Structural Outlooks for the OTIS-Arrangement Network

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ABSTRACT

Recent studies have revealed that the Optical Transpose Interconnection Systems (OTIS) are promising candidates for future high-performance parallel computers. This paper presents and evaluates a general method for algorithm development on the OTIS-Arrangement network (OTIS-AN) as an example of OTIS network. The proposed method can be used and customized for any other OTIS network. Furthermore, it allows efficient mapping of a wide class of algorithms into the OTIS-AN. This method is based on grids and pipelines as popular structures that support a vast body of parallel applications including linear algebra, divide-and-conquer types of algorithms, sorting, and FFT computation. This study confirms the viability of the OTIS-AN as an attractive alternative for large-scale parallel architectures.

Keywords: Arrangement-Star Network, Grid structure, Interconnection Networks, OTIS, Pipeline Structure

INTRODUCTION

The choice of network topology for parallel systems is a critical design decision that involves inherent trade-offs in terms of efficient algorithms support and network implementation cost. For instance, networks with large bisection width allow fast and reliable communication. However, such networks are difficult to implement using today’s electronic technologies that are two dimensional in nature (Wang & Sahni, 2002). In principle, free-space optical technologies offer several fronts to improve this trade-off. The improved transmission rates, dense interconnects, power consumption, and signal interference are few examples on these fronts (Agelis, 2005; Akers et al., 1977; Dally, 1988; Day & Tripathi, 1990; Hendrick et al., 1959; Wang & Sahni, 2001; Yayla et al., 1998).

In this paper, we focus on Optical Transpose Interconnection Systems Arrangement Networks-(OTIS-AN) which was proposed by Al-Sadi that can be easily implemented using free-space optoelectronic technologies (Agelis, 2005; Al-Sadi & Awwad, 2010). In this model, processors are partitioned into groups, where each group is realized on a separate chip with electronic inter-processor connects. Processors
on separate chips are interconnected through free space interconnects. The philosophy behind this separation is to utilize the benefits of both the optical and the electronic technologies.

The advantage of using OTIS as optoelectronic architecture lies in its ability to maneuver the fact that free space optical communication is superior in terms of speed and power consumption when the connection distance is more than few millimeters (Dally, 1988). In the OTIS-AN, shorter (intra-chip) communication is realized by electronic interconnects while longer (inter-chip) communication is realized by free space interconnects.

Extensive modeling results for the OTIS have been reported in (Day & Tripathi, 2002). The achievable Terra bit throughput at a reasonable cost makes the OTIS-AN a strong competitive to the to its factor network (Dally, 1988; Krishnamoorthy et al., 1992; Marsden et al., 1993).

These encouraging findings prompt the need for further testing of the suitability of the OTIS-AN for real-life applications. A number of recent studies have been conducted in this direction (Al-Sadi, 2004; Awwad & Al-Ayyoub, 2001; Chatterjee & Pawlowski, 1999; Day & Al-Atiyoub, 2002). Awwad (2005) have presented and evaluated various algorithms on OTIS-networks such as basic data rearrangements, routing, selection and sorting. They have also developed algorithms for various matrix multiplication operations and image processing (Sahni & Wang, 1997; Wang & Sahni, 2000). Zane et al. (2000) have shown that the OTIS-mesh efficiently embeds four-dimensional meshes and hypercubes.

Aside from the above mentioned works, the study of algorithms on the OTIS is yet to mature (Sahni, 1999). In this paper we contribute towards filling this gap by presenting a method for developing algorithms on the OTIS-AN. These methods is based on grid and pipeline as popular a structure that supports a vast body of applications ranging from linear algebra to divide-and-conquer type of algorithms, sorting, and FFT computation. The proposed methods are discussed in the sequel, but first we give the necessary definitions and notation.

**PRELIMINARY NOTATIONS AND DEFINITIONS**

Let \( n \) and \( k \) be two integers satisfying \( 1 \leq k \leq n-1 \) and let us denote \(<n> = \{1, 2, ..., n \} \) and \(<k> = \{1, 2, ..., k \} \). Let \( P_1^n \) taken \( k \) at a time, the set of arrangements of \( k \) elements out of the \( n \) elements of \(<n>\). The \( k \) elements of an arrangements \( p \) are denoted \( p_1, p_2, ..., p_k \).

**Definition 1 (Arrangement Graph):**

The \((n,k)-arrangement graph\) \( A_{n,k} = (V, E) \) is an undirected graph given by:

\[
V = \{ p, p_2, ..., p_i \mid p_i \text{ in } <n> \text{ and } p_i \neq p_j \text{ for } i \neq j \} = P_k^n,
\]

and

\[
E = \{(p,q) \mid p \text{ and } q \text{ in } V \text{ and for some } i \text{ in } <k>, p_i \neq q_i \text{ and } p_j = q_j \text{ for } j \neq i \}
\]

That is the nodes of \( A_{n,k} \) are the arrangements of \( k \) elements out of \( n \) elements of \(<n>\), and the edges of \( A_{n,k} \) connect arrangements which differ exactly in one of their \( k \) positions. For example in \( A_{23} \), the node \( p=23 \) is connected to the nodes \( 21, 24, 25, 13, 43, \) and \( 53 \). An edge of \( A_{n,k} \) connecting two arrangements \( p \) and \( q \) which differ only in one position \( i \), it is called \( i \)-edge. In this case, \( p \) and \( q \) is called the \((i,q)\)-neighbour of \( p \). \( A_{n,k} \) is therefore a regular graph with degree \( k(n-k) \) and \( n!(n-k)! \) nodes. As an example of this network Figure 1 shows \( A_{4,3} \) arrangement with size of 12 nodes and a symmetric degree of 4.

Since OTIS-networks are basically constructed by “multiplying” a known topology by itself. The set of vertices is equal to the Cartesian product on the set of vertices in the factor network. The set of edges consists of
edges from the factor network and new edges called the transpose edges. The formal definition of OTIS-networks is given below.

**Definition 2 (OTIS-Network):**

Let $G_0 = (V_0, E_0)$ be an undirected graph representing a factor network. The OTIS-$G_0 = (V, E)$ network is represented by an undirected graph obtained from $G_0$ as follows $V = \{ \langle x, y \rangle | x, y \in V_0 \}$ and $E = \{ (\langle x, y \rangle, \langle x, z \rangle) \mid if (y, z) \in E_0 \} \cup \{ (\langle x, y \rangle, \langle y, x \rangle) \mid x, y \in V_0 and x \neq y \}$.

The set of edges $E$ in the above definition consists of two subsets, one is from $G_0$, called $G_0$-type edges, and the other subset contains the transpose edges. The OTIS-AN approach suggests implementing Arrangement-type edges by electronic links since they involve intra-chip short links and implementing transpose edges by free space optics. Throughout this paper the terms “electronic move” and the “OTIS move” (or “optical move”) will be used to refer to data transmission based on electronic and optical technologies, respectively.

**Definition 3 (Cross Product):**

The cross product $G = G_1 \otimes G_2$ of two undirected connected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the undirected Graph $G = (V, E)$, where $V$ and $E$ are given by:

$$V = \{ \langle x_1, y \rangle \mid x_1 \in V_1 and y \in V_2 \} and$$

$$E = \{ (\langle x_1, y \rangle, \langle y_1, y \rangle) \mid (x_1, y_1) \in E_1 \} \cup \{ (\langle x, x_2 \rangle, \langle x, y_2 \rangle) \mid (x_2, y_2) \in E_2 \}.$$

So for any $u = \langle x_1, x_2 \rangle$ and $v = \langle y_1, y_2 \rangle$ in $V$, $(u, v)$ is an edge in $E$ if, and only, if either $(x_1, y_1)$ is an edge in $E_1$ and $x_2 = y_2$, or $(x_2, y_2)$ is an edge in $E_2$ and $x_1 = y_1$. The edge $(u, v)$ is called a $G_1$-edge if $(x_1, y_1)$ is an edge in $E_1$, and it is called $G_2$-edge if $(x_2, y_2)$ is an edge in $E_2$. The size, degree, diameter and number of links of the cross product of two networks are defined next.

**Definition 4 (Topological properties of cross product networks):**

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Figure 1. The arrangement graph $A_{4,2}$
If $G_1$ and $G_2$ are two undirected connected graphs of respective size $s_1$ and $s_2$ and have respective diameters $\delta_1$ and $\delta_2$, then (Day & Al-Ayyoub, 2002):

1. $G_1 \otimes G_2$ connected.
2. The diameter $\delta$ of $G_1 \otimes G_2$ is $\delta = \delta_1 + \delta_2$.
3. The size $s$ of $G_1 \otimes G_2$ is given by $s = s_1 \cdot s_2$.
4. The degree of a node $u = \langle x_1, x_2 \rangle$ in $G_1 \otimes G_2$ is equal to the sum of the degrees of vertices $x_1$ and $x_2$ in $G_1$ and $G_2$, respectively.
5. Number of links for the product network, is given by $(\text{size} \cdot \text{degree})/2$.

**TOPOLOGICAL PROPERTIES OF OTIS-AN**

This section reviews some of the basic topological properties of the OTIS-Arrangement network including size, degree, diameter, number of links, and shortest distance between 2 nodes (Day & Tripathi, 1992; Al-Sadi & Awad, 2010).

The topological properties of the OTIS-Arrangement network along with those of the Arrangement network are discussed below.

We will refer to $g$ as the group address and $p$ as the processor address. An intergroup edge of the form $(\langle g, p \rangle, \langle p, g \rangle)$ represents an optical link and will be referred to as OTIS or optical move. Note that also we will be using the following notations are defined:

- $|A_{n,k}| = \text{size of the graph } A_{n,k}$.
- $|\text{OTIS- } A_{n,k}| = \text{size of the graph OTIS- } A_{n,k}$.
- Deg. $A_{n,k}(p)$ = Degree of the graph $A_{n,k}$ at node $p$.
- Deg. OTIS- $A_{n,k}(g,p)$ = Degree of the graph OTIS- $A_{n,k}$ at node address $\langle g, p \rangle$.
- Dist-$A_{n,k}(p_1, p_2)$ = The length of a shortest path between the two nodes $p_1$ and $p_2$ in Arrangement graph.
- Dist. OTIS- $A_{n,k}(p_1, p_2)$ = The length of a shortest path between the two nodes $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ in OTIS-Arrangement.

In the OTIS-Arrangement the notation $\langle g, p \rangle$ is used to refer to the group and processor addresses respectively. Figure 2 shows that as an example of OTIS-$A_{n,k}$. The figure shows that two nodes $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ are connected if and only if $g_1 = g_2$ and $(p_1, p_2) \in E_0$ (such that $E_0$ is the set of edges in Arrangement network) or $g_1 = p_1$ and $g_2 = p_2$, in this case the two nodes are connected by transpose edge. The distance in the OTIS-Arrangement is defined as the shortest path between any two processors, $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$, and involves one of the following forms:

1. When $g_1 = g_2$, then the path involves only electronic moves from source node to destination node.
2. When $g_1 \neq g_2$, and if the number of optical moves is an even number of moves and more than two, then the paths can be compressed into a shorter path of the form:

$$\langle g_1, p_1 \rangle \xrightarrow{E} \langle g_1, p_2 \rangle \xrightarrow{O} \langle p_2, g_2 \rangle$$

Then the size, the degree, the diameter, number of links, and the shortest distance of OTIS-Arrangement network are as follows:
• Size of $|\text{OTIS-}A_{n,k}| = \frac{n!}{(n-k)!^2}$.
• Degree of $\text{OTIS-}A_{n,k} = \text{Deg}(A_{n,k})$, if $g = p$.
• Deg. $(A_{n,k}) + 1$, if $g \neq p$.
• Diameter of $\text{OTIS-}A_{n,k} = 2 \left\lfloor\frac{1.5k}{2}\right\rfloor + 1$.
• Number of Links: Let $N_o$ be the number of links in the $A_{n,k}$ and let $M$ be the number of nodes in the $A_{n,k}$. The number of links in the $\text{OTIS-}A_{n,k}$ consisting of 144 processors is $= \frac{(12^2 - 12)}{2} + \frac{23^2}{2} = 595$
• Dist. of $\text{OTIS-}A_{n,k}$ = 
  \[
  \begin{cases} 
    \min(d(p_1, g_2) + d(g_1, p_2) + d(g_1, g_2) + 2) & \text{if } g_1 \neq g_2 \\
    d(p_1, p_2) & \text{if } g_1 = g_2 
  \end{cases}
  \]

Theorem 1:

The length of the shortest path between any two processors $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ in $\text{OTIS-}\text{Arrangement}$ is $d(p_1, p_2)$ when $g_1 = g_2$ and $\min\{d(p_1, p_2) + d(g_1, g_2) + 2, d(p_1, g_2) + d(g_1, p_2) + 1\}$ when $g_1 \neq g_2$, where $d(p, g)$ stands for the shortest distance between the two processors $p$ and $g$ using any of the possible shortest paths as seen in the above forms 1, 2 and 3 (Awad, 2005).

It is obvious from the above theorem that when $g_1 = g_2$, then the length of the path between the two processors $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ is $d(p_1, p_2)$. From the shortest path construction methods in (2) and (3) above, it can be easily verified that the length of the path equal $\min\{d(p_1, p_2) + d(g_1, g_2) + 2, d(p_1, g_2) + d(g_1, p_2) + 1\}$ when $g_1 \neq g_2$.

To send a message $M$ from the source node $\langle g_1, p_1 \rangle$ to the destination node $\langle g_2, p_2 \rangle$ it must follow a route along one of the three possible paths 1, 2, and 3. The length of the shortest path between the nodes $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ is one of the forms (as seen in Box 1).

Where $d(p_1, p_2)$ is the length of the shortest path between any two processors $\langle g_1, p_1 \rangle$.
and \( \langle g_1, p_2 \rangle \). If \( \delta_0 \) is the diameter of the fac-
tor network \( A_{n,k} \), then from (11) it follows that
the diameter of the OTIS-\( A_{n,k} \) is \( 2\delta_0 + 1 \). The
diameter of OTIS-\( A_{n,k} \) is the \( \text{Max} (\delta_0, 2\delta_0 + 1) \)
which is equal to \( 2\delta_0 + 1 \). The proof of the above
theorem is a direct result from (I).

GRID STRUCTURAL
OUTLOOK FOR
OTIS-ARRANGEMENT
NETWORK

In this section the hierarchical structure of the
OTIS-AN is discussed. The properties of a
new decomposition method for the OTIS-AN
presented and proved. These properties are
then used in the subsequent sections to develop
grids and pipelines as methods for developing
various parallel algorithms on the OTIS-AN.

An OTIS-AN based computer contains
\( N^2 \) processors partitioned into \( N \) groups with
\( N \) processors each. A processor is indexed by
a pair \( \langle x, y \rangle \), \( 0 \leq x, y < N \) where \( x \) is the group
index and \( y \) is the processor index. Processors
within a group are connected by a certain inter-
connecting topology; while inter-group links are
achieved by transposing group and processor
indexes (Sahni, 1999; Wang & Sahni, 2002).

The OTIS-AN constructed by “multiply-
ing” the arrangement factor topology by itself.
The vertex set is equal to the Cartesian
product on the vertex set in the arrangement network.
The edge set consists of edges from the ar-
rangement network and new edges called the
transpose edges.

The address of a node \( u = \langle x, y \rangle \) from
\( V \) is composed of two components: the first,
denoted by \( y(u) = x \), designates the group address
and the second, denoted by \( \rho(u) = y \), designates
the processor address within that group.

The network OTIS-AN can be decomposed
into \( |V_0| \) disjoint copies of \( AN \). This decomposi-
tion can be achieved by fixing the group address
and varying the processor address. Another
way of decomposing the OTIS-AN is by fix-
ing the processors \( address \) and varying the group
address. These two decomposition methods
are given below.

Definition 5 (row-subgraph):

Let \( \Psi_i \), for all \( i \in V_0 \), be the subgraph in-
duced by the set of nodes from \( V \) having the
form \( \langle i, x \rangle \) \( \forall x \in V_0 \).

Definition 6 (column-subgraph):

Let \( \Phi_j \), for all \( j \in V_0 \), be the subgraph in-
duced by the set of nodes from \( V \) having the
form \( \langle x, j \rangle \) for all \( x \in V_0 \).

Definition 7 (perfectly matching):

Let \( G_\Psi = (V_\Psi, E_\Psi) \) be the graph obtained
from OTIS-\( AN \) by clustering \( \Psi_i \) into a single
vertex labeled by \( i \) and having a link between
\( i \) and \( j \) if \( \Psi_i \) and \( \Psi_j \) share a perfect matching,
i.e. \( V_\Psi = V_0 \) and \( E_\Psi = \{ (i, j) \mid \Psi_i \text{ perfectly
matches } \Psi_j \} \).

\[
\text{Length} = \begin{cases} 
  d(p_1, p_2) & \text{if } g_1 = g_2 \\
  \min( d(p_1, g_2) + d(g_1, p_2) + 1, d(p_1, g_2) + d(g_1, g_2) + 2) & \text{otherwise}
\end{cases}
\]
Theorem 2:

The two $\Psi$ and $\Phi$ decomposition methods of the OTIS-AN$_0$ have the following properties:

1. $\Psi_i$ is isomorphic to $AN_0$.
2. $V_{\Psi_i} \cap V_{\Phi_j} = \{ \langle i, j \rangle \}$.
3. $\Psi_i$ and $\Phi_i$ share perfect matching for all $i$ values.
4. $\Psi_i$ and $\Psi_j$ share perfect matching for all $i$ and $j$ values and hence $G_{\Psi}$ is a complete graph. (Figure 3)

Proof:

Property 1 is a direct consequence of Definition 7. The function $\rho$ maps nodes from $V_{\Psi_i}$ to $V_0$. In fact, the set $\{\rho(u) \mid u \in \Psi_i\}$ is equal to $V_0$ for any $i$. Since any two neighboring nodes $u$ and $v$ in $\Psi_i$ should have $\gamma(u) = \gamma(v)$ and since $(\rho(u), \rho(v))$ is an edge in $E_0$; the subgraph $\Psi_i$ is isomorphic to $AN_0$.

Property 2 states that for any two labels $i$ and $j$ from $V_0$, the two subgraphs $\Psi_i$ and $\Phi_j$ have exactly one node in common. Since, $V_{\Psi_i} = \{ \langle i, x \rangle \mid x \in V_0 \}$ and $V_{\Phi_j} = \{ \langle x, j \rangle \mid x \in V_0 \}$, the intersection $V_{\Psi_i} \cap V_{\Phi_j}$ contains only the node $\langle i, j \rangle$.

Let $f_i: V_{\Psi_i} \rightarrow V_{\Phi_j}$ be a function that maps nodes form $\Psi_i$ into $\Phi_j$ for all $i$ values defined as follows: $f_i(\langle x, y \rangle) = \langle y, x \rangle$. First we have $|V_{\Psi_i}| = |V_{\Phi_j}|$ for all $i$ and $j$. For any two distinct nodes $u$ and $v$ in $V_{\Psi_i}$ we have $f_i(\langle \gamma(u), \rho(u) \rangle) = \langle \rho(u), \gamma(u) \rangle$, because $\rho(u) \neq \rho(v)$. Hence the function $f_i$ is on-to-one and onto. Thus property 3 follows.

Let $t_i: V_{\Psi_i} \rightarrow V_{\Phi_j}$ be a function that maps nodes form $\Psi_i$ into $\Psi_j$, for any $i$ and $j$, as fol-
follows: \( t_p(\langle i, x \rangle) = \langle j, x \rangle \). For any two distinct nodes \( u \) and \( v \) from \( V_\Psi \), we have \( t_p(\langle i, \rho(u) \rangle) = \langle j, \rho(u) \rangle \neq t_p(\langle i, \rho(v) \rangle) = \langle j, \rho(v) \rangle \). Since \( |V_\Psi| = |V_\pi| \) it follows that \( \Psi \) and \( \Psi \) share perfect matching for all \( i \) and \( j \) values and hence \( G_\Psi \) is a complete graph.

**Lemma 1:** \( G_\Psi \) can be embedded into OTIS-\( AN_0 \) with dilation \( \delta_{AN0} + 2 \).

**Proof:**

Since \( G_\Psi \) is complete, any two distinct nodes \( i \) and \( j \) in \( V_\Psi \) are neighbors. The "virtual" path between \( \langle i, x \rangle \) and \( \langle j, x \rangle \) in OTIS-\( AN_0 \) that corresponds to the edge \((i, j)\) in \( E_\Psi \) is constructed as follows: \( \langle i, x \rangle \rightarrow \langle x, i \rangle \parallel \pi_{\Psi0}(i, j) \parallel \langle x, j \rangle \rightarrow \langle j, x \rangle \). An arrow represents an edge connecting the two nodes and the operation "\( \parallel \)" means appending two paths (i.e., connecting the last node in the left path to first node in the right path). Notice that the choice of \( x \) from \( V_\rho \) does not affect the construction of this path nor its length. The path segment \( \pi_{\Psi0}(i, j) \) is an isomorphic copy to the optimal length path from \( i \) to \( j \) in \( AN_0 \). It can be verified that the above constructed path is of optimal length equal to \( d_{AN0}(i, j)+2 \). Hence, the longest such path cannot exceed \( \delta_{AN0} + 2 \).

**PIPELINE STRUCTURE FOR THE OTIS-ARRANGEMENT NETWORK**

The structural outlooks are based on grid and pipeline views as popular structures that support a vast body of applications that are encountered in many areas of science and engineering, including matrix computation, divide-and-conquer type of algorithms, sorting, and Fourier transforms. The proposed structural outlooks are applied to the OTIS, notably the OTIS-AN network.

The pipeline structure is a well-known structural outlook that is suitable for real applications. It is known from the literature that the Arrangement graph can be structured according to a pipelined view where the OTIS-AN can be arranged as a sequence of \( (n) \)-Arrangement forming an \( n \)-stage pipeline. However, the structural outlook based on the pipeline view for the Arrangement graph is insufficient as it generates a large number of nodes in each pipeline stage (Saika & Sen, 1995, 1996).

The OTIS-AN graph possesses a structural outlook that provides a pipelined structure in a more balanced manner. From the literature (Day & Tripathi, 1992; Jwo et al., 1991), we know that the arrangement graph is Hamiltonian. Therefore, the pipeline structure can be issued for the OTIS-AN graph in: by the graph \( A_{\Psi,k} \) and the Hamiltonian path OTIS or vice versa (Day & Al-Ayyoub, 1992). We have full control over the number of stages and the size of each stage by tuning the parameters \( n, m, k \).

The broadcasting across stages is the cost in one stage plus the cost of shifting the data to the next stage for the pipeline structure as a performance measure. The results will be reported for different network sizes and a fixed message of length \( M = 1024 \) byte (Graham & Seidel, 1993).

For the structure, we estimate the communication cost of broadcasting across a row plus the communication cost across a column (Day & Tripathi, 1992; Jwo et al., 1991; Graham & Seidel, 1993). To estimate the broadcasting in both directions we use the lower bound formula that has been extensively used in existing similar studies (Al-Ayyoub & Day, 1997; Graham & Seidel, 1993):

\[
\left\{ \frac{M \cdot a}{\beta \Delta} + \sqrt{\delta - 1} \right\} \beta.
\]

The parameters \( \Delta \) and \( \delta \) are the degree and diameter of the graph respectively and the symbols \( M, a, \) and \( \beta \) are the message length, unit transmission cost and the message latency cost. The values of these parameters are set to 1024 byte, 1 μs and 1000 μs respectively.

The broadcasting cost based on the pipeline structure can be estimated in two different ways; by shifting the data through the Hamiltonian...
cycle of $A_{n,k}$ or by shifting the data through the Hamiltonian cycle of OTIS. The lower bound of the broadcasting cost in the pipelined structure is equal to the lower bound of the broadcasting cost in the arrangement graph across one stage plus the cost of shifting the data to the next stage or vice versa. The cost of shifting the data to the next stage is equal to $δ(β + Ma)$, where the parameters $M$, $β$ and $a$ are as defined previously (Al-Ayyoub & Day, 1997).

**Theorem 3:** If $A_{n,k}$ is Hamiltonian, OTIS- $A_{n,k}$ embeds a pipeline consisting of $|V_o|$ stages of size $|V_o|$ nodes each. Stages are $A_{n,k}$-configured and interstage distance is 3.

**Proof:** OTIS- $A_{n,k}$ network can be decomposed into $|V_o|$ disjoint copies of $Ψ_x$ sub-networks. The $Ψ_x$ subnetworks form the different pipeline stages. By Theorem 2 each of the $Ψ_x$ is isomorphic to $A_{n,k}$. We arrange the pipeline stages (the $Ψ_x$'s) according the rank of $x$ in a Hamiltonian cycle of $A_{n,k}$. Let $h: V_o → \{1, 2, ..., |V_o|\}$ be a function that defines the node's rank in the Hamiltonian cycle of $A_{n,k}$. So, the $j^{th}$ stage consists of the set of nodes $\{h^{-1}(y) | y ∈ V_o\}$. The node $⟨h^{-1}(j), y⟩$ in the $j^{th}$ stage is coupled with the node $⟨h^{-1}(j+1), y⟩$ in the $(j+1)^{th}$ stage of the pipeline. These two nodes are connected by the path $⟨h^{-1}(j), y⟩ → ⟨y, h^{-1}(j)⟩ → ⟨y, h^{-1}(j+1)⟩ → ⟨h^{-1}(j+1), y⟩$. Notice that $h^{-1}(j)$ and $h^{-1}(j+1)$ are neighbours in $A_{n,k}$.

**Corollary 1:** If $A_{n,k}$ is Hamiltonian, OTIS- $A_{n,k}$ can embed a two-dimensional wraparound mesh with dilation 3.

The above pipeline structural outlook exemplifies circular pipelines. Changing the number of stages or stage configuration (linear, circular, tree, etc.) in the above pipeline structure is straightforward. This can be done by characterising a path, cycle, or tree of size equal to the required number of stages in the new pipeline. Stages in the new pipeline are then ordered according to the ranks of the nodes in the characterised path, cycle, or tree. The interstage distance is 3 in all these cases. In fact, the result in Theorem 3 can be extended so that we have control over the stage structure as well. The stage can be $A_{n,k}$ or any network embedded in $A_{n,k}$.

**CONCLUSION**

The study of algorithms on the Optical Transpose Interconnection Systems (OTIS) is still far from being matured. In this paper, we have contributed towards filling this gap by proposing a method for algorithm development on OTIS-AN network. This method is based on the grid and pipeline structure as popular framework for supporting vast body of important real-world parallel applications. Utilizing this method to develop parallel algorithms for linear algebra will be discussed as a future case study.

Several topological properties including size, degree, diameter, number of links and shortest distance between any two nodes have been discussed. The proposed OTIS-AN shown to be an attractive alternative for its factor network in terms of routing by utilizing both electronic and optical technologies. As a future research work we could utilize the proposed framework in solving real life problems on OTIS-AN including Matrix problems and Fast Fourier transforms.

**REFERENCES**


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