Slide-Mode Fuzzy Controller for a Liquid Level System

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Goals

◆ To design a Fuzzy Controller (FC) based on Variable Structure Controller (VSC) for fuzzy systems represented by Takagi-Sugeno (T-S) affine fuzzy model.

◆ To obtain an improved performance of a non-linear system represented by a liquid level system. It is aimed to control level set point by adjusting the flow rate of a liquid entering the tank.
Summary

- Review of T-S fuzzy affine model and brief review of some aspects of VSC
- The proposed algorithm of the FLC-VSC
- Application of the proposed controller to the control of a liquid-level control system.

T-S Affine Fuzzy Model

- The local dynamics in various equilibrium states are represented by input-output linear subsystems

\[ S^{(i_1 \ldots i_n)}: \text{if } x \text{ is } M_1^{i_1} \text{ and } \dot{x} \text{ is } M_2^{i_2} \text{ and } \ldots x^{(n-1)} \text{ is } M_n^{i_n} \text{ then} \]

\[ \dot{x} = a_0^{(i_1 \ldots i_n)} + A^{(i_1 \ldots i_n)} x + B^{(i_1 \ldots i_n)} u \]

\[ \dot{x} = \frac{\sum_{i_1=1}^{n_1} \ldots \sum_{i_n=1}^{n_n} w^{(i_1 \ldots i_n)}(x) \dot{x}^{(i_1 \ldots i_n)}}{\sum_{i_1=1}^{n_1} \ldots \sum_{i_n=1}^{n_n} w^{(i_1 \ldots i_n)}(x)} \]
Variable Structure Control

- Variable Structure Control System is a combination of subsystems together with a suitable switching logic
- In VSC, the design algorithm includes choosing the desired sliding functions which are formed by a choice of their parameters
- Then a discontinuous control is found which assures the existence of the sliding modes at each point of the sliding plane \( s(x) = 0 \)
- In the final stage, the control should drive the system states to the sliding plane

Variable Structure Control (2)
Variable Structure Control (3)

Given the $n$-order system with $m$ inputs

$$\dot{x} = Ax + Bu$$

define a $m$-dimensional function $s(x)$

$$s(x) = Cx$$

where $C$ is a $(m \times n)$ matrix chosen such that

$$s(x) = 0$$

determines a stable dynamic system of reduced order

Variable Structure Control (4)

The next step in the design of the VSC includes choosing the structure of the control to satisfy a reaching condition, e.g.

- when $s_i(x) > 0$ choose $u(x)$ such that $\dot{s}_i(x) < 0$
- when $s_i(x) < 0$ choose $u(x)$ such that $\dot{s}_i(x) > 0$
Variable Structure Control (5)

In this paper, a linear feedback with switched gains is used.

\[ u(x) = -\Psi(x)x \]

\[ \Psi_{ij}(x) = \begin{cases} \alpha_{ij} \text{ when } s_i(x)x^{(j-1)}>0 \\ \beta_{ij} \text{ when } s_i(x)x^{(j-1)}<0 \end{cases} \]

Variable Structure Control (6)

\[ \ddot{x} - 2\dot{x} + 2x = u \]

\[ s(x) = [1 \ 1] \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \]

\[ \alpha_1 = 0 \quad \beta_1 = 4 \]
\[ \alpha_2 = -4 \quad \beta_2 = 0 \]
Design of FLC-VSC

![Diagram of FLC-VSC](image)

**Design of FLC-VSC (2)**

- **The T-S model**

  \[ S^{(l_1 \ldots l_n)}: \text{if } x \text{ is } M_1^{(l_1)} \text{ and } \dot{x} \text{ is } M_2^{(l_2)} \text{ and } \ldots \text{ and } x^{(n-1)} \text{ is } M_n^{(l_n)} \]

  \[ \text{then } \dot{x} = a_0^{(l_1 \ldots l_n)} + A^{(l_1 \ldots l_n)} x + B^{(l_1 \ldots l_n)} u \]

- **The fuzzy controller**

  \[ C^{(j_1 \ldots j_n)}: \text{if } x \text{ is } N_1^{(j_1)} \text{ and } \dot{x} \text{ is } N_2^{(j_2)} \text{ and } \ldots \text{ and } x^{(n-1)} \text{ is } N_n^{(j_n)} \]

  \[ \text{then } u = r - (d_0^{(j_1 \ldots j_n)} + D^{(j_1 \ldots j_n)} x) \]
Design of FLC-VSC (3)

Substituting we get the feedback system

\[
SC^{(i_1\ldots i_n, j_1\ldots j_n)} : \text{if } x \text{ is } (M_1^{j_1} \text{ and } N_1^{j_1}) \text{ and } \\
\dot{x} \text{ is } (M_2^{j_2} \text{ and } N_2^{j_2}) \text{ and } \ldots \\
x^{(n-1)} \text{ is } (M_n^{j_n} \text{ and } N_n^{j_n}) \text{ then} \\
\dot{x} = a_0^{(i_1\ldots i_n)} + A^{(i_1\ldots i_n)} x + B^{(i_1\ldots i_n)} \left[ r - (d_0^{(j_1\ldots j_n)} + D^{(j_1\ldots j_n)} x) \right]
\]

Design of FLC-VSC (4)

Choosing \( r = 0 \) and \( d_0^{(j_1\ldots j_n)} \) such that

\[
a_0^{(i_1\ldots i_n)} - B^{(i_1\ldots i_n)} d_0^{(j_1\ldots j_n)} = 0
\]

the controlled system will be simplified

\[
SC^{(i_1\ldots i_n, j_1\ldots j_n)} : \text{if } x \text{ is } (M_1^{j_1} \text{ and } N_1^{j_1}) \text{ and } \\
\dot{x} \text{ is } (M_2^{j_2} \text{ and } N_2^{j_2}) \text{ and } \ldots \\
x^{(n-1)} \text{ is } (M_n^{j_n} \text{ and } N_n^{j_n}) \text{ then} \\
\dot{x} = A^{(i_1\ldots i_n)} x - B^{(i_1\ldots i_n)} D^{(j_1\ldots j_n)} x
\]
Design of FLC-VSC (5)

The FLC will be

\[
\dot{x} = \frac{\sum_{i_1} \cdots \sum_{j_n} \sum_{j_1} \cdots \sum_{j_n} w_{ij_1 \cdots j_n}(x) A_{ij_1 \cdots j_n}(x)}{\sum_{i_1} \cdots \sum_{j_n} \sum_{j_1} \cdots \sum_{j_n} w_{ij_1 \cdots j_n}(x)} - \frac{\sum_{i_1} \cdots \sum_{j_n} \sum_{j_1} \cdots \sum_{j_n} w_{ij_1 \cdots j_n}(x) B_{ij_1 \cdots j_n} D_{ij_1 \cdots j_n} x}{\sum_{i_1} \cdots \sum_{j_n} \sum_{j_1} \cdots \sum_{j_n} w_{ij_1 \cdots j_n}(x)}
\]

Design of FLC-VSC (6)

Comparing now VSC with FLC we obtain

\[
\psi_{ij} = d_{ij}^{(j_1 \cdots j_n)}
\]

The discontinuous controller, which guarantees the existence of the sliding plane, must be found by solving the mentioned condition:

\[
d_{ij}^{(j_1 \cdots j_n)}(x) = \begin{cases} 
\alpha_{ij}^{(j_1 \cdots j_n)} & \text{when } s_i(x)x^{(j-1)} > 0 \\
\beta_{ij}^{(j_1 \cdots j_n)} & \text{when } s_i(x)x^{(j-1)} < 0
\end{cases}
\]
LEVEL CONTROL SYSTEM

It pumps out the liquid as an inlet flow to reach a predetermined level inside a tank with a horizontal cross-section, which discharges liquid through a restriction in its base.

LEVEL CONTROL SYSTEM (2)

The feed pump can be described as:

\[
\frac{Q(s)}{U(s)} = \frac{K_p}{(1+T_1 s)(1+T_2 s)} = \frac{1.3}{(1+0.36 s)(1+0.71 s)}
\]  

(1)

\(K_p\): s.s. gain  
\(T_1\) and \(T_2\): time constants of the pump.  
Their values are identified using the data obtained from experimental tests.
LEVEL CONTROL SYSTEM (3)

The relation between I/P voltage applied to the pump and the I/P flow rate \( q_i \) to the tank:

\[
A \dot{h} = q_i(t) - q_o(t) \quad (2)
\]

The O/P flow rate \( q_o \) is proportional to the square root of the decreasing level:

\[
q_o(t) = k \sqrt{h(t)} \quad (3)
\]

Substituting Eq. (3) in Eq. (2) we obtain,

\[
A \dot{h} + k \sqrt{h(t)} = q_i(t) \quad (4)
\]

k is related to the restriction of the valve at the o/p of the tank.

LEVEL CONTROL SYSTEM

Linearizing the resulted non-linear system Eq. (4) and substituting in Eq. (1):

\[
\begin{align*}
\dot{h} + & \left[ \frac{A(T_1 + T_2) + \frac{kT_2}{2\sqrt{h_0}}}{AT_2} \right] \dot{h} + \left[ \frac{A + \frac{k}{2\sqrt{h_0}} (T_1 + T_2)}{AT_2} \right] h + \\
& \left[ \frac{-\frac{k}{2\sqrt{h_0}} h_o + q_{io}}{AT_2} \right] = \left[ \frac{K_p}{AT_2} \right] u
\end{align*}
\]
Firstly, a T-S affine model is used to represent the process which is linearized as shown in Figure.

\[
\begin{bmatrix}
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
-6.5 & -13.5 & -8.7 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
a_0^1 \\
0 \\
0.08 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
B^1 \\
0 \\
26.5 \\
\end{array}
\end{bmatrix}, \quad d_0^1 = 0.08
\]

\[
\begin{bmatrix}
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
-5.3 & -8.2 & -3.9 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
a_0^2 \\
0 \\
0.2 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
B^2 \\
0 \\
8 \\
\end{array}
\end{bmatrix}, \quad d_0^2 = 0.2
\]

\[
\begin{bmatrix}
\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
-5 & -7.1 & -2.9 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
a_0^3 \\
0 \\
0.26 \\
\end{array}
\end{bmatrix}, \quad \begin{bmatrix}
\begin{array}{c}
B^3 \\
0 \\
16.7 \\
\end{array}
\end{bmatrix}, \quad d_0^3 = 0.26
\]
Assuming that the desired SM equation is
\[ s = C_1 x + C_2 \dot{x} + C_3 \ddot{x} = 2.89 x + 3 \dot{x} + \ddot{x} \]
Now the parameters \( \alpha_{ij}^{(k-\mu)} \) and \( \beta_{ij}^{(k-\mu)} \) are calculated for each subsystem:
\[ \alpha_{ii}^{(1)} \geq 0.7 \text{ and } \beta_{ii}^{(1)} \leq -0.1 \]
\[ \alpha_{ii}^{(2)} \geq 0.5 \text{ and } \beta_{ii}^{(2)} \leq 0.1 \]
\[ \alpha_{ii}^{(3)} \geq 0.8 \text{ and } \beta_{ii}^{(3)} \leq 0.1 \]
Conclusions

- A FLC-VSC has been designed for the control of liquid level system represented by the affine T-S fuzzy model
- The fuzzy system and controller have been represented by the affine T-S model
- It was aimed to control level inside a tank by controlling the flow rate of liquid entering the tank through a feed pump.
- The results obtained in this paper have shown a fast response with small overshoot in the transient response and a well damped oscillations with zero steady state error.