in the $j^{th}$ period. It can be expressed as:

$$R_{0,j}^i = R_{f,j}^i + m_2 (R_0 - R_{f,j-1}^i) \quad (17)$$

• $R_{f,j-1}^i$ indicate the final reliability of component $i$ in the $(j - 1)^{th}$ period.

### 3.1.3 Replacement preventive (RP)

Normally, RP would restore the component to its original condition. The reliability of component $i$ in the $j^{th}$ preventive maintenance stage is expressed as:

$$R_j^i(t) = R_0 R_i(t - (j - 1)T); \quad (j - 1)T \leq t \leq jT \quad (18)$$

### 3.2 Preventive maintenance scheduling

We consider a multi-component system with $n$ components and we denote by $(d_i^*, T_i^*)$ the optimal ordering and replacement times of component $i$ which can be derived from section 2. According
to the basic preventive maintenance policy, a PM action must be performed for each component \(i\) \((i \in 1...n)\) at preventive maintenance time \(T^*_i\). Whenever a PM action is performed, the whole system has to stop. During this shut down PM opportunities arise for all the other components in the system because combining PM activities can lead to a saving on the system dependent cost (i.e., Down time cost) which is called set-up cost by Wildeman and al. [17].

Therefore, we choose the minimum one among all the \(T^*_i\) of the component as the PM interval of the system \(T_s\), i.e. \(T_s = \min(T^*_i \ (i \in 1...n))\) and the scheduled time for spare ordering is \(d_s = \min(d^*_i \ (i \in 1...n))\). On the other hand, the component which \(T^*_i > T_s\) are taken with maintenance action from the set MS; MR; RP; No action in this time. The selection of PM actions is based on the minimization of maintenance cost and the maximization of the reliability using the maintenance benefit of the component.

In this work, the preventive maintenance scheduling for system can be described as follows:

**Step 1:** For each component \(i\), we calculate \(T^*_i\) and \(d^*_i\) by using the joint optimization of spare parts inventory and maintenance policy proposed in Section2.

**Step 2:** Determining the PM interval of system \(T_s\) and the scheduled time for spares ordering \(d_s\):

\[
T_s = \min(T^*_i \ (i \in 1...n))
\]

\[
d_s = \min(d^*_i \ (i \in 1...n))
\]

Put \(j = 1\);

**Step 3:** Put \(T_j = j \ast T_s\);

For each component \(i\) which \(T^*_i > T_s\), there are two cases:

**Case 1:** \(R_i(2 \ast T_s) \geq R_{min}\), we won’t take any action

**Case 2:** \(R_i(2 \ast T_s) \leq R_{min}\), we calculate the maintenance benefit of component \(i\) in the \(j^{th}\) stage using the following relation:

\[
B_{i,k} = \frac{\int_{j \ast T_s}^{\infty} R^i_{j+1}(t) \, dt - \int_{j \ast T_s}^{\infty} R^i_{j}(t) \, dt}{C_{i,k}}
\]

(19)

Where \(C_{i,k}\) is maintenance cost of action \(k\). The numerator indicates the extended life of component \(i\) by action \(k\).

The action which leads to the maximum maintenance benefit, i.e. \(B^*_i = \max(B_i, k)\); would be selected for the component \(i\).
Step 4: Put \( j = j + 1 \),

While \( T_j \leq T_L \): go to step 3.

The PM scheduling ends when \( T_j \geq T_L \)

4 Numerical example

In order to progress example analysis, we suppose that the improvement factors, the maintenance costs can be identified and the lifetime distribution of each component can be determined.

In this example, a system consists of four components in series. The lifetime of each component has a Weibull distribution. The reliability parameters of the component parameters and the maintenance costs are listed in Table 2. The expected life of system is set to \( T_L = 4500h \). The initial reliability of each component is set to \( R_0 = 0.999 \). The minimum reliability for judging whether to maintain or not is set to \( R_{min} = 0.8 \).

<table>
<thead>
<tr>
<th>Components</th>
<th>( \theta_i )</th>
<th>( \beta_i )</th>
<th>( c_p )</th>
<th>( c_r )</th>
<th>( c_s )</th>
<th>( c_h )</th>
<th>( m_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1800</td>
<td>1.8</td>
<td>240</td>
<td>480</td>
<td>120</td>
<td>30</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>2200</td>
<td>2.8</td>
<td>360</td>
<td>720</td>
<td>144</td>
<td>40</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>2600</td>
<td>3</td>
<td>400</td>
<td>800</td>
<td>140</td>
<td>50</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>3200</td>
<td>3.2</td>
<td>280</td>
<td>560</td>
<td>70</td>
<td>50</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\[ c_f = 0.6 \times c_p; \quad c_{MS} = 0.3 \times c_p; \quad L = 30; \quad p = 0.6; \quad m_1 = m_2 \]

Table 2: The supposed parameters of the components in the example

According to the given parameters in Table 2, the optimal ordering and replacement times \( (d_i^*, T_i^*) \) of the components can be obtained by the optimization procedure. They are \( d^* = \{678; 920; 1190; 1633\} \) and \( T^* = \{708; 950; 1220; 1663\} \).

The PM interval of the system would be \( T_s = 708 \) and the ordering time of space parts \( d_s = 678 \).

Next, the maintenance benefits of different PM actions are calculated for choosing the PM actions by Eq.19 (Table 3). Along the lifetime of the system, the optimal PM actions of components are reported in Table 4.
Table 3: The maintenance benefits of the components

The calculated results show that the total maintenance cost under the proposed PM scheduling policy is 10.5% less expensive than under the basic preventive maintenance policy of each component based on the model suggested in section 2.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Actions</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>j=1</td>
<td>MS</td>
<td>1.74 2.26</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 1.36</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 1.94</td>
</tr>
<tr>
<td>j=2</td>
<td>MS</td>
<td>1.74 3.19</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 3.21</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 3.11</td>
</tr>
<tr>
<td>j=3</td>
<td>MS</td>
<td>1.74 2.25</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 2.17</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 2.47</td>
</tr>
<tr>
<td>j=4</td>
<td>MS</td>
<td>1.74 2.26</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 1.36</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 1.94</td>
</tr>
<tr>
<td>j=5</td>
<td>MS</td>
<td>1.74 3.19</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 3.21</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 3.11</td>
</tr>
<tr>
<td>j=6</td>
<td>MS</td>
<td>1.74 2.25</td>
</tr>
<tr>
<td></td>
<td>MR</td>
<td>2.08 2.17</td>
</tr>
<tr>
<td></td>
<td>RP</td>
<td>2.76 2.47</td>
</tr>
</tbody>
</table>

Table 4: The optimal PM actions of components

<table>
<thead>
<tr>
<th>Stage</th>
<th>Time(h)</th>
<th>PM actions</th>
<th>System reliability</th>
<th>Maintenance cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j=1</td>
<td>708</td>
<td>RP MS</td>
<td>0 0</td>
<td>0.93</td>
</tr>
<tr>
<td>j=2</td>
<td>1416</td>
<td>RP MR RP MS</td>
<td>0.96</td>
<td>869</td>
</tr>
<tr>
<td>j=3</td>
<td>2124</td>
<td>RP RP</td>
<td>0 0</td>
<td>0.97</td>
</tr>
<tr>
<td>j=4</td>
<td>2832</td>
<td>RP MS RP</td>
<td>0 0</td>
<td>0.91</td>
</tr>
<tr>
<td>j=5</td>
<td>3540</td>
<td>RP MR 0</td>
<td>MS 0.96</td>
<td>510</td>
</tr>
<tr>
<td>j=6</td>
<td>4248</td>
<td>RP RP RP RP</td>
<td>0.99</td>
<td>1176</td>
</tr>
</tbody>
</table>

0: No action; Total maintenance cost = $4405

Table 4: The optimal PM actions of components
Moreover, Fig.1 gives the reliability changing for the system under the proposed policy and the basic policy. This figure shows that the introduction of two PM actions (MS & MR) improves the degradation of reliability. Based on these results, the PM schedule suggested ensures the availability of the system at a low maintenance cost.

5 Conclusion

Most published researches treat the maintenance and spare parts inventory policies separately or sequentially. However, since the space parts management is often dependent on the maintenance policies, it is a better practice to deal with these problems simultaneously. Therefore, this paper deals with joint optimization of spare parts inventory and maintenance strategies.

In general, most PM models deals with single-unit systems. However, in industry, most of systems are multi-component systems. This paper proposes an opportunistic PM scheduling policy for multi-component series systems. This policy is based on minimization of maintenance cost and maximization of the reliability of the system with three PM actions integration. Numerical example shows that the proposed PM policy is better than the individual maintenance policy. However, the preventive maintenance policy depends on the production management. Therefore, further research will treat coordinating production, inventory and preventive maintenance operations simultaneously.

References


