Coolant optimization of a gas turbine engine

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This paper describes an analysis used to study the performance of the gas turbine engine with particular emphasis on the optimum amount of coolant required for maximum overall efficiency of the engine. The effect of pre-bled air, as well as that drawn from the exit of the compressor, is also studied.

The optimum amount of coolant for the engine is found to depend on the effectiveness of the heat extraction parameter $A$, component efficiencies of the engine and the compressor pressure ratio of the engine.

NOTATION

- $A$: surface area or heat extraction factor as defined by equation (25)
- $b$: function of fluid properties, gas mass flow and blade dimensions in equation (23)
- $c$: function of fluid properties, gas mass flow and blade dimensions in equation (23)
- $C$: specific heat of fluid
- $f$: fuel–air ratio
- $h$: heat-transfer coefficient
- $h_1$: mass flowrate
- $N$: number of blades
- $N_r$: number of rows of blades being cooled
- $P$: absolute pressure
- $r$: compressor pressure ratio
- $R$: specific gas constant
- $SWO$: specific work output
- $T$: absolute temperature
- $\gamma$: isentropic index
- $\varepsilon_1$: mass fraction of coolant to generator turbine
- $\varepsilon_2$: mass fraction of coolant to free power turbine
- $\varepsilon_{2p}$: mass fraction of pre-bled coolant to free power turbine
- $\eta$: component isentropic efficiency
- $\eta_{comp}$: compressor efficiency
- $\eta_{isent}$: isentropic efficiency of both turbines
- $\eta_{generator turb}$: gas generator turbine efficiency
- $\eta_{power turb}$: power turbine efficiency
- $\eta_{o}$: overall engine efficiency based on heat rejected
- $\eta_{w}$: overall engine efficiency based on work output
- $\lambda$: calorific value of fuel

Subscripts

1: air entering the compressor or air bled to cool generator turbine
2: air leaving the compressor
2p: pre-bled air leaving the compressor
3: gas generator turbine inlet
4: mainstream gas immediately before mixing with bled air
5: mainstream gas immediately after mixing with bled air
6: bled air at turbine blade roots
7: bled air at blade tips after cooling all components
8: gas generator turbine exit and free power turbine inlet
9: free power turbine exit
b: based on average temperature
bm: blade mean condition
c: coolant condition
g: gas condition
max: maximum value
p: condition in power turbine corresponding to that in compressor turbine
u: uncooled condition

1 INTRODUCTION

Efforts to cool effectively the blades in the high-pressure turbine of gas turbines have been developing for more than thirty years. The advantages of blade cooling in a gas turbine engine are well known; such cooling permits increases in turbine entry temperature while maintaining sufficiently low blade metal temperature to preserve the desired material properties. However, a number of performance penalties are thereby introduced which detract from the overall effectiveness. With air cooling, the amount of compressed air required as the coolant can be in the region of 10 per cent of the intake mass flow in a modern high-performance engine. This coolant air is bled from the compressors and, after
cooling the turbine components, is fed back into the mainstream. In thus bypassing the combustion chamber, its full work potential is not realized in the turbine, and, in addition, energy is used in first compressing and then pumping the coolant air. On mixing with the mainstream, the latter loses enthalpy and stagnation pressure, and in multi-stage turbines there is a negative reheat effect. With increases in coolant air, these penalties are likely to become so severe as ultimately to more than offset the gains in work output and thermal efficiency resulting from higher turbine entry temperatures. To avoid such deterioration, there is a need to be able to determine the amount of coolant required for optimum performance.

Although many studies have been conducted on cooling gas turbine blades, especially for obtaining the best configurations and blowing rates for convection, film and impingement cooling such as described by Goldstein (1), Esgar (2), Goldstein and Chen (3) and Metzger and Bunker (4), very little has been published about optimizing the process of blade cooling. Details of some of the optimization procedures of blade cooling studies are reported by Hedrick et al. (5), Ayache et al. (6) and Byerley (7) based on the methodology of Oates (8). In these studies the amount of coolant was assumed to be constant while the other parameters were optimized. The present treatment differs from the previous investigations in that it attempts to predict the optimum amount of coolant air to give best engine performance for given conditions. This is done for the case of a gas generator feeding a free power turbine, rather than a propelling nozzle for aircraft propulsion, the choice making for readier expression of the performance characteristics of the system, bearing in mind the likely need to cool the blades of both turbines.

It will be seen from the schematic arrangement in Fig. 1 that the analysis allows for withdrawal of coolant air from the compressor both during and at the end of compression, in the former case air is used to cool the latter stages of expansion, assumed to be wholly located in the free power turbine. The analysis also covers the case where coolant is preheated before it reaches the blades to be cooled, as is almost invariably the case in its passage from the compressor to the turbine, though the computer studies of Hay and Taylor (9) indicate the need to minimize coolant delivery temperatures. Byerley (7), who optimizes for the compressor pressure ratio, assumes constant coolant temperature. The assumed mainstream gas–blade–coolant heat-transfer process, through which the mean blade temperature is related to other cycle temperatures, employs a simple linear theory, based on infinite conductivity of the blade material, in the application of standard forced convective data for coolant flow through spanwise passages from the blade root to blade tip. Subsequent mixing with the mainstream is assumed for the purpose of analysis to be concentrated at a predetermined point in the expansion process.

The treatment is one-dimensional in that uniform properties are assumed across any station in the mainstream and coolant flow paths. The thermodynamic conditions to be satisfied include continuity, component isentropic efficiency relationships, the appropriate forced convection heat-transfer correlations and equations for turbulent flow in the coolant passages and from mainstream gas to the blade, together with the combustion equation, the mixing energy balance following cooling and the power balance equation for the gas generator. Stagnation pressure losses during combustion, mixing and in the interconnecting ducting, and mechanical losses, are neglected. The performance parameters to be computed are the specific work output (SWO) and the cycle thermal efficiency. This may be determined either from the SWO as $\eta_w$ or from the difference between the heat input and the heat finally rejected, as $\eta_w$. The extent to which the above two measures agree is an important test of the validity and accuracy of the prediction procedure.

This has been carried out for ambient conditions of 1 bar and $T_1 = 288 \, \text{K}$, compressor pressure ratios $r_1$, representative of current gas turbine practice ranging from 12 to 32, compressor and turbine component isentropic efficiencies $\eta$ from 0.8 to 1.0, a maximum mean blade temperature $T_{bm}$ of 1125 K, with cooling as necessary to maintain this value. The reference fuel assumed in the analysis is a hypothetical liquid hydrocarbon, approximating to kerosine, containing 13.92 per cent hydrogen and 86.08 per cent carbon, for which the stoichiometric fuel-air ratio is 0.0683 and the reaction enthalpy per unit mass of fuel is 43100 KJ/kg, as adopted by Cohen et al. (10). Full account is taken, at all stages of the iteration process outlined below, of changes in fluid property values (including specific heat, viscosity, thermal conductivity and Prandtl number) resulting from changes in composition due to both combustion and mixing, as well as from changes in temperature, together with the reductions in the effective air–fuel ratio due to mixing.

![Fig. 1](image-url)
Since the property values used, particularly those for specific heats, are believed to be among the most reliable available (11), it was important to attempt some evaluation of the effects of dissociation on heat release and hence turbine entry temperature for the above fuel over the range of compressor pressure ratios considered.

Under stoichiometric conditions, the heat released results in temperatures around 2200 K for pressure ratios between 12 and 32. From the dissociation constants arising from the constituent partial pressures for this temperature, it was estimated that dissociation reduces heat release by about 1.8 per cent for a compressor pressure ratio of 12 and 1.3 per cent for a pressure ratio of 32, the corresponding temperature reductions being 28 and 18°C respectively.

For non-stoichiometric conditions, with high excess air and much lower maximum turbine entry temperatures, typically in the range 1700–1800 K, dissociation is likely to be even less influential, with temperature reductions probably no more than 10–18°C, and therefore not warranting further consideration.

A computer program has been written to accommodate an iterative procedure for solving the equations representing the conditions to be satisfied. The program consists basically of five sub-routines covering air properties, gas properties, the gas generator turbine, the power turbine and thermal efficiency. The procedure starts by validating the initial guess for the compressor outlet temperature for given initial conditions, with iteration until there is agreement between the two values within the chosen error criterion. The iteration procedure then continues to calculate temperatures at the various stations in the engine cycle, and the associated amounts of coolant, for the chosen mean blade temperature limit of 1125 K. The SWO and the thermal efficiencies are then determined and compared with those based on averaged fluid property values.

The treatment indicates an optimum coolant flow for maximum thermal efficiency for all compressor pressure ratios considered. Because of the approximate technique used, the thermal efficiency based on heat rejected is generally less than when based on the work output, but the difference is very small for the condition of maximum efficiency. The advantage of using pre-bled air, rather than fully compressed air, on thermal efficiency is generally slight, even negligible, up to the optimum amount of coolant, above which it can become markedly negative. This is so for all pressure ratios, degrees of coolant preheating and compressor efficiencies. Optimum coolant flow increases nearly linearly with the overall pressure ratio and correspondingly higher generator turbine entry temperatures, but the optimum amount required is naturally less if there is little coolant preheating. The SWO increases almost linearly with the maximum cycle temperature for a given overall pressure ratio. The temperature dependence of fluid properties significantly affects overall engine efficiency, which is underpredicted by calculations based on average properties.

2 ANALYSIS

A schematic of the gas generator and free power turbine is shown in Fig. 1. The arrangement permits pre-bleeding of coolant air for the free power turbine at some intermediate stage of the compression process (in order to save some compression work), as well as at the end of compression; in both cases the final temperature, after iteration from the initial guess, is

$$T_2 = T_1 \left\{ \frac{C_1}{C_2} + \frac{1}{\eta_{12}} \left( r_0^{0.574} (c_1 + c_2) - \frac{C_1}{C_2} \right) \right\}$$ (1)

where the exponent is based on the arithmetic mean specific heat, with $2R = 0.574$ kJ/kg K for air. This equation involves the sub-routine for air properties.

The temperatures in the compressor turbine were considered next, where the gas entry temperature is $T_3$. For the purposes of analysis it was assumed that the blade cooling process is concentrated at a lower temperature $T_4$ in the gas stream, resulting in a reduced stagnation temperature $T_5$ after cooling and mixing with the coolant (of mass fraction $\epsilon_1$), whose temperature increases from $T_4$ at entry to the blade coolant passages to $T_5$ at exit. The mainstream gas temperature at the end of expansion in the compressor turbine is $T_5$.

Irrespective of whether the coolant for the power turbine is pre-bled or not, the mixing energy balance in the compressor turbine for coolant and mainstream gas then is

$$(1 - \sum \epsilon)(1 + f)C_4 T_4 + \epsilon_1 C_7 T_7 = C_5 T_5 \{ (1 - \sum \epsilon)(1 + f) + \epsilon_1 \}$$ (2)

where $f$ is the fuel–air ratio. This equation yields $T_5$ as

$$T_5 = \frac{C_4 T_4}{C_5} + \frac{\epsilon_1 (C_7 T_7 - C_4 T_4)}{C_5 \{ (1 - \sum \epsilon)(1 + f) + \epsilon_1 \}}$$ (3)

and involves the sub-routine for gas properties.

In the first case, where all coolant is bled from the exit of the compressor, the power balance equation for the gas generator is given by

$$(1 - \sum \epsilon)(1 + f)(C_3 T_3 - C_4 T_4) + \{(1 - \sum \epsilon)(1 + f) + \epsilon_1\}(C_5 T_5 - C_8 T_8) = C_2 T_2 - C_1 T_1$$ (4)

and hence solving for the gas outlet temperature from the generator turbine yields

$$T_8 = \frac{C_2 T_2 - C_3 T_3 - C_4 T_4}{C_8} - \frac{\epsilon_1 (C_3 T_3 - C_4 T_4) + (C_2 T_2 - C_1 T_1)}{C_8 \{ (1 - \sum \epsilon)(1 + f) + \epsilon_1 \}}$$ (5)

In the absence of cooling, equation (4) reduces to

$$(1 + f_0)(C_3 T_3 - C_4 T_4) = C_2 T_2 - C_1 T_1$$ (6)

whence

$$T_{8u} = \frac{C_2 T_{3u} - C_1 T_1}{C_8} - \frac{C_2 T_2 - C_1 T_1}{C_8 (1 + f_0)}$$ (7)

To proceed further it was necessary to define where the representative temperature $T_a$ occurs in the expansion in the compressor turbine. Clearly it should correspond to the average amount of cooling. Therefore $T_a$ was assumed to be the arithmetic mean of the inlet and outlet temperatures in the compressor turbine in the
uncooled condition, that is

\[ T_4 = \frac{T_{3u} + T_{8u}}{2} \]  

(8)

Combining equations (7) and (8) yields

\[ T_{3u} = \frac{2T_4}{1 + C_3/C_8 + \frac{C_4}{C_8}(1 + f_\text{c})} \]

(9)

The combustion equation for the uncooled case is

\[ \dot{\lambda}_{\text{c}} = (1 + f_\text{c})C_3 T_{3u} - C_2 T_2 \]

(10)

and eliminating \( f_\text{c} \) between equations (9) and (10) gives

\[ T_{3u} = \frac{T_4 + \left\{ \lambda(C_2 T_2 - C_1 T_1) \right\}/\{2C_4(\lambda - C_2 T_3)\}}{(1 + C_3/C_8)\{2\} + \{C_3(C_2 T_2 - C_1 T_1)\}/\{2C_4(\lambda - C_2 T_3)\} \}

(11)

In the second, or pre-bled, case the power balance energy equation for the gas generator is

\[ (1 - \sum \epsilon)(1 + f)\left\{ (C_3 T_3 - C_4 T_4) \right\} \]

\[ + \left\{ (1 - \sum \epsilon)(1 + f) + \epsilon_1 \right\}(C_3 T_3 - C_4 T_4) \]

\[ = (1 - \epsilon_{\text{c}})C_3 T_3 + \epsilon_{\text{c}} C_2 C_3 T_3 - C_1 T_1 \]

(12)

where \( \epsilon_{\text{c}} \) is the mass fraction bled off at the intermediate location 2p in the compression process. Solving for: \( T_3 \) as before, then

\[ T_3 = \frac{C_3 T_3}{C_3} + \frac{C_2 T_2 - C_4 T_4}{C_4} \]

\[ \epsilon_1 \left\{ (C_3 T_3 - C_4 T_4) \right\} + \left\{ (1 - \epsilon_{\text{c}})C_3 T_3 + \epsilon_{\text{c}} C_2 C_3 T_3 - C_1 T_1 \right\} \]

\[ C_4\{1 - \sum \epsilon\}(1 + f) + \epsilon_1 \}

(13)

Proceeding as in the first case, it was first found that, for the uncooled gas generator turbine, \( T_{3u} \) is now given by

\[ T_{3u} = \frac{C_3 T_{3u} - C_2 T_2(1 - \epsilon_{\text{c}}) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_1 T_1}{C_4(1 + f_\text{c})} \]

(14)

and that consequently

\[ T_{3u} = \frac{T_4 + \theta_1}{\left\{ (1 + C_3/C_8)\{2\} + \theta_2 \right\}} \]

(15)

where

\[ \theta_1 = \lambda \left\{ \frac{C_3 T_3(1 - \epsilon_{\text{c}}) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_1 T_1}{C_3} \right\} \]

(16)

\[ \theta_2 = \frac{C_3}{C_4} \left\{ \frac{C_3 T_3(1 - \epsilon_{\text{c}}) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_1 T_1}{C_4} \right\} \]

(17)

Preheating of the coolant from \( T_{3u} \) to \( T_{\text{opt}} \), or \( T_1 \) to \( T_6 \) and \( T_{\text{opt}} \), in its passage from the compressor to one or other turbine, as indicated earlier, is assumed to correspond to a reduction in compressor turbine entry temperature from the uncooled value \( T_{3u} \) to the actual value \( T_5 \), according to the equation

\[ (1 - \sum \epsilon)(1 + f)C_3m(T_{3u} - T_5) \]

\[ = \epsilon_1(C_6 T_6 - C_2 T_2) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_2 T_2 \]

for the pre-bled case, and

\[ (1 - \sum \epsilon)(1 + f)C_3m(T_{3u} - T_3) \]

\[ = \epsilon_1(C_6 T_6 - C_2 T_2) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_2 T_2 \]

(19)

where all cooling air is withdrawn at the end of compression. The basis of the assumption is, if course, that any preheating derives in the last resort from reaction enthalpy or calorific value of the fuel, \( \lambda \), thereby reducing its capacity to heat the compressed air. Solving equation (18) for \( T_3 \), then

\[ T_3 = T_{3u} - \epsilon_1(C_6 T_6 - C_2 T_2) + \epsilon_{\text{c}} C_2 C_3 T_3 - C_2 T_2 \]

\[ \left\{ (1 - \sum \epsilon)(1 + f)C_3m \right\} \]

(1 - \sum \epsilon)(1 + f)C_3m

(20)

Here \( \epsilon \) is taken as the uncooled value and \( C_{3b} \), as the arithmetical average of the values for \( T_3 \) and \( T_{3u} \).

The effect of coolant on other temperatures such as \( T_4 \) and \( T_5 \) can be found from consideration of heat transfer before and after the coolant is introduced at the temperatures in question. The gas-blade coolant heat-transfer process must be considered if the mean temperature of the turbine blades is to be related to other temperatures used in the thermodynamic analysis. Using a simple linear theory and assuming convective heat transfer and infinite conductivity of blade material the representative heat balance for the compressor turbine may be written as follows:

\[ \epsilon_1 \theta C_3m(T_7 - T_6) = h\frac{T_{\text{m}} - \frac{T_6 + T_7}{2}}{2} \]

\[ = h\frac{\frac{T_6 + T_7}{2} - T_{\text{m}}}{2} \]

(21)

where \( A \) is the surface area and the subscripts \( c \) and \( g \) refer to the coolant and hot gas side respectively. Solving equation (21) for the temperature difference ratio yields

\[ \frac{T_6 - T_7}{(T_4 + T_5)/2 - T_6} = 2 \]

\[ = 1 + \frac{2\epsilon_1 \theta C_3m(T_7 - T_6)}{1 + h\frac{h}{\epsilon_{\text{c}}}} \]

(22)

Apart from the right-hand side being a function of the fluid properties, which may to a first-order approximation be assumed to be the same function for all engines, \( h \) is a function of \( \theta \), say \( (\theta)^\beta \), where \( \beta \approx 0.3 \), and \( \epsilon \) is a similar function of \( \epsilon_1 \theta \), that is \( (\epsilon_1 \theta)^\gamma \), and both are functions of \( N \), the number of blades in a row of blades, and \( N_r \), the number of rows of blades being cooled. The surface areas \( A_\text{c} \) and \( A_g \) are both functions of \( N \) and \( N_r \). Thus, for a given engine,

\[ T_6 - T_7 = \frac{(T_4 + T_5)/2}{\gamma \left( \frac{T_4 + T_5}{2} \right)} - \frac{2}{1 + b\epsilon_1/\sqrt{N}} \]

(23)

Similarly,

\[ \frac{T_{\text{m}} - T_6}{(T_4 + T_5)/2 - T_6} = 1 + b\left( \frac{\epsilon_1}{N} \right)^{1/2} \]

\[ = \frac{T_7 - T_6}{(T_4 + T_5)/2 - T_6} \]

(24)

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where $b$ and $c$ are functions of fluid properties, gas mass flow rate and blade dimensions.

Preheating of the coolant to $T_0$ or $T_{0p}$ before it begins to cool the turbine components can be taken into account by defining a temperature difference ratio $A$, which for the compressor turbine is given by

$$A = \frac{T_2 - T_s}{T_7 - T_2}$$

(25)

Thus it can be shown that

$$T_6 = (1 - A)T_1 + AT_2$$

(26)

and with equation (23) that

$$T_7 = \frac{2T_{bm} + AT_2(b\epsilon_1/NN)^{0.2} - 1)}{2 + A(b\epsilon_1/NN)^{0.2} - 1}$$

(27)

$$T_4 = T_{bm} + \frac{(bc/N)^{0.5} \epsilon_1/N}(T_{bm} - T_2)}{2 - A - 1 + b(\epsilon_1/NN)^{0.2}}$$

(28)

Clearly, less coolant is required as $A$ increases towards unity.

Having obtained sufficient equations to determine all relevant temperatures for the compressor turbine, the power turbine is next considered, for which an amount of coolant $\epsilon_2\dot{m}_k$ is bled at the end of compression, or $\epsilon_2\dot{m}_k$ for some intermediate compression stage; thus the total coolant is either $(\epsilon_1 + \epsilon_2)\dot{m}_k$ or $(\epsilon_1 + \epsilon_2)\dot{m}_k$. Proceeding just as before for the gas generator turbine, first to determine $T_{sp}$ from the mixing energy balance, then

$$T_{sp} = \frac{C_{sp} T_4 + \epsilon_2}{C_{sp}} \left[ \frac{C_{sp} T_2p - C_{sp} T_4 p}{(1 - \sum \epsilon)(1 + f) + \epsilon_2} \right]$$

(29)

The power turbine entry temperature $T_8$ is already known, and, as for the compressor turbine,

$$T_{8p} = (1 - A)T_8 + AT_{8p}$$

(30)

with similar equations to (27) and (28) for $T_{8p}$ and $T_{4p}$.

In order to complete the above system of equations, equation (1), as well as the relationships between temperature and pressure, and introducing the component isentropic efficiencies, were used as follows:

$$T_8 = \frac{C_8}{C_4} \left[ \frac{C_8 - \eta_{sp} \left( C_8 - \frac{1}{T_1 (SWO)} \right)(C_8 - C_4)}{T_1 - \eta_{sp} \left( C_8 - \frac{1}{T_1 (SWO)} \right)(C_8 - C_4)} \right]$$

(31)

$$T_{8p} = \frac{C_{8p}}{C_{4p}} \left[ \frac{C_{8p} - \eta_{sp} \left( C_{8p} - \frac{1}{T_1 (SWO)} \right)(C_{8p} - C_{4p})}{T_1 - \eta_{sp} \left( C_{8p} - \frac{1}{T_1 (SWO)} \right)(C_{8p} - C_{4p})} \right]$$

(32)

where $T_1 = T_{T1}/T_1$. In order to compute the pressure ratios for the gas generator turbine, the following relationships were used:

$$r_{34} = \frac{1}{((C_3/C_4)(1 - (1/\eta_{34}) + (T_{04}/T_3 \eta_{34}))^{(C_3 - C_4)/0.574})}$$

(33)

$$r_{58} = \frac{1}{((C_5/C_8)(1 - (1/\eta_{58}) + (T_{08}/T_5 \eta_{58}))^{(C_5 - C_8)/0.574})}$$

(34)

The pressure ratio $r_{99}$ across the power turbine is assumed to be related to these gas generator pressure ratios by

$$r_{99} = \frac{r_{12}}{r_{34} r_{58}}$$

(35)

that is it is assumed there are no losses of pressure in connecting ducts, combustion chambers or in mixing processes. The pressure ratio $r_{8+}$ between the generator turbine exit and part-way through the power turbine is related to the component efficiency $\eta_{8+}$ by

$$r_{8+} = \frac{1}{((C_5/C_4)[1 - (1/\eta_{8+})] + (T_{04}/T_6 \eta_{8+}))^{(C_5 - C_4)/0.574}}$$

(36)

Similarly,

$$r_{5+} = \frac{1}{((C_5/C_4)[1 - (1/\eta_{5+})] + (T_{04}/T_5 \eta_{5+}))^{(C_5 - C_4)/0.574}}$$

(37)

from which the temperature at the outlet of the power turbine is related to the pressure ratio $r_{5+}$ by

$$T_p = r_{5+} T_{5p} C_{5p}$$

$$\times \left[ 1 - \frac{1}{1 - \eta_{5+} \left( 1 - C_9 \left( \frac{1}{T_{5+} \eta_{5+}} \right)^{0.574} \right) (C_5 - C_4)} \right]$$

(38)

If $W$ is the work output for the mass flow rate $\dot{m}$ through the engine then the specific work output (SWO) is given in dimensionless form as

$$SWO = \frac{(1 - \sum \epsilon)(1 + f) + \epsilon_1[C_6 T_6 - C_{4p} T_{4p})]}{C_1 T_1} + \frac{(1 - \sum \epsilon)(1 + f) + \sum \epsilon[C_5 T_{5p} + C_9 T_9 - C_1 T_1]}{C_1 T_1}$$

(39)

The overall engine efficiency based on heat rejected is calculated from

$$\eta_w = \frac{1 - \frac{(1 - \sum \epsilon)(1 + f) + \sum \epsilon[C_9 T_9 - C_1 T_1]}{1 - \sum \epsilon}}{1 - \sum \epsilon f}$$

(40)

and that based on the work output by

$$\eta_w = \frac{C_1 T_1 (SWO)}{1 - \sum \epsilon}$$

(41)

As previously stated, a computer program has been written to accommodate an iterative procedure for solving the above equations. It consists basically of five sub-routines, namely the air properties, gas properties, gas generator turbine, power turbine and efficiency sub-routines. Account is taken of temperature effects on properties and also the effects of the constituent combustion products.
Briefly, the computer sub-routine for the compressor turbine initially involves a first estimate for \( T_4 \) from heat-transfer data, and hence \( T_{3u} \), \( T_3 \) and \( T_{8u} \), from the energy balances. From the coolant temperature rise, \( T_3 \) and \( T_4 \) can be calculated and \( T_3 \) and \( T_8 \) redetermined. The sub-routine for the power turbine with no pre-bleeding first determines \( T_{sp} \) based on \( T_8 \) and \( r_{99} \) and zero coolant. If it is required, \( T_{sp} \) is found from the heat-transfer data, and also \( T_{6p} \) and \( T_{pp} \). All gas generator temperatures, pressures and values of \( f \) are then redetermined for the required \( e_2 \) and \( T_{6p} \), followed by redetermination of \( T_{sp} \) based on \( T_8 \) and \( r_{99} \). By accounting for the increase in coolant temperature from \( T_{sp} \) to \( T_{pp} \) and \( T_{pp} \) can be computed, and finally \( T_{5p} \) and \( T_5 \), prior to evaluating work output and thermal efficiency.

All quantities were calculated within a chosen error criterion of the form \( \epsilon_p = \left( \frac{T_{sp} - T_{tp}}{T_{sp}} \right)^2 \) ranging from \( 10^{-4} \) to \( 10^{-10} \), where \( T_{tp} \) is the value of \( T_{sp} \) at the previous iteration. Values of 100, 2, 5.04 and 615.9 were assigned to \( N, N_r, b \) and \( c \) respectively, and, as already stated, full account is taken of changes in fluid properties values arising from changes in constituents and temperature and the reduction in the effective air–fuel ratio due to mixing. The values of \( N \) and \( N_r \) are clearly reasonable and self-justifying; however, a little explanation is necessary for the choice of values for \( b \) and \( c \). It was assumed that the blades were forced convection cooled and that the heat transfer in the blade cooling passages was fully turbulent and described by the Dittus–Boelter equation. In the case of heat transfer between the hot gases and the blade external surfaces, it was assumed that the boundary-layer flow was turbulent over the full extent of the surfaces, and averaged values of heat-transfer coefficients were used based on the McAdams equation. Fairly typical values of blade geometries are used representative of high-pressure (HP) and low-pressure (LP) turbines.

All the computations are for ambient conditions of 1 bar and \( T_2 = 288 \) K, and for the values assigned above to the turbine parameters \( N, N_r, b \) and \( c \), together with the reference fuel with properties as specified in the introduction. In addition, cooling air is provided to the extent necessary to prevent the mean blade temperature \( T_{bm} \) exceeding 1125 K in either turbine, whatever the maximum cycle temperature. At its lower values, with \( T_{bm} \) below 1125 K, no coolant is required and \( \epsilon \) is zero.

Each computer run also involves the specification of the overall pressure ratio \( r_{12} \) at one of six values between 12 and 32, the compressor and turbine isentropic efficiencies as conventionally defined above, each for a value between 0.8 and 0.95 for the turbines, and up to unity for the compressor, but with emphasis on representative values of 0.85 for the compressor and 0.88 for the turbines, and the heat extraction factor \( A \), either 0.25 or 0.5. Given these constraints, the performance of the arrangement may be computed as outlined above.

### 3 RESULTS AND DISCUSSION OF RESULTS

The computer program was run initially to investigate the effect of the two methods used to calculate the thermal efficiency, namely the heat rejected approach required by equation (40) and the work-done approach represented by equation (41). Figure 2 shows the variations of the efficiencies against the total amount of coolant when the compressor ratio is 16 and the effectiveness cooling parameter \( A \) is 0.5. While the curves clearly demonstrate the existence of an optimum (and relatively quite small) percentage of coolant for maximum thermal efficiency, that calculated using a heat-rejected approach is clearly less than that obtained from the work-done approach, except for very small amounts of coolant. The difference may reasonably be ascribed to the approximate technique used in the calculation of the work output. The effect of changing the error criterion \( \epsilon_r = \left( \frac{T_{sp} - T_{tp}}{T_{sp}} \right)^2 \) on the prediction was examined; a typical example in Fig. 3 shows the overall engine efficiency plotted against coolant quantity for a compressor pressure ratio of 16 and \( A = 0.25 \) for \( \epsilon_r \) of \( 10^{-4} \) and \( 10^{-10} \). In the range investigated the change in error criterion has a negligible effect, and in what follows \( \eta_c \) rather than \( \eta_w \) is used and, as in Fig. 3, \( \eta_w \) has the same value in both turbines, except where specifically stated as otherwise.

Having regard to the assumption in the cycle analysis of no pressure losses, the next part of the program was designed largely to compare the effects on performance of all coolant withdrawn at the compressor exit with that of pre-bleeding of coolant for the power turbine at position 2p, where \( P_{2p} = P_{ap} \), so that the pressure difference between the positions is minimized. For these

![Fig. 2 Variations of the efficiencies versus \( \Sigma \epsilon \) using heat-rejected approach (\( \eta_c \)) and work-done approach (\( \eta_w \)) for a compressor pressure ratio of 16 and \( A = 0.5 \)](image)
two cases, Figs 4 and 5 show the variations in thermal efficiency with coolant quantity and the variation of dimensionless specific work output with generator turbine entry temperature $T_3$ for $A$, the other factor to be considered, of 0.25 and 0.5 respectively, at a pressure ratio $r_{12}$ of 16. A compressor efficiency of 0.85 and turbine efficiencies of 0.88 have been assumed.

It is clear from Fig. 4 that the pre-bled case gives slightly better thermal efficiency up to the optimum amount of coolant, which is 3 per cent, after which the reverse is true. This is partly mirrored in the corresponding curve of SWO versus $T_3$, the near linearity of which is a characteristic feature of the entire investigation. Though for the optimum coolant of 1–3 per cent in Fig. 5, $A = 0.5$ yields a greater $\eta_w$ (approaching 0.42) than $A = 0.25$ in Fig. 4, the effect on thermal efficiency of pre-bleeding is more significant than compressor exit bleeding for $A = 0.25$, which is to be expected as less heat is then transferred to the coolant before entry to the turbine components.

Figures 6 and 7 repeat the predictions in Figs 4 and 5 for the higher pressure ratio of 32. In this case the effect of pre-bleeding on $\eta_w$ is almost negligible for coolant quantities below the optimum, which falls from over 7 per cent for $A = 0.25$ to between 2 and 3 per cent for $A = 0.5$, with a corresponding further improvement in $\eta_w$ to nearly 0.48. In fact, coolant bled from the compressor exit gives the greater $\eta_w$ for $A = 0.5$, though for greater optimum coolant than the pre-bled case, which always results in a sudden drop in $\eta_w$ for coolant flows above the optimum. Thus while the thermodynamic advantages of pre-bleeding over the alternative are by no means decisive, the benefits of high $A$ and low coolant delivery temperatures are clear, both in terms of lower coolant requirements and greater thermal efficiency, as noted by Hay and Taylor (9).

Comparison of Figs 4 to 7 illustrates the comparative insensitivity of SWO to changes in $A$, $r_{12}$ or whether the...
Fig. 5 The effects of coolant pre-bleeding on $\eta_w$ and SWO for compressor pressure ratio of 16 and $A = 0.5$

Fig. 6 The effects of coolant pre-bleeding on $\eta_w$ and SWO for compressor pressure ratio of 32 and $A = 0.25$
coolant to the power turbine is pre-bled or compressor-exit bled. For instance, at \( T_3 = 1800 \text{ K} \), the value of SWO differs little from 2.5. In fact, the relation is quite well represented by the equation

\[
\text{SWO} = \frac{T_3}{400} - 2
\]  

(42)

where \( T_3 \) is in degrees kelvin. These findings tend to support Byerley's (7) analogous computations of specific thrust for an aircraft gas turbine at a flight condition of Mach 0.7 and a temperature corresponding to the standard altitude at 8840 m. Over the same range of compressor pressure ratios from 12 to 32, his specific thrust curves become increasingly flat with an increase in coolant quantity. For example, at a turbine entry temperature of 1772 K and 8 per cent coolant, the specific thrust varies by less than 3 per cent, irrespective of whether the film coolant is assumed to be added to the core flow upstream or downstream of the rotor.

Having indicated how \( \eta_w \) varies with coolant quantity for \( r_{12} \) of 16 and 32, Figs 8, 9 and 10 illustrate optimum conditions and maximum thermal efficiency for the six values of \( r_{12} \) investigated, the scales masking the slight differences between compressor-exit bleeding and the pre-bled case. As before, compressor and turbine efficiencies are 0.85 and 0.88 respectively. Figure 8 shows the optimum total coolant to be an essentially linear function of the overall pressure ratio for both \( A = 0.25 \) and \( A = 0.5 \). The optimum coolant for the latter value is, as expected, always less than the former, but by a factor near 4 at \( r_{12} = 32 \), again emphasizing the importance of a low coolant delivery temperature; contemporary knowledge of the amount of compressed air required as coolant for various pressure ratios suggests

Fig. 7 The effects of coolant pre-bleeding on \( \eta_w \) and SWO for compressor pressure ratio of 32 and \( A = 0.5 \)

Fig. 8 The effect of compressor pressure ratio on the optimum amount of coolant for \( A = 0.25 \) and 0.5
that $A = 0.5$ is a target to be aimed at rather than one already achieved.

Figure 9 shows the corresponding curves for maximum thermal efficiency. These increase non-linearly from around 39 per cent at $r_{12} = 12$ to approximately 47 per cent at $r_{12} = 32$, with the larger $A$ yielding the greater thermal efficiency, though perhaps surprisingly, by less than 2 per cent, in view of the large differences in optimum coolant. Increases in optimum total coolant with $T_3$ are depicted in Fig. 10 for the same two values of $A$, the rates of increase being greater for $A = 0.25$. Again as expected, the less the coolant is preheated, the smaller the amount needed to sustain a given $T_3$ over the entire range of $r_{12}$ examined. That $T_3$ does not exceed about 1775 K helps validate the introductory statement that dissociation effects on heat release can be disregarded.

The effects of component isentropic efficiencies on cycle performance are now considered. The influence of compressor efficiency on overall engine thermal efficiency, and the optimum amount of cooling for $r_{12} = 16$ and turbine efficiencies of 0.88, can be studied by comparing Figs 4 and 5 and 11 to 14, where the compressor efficiency varies from 0.8 to unity, for both $A$ values, and compressor exit and pre-bleeding. As the compressor efficiency approaches unity, the difference in thermal efficiency for these two cases becomes vanishingly small for all coolant flows below the optimum. This is especially true of $A = 0.5$. Only as the compressor efficiency falls from 0.85 to 0.8 for $A$ of 0.25 does the pre-bleed case give a significant increase in thermal efficiency over compressor exit bleeding at the optimum coolant value. Otherwise, for both $A$ values, increases in compressor efficiency always lead, as anticipated, to reductions in optimum coolant and increases in thermal efficiency. Values around 0.45 could be achieved for $r_{12}$ of 16 were compressor efficiencies near unity ever to become attainable.

Figures 4 and 5 and 15 to 17 display the effects of varying the gas generator isentropic efficiency from 0.8 to 0.95 for $r_{12} = 16$, the compressor efficiency being 0.85 and the power turbine efficiency 0.88. The pre-bleed case still gives marginally better engine thermal efficiencies, up to the optimum coolant, but only for $A = 0.25$; for $A = 0.5$, predictions are identical. The common feature, for reasons not entirely clear, is that the optimum coolant remains at about 3 per cent for $A = 0.25$ and 1 per cent for $A = 0.5$, despite the variation in gas values.
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Fig. 11  The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{comp}} = 80$ per cent

Fig. 12  The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{comp}} = 90$ per cent
Fig. 13 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{comp} = 95$ per cent

(a) $A = 0.25$

(b) $A = 0.5$

Fig. 14 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{comp} = 100$ per cent

(a) $A = 0.25$

(b) $A = 0.5$
Fig. 15 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{generator turb}} = 80$ per cent.

Fig. 16 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{generator turb}} = 90$ per cent.
generator efficiency. However, the maximum thermal efficiency increases from around 0.39 to over 0.43 for $A = 0.25$, and from nearly 0.40 to almost 0.44 for $A = 0.5$.

The effects of varying the power turbine efficiency from 0.8 to 0.95 for $r_{39}$ of 16, a compressor efficiency of 0.85 and a generator turbine efficiency of 0.88, are shown in Figs 4 and 5 and 18 to 20. As before, pre-bleeding gives slightly better thermal efficiencies up to the optimum coolant, but only for $A = 0.25$. At $A = 0.5$, compressor exit bleeding gives the same thermal efficiency. However, not only does the optimum coolant remain the same at around 3 per cent for $A = 0.25$ and 1 per cent for $A = 0.5$, but the maximum thermal efficiency follows suit, at just over 0.41 for $A = 0.25$ and approaching 0.42 for $A = 0.5$, whatever the power turbine efficiency within the above range.

The following conclusions may therefore be drawn about the influence of component efficiencies on optimum coolant and maximum thermal efficiency. While an increase in $A$ benefits performance irrespective of which component is considered, an increase in compressor efficiency benefits both optimum coolant and maximum thermal efficiency; an increase in gas generator turbine efficiency improves only the latter, while an increase in power turbine efficiency affects neither, at least over the ranges examined. These conclusions would seem to point the direction in which improvements to component efficiencies should be concentrated.

Figure 21 shows the predicted overall thermal efficiency against the amount of coolant for a compressor pressure ratio of 16 and isentropic efficiency of 0.85, both turbine isentropic efficiencies of 0.88 and $A = 0.25$, when average fluid properties are used, compared to that when properties are taken as functions of temperature. The temperature dependence of fluid properties evidently has a significant effect on the calculations, the use of average properties underpredicting engine performance by 10 per cent or more in this case, irrespective of the amount of coolant. Moreover, using average properties does not indicate the existence of optimum coolant conditions for maximum engine thermal efficiency.

Throughout the analysis pressure losses have been ignored. Pressure losses will occur in the mixing processes, ducting and in combustion. Combustion pressure losses will be the most significant, being of the order of 5–7 per cent of compressor delivery pressure. The authors are of the opinion that the effects of pressure losses will be to reduce the values of specific work output and engine efficiency, but the general conclusions regarding the effects of component efficiency, pressure ratio and heat extraction factor will be unchanged. For example, the effect of combustion pressure losses will be to reduce the pressure at entry to the gas generator turbine and, therefore, the pressure ratio $r_{39}$ will be reduced with a consequential reduction in specific work output and engine efficiency. Note that equation (35) is based on zero pressure losses and equates $r_{12}$ to $r_{39}$.

Having regard to the nature and extent of the assumptions made in this analysis, it is encouraging that predictions of the percentage of coolant required for
Fig. 18 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{power turb}} = 80$ per cent

Fig. 19 The effects of coolant pre-bleeding on $\eta_w$ for compressor pressure ratio of 16 and $\eta_{\text{power turb}} = 90$ per cent
maximum engine efficiency appear to be representative of current practice for the range of compressor pressure ratios examined. The findings also illustrate the sometimes severe performance penalties that can result from comparatively minor departures from optimum coolant conditions. The effects of changes in component isentropic efficiencies in general also accord with what might be expected. It is hoped that where direct application or interpretation of present results is neither possible nor appropriate, the approaches and procedures reported herein will nevertheless be found useful in investigating other configurations and arrangements.

4 CONCLUSIONS

An iterative procedure has been developed to optimize the amount of cooling air required for high-temperature gas turbine blades, if maximum thermal efficiency is to be achieved, in a system of a gas generator feeding a free power turbine. The analysis covers the cases where coolant is extracted both during and after compression, with due allowance for preheating before it reaches the coolant passages in the blades, prior to mixing with mainstream gas.

Over extensive ranges of compressor pressure ratios and component isentropic efficiencies, with full account taken of temperature-dependent fluid property values and changes in composition and air-fuel ratio due to combustion and mixing, an optimum coolant flow is predicted, up to which the advantage of using pre-bled air is generally slight and beyond which it can become markedly negative. Optimum coolant flow increases with overall pressure ratio and higher generator turbine entry temperature, to yield greater engine thermal efficiency, but less coolant is required if preheating is minimized. The specific work output is a nearly linear function of maximum cycle temperature. An increase in any component efficiency generally has a beneficial
Effect on engine performance, though the improvement is sometimes small or even negligible.

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