STUDY ON PREDICTION METHODS FOR DYNAMIC SYSTEMS OF NONLINEAR CHAOTIC TIME SERIES *

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Abstract: The prediction methods for nonlinear dynamic systems which are decided by chaotic time series are mainly studied as well as structures of nonlinear self-related chaotic models and their dimensions. By combining neural networks and wavelet theories, the structures of wavelet transform neural networks were studied and also a wavelet neural networks learning method was given. Based on wavelet networks, a new method for parameter identification was suggested, which can be used selectively to extract different scales of frequency and time in time series in order to realize prediction of tendencies or details of original time series. Through pre-treatment and comparison of results before and after the treatment, several useful conclusions are reached: High accurate identification can be guaranteed by applying wavelet networks to identify parameters of self-related chaotic models and more valid prediction of the chaotic time series including noise can be achieved accordingly.

Key words: nonlinear self-related chaotic model; wavelet neural network; parameter identification; time series prediction

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Introduction

Development in modeling, predicting and macro controlling of chaotic time series has gained good results in the past decades. In process of studying chaotic time series modeling and predicting technology, scholars overseas and domestic have done a lot of fruitful researches, put forward enormous valuable and applicable models and modeling methods\(^1\)\(^{11} \sim 12\). Shi Zhong-zi and his colleague have been engaging in study of nonlinear time series by neural networks and achieved certain positive results, so have Liang Yue-cao\(^1\), Zhang Qing-hua\(^2\) but by applying

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theory of wavelet. E. Castillo, J. M. Gutierrez, Kevin Judd and Alistair, Christian G. Shroer have studied on chaotic time series modeling and predicting, they have obtained several valuable results. The answer of which specific nonlinear model or method shall be chosen is subject to results of practical background analysis.

Theory of wavelet analysis has been established in recent decades. It is especially useful in treatment of nonlinear chaotic phenomenon and a kind of excellent tool for time frequency analysis whether it is a time or frequency domain/field. The theory can give good local representation of signals and particularly fits for signal analysis and characteristic extraction. It just suits the conditions of intrinsic chaotic data including noise and data showing chaotic characteristics. Results of researches indicate that prediction of chaotic time series (with noise) by nonlinear self-related chaotic models is comparably effective.

I Structure of Nonlinear Self-Related Chaotic Model

Resume: \[ y_i \ (i = 1, 2, 3, \ldots, N) \] is the time series under our observation, phase randomization proves it chaotic

\[ x_n = G[ x_{n-1}, x_{n-2}, \ldots, x_{n-M} ] + k\epsilon_n, \] (1)

where \( G \) is nonlinear function, \( M \) is optimum embedding dimension. The following nonlinear self-related chaotic model is obtained:

\[ x_n = \sum_{m=0}^{M} \alpha_m P_m(n) + \epsilon_n. \] (2)

For \( P_0(n) = 1, m \geq 0, P_m(n) = x_{n-i_m}, x_{n-i_m}, \ldots, x_{n-i_m}, \{i_m\} \) are determined integers. In Eq. (2), the order is \( \max_{m_j}(i_m) \).

Parameters in Eq. (2) are computed with algorithm of fast perpendicularity and wavelet net works. Let

\[ x_n = \sum_{m=0}^{M} g_m W_m(n) + k\epsilon_n, \] (3)

where \( W_m(n) \) are reconstructive \( P_m(n) \) orthogonal functions, and \( W_0(n) = 1, g_0 = x \), spread coefficient \( g_m \) is decided by object function (4)

\[ J = \sum_{n=0}^{N} \left( x_n - \sum_{m=0}^{M} g_m W_m(n) \right)^2. \] (4)

Let

\[ W_m(n) = P_m(n) - \sum_{r=0}^{m-1} \alpha_{mr} W_r(n), \] (5)

where

\[ \alpha_{mr} = \frac{\sum_{n=0}^{N} P_m(n)}{\sum_{n=0}^{N} W_r(n)^2}, \quad g_m = \frac{\sum_{n=0}^{N} W_m(n)}{\sum_{n=0}^{N} (W_m(n))^2}, \] (6)

\[ \alpha_m = \sum_{i=m}^{M} \alpha_i V_i, \quad V_m = 1, \quad V_i = - \sum_{r=m}^{i-1} \alpha_{ir} V_r \quad (i = m + 1, m + 2, \ldots, M). \] (7)

Let

\[ Q(m) = g_m^2 \sum_{n=0}^{N} W_m^2(n). \] (8)
Order of the model is decided by Eq. (9)

\[
Q(M) \left[ \sum_{n=0}^{N} v_n^2 - \sum_{m=0}^{M-1} \sum_{n=0}^{M} D(m, m) \right]^{1/2} > \frac{2}{\sqrt{N}}.
\]  

In Eq. (4) ,

\[
J = \sum_{n=0}^{N} x_n - \sum_{m=0}^{M} \sum_{n=0}^{M} W_m(n)^2 = f(\alpha_{1,0}, \alpha_{2,0}, \alpha_{2,1}, \ldots, \alpha_{m,m-1}, \ldots, \alpha_{M,M-1}).
\]  

Process of identifying parameters \( \alpha_{m,m-1}, m = 1, 2, \ldots, M \), of the model with wavelet neural network as follows.

2 Structure of the Neural Network

Assume \( \Psi(\cdot) \) satisfies admissible condition:

\[
\int_{R} (|\Psi(w)| / |w|) \, dw < \infty,
\]

then countable set \( \Phi = \{ \sqrt{a_k} \Psi(a_k x - b_k) : a_k \in \mathbb{R}, b_k \in \mathbb{R}, k \in \mathbb{Z} \} \) satisfies frame property, namely, existing two constants \( A > 0 \) and \( B < \infty \), for any arbitrary \( f \in L^2(\mathbb{R}) \), the under is obtained

\[
A \| f \|^2 \leq \sum_{\Phi} \| f, \varphi \|^2 \leq B \| f \|^2.
\]  

This indicates frame \( \Phi \) is dense in \( L^2(\mathbb{R}) \), namely, set of all the linear combination of elements is frame \( \Phi \)

\[
g(x) = \sum_{i=1}^{L} w_i \sqrt{a_i} \Psi(a_i x - b_i) = \sum_{i=1}^{L} w_i \sqrt{a_i} D_i(t - t_i) f + \bar{g},
\]  

where \( t_i \) is a translation vector, \( a_i \) is an arbitrary magnification parameter, \( b_i \) is an arbitrary translation parameter. \( D_i \) is diagonal matrix, \( D_i = \text{diag} \{ d_i \} \), \( \bar{g} \) is evaluation of means of function \( g(x) \). Eq. (12) is called wavelet network. Where \( L \) is given the value of the optimum embedding dimension. The wavelet function \( \Psi \) is product of several “Mexico hut”

\[
w(s) = (1 - s^2)e^{-s^2/2},
\]  

where in the wavelet network, see Fig.1, \( w_i, a_i, b_i \) are all adjustable parameters, so this can make the network learn the nonlinear functions more flexibly in order to satisfy higher precision.

Computation shows optimum embedding dimension for the chaotic time series \( x_i (i = 1, 2, 3, \ldots, N) \) is \( m \), let \( L = m \) in Eq. (12),

\[
a = \min \{ x(i), 1 \leq i \leq N \}, b = \max \{ x(i), 1 \leq i \leq N \}, \bar{g} = \sum_{i=1}^{N} x(i)/N, \text{ then we choose wavelet function } \Psi \text{ as}
\]

\[
\Psi(x) = w(x_1) w(x_2) \ldots w(x_m).
\]  

For \( m = 2 \),
\[
\begin{align*}
  t_1(1) &= t_1(2) = (b + a)/2, \\
  d_1(1) &= d_1(2) = 2/(b - a), \\
  D_1 &= \text{diag} \{ d_1(1), d_1(2) \}, \\
  d_2(1) &= d_2(2) = 2^2/(b - a), \\
  D_2 &= \text{diag} \{ d_2(1), d_2(2) \}, \\
  d_3(1) &= d_3(2) = 2^3/(b - a), \\
  D_3 &= \text{diag} \{ d_3(1), d_3(2) \}.
\end{align*}
\]

3 Learning Algorithm of Wavelet Neural Network

Assume there are \( M \) learning sample pairs \( (x_m, y_m) : y_m = f(x_m), m = 1, 2, \ldots, M \), let \( \theta \) be the vector of all parameters of \( w_i, a, \) and \( b_i (i = 1, 2, \ldots, N) \), \( g(\theta) \) is the expression of Eq. (12), objective function of the network is

\[
E(\theta) = \frac{1}{2} \sum_{m=1}^{M} (g(\theta(x_m)) - y_m)^2.
\]

By algorithm of gradient descent to minimize the above objective function, the optimum parameter vector \( \theta^* \) can be gained, process of iteration,

\[
\theta(k+1) = \theta(k) - \lambda \nabla E(\theta(k)),
\]

where \( \lambda > 0 \) is the learning step, \( \nabla E(\theta) \) is the gradient of the function \( E(\theta) \) to \( \theta \), namely,

\[
\nabla E(\theta) = \sum_{m=1}^{M} (g(\theta(x_m)) - y_m) \cdot \frac{\partial}{\partial \theta} g(\theta(x_m)).
\]

To speed up convergence, comparably optimum length of learning step shall be chosen. Simulation experiment shows wavelet neural network can increase approximate precision a great deal in contrast with traditional feed forward neural network.

4 Pretreatment of Data

In order to build model based on dynamic data and identify its parameters with wavelet neural networks, it is necessary to treat the data in advance:

\[
x_i = (x_i - \bar{x})/\sigma (i = 0, 1, 2, \ldots, N).\]

5 Results

The Henon map and two groups of experimental data from nonlinear mechanical vibration model, built by Tianjin University are chosen to test the algorithm discussed in this article. According to Refs. [10, 13], two groups of data are proven chaotic. Comparison of parameters in model of Henon map, which are obtained with algorithm given by this article and wavelet is listed in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>Model coefficient</th>
<th>( y_{n-1} )</th>
<th>( y_{n-1}^2 )</th>
<th>( y_{n-2} )</th>
<th>( y_{n-2}^2 )</th>
<th>( y_{n-3} )</th>
<th>( y_{n-3}^2 )</th>
<th>( y_{n-4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model coefficient</td>
<td>3.168</td>
<td>-1.000</td>
<td>0.300</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Algorithms of this article</td>
<td>3.168</td>
<td>-1.000</td>
<td>0.300</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Algorithm of wavelet</td>
<td>3.168</td>
<td>-1.000</td>
<td>0.300</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Time course graph of the first group experimental data is shown in Fig. 2, related phase plane plot in Fig. 3.
Table 2 Comparison among model coefficients of Henon map, parameters estimated with the foresaid two methods in case 2% white noise added to the Henon map

<table>
<thead>
<tr>
<th>Model</th>
<th>$y_{n-1}$</th>
<th>$y_{n-1}^2$</th>
<th>$y_{n-2}$</th>
<th>$y_{n-2}^2$</th>
<th>$y_{n-3}$</th>
<th>$y_{n-3}^2$</th>
<th>$y_{n-4}$</th>
<th>$y_{n-4}^2$</th>
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<tbody>
<tr>
<td>Model coefficient</td>
<td>3.168</td>
<td>-1.000</td>
<td>0.300</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Algorithms of this article</td>
<td>2.869</td>
<td>-0.953</td>
<td>0.293</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Algorithm of wavelet</td>
<td>2.975</td>
<td>-0.980</td>
<td>0.294</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Time course graph of the second group experimental data is shown in Fig. 4. Its $x\,y$, $x\,z$, $y\,z$ and $x\,y\,z$ phase plane plots, are shown in Fig. 5, Fig. 6, Fig. 7, and Fig. 8, respectively.

Fig. 2 Time course graph of the first group chaotic experimental data

Fig. 3 Phase plane plot of the first group chaotic experimental data

Fig. 4 Time course graph of the second group chaotic experimental data

Fig. 5 $x\,y$ phase plane plot of the second group chaotic experimental data

Fig. 6 $x\,z$ phase plane plot of the second group chaotic experimental data
Depending on results computed from Eq. (9), the optimum embedding dimensions for the above experimental data are all 5, so we can choose 5 as the order for the Eq. (2).

Among the experimental data, 60 data are selected as sample points, with Eq. (2), 5 being its order, and parameters estimated with wavelet neural networks, we obtain the mean and standard variance of the first group data: the mean, \( \bar{x} = -7.7333 \); standard variance, \( \sigma = 6.406741 \) as well as the model is

\[
\frac{X_{t+n} - \bar{x}}{\sigma} = -3.2223 317X_{t+n-1} + 2.813152X_{t+n-2} + 0.2659X_{t+n-3} + 0.07835X_{t+n-4} + 7.740718X_{t+n-5} + 11.5825X^2_{t+n-1} - 11.7473X_{t+n-1}X_{t+n-2} - 2.1928X_{t+n-1}X_{t+n-3} + 0.1796X_{t+n-1}X_{t+n-4} - 6.6741X^2_{t+n-5}.
\]

For the second group data: the mean, \( \bar{x} = 0.0442 \); standard variance, \( \sigma = 0.097537 \), as well as the model is

\[
\frac{X_{t+n} - \bar{x}}{\sigma} = 1.1166 6X_{t+n-1} - 0.441X_{t+n-2} + 0.6004X_{t+n-3} - 0.5741X_{t+n-4} + 0.12737X_{t+n-5} - 3.1028X^3_{t+n-1} + 4.5257X_{t+n-1}X_{t+n-2} - 6.9259X_{t+n-1}X_{t+n-3} + 9.3423X_{t+n-1}X_{t+n-4} + 0.2018X^2_{t+n-1}X_{t+n-5}.
\]

6 Conclusions

Establishing nonlinear self-related chaotic model on dynamic data, following the example given in this article, pre-treatment of data is needed. Model parameters estimated with wavelet neural networks algorithm has good identification grade. In contrast, the two algorithms are much consistent with each other in computing speed, precision of parameter identification in case of white noise the result also keeps.

Computation shows: For the first and second groups of chaotic data, the established chaotic models have good short-term prediction, and the length of prediction is no more than 10 points, otherwise relative error will be very large. Further prediction needs more experimental chaotic data to renew model parameters.

The order of the chaotic model decided by Eq. (9) is comparably accurate.
The time for computing value of the predicted points is usually longer than the time experienced by time series practically. This is also a difficult point which shall be resolved in the future study.

References:


