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New class of generalized photon-added coherent states and some of their non-classical properties

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Abstract
In this paper, we construct a new class of generalized photon added coherent states (GPACSs), \(|z, m\rangle\), by excitations on a newly introduced family of generalized coherent states (GCSs) \(|z, r\rangle\) (A Dehghani and B Mojaveri 2012 J. Phys. A: Math. Theor. 45 095304), obtained via generalized hypergeometric type displacement operators acting on the vacuum state of the simple harmonic oscillator. We show that these states realize resolution of the identity property through positive definite measures on the complex plane. Meanwhile, we demonstrate that the introduced states can also be interpreted as nonlinear coherent states (NLCSs), with a spacial nonlinearity function. Finally, some of their non-classical features as well as their quantum statistical properties are compared with Agarwal’s photon-added coherent states (PACSs), \(|z, m\rangle\).

Keywords: photon-added coherent states, sub-Poissonian statistics, squeezing effect

(Some figures may appear in colour only in the online journal)

1. Introduction
In 1991, Agarwal et al [1] introduced photon-added coherent states (PACSs), \(|z, m\rangle\), which are obtained by the repeated action of the photon creation operator on a coherent state, i.e.

\[ |z, m\rangle = a^m |z\rangle, \]

where \(m\) is a non-negative integer. PACSs are intermediate between a single-photon Fock state \(|n\rangle\) and a coherent one \(|z\rangle\). These states offer the opportunity to closely follow the smooth transition between the particle-like and the wavelike behavior of light. Their physical and statistical properties have been studied in detail and, for instance, they exhibit phase squeezing and sub-Poissonian statistics of the field. Also, an interesting theoretical framework has been proposed about how such states can be generated in nonlinear processes in cavities. Dynamical squeezing of these states and their classification in a special class of non-linear coherent states have been done in [2, 3]. Using the Stieltjes power-moment problem, the over-completeness of PACSs is explicitly shown in [4]. Higher-order squeezing and higher-order sub-Poissonian statistics of the PACSs have been studied in [5]. Fortunately, their aspiration became a reality and in 2004 Zavatta et al [6] set out the experimental generation of single-photon-added coherent states as well as their complete characterization by quantum tomography. Photon-added and photon-subtracted coherent states associated with inverse q-boson operators were introduced in [7]. Similarly, physical and statistical properties of states generated by excitations on squeezed vacuum states [8], even (odd) coherent states [9], displaced states [10], displaced squeezed states [11], thermal states [12] and generalized pair coherent states [13] are studied. Generalized hypergeometric photon-added and photon-depleted coherent states and deformed photon-added non-linear coherent states were introduced, respectively, in [14] and [15]. In fact, the PACSs and the lower truncated coherent states are the limiting cases of a suitably deformed PACSs [16]. Photon-added Gazeau-Klauder and Klauder-Perelomov coherent states for exactly solvable Hamiltonian were studied in [17] and photon-added Barut-Girardello coherent states of the pseudo harmonic oscillator were constructed in [18]. Also,
excited coherent states corresponding to the Morse potential have been constructed and some of their statistical properties have been investigated [19, 20].

Recently, we introduced a new kind of GCS, $|z\rangle$, for harmonic, pseudo harmonic oscillator and Landau levels based on the generalization of the boson displacement operator associated with the Heisenberg-Weyl, $su(1, 1)$ and $su(2)$ Lie algebras, respectively [21–23]. It has been shown that they are new class of nonlinear coherent states (NLCSs) [24–26] and include some interesting features such as temporal stability and non-classical properties. A considerable feature of the above-mentioned approach is due to the fact that, contrary to the Klauder-Perelomov and Barut-Girardello approach, it does not require the existence of the dynamical symmetries associated with the considered systems. In order to construct such states, we need only a raising operator in the framework of supersymmetric quantum mechanics. In the present paper, we have defined a new kind of GPACS by excitations on the GCSs for a harmonic oscillator [21]. We show that they are eigenstates of $f$-deformed annihilation operators $f(\hat{a}, m)\hat{a}$. Furthermore, some of the non-classical features and quantum statistical properties of these states have been studied and compared with the PACSs.

This paper is organized as follows: in section 2, we introduce a new kind of GPACS by excitations on the GCSs for harmonic oscillator $|z\rangle$. In order to realize the resolution of the identity property, we have found positive definite measures on the complex plane. By construction of the explicit form of the operator-valued nonlinearity function associated with GPACSs, we show that these states can be interpreted as NLCSs. In section 3, by evaluating some physical quantities, we discuss their statistical and non-classical properties. Finally, we conclude the paper in section 4.

2. New generalized photon-added coherent states

In accordance with equation (1), we introduce GPACSs $|z, m\rangle$, as

$$|z, m\rangle = a^m |z\rangle,$$  

where $r$ refers to the deformation parameter and $|z\rangle$, recalls GCSs for the harmonic oscillator

$$|z\rangle := F_r \left( 10, 0, 1, ..., r - 1 \right) |z\rangle, \quad r \geq 1.$$  

It is obvious that the states $|z, m\rangle$, reduce to Agarwal’s PACSs when $r$ tends to unity. Inserting equation (3) into equation (2) and taking into account the action of the raising operator on the Fock states $|n\rangle$, the states $|z, m\rangle$, can be written as

$$|z, m\rangle = M_r^{-\frac{1}{2}} \left( |k\rangle \right) \sum_{n=0}^{\infty} \frac{z^2}{n!} \left( \sum_{k=0}^{n} \frac{\Gamma(k)}{\Gamma(n+k)} \right) |n+m\rangle, \quad r \geq 2,$$  

where the normalization constant $M_r (|z\rangle)$ is chosen so that $|z, m\rangle$, is normalized to unity, i.e. $\langle z, mlz, m\rangle = 1$, then

$$M_r (|z\rangle) = f_{2r-1} (|m+1\rangle,$$

$$|1, 1, 1, 2, 2, 3, 3, ..., r - 1, r - 1\rangle, |z\rangle).$$  

It follows that, due to the orthogonality relation of the Fock space basis, overlapping of two different kinds of these normalized states must be nonorthogonal in the following sense

$$\langle z_1, m|z_2, m\rangle = \left( \int F_{2r-1} (|m+1\rangle, |1, 1, 1, 2, 2, 3, 3, ..., r - 1, r - 1\rangle, |z\rangle) \right)^{1/2}.$$  

Now, we should check the realization of the resolution to identity for the states $|z, m\rangle$, in the Hilbert space $H_m = \text{span} \{ |n+m\rangle \}_{n=0}^{\infty}$

$$\int_{0}^{\infty} d\|z\| |z, m\rangle \langle z, m| = \sum_{n=0}^{\infty} |n+m\rangle \langle n+m|,$$

where $K_r (|z\rangle)$ is a positive definite measure that we search for when obtaining it. Substituting equation (4) in equation (7) leads us to the following integral relation

$$\int_{0}^{\infty} d\|z\| |z, m\rangle \langle z, m| = \left( \int F_{2r-1} (|m+1\rangle, |1, 1, 1, 2, 2, 3, 3, ..., r - 1, r - 1\rangle, |z\rangle) \right)^{1/2} m! (n!)^2 \frac{\pi n (n+m)!}{2\pi n (n+m)!}.$$  

By using the integral relation for the Meijers G-functions (see $\zeta^{-\frac{8n+1}{4}}$ in [27]), the positive definite and non-oscillating measure $K_r (|z\rangle)$ is obtained as
The measure $K_r(|z|)$ in terms of $|z|^2$ for different values of $m$ and a fixed value of $r = 2$ which signify the positivity of $K_r(|z|)$.

2.1. GPACSs as nonlinear coherent states

In this section we construct an explicit form of an operator valued nonlinearity function associated to GPACSs. As seen in [21], the GCSs $|z⟩$ satisfy the following eigenvalue equation

$$\Gamma(N + r) \hat{a} |z⟩ = z |z⟩.$$  \hspace{1cm} (10)

Now, multiplying both sides of this equation by $a^†_m$ yields

$$a^†_m \Gamma(N + r) \hat{a} |z⟩ = z a^†_m |z⟩,$$  \hspace{1cm} (11)

which, making use of the commutation relation

$$a^†_m a^† = f(N) f(N - m) a^†_m a^†_m a^†_m = a^†_m - m a^†_m a^†_m,$$  \hspace{1cm} (12)

leads to

$$\left[ \frac{\Gamma(N + r)}{\Gamma(N + m + 1)} \right] \frac{\Gamma(N - m + r)}{\Gamma(N - m + 1)} a^†_m |z⟩ = z |z⟩.$$  \hspace{1cm} (13)

The above eigenvalue relation shows that $|z, m⟩$ can be identified as new classes of NLCSs with characterized non-linearity functions, $\frac{(N - m + 1) Γ (N - m + r)}{(N + 1) Γ (N + r)}$. Obviously, in the case $r = 1$, equation (13) reduces to the eigenvalue equation for PACSs.

3. Statistical properties of the GPACSs

In this section, some quantum statistical properties including quadrature squeezing and photon statistics of the constructed GPACSs will be studied. These properties are helpful criteria for investigating the non-classicality exhibition of a general state. We should indicate that if a state is squeezed or obey from sub-Poissonian statistics, in a region of allowed space, it belongs to a family of non-classical states [28].

3.1. Quadrature squeezing

By using the definitions of position and momentum quadratures in terms of the creation and annihilation operators as,

$$x = a + \frac{a^†}{\sqrt{2}}, \quad p_i = \frac{a - a^†}{i\sqrt{2}},$$  \hspace{1cm} (14)

it is straightforward that squeezing respectively, occurs in $(x)$ or $p_i$ components if $S_x = 2 \langle (Δx)^2 \rangle - 1 < 0$ or $S_{p_i} = 2 \langle (Δp_i)^2 \rangle - 1 < 0$ [29, 30]. The quadrature variances $\langle (Δx)^2 \rangle$ and $\langle (Δp_i)^2 \rangle$ are described in terms of the operators $a$ and $a^†$ as follows

$$\langle (Δx)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2} \left[ 1 + 2 \langle a^† a \rangle - \langle a^† a^† \rangle - 2 \langle a \rangle \langle a^† \rangle \right]$$

$$\langle (Δp_i)^2 \rangle = \langle p_i^2 \rangle - \langle p_i \rangle^2 = \frac{1}{2} \left[ 1 + 2 \langle a^† a \rangle - \langle a^† a^† \rangle + \langle a^† \rangle^2 + \langle a \rangle^2 - 2 \langle a \rangle \langle a^† \rangle \right].$$  \hspace{1cm} (17)

Now, with the help of following mean values of the relevant operators in the state $|z, m⟩$,

$$\langle a \rangle_r = \frac{\langle a^† \rangle_r}{\Gamma (r)} = \frac{(m + 1) z}{r} \left[ f_{2,r+1}([1, m + 2, r], [2, 1, 1, 2, 2, ..., r, r, |z|^2]) \right]^{\frac{1}{2}}$$

$$\langle a^† \rangle_r = \frac{\langle a \rangle_r}{\Gamma (r)} = \frac{(m + 1) z}{r} \left[ f_{2,r+1}([1, m + 1, |z|^2]) \right]^{\frac{1}{2}}.$$  \hspace{1cm} (18a)
squeezing parameters can be easily evaluated for GPACSSs. In figure 2, we have plotted $S_x$ and $S_p$ versus $|z|^2$ for different values of $m$ associated with the GPACSSs for fixed $\phi = 0$ and $r = 2$. From these figures, we find that while quadrature squeezing in the $x$ component occurs for $\phi = 0$, $\phi = \pi/16$ and $\phi = \pi/8$, there is a quadrature squeezing effect in the $p$ component for $\phi = \pi/4$ and $\phi = 7\pi/16$.

3.2. Photon statistics

We now consider the Mandel parameter $Q$ which yields information about photon statistics of the quantum states defined as [31]

$$Q = \frac{\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2}{\langle \hat{N} \rangle},$$

where $\hat{N} = a^\dagger a$ is the number operator. In fact, a quantum state exhibits super-Poissonian (photon-bunching), Poissonian and sub-Poissonian (photon-antibunching) statistics, respectively, if $Q > 1$, $Q = 1$ and $Q < 1$. Now, evaluating the
for different values of $Q_r$ and fixed $r = 2$. The mean value of $\hat{N}^2$ with respect to the states (4) yields

$$\langle \hat{N}^2 \rangle = \frac{m}{m} \left[ \frac{[F_{2m+1}(2m+1, m+1, m+1), |z|^2]}{[F_{2m}(2m+1, m+1, m+1), |z|^2]} \right],$$

By using equations (18c) and (20), the Mandel parameter $Q_r$ can be calculated for GPACSs. In figure 4, the Mandel parameter $Q_r$ has been plotted in terms of $|z|^2$ for $r = 2$ and different values of $m(=1, 2, 3$ and 4). Evidently, $Q_r$ is less than one for any values of $m$. This indicates that the GPACSs have sub-Poissonian statistics. In comparison with the analogous numerical results of PACSs (figure 3 of [1]), we find that GPACSs show more non-classicality than PACSs. Besides, figure 4 shows that by increasing $m$, the Mandel parameter $Q_r$ tends to zero. Therefore, increasing $m$ results in an increase of the non-classicality of GPACSs analogously to the numerical results of PACSs.

4. Summary and conclusion

In summary, by the repeated action of the photon creation operator on the GCSs, new kinds of GPACSs, $|z, m\rangle$, are introduced. We have shown that these states satisfy the resolution of identity property with positive definite measures. As in the case of PACSs, we have established that the GPACSs are NLCSs with a spacial nonlinearity function. Then, some of the non-classical properties such as quadrature squeezing, photon statistics for these states were studied in detail. It was shown that GPACSs exhibit the squeezing effect and posses sub-Poissonian statistics for any value of $m$. We observed that the depth of the non-classicality of GPACSs is more than PACSs.

References