This paper discusses the problem of capital budgeting in the situation where only some model parameters are well described by their past data and therefore are specified by random variables, whereas the remaining model parameters can hardly be predicted by historical data and therefore they are described by means of fuzzy variables. In order to be able to process such hybrid data, a model of the problem is proposed. The model takes into account both stochastic and economic interdependency between projects. Additionally, a new hybrid method for solving this model is developed. The method combines stochastic simulation with arithmetic on interactive fuzzy numbers and nonlinear programming. As a result a set of Pareto-optimal alternatives is obtained. In order to illustrate the performance of the proposed hybrid method, an example from metallurgical industry is provided.

1. INTRODUCTION

Accurate prediction of investment outlays is very difficult in an uncertain economic environment. Therefore, decisions related to the realization of investment projects are unmanageable without appropriate description of investments and identification of financial and material problems. The allocation of a company’s capital to a combination of investment projects, which brings the company a maximal total profit is referred to as the capital budgeting problem (CBP) [5].

An important aspect of the capital budgeting problem is the analysis of dependency, which is characteristic for many economic problems. In the frames of the CBP, generally the two kinds of dependency can be distinguished, the one between model parameters and the one between projects. The economic interdependence
between projects shows that one project is influencing the effects generated by other projects. Whereas, the statistical dependency is used to measure the strength of dependencies (associations) between probability distributions or fuzzy variables [9,11].

This paper presents a solution to the capital budgeting problem, where projects are statistically and economically dependent. It is assumed that uncertainty attached to model parameters originates from two sources: randomness and imprecision. Distinct approaches are used to adequately represent these two kinds of uncertainty. The objective uncertainty (randomness, variability) is represented by a probability distribution, whereas the subjective uncertainty (imprecision) is described by a possibility distribution (fuzzy numbers). In order to be able to jointly propagate both kinds of uncertainty a new procedure is proposed. The procedure also enables to take into account the two types of dependencies listed above.

The remaining part of the paper is organized as follows. Section 2 presents a capital budgeting problem (CBP). In Section 3, a hybrid method for solving the CBP is described. Section 4 presents a numerical example that illustrates the performance of the proposed hybrid method.

2. THE CAPITAL BUDGETING PROBLEM

2.1. INTRODUCTION

The problem of building an effective capital budget is divided here into two models – the portfolio selection model (PSM) and portfolio evaluation model (PEM). The purpose of the first model is to find an efficient portfolio of investment projects. Whereas the second model is used to determine efficiency parameters for the set of investment projects obtained from the PSM model. The PSM is a multi-criteria binary linear programming model, while PEM is a non-linear programming model combined with stochastic simulation.

2.2. PORTFOLIO SELECTION MODEL

Let us consider a company in which there are \( m \) potential interdependent projects available for realization. Let us also assume, that each project creates new or modifies existing process steps within a primary production process. A portfolio of investment projects is defined as \((y_1, \ldots, y_w)\), where \( y_i = 1 \) if project \( i \) (\( i \in W = \{1, \ldots, m\} \)) is selected for realization, and 0 otherwise. Let \( \text{fin}(y_1, \ldots, y_w) \) denotes a financial evaluation parameter for a given portfolio of investments. The performance of the selected portfolio is measured by the expected value \( E(\text{fin}(y_1, \ldots, y_w)) \) and the standard deviation
\( \sigma(\text{fin}(y_1, \ldots, y_w)) \). Then, the problem of selecting the efficient portfolio of investment projects can be defined as follows.

Find \((y_1, \ldots, y_w)\) that maximizes the expected value of \(\text{fin}(y_1, \ldots, y_w)\) and minimizes the standard deviation of \(\text{fin}(y_1, \ldots, y_w)\).

In order to solve the PSM problem, the PEM model must be first invoked for each portfolio of investment projects in order to obtain the respective value of the financial evaluation parameter.

### 2.3. PORTFOLIO EVALUATION MODEL

The value of financial evaluation parameter of a portfolio of investment projects is estimated by solving a model which consists of two groups of equations. The first group describes balances of a company’s manufacturing capacities and material balances, and the second one consists of financial equations.

Equations of manufacturing capacities balance for primary production process are the following:

\[
\sum_{i \in I} X_{ijw}^{rt} \leq v_{jw}^{*} \cdot \Delta_{jw}^{*} \quad \text{for} \quad \tau = 0, 1, \ldots, \bar{\tau}, \tau \leq t, j \in J, w \in W_j, t = \tau, \tau + 1, \ldots, \tau + \bar{t}_{jw}
\]

\[
X_{ijw}^{rt} \geq 0, \quad \varsigma = t - \tau,
\]

\[
\Delta_{jw}^{*} = \begin{cases} 
1, & \text{for } w \in \bar{W} \\
0, & \text{for } w \in W - \bar{W}.
\end{cases}
\]

\[
\kappa(\bar{W}) = 1,
\]

\[
\eta^{*}(\bar{W}) \leq \bar{\eta}^{*}, \quad \text{for} \quad \tau = 0, 1, \ldots, \bar{\tau}
\]

whereas the equations for the enterprise material balance are as follows:

\[
\sum_{j \in J} \sum_{w \in W_j} \sum_{r=1}^{\bar{r}} X_{ijw}^{rt} - \sum_{j \in J} \sum_{w \in W_j} \sum_{z \in I} \sum_{r=1}^{\bar{r}} m_{izjw} X_{izw}^{rt} = G_i^{t} \quad \text{for} \quad t = 0, 1, 2, \ldots, T,
\]

\[
G_i^{t} \leq \bar{G}_i^{t}(\bar{W}) \quad \text{for} \quad t = 0, 1, 2, \ldots, T,
\]
where:

\( X'_{i,jw} \) – variable determining the quantity of the gross output of the product \( i \) produced in the department \( j \) in year \( t \), in case of qualifying the project \( w \) for the realization in year \( \tau \),

\( \tau \) – capital budgeting period

\( T \) – time horizon of the optimization

\( G_i^t \) – variable determining the size of sale of the product \( i \) in year \( t \),

\( I \) – set of product indexes,

\( I_j \) – set of indexes of products produced in the department \( j \),

\( W \) – set of project indexes,

\( W_j \) – set of indexes of projects connected with the department \( j \),

\( \bar{W} \) – set of indexes of projects qualified to realization,

\( J \) – set of primary production department indexes

\( v_{jw}^\zeta \) – manufacturing capacity of the department \( j \) after realization of the project \( w \) in \( \zeta \) year of the duration,

\( \bar{\eta}^\tau \) – limit of investment outlays in the year \( \tau \),

\( m_{izjw} \) – consumption per unit of the product \( i \) for producing the product \( z \) in the department \( j \) after realizing the project \( w \),

\( \bar{t}_{jw} \) – duration of the project \( w \) being realized in the department \( j \)

\( c_i^t \) – selling price for the product \( i \) in year \( t \)

\( r_d \) – long-term interest rate

\( r_k \) – short-term interest rate

\( \vec{k} : 2^W \rightarrow \{0,1\} \) – function determining sets of projects being possible for the realization, value 1 means a set possible for the realization, value 0 means set impossible for the realization,

\( \eta^\tau : 2^W \rightarrow R \) – function assigning to \( \bar{W} \) set of the projects an investment outlay for realization of this set in \( \tau \) year of capital budgeting period

\( \bar{g}_i^t : 2^W \rightarrow R \) – function assigning to \( \bar{W} \) set of the projects possible sale of the product in the \( t \) year

The set of financial equations express commonly known dependencies such as balance sheet, P&L account and net cash flows (NCF). Their detailed presentation would considerably increase the volume of the article, therefore, they are omitted.
These two groups of equations are constraints and the goal function is defined as the financial efficiency parameter $\text{EffPar}$. Usually, the net present value ($\text{NPV}$) is used as a financial evaluation parameter. In this article, a company's gross profit ($\text{GP}$) is used instead. This approach simplifies the model and allows us to focus on presenting a hybrid method for processing possibilistic and probabilistic variables. The company gross profit ($\text{GP}$) is presented below:

$$\text{GP}^t = \sum_{j=J} \sum_{i=I} c^t_i G^t_i - \sum_{r=1}^T \sum_{w \in W} \sum_{j=J} \sum_{i=I} k^t_{ijw} X^t_{ijw} - r^t_s \text{STC}^t - r^t_d \text{LTC}^t - \chi^t(\bar{W}) + \xi^t(\bar{W})$$  \hspace{0.5cm} (8)

where:

$\text{STC}^t$ – variable determining the value of short-term credit in year $t$,

$\text{LTC}^t$ – variable determining the value of long-term credit in year $t$,

$\text{GP}^t$ – variable determining the gross profit in year $t$,

$k^t_{ijw}$ – cost of processing the product $i$ by the department $j$ after realization of the project $w$ in $\zeta$ year of the duration

$\chi^t(\bar{W})$ – function assigning a company's fixed costs without amortization in year $t$ to a project portfolio

$\xi^t(\bar{W})$ – function assigning the value of amortization in year $t$ to a project portfolio

3. SOLUTION METHOD

3.1. DESCRIPTION OF UNCERTAINTY IN CAPITAL BUDGETING PROBLEM

Risk analysis recognizes two types of uncertainty – aleatory and epistemic. The aleatory uncertainty is due to variability or randomness, whereas the epistemic uncertainty comes from the ignorance or the lack of knowledge. In the case of economic calculus, data may come from a variety of sources, and therefore it is usually heterogeneous, i.e., both random and imprecise. The most common situation in practice is when for some parameters it is possible to determine probability distributions (there is a sufficient enough amount of historical data), while some information is available in the form of possibility distributions (obtained from subjective assessments of phenomena made by experts) [2,7,8]. These two methods for description of uncertainty of parameters of economic calculus are used usually as alternatives. There are few studies which describe the use of hybrid data – data partially described by probability distributions, and partially by possibility
distributions. The use of such (hybrid) data allows to reflect more properly the knowledge on parameters of economic calculus [1,3,4,7].

The most common framework for representing and reasoning with uncertain knowledge is the Dempster–Shafer (D–S) theory of evidence. The D–S theory allows to treat variability and imprecision together in single framework. This theory is based on D–S probability mass structures. A D–S structure is a mass function which is much the same like a discrete probability distribution except that probability is attached to a set of values (intervals) instead of single points. Dempster–Shafer theory is widely used by many authors (Guyonnet et al. [4], Ferson [3], Baudrit [1] Cooper et al. [11] and others) to develop techniques of uncertainty propagation. On the other hand, many authors present methods for calculating measures of risks (for example the standard deviation of NPV) on the basis of the D–S theory, random fuzzy set theory and credibility theory [6].

3.2. SOLUTION OF PORTFOLIO EVALUATION MODEL

To solve the Portfolio Evaluation Model (PEM) it has been decided to use the hybrid propagation method proposed by Baudritt et al. [1]. The hybrid propagation method combines a Monte Carlo simulation with the extension principle of the fuzzy set theory.

Consider the PEM model which measures efficiency of a given portfolio of projects \(y_1, \ldots, y_m\). PEM is a linear programming model with \(p\) parameters. These parameters can be divided into two groups based on the type of uncertainty attached to them. The first group consists of probabilistic parameters and the second one of possibilistic parameters. In the model considered in this paper, probability distributions are used to describe demand and selling prices, whereas material consumption and product prices are described by possibilistic variables.

The overall computational procedure can be summarized as follows. In order to solve the PEM model, possibility distributions are first divided into a set of intervals using the \(\alpha\)-cuts approach. For each interval (\(\alpha\)-level) a Monte Carlo sampling of random variables is performed taking into account dependencies between those variables. The dependencies are processed using a method presented by Yang [10], which employs the Cholesky decomposition of the correlation matrix. The Cholesky decomposition is commonly used in Monte Carlo-based methods for simulating systems with multiple correlated variables. Obtained realizations of random variables are put into the PEM model. Moreover, for each \(\alpha\)-level, interval parameters are converted into variables in the PEM model and the following constraint is added to the model for each converted parameter:

\[
\inf \left( X_i \right)_\alpha \leq x_i \leq \sup \left( X_i \right)_\alpha ,
\] (9)
where $\inf(\tilde{X}_i)_{\alpha}$, $\sup(\tilde{X}_i)_{\alpha}$ are respectively lower and upper bounds of the respective $\alpha$-level of a fuzzy parameter $\tilde{X}_i$.

Dependencies between fuzzy parameters are described by means of interval regression. Interval regression parameters are reflected by two additional constraints:

$$x_i \geq \inf\left(a_i^{\varepsilon}\right) \cdot x_z + \inf\left(a_z^{\varepsilon}\right),$$

(10)

$$x_i \leq \sup\left(a_i^{\varepsilon}\right) \cdot x_z + \sup\left(a_z^{\varepsilon}\right),$$

(11)

where $\sup\left(a_i^{\varepsilon}\right)$, $\inf\left(a_i^{\varepsilon}\right)$, $\sup\left(a_z^{\varepsilon}\right)$, $\inf\left(a_z^{\varepsilon}\right)$ are respectively lower and upper bounds of the interval regression coefficients describing the dependency between parameters $\tilde{X}_z$ and $\tilde{X}_i$.

Next, in order to determine the lower and upper bounds of the respective $\alpha$-level of the financial efficiency parameter, the following constrained optimization problems must be solved:

$$EffPar_{\alpha} \rightarrow \min$$

(12)

for the definition of the lower bound of the $\alpha$-level of the $EffPar$, and

$$EffPar_{\alpha} \rightarrow \max$$

(13)

for the definition of the upper bound of the $\alpha$-level of the $EffPar$.

The optimization problems (12) and (13) are solved for each $\alpha$-cut and as a result a fuzzy number $\mu_{EffPar}$ is found. Drawing probabilistic values and determining a realization of $\mu_{EffPar}$ is repeated $\bar{n}$ times. As a result, $\bar{n}$ a family of fuzzy numbers $(\mu_{EffPar}^1, \ldots, \mu_{EffPar}^\bar{n})$ are obtained. Based on the vector $(\mu_{EffPar}^1, \ldots, \mu_{EffPar}^\bar{n})$, the mean value and standard deviation of $fin(y_1, \ldots, y_w)$ are calculated.

3.3. SOLUTION OF PORTFOLIO SELECTION MODEL

The hybrid method for uncertainty propagation and fuzzy simulation described above is used to obtain the mean value and standard deviation of a financial efficient parameter for a given portfolio of project. To find an efficient project portfolio, the $PSM$ model is used. To solve this model, a lexicographic approach is used. As a result, a set of Pareto-optimal solutions is obtained.
4. NUMERICAL EXAMPLE

The capital budget was determined for the production process presented in Fig. 1. This setup includes the production cycle in a steel industry, starting from the pig iron production, through the production of steel, hot rolling products to the production of products coated with metal and plastic.

Five investment projects are taken into consideration: steel making plant, hot rolled sheet mill, cold-rolled sheet mill, hot-dip galvanizing sheet plant and sheet organic coating plant. In Fig. 1 they are denoted with the suffix “-project”. The gross profit (12) is used as a measure of efficiency of a portfolio of investment projects.

Decision variables for the estimation of the efficiency and risk of investment projects in the case of the iron metallurgy are the following: quantity and selling prices, costs of materials and quantity of investment outlays. The model parameters such as quantity of sale for each of products ranges being produced by the company, prices of these products, prices of metallurgic raw materials (prices of iron ores and the pellets), consumption per unit indexes and quantity of investment outlays are assumed to be uncertain, which makes the model closer to reality. The remaining parameters of the efficiency calculus are assumed to be deterministic. Moreover, it is assumed prices of individual assortment of metallurgical products and metallurgical raw materials are correlated. The same concerns sale quantities of each assortment of metallurgical products. These dependencies are taken into account when processing the values of uncertain parameters of the efficiency calculus.

Table 1. Trapezoidal fuzzy numbers (TFN) indicating material consumption

<table>
<thead>
<tr>
<th>Material consumption</th>
<th>TFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel half-products – molten iron</td>
<td>(0.855; 0.860; 0.870; 0.875)</td>
</tr>
<tr>
<td>half-products – hot rolled steel sheets</td>
<td>(1.058; 1.064; 1.075; 1.078)</td>
</tr>
<tr>
<td>hot rolled steel sheets – cold rolled sheets</td>
<td>(1.105; 1.111; 1.124; 1.130)</td>
</tr>
<tr>
<td>cold rolled sheets – dip galvanized sheets</td>
<td>(1.010; 1.020; 1.026; 1.031)</td>
</tr>
<tr>
<td>dip galvanized sheets – organic coated sheets</td>
<td>(0.998; 0.999; 1.000; 1.001)</td>
</tr>
</tbody>
</table>
Material consumption as well as product and half-product prices are given in the form of fuzzy numbers. They are presented, respectively, in Table 1 and Table 2. Sale parameters are described by normal probability distributions given in Table 3. The Cholesky matrix which is used to process dependencies between sale parameters is the following:

\[
\begin{pmatrix}
1.00000 & 0.87786 & 0.91142 & 0.86321 \\
0.00000 & 0.47891 & 0.24007 & 0.27276 \\
0.00000 & 0.00000 & 0.33418 & 0.34165 \\
0.00000 & 0.00000 & 0.00000 & 0.25249
\end{pmatrix}
\]

In this computational example, the number of \( \alpha \)-levels of fuzzy variables is set at 10 and the number of simulation is set to 100. The results obtained are presented in Fig. 2.
5. CONCLUSION

This paper presents a new method for selecting an effective portfolio of investment project. The presented concept of the mathematical model and numerical algorithm make it possible to generate a set of Pareto optimal solutions. These solutions are different variations of a company's acceptable capital budgets along with the estimation of their effectiveness (expected value of financial evaluation parameter) and risk (standard deviation of financial evaluation parameter). The method allows to take into account statistical as well as economic dependencies between projects. It also allows for flexible definition of uncertainty of the parameters using probability distribution or fuzzy numbers.

REFERENCES


