Optimized Hybrid Fuzzy Fed PID Control of Nonlinear Systems

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**Abstract.** The design of controllers for nonlinear systems in industry is a complex and difficult task. One approach which has shown promise for solving nonlinear control problems is the use of fuzzy logic control. This paper proposes a new method utilizing proportional–integral-derivative (PID) control as a hybrid fuzzy PID controller for nonlinear system. The salient feature of the proposed approach is that it combines the fuzzy gain scheduling method and a fuzzy Fed PID controller to solve the nonlinear control problem. The resultant fuzzy rule base of the proposed controller contains one part for a non-optimized controller. This single part of the rules uses the Takagi-Sugeno method for solving the nonlinear problem and compares it to the mamdani method. The number of fuzzy rules are minimized using a method of series reduction fuzzy rule base. The simulation results of a nonlinear system show that the performance of a Fed PID Hybrid Takagi-Sugeno fuzzy controller is better than that of the conventional fuzzy PID controller or Hybrid Mamdani fuzzy Fed PID controller, especially using the reduction of the number of fired rules.

**I Introduction**

Fuzzy control is considered as one of the most important sciences in our industrial revolution nowadays. The progress in the automation fields makes fast steps by using robotics and controllable machines.

A scientific research in the area of Hybrid Fuzzy Fed PID Control will be proposed to increase the knowledge in this field, so the best efforts will be held to collect data from books and internet to support this research.

PID control is widely used in industrial applications because of its simplicity. Stability of PID controller can be guaranteed theoretically, and zero steady-state tracking error can be achieved for linear plant in steady-state phase. Computer simulations of PID control algorithm have revealed that the tracking error is quite often oscillatory, however, with large amplitudes during the transient phase. To improve the performance of the PID controllers, several strategies have been proposed, such as adaptive and supervising techniques.

Fuzzy control methodology is considered as an effective method to deal with disturbances and uncertainties in terms of ignorance and ambiguity. Fuzzy PID controller combining fuzzy technology with traditional PID control algorithm has become the most effective domain in artificial intelligence control [1],[2].

The most common problem resulted early depending on the complexity of Fuzzy Logic Control (FLC) is the tuning problem. It is hard to design and tune FLCs manually for most machine problems especially nonlinear industrial systems. For alleviation of difficulties in constructing the fuzzy rule base, there is the conventional nonlinear design method which was inherited in the fuzzy control area, such as fuzzy sliding mode control, fuzzy gain scheduling [3],[4], and adaptive fuzzy control [5],[6]. The error signal for most control systems is available to the controller if the reference input is continuous. The analytical calculations present two-inputs for FLC to employ.
which are proportional error signal and velocity error signal. PID controller is the most common controller used in industries, most of development of fuzzy controllers revolve around fuzzy PID controllers to insure the existence of conventional controllers in the overall control structure, simply called Hybrid Fuzzy Controllers [7],[8].

The key idea of the proposed method is as follows: First, the fuzzy gain scheduling method is applied to linearize the nonlinear system at frozen times. A fuzzy Fed PID controller is designed for each frozen system by replacing the conventional PID controller by an incremental FLC, the integral part of the PID controller is fed by a differentiated feedback gain, this Fed PID controller is the new method used in this paper and it gives the best results anyway. Second, fuzzification of the reference input is performed for the system, while the control space of error signals is linearly partitioned after normalization. Third, the fuzzy rule base is constructed in a recursive way to obtain better nonlinear control as well as to guarantee closed-loop stability of the frozen system.

The gain scheduling method is introduced as an effective nonlinear control method for nonlinear systems. Finally a novel fuzzy Fed PID controller is proposed. We show that recursive design of the fuzzy rule base can guarantee stability of local closed-loop systems. Then, control of a pole-balancing robot illustrates how the proposed design method can be easily applied to a nonlinear robotics system.

II Nonlinear Control Problem

Gain Scheduling Method. Nonlinear systems can be generally expressed by the following nonlinear autonomous system equation:

\[
\dot{x} = f(x) + g(x)u
\]

(1)

Where \(x = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n\) is the state vector, \(u = [u_1, u_2, ..., u_m]^T \in \mathbb{R}^m\) is the control input vector, \(f(x)\) and \(g(x)\) are vector functions of states.

Assume \(x^d(t) \in \mathbb{R}^n\) is the given reference trajectory whose corresponding reference input is \(u^d(t)\)

\[
x^d = f(x^d) + g(x^d)u^d
\]

(2)

Taking Lyapunov linearization around the operating points \((x^d, u^d)\), we have

\[
\dot{x} = x + A(x^d)(x - x^d) + B(x^d)(u - u^d)
\]

(3)

Where

\[
A(x^d) = \left. \frac{df}{dx} \right|_{x=x^d}, \quad B(x^d) = g(x^d)
\]

(4)

Let \(e = x - x^d\), \(\dot{e} = x - x^d\) and

System Eq.(3) is equivalent to

\[
\dot{e} = A^d e + B^d u^d
\]

(5)

where \(A^d\) and \(B^d\) are assumed to be transformed into diagonal CCF, which is valid for many robotics systems. Because the reference trajectory \(x^d(t)\) is a function of time, the nonlinear system Eq.(1) can be linearized at frozen time \(\tau\) so that the tracking problem of the nonlinear system is transformed into a stabilization problem of the linear system Eq.(6) in the error state space. The equilibrium points are shifted from the reference trajectory points \(x^d(\tau)\) to the origin. However, the aforementioned conventional gain-scheduling method employs linearization between two consecutive operating points. If the system states vary significantly along the time axis, global
stability will be a problem. An alternative solution is to utilize fuzzy rules containing expert knowledge to perform smoother interpolation of all the operating points in the control envelope [9].

**Fuzzy Gain Scheduling.** At some frozen times \( \tau_i \) the corresponding control input can be approximated by Eq.(2), which is \( x^d(\tau_i) \) or \( x^i \) shortly. In partitioning the state space, this \( x^i \) will be the centers of membership functions (MFs), \( LX^i \) [10]. The nonlinear system given by Eq.(1) can, therefore, be transformed into several local linearized systems

\[
R^i : IF \ x^d \ is \ LX^i, THEN \ \dot{e} = Ae + Bu^e
\]

where \( A^i \) and \( B^i \) are system state matrices corresponding to \( x^i \).

The control law to be designed is

\[
R^i : IF \ x^d \ is \ LX^i, THEN \ u = u^d + u^e
\]

where \( u^d \) is the control input corresponding to the reference input \( x^d \) and \( u^e \) is the control input derived from error inputs.

The conventional approach of using the gain scheduling method is to design a linear controller for each local system in Eq.(7). The main advantage of this approach is that the powerful linear control theory may be applied. However, some simple nonlinear controllers like fuzzy PID controllers could be a better choice for handling the system nonlinearities.

**III Hybrid Fuzzy Control Problem**

A fuzzy PID controller is proposed by discretizing the conventional PID controller and constructing from simple linear fuzzy rules in an incremental way. However in this chapter, a new type of fuzzy PID controller is proposed based on fuzzy Fed PID control structure using Mamdani versus the Takagi-Sugeno method [11].

The fuzzy Fed PID controller is constructed in an incremental way by employing both error signals and recursive feedback signals as inputs to Fed PID. The main idea is found in the integral side, where the integral side when it is fed by a deferential feedback gives us a null overshoot and a null steady state error, the enhancement is very significant using Fuzzy Fed PID controller. The most widely adopted conventional PID controller structure used in industrial applications is the following structure [12]:

\[
u_{PID}(t) = K_P^C e_v(t) + K_I^C e_p(t) + K_D^C e_a(t)
\]

where \( K_P, K_I, \) and \( K_D \) are the conventional proportional, integral, and derivative gains, respectively, and \( u_{PID}(t) \) is the controller output and \( e_v(t) \) is the velocity error signal, \( e_p(t) = \int e_v(t) \) is the proportional error signal and \( e_a(t) = de_v(t)/dt \) is the acceleration error signal.

The items in Eq.(9) form the PID controller and can be replaced by the following linear fuzzy rules:

\[
R^j : IF \ e_p \ is \ LE_j^p AND \ e_v \ is \ LE_j^v, THEN u_{PID} \ is \ DU_j^{PID}
\]

Where \( LE_j^p \) and \( LE_j^v \) are the linguistic values of the error signals of the \( j^{} \)th fuzzy rule and \( DU_j^{PID} \) is the desired function value of the output \( u_{PID}(t) \)

The first look to the Fed PID gives the following equation:

\[
u_{PID}(t) = K_P^C e_v(t) + (0.5)K_I^C e_p(t) + K_D^C e_a(t)
\]
Note that the output feedback from the integrator is taken from the output of the defuzzification process which gives the best results showing in the illustrative example.

IV Illustrative example

The example will be illustrated to make sure of the proposed results, which gives the minimum overshoot and minimum steady state error. In the example, the proposed controller is used in Mamdani and Takagi-Sugeno fuzzy control with an inverted pendulum robot, that robot is used in the most of our applications because of nonlinearity problem and marginally stability. The dynamic equation of the inverted pendulum robot is given by

\[
\ddot{\theta} = \frac{(m_p + m_c)g \sin \theta - m_p \dot{\theta}^2 l \sin \theta - F \cos \theta}{(m_p + m_c)(4/3 - m_p \cos^2 \theta)}
\]  

(12)

Where \( \theta \) is the angle between the pendulum and the vertical, the angular velocity is expressed by \( \dot{\theta} \), the force which acts on the cart is \( F \), the gravity acceleration \( g \) is 9.8m/sec\(^2\), \( m_c \) and \( m_p \) are the mass of cart and the mass of pole respectively, and \( l \) is the half length of the pendulum. The system equation is written as follow:

\[
\dot{x} = f(x) + g(x)u
\]  

(13)

Where

\[
f(x) = \begin{bmatrix}
\dot{\theta} \\
\frac{(m_p + m_c)g \sin \theta - m_p \dot{\theta}^2 l \sin \theta \cos \theta}{(m_p + m_c)(4/3 - m_p \cos^2 \theta)}
\end{bmatrix},
g(x) = \begin{bmatrix}
0 \\
\frac{-F \cos \theta}{(m_p + m_c)(4/3 - m_p \cos^2 \theta)}
\end{bmatrix}
\]
The last two equations are used for simulation without a previous technique of linearization because of two methods are used, the first one is the gain scheduling method which divides the system into small areas to relent using of iterations, the second method is the fuzzy PID controller that uses the linguistic formulas and by default it makes a linearization of the nonlinear system. The addition of the two methods is called hybrid fuzzy PID controller [13],[14].

Let us discuss briefly the pendulum and give the numerical calculations and membership functions equations. As discussed, the angular position is $\theta$, the angular velocity is $\dot{\theta}$, the external force $F$ is applied to the cart. The gravity acceleration, $g$ is 9.8 m/s$^2$, the mass of the cart, $m_c$ is 1.0kg, the mass of the pole, $m_p$ is 0.1kg and the half length of the pole, $l$ is 0.5m. Say that $x=[\theta \hspace{1cm} \dot{\theta}]^T$ and $u=F$. Assume that the pole angle is required to follow a particular trajectory $\theta^d$, now we can calculate the corresponding control input, $u^d$, at a frozen times. Then the system can then be linearized to

$$\dot{x} = x^d + A^d (x - x^d) + B^d (u - u^d) \quad \text{where} \quad x^d = [\theta^d \hspace{1cm} \dot{\theta}^d]^T, \quad A^d = df/dx|_{x=x^d} \quad \text{and} \quad B^d = g(x^d).$$

Now we will find the gain of oscillation $K_u$ and the frequency of oscillation $T_u$ from Nyquist diagram of the inverted pendulum:

Frequency of oscillation is $T_u = 3.5867e^{-7}$ Hz and the gain of oscillation is $K_u = 27$.

$$\mu_{pos}(\theta) = \exp\left(-(75278110.8)(\theta - 1.5)^2\right) \quad \text{(Function 1)}$$

$$\mu_{zero}(\theta) = \exp\left(-(75278110.8)(\theta)^2\right) \quad \text{(Function 2)}$$

$$\mu_{neg}(\theta) = \exp\left(-(75278110.8)(\theta + 1.5)^2\right) \quad \text{(Function 3)}$$

Figure 2: Nyquist diagram of the open loop inverted pendulum

Beside the point of the amount of masses and measurements of the pendulum, the most point to be focused is the Fed Fuzzy PID controller that makes lower overshoot and minimum steady state error. This technique always makes the best results shown in Figure 3, the fuzzy rules of the Fed PID controller using Takagi-Sugeno shown below is better than the results of Hybrid fuzzy Fed PID controller:
For the fuzzy proportional integrator differentiator:

1. If (input1 is -ve) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
2. If (input1 is -ve) and (input2 is -ve) and (input3 is zero) then (output1 is Function1)
3. If (input1 is -ve) and (input2 is -ve) and (input3 is +ve) then (output1 is Function1)
4. If (input1 is -ve) and (input2 is zero) and (input3 is -ve) then (output1 is Function1)
5. If (input1 is -ve) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
6. If (input1 is -ve) and (input2 is zero) and (input3 is +ve) then (output1 is Function2)
7. If (input1 is -ve) and (input2 is +ve) and (input3 is -ve) then (output1 is Function2)
8. If (input1 is -ve) and (input2 is +ve) and (input3 is zero) then (output1 is Function3)
9. If (input1 is -ve) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)
10. If (input1 is zero) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
11. If (input1 is zero) and (input2 is -ve) and (input3 is zero) then (output1 is Function2)
12. If (input1 is zero) and (input2 is -ve) and (input3 is +ve) then (output1 is Function2)
13. If (input1 is zero) and (input2 is zero) and (input3 is -ve) then (output1 is Function2)
14. If (input1 is zero) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
15. If (input1 is zero) and (input2 is zero) and (input3 is +ve) then (output1 is Function2)
16. If (input1 is zero) and (input2 is +ve) and (input3 is -ve) then (output1 is Function2)
17. If (input1 is zero) and (input2 is +ve) and (input3 is zero) then (output1 is Function2)
18. If (input1 is zero) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)
19. If (input1 is +ve) and (input2 is -ve) and (input3 is -ve) then (output1 is Function1)
20. If (input1 is +ve) and (input2 is -ve) and (input3 is zero) then (output1 is Function2)
21. If (input1 is +ve) and (input2 is -ve) and (input3 is +ve) then (output1 is Function3)
22. If (input1 is +ve) and (input2 is zero) and (input3 is -ve) then (output1 is Function2)
23. If (input1 is +ve) and (input2 is zero) and (input3 is zero) then (output1 is Function2)
24. If (input1 is +ve) and (input2 is zero) and (input3 is +ve) then (output1 is Function3)
25. If (input1 is +ve) and (input2 is +ve) and (input3 is -ve) then (output1 is Function3)
26. If (input1 is +ve) and (input2 is +ve) and (input3 is zero) then (output1 is Function3)
27. If (input1 is +ve) and (input2 is +ve) and (input3 is +ve) then (output1 is Function3)

Figure 3 illustrates the membership functions of the inputs and outputs of the desired controller, the blue color (left) for the membership function point to the negative input, the green (mid) one point to the zero membership and the red (right) point to the positive membership for each input.
Figure 4 illustrates the step response of hybrid fuzzy Fed PID controller versus conventional PID controller using Mamdani technique, the results are shown in Figure 4 clearly give the best steady state error and the best overshoot but give a delay:

As illustrated in the Figure 4, the overshoot of Hybrid Fuzzy Fed PID using the Mamdani method is less value than the overshoot of the conventional PID controller that satisfy the idea of using the fuzzy control is better than conventional PID in maximum overshoot and the steady state error.

Figure 5 illustrates the step response of hybrid fuzzy Fed PID controller (Takagi-Sugeno) versus conventional PID controller:
Figure 5 illustrates the Mamdani versus Fed Sugeno Hybrid Fuzzy PID controller where the Fed Takagi-Sugeno achieves the zero overshoot but the Mamdani makes some overshoot, in addition the Fed Takagi-Sugeno has steady state error less value than Mamdani method.

![Graph showing voltage over time](image)

**Figure 6:** Stabilization control of the Fed PID (Mamdani) versus Fed PID (Takagi-Sugeno)

Figure 6 shows the Fed Mamdani versus Fed Sugeno Hybrid Fuzzy PID controller where the Fed Takagi-Sugeno achieves the zero overshoot but the Fed Mamdani makes some overshoot, in addition the Fed Takagi-Sugeno has steady state error less value than Mamdani method. Anyway, when the Fed theorem is used the minimum steady state error and the minimum overshoot will be achieved.

The 27 rules used in the inverted pendulum example can be reduced using the series method shown in the Figure 7.

![Diagram showing rule reduction](image)

**Figure 7:** Reduction of the inverted pendulum Fed PID controller rules using series method
The fuzzy PID controller is divided into two main rules the PD rules and the Integrator rules, I divided the integrator rules to make a feedback from the output of the integrator with a deferential feedback to the input of the integrator, this technique always makes the best results referred to optimization in the direction of reducing the number of rules, the fuzzy rules of the Fed PID controller shown bellow:

For the fuzzy Fed integrator:
IF \( e_p \) is −ve ) THEN \( u_{PID} \) is −ve )
IF \( e_p \) is zero) THEN \( u_{PID} \) is zero)
IF \( e_p \) is +ve ) THEN \( u_{PID} \) is +ve )

For the fuzzy proportional differentiator:
IF \( e_v \) is −ve ) AND \( e_a \) is −ve ) THEN \( u_{PID} \) is −ve )
IF \( e_v \) is −ve ) AND \( e_a \) is zero) THEN \( u_{PID} \) is −ve )
IF \( e_v \) is −ve ) AND \( e_a \) is +ve ) THEN \( u_{PID} \) is zero)
IF \( e_v \) is zero) AND \( e_a \) is zero) THEN \( u_{PID} \) is zero)
IF \( e_v \) is zero) AND \( e_a \) is +ve ) THEN \( u_{PID} \) is +ve )
IF \( e_v \) is +ve ) AND \( e_a \) is −ve ) THEN \( u_{PID} \) is zero)
IF \( e_v \) is +ve ) AND \( e_a \) is zero) THEN \( u_{PID} \) is +ve )
IF \( e_v \) is +ve ) AND \( e_a \) is +ve ) THEN \( u_{PID} \) is +ve )

The results of the reduced rules are shown in Figure 8 that illustrates a good overshoot and a good steady state error which nearly equals to that used in 27 rules but don’t forget that the optimization in rules numbers qualifies big processes to be reserved. The 27 rule TS Fed PID achieves null overshoot and very small steady state error, and the 12 rule TS Fed PID has some overshoot and small steady state error. But the 27 rule wastes time, processing and money where that is not achieved in 12 rule TS Fed PID.

![Figure 8: Fuzzy (TS) Fed PID controller with only 12 rule step response versus 27 rules step response,](image-url)
V Conclusion

In this paper, the new approach of control design of a hybrid fuzzy PID controller is proposed. Instead of analyzing the fuzzy controller by numerical calculations, the proposed design method focuses on constructing the fuzzy rule base. The proposed controller demonstrates excellent control performance for nonlinear robot which depends on the hybridizing of the gain scheduling method and fed PID controller which gives the best control specifications towards the conventional PID, fuzzy PID and hybrid fuzzy PID. The proposed problem is considered one of the most hot and useful topics in the area of fuzzy control field related with robotics systems.

VI References