Fractal dimensions of cohesive sediment during settling in steady state flow with different initial sediment concentrations

M. Stone and B.G. Krishnappan

Abstract: Morphology of particle populations of cohesive sediment were examined during settling experiments in an annular flume with different initial sediment concentrations (200 and 350 mg/L) at constant bed shear stress (0.121 N/m^2) using fractal dimensions. The area, longest axis, and perimeter of suspended solids were measured with light microscopy and an image-analysis system to determine three fractal dimensions (D, D_1, D_2). The ratio between the initial and steady state (time T = 300 min) sediment concentration was 0.54 for both experimental runs and is a function of bed shear stress, not the initial sediment concentration. The fractal dimension D changed from 1.32 at the start of the experiment to 1.36 at steady state, which represents an increase in shape irregularity of larger particles over time compared with smaller particles. At steady state, D_1 and D_2 were 1.19 and 1.66, respectively. Small increases in D_1 and D_2 over time indicated a change in morphology towards longer and more elongated particles. The D_2 measurements in the present study indicate that differential sedimentation is the predominant flocculation mechanism of cohesive sediments in the flume settling experiments. Fractal dimensions of suspended solids were not significantly different at steady state as a function of initial sediment concentration.

Key words: particle morphology, fractal dimensions, cohesive sediment, flocculation, deposition, annular flume.

Résumé : La morphologie des particules dans des sédiments cohésifs a été examinée durant les expériences de sédimentation dans un panache annulaire avec différentes concentrations initiales de sédiments (200 mg/L^{-1} et 350 mg/L^{-1}) et une contrainte de cisaillement du lit constante (0,121 N/m^{-2}) en utilisant des dimensions fractales. La superficie, le plus long axe et le périmètre des solides en suspension ont été mesurés par microscopie optique et par un système d'analyse d'images afin de déterminer trois dimensions fractales (D, D_1, D_2). Le rapport entre la concentration des sédiments à l'état initial et celle à l'état stable (T = 300 min) était de 0,54 pour les deux essais expérimentaux et il est fonction de la contrainte de cisaillement dans le lit, non pas de la concentration initiale des sédiments. La dimension fractale D a changée de 1,32 au départ de l'expérience, à 1,36 à l'état stable, ce qui représente une augmentation de l'irrégularité de la forme des grosses particules dans le temps par rapport aux plus petites particules. À l'état stable, D_1 et D_2 étaient de 1,19 et de 1,66 respectivement. Des petites augmentations de D_1 et D_2 dans le temps indiquent un changement dans la morphologie, vers des particules plus longues et élancées. Les mesures D_2 de la présente étude indiquent que la sédimentation différentielle est le mécanisme de floculation prédominant des sédiments cohésifs dans les expériences de sédimentation du panache. Les dimensions fractales des solides en suspension n'étaient pas significativement différentes à l'état stable en tant que fonction de la concentration initiale de sédiments.

Mots clés : morphologie des particules, dimensions fractales, sédiment cohésif, floculation, déposition, panache annulaire.

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Introduction

In aquatic systems, physical and electrochemical forces and biological factors cause cohesive materials to form flocs (Droppo et al. 1997) that settle at rates different from those of their constituent primary particles (Lau and Krishnappan 1992). Flocs have relatively low densities, large pore spaces, and reactive surfaces that remove contaminants from the water column (Droppo et al. 2000). Consequently, flocculation influences the transport behaviour of suspended particles in aquatic systems (Lick et al. 1993; Krishnappan 2000), and knowledge of how cohesive sediments settle and deposit is necessary to model the pathways and fate of sediment-associated contaminants (Ongley et al. 1992).

Studies on the settling of cohesive sediment suspensions in turbulent flows show that all initially suspended materials deposit on the bed if the shear stress is less than a critical

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value, defined as the critical shear stress for deposition (Lau 1996). For a given steady state bed shear stress above this critical condition, a portion of the material will settle out, but the remainder will stay in suspension. The fraction of solids remaining in suspension is referred to as the steady state sediment concentration. For a given sediment type, the steady state sediment concentration was found to be a function of the bed shear stress and the initial sediment concentration (Partheniades and Kennedy 1967; Lau and Krishnappan 1992). Stone and Krishnappan (2003) investigated the influence of bed shear stress on the size distribution and morphology of cohesive sediment at steady state flow. They reported that small particle clusters (micro-flocs) are the building blocks of larger flocs, and the stability of flocs is a function of shear stress, which influences floc formation and breakup. In the present paper, the influence of the initial concentration on floc morphology is examined.

Fractal dimensions have been used to quantify the morphology of particle populations formed in different fluid mechanical (Li and Ganczarczyk 1989; Jiang and Logan 1991; Logan and Kilps 1995), stream (De Boer 1997; De Boer et al. 2000), and marine environments (Logan and Wilkinson 1990, 1991; Kilps et al. 1994). Fractal dimensions reflect the nature of particles and their mechanism of formation (Meakin 1989). Particle properties such as settling velocity and density are a function of their fractal dimensions (Logan and Kilps 1995). The objective of this study is to determine fractal dimensions of particle populations of cohesive sediment during settling experiments in an annular flume with different initial sediment concentrations at a particular bed shear stress.

Methods

Theoretical

The application of fractals and fractal dimensions (Mandelbrot 1983) to the analysis of the geometrical properties of aggregates has been described in detail (Jiang and Logan 1991; Logan and Kilps 1995). Relevant aspects of the fractal approach to this paper are reviewed in the following.

Fractal dimensions relate aggregate size to some property in n dimensions, where n = 1, 2, 3, and D_n is the fractal dimension in n dimensions. Collections of similar, natural objects have area–perimeter relationships described by a power function (Logan and Kilps 1995):

$$[1] \qquad P \propto A^{D/2}$$

where P is the perimeter, A is the projected area, and D is the fractal dimension of the objects. For Euclidean objects, $D = 1$, but values of $D > 1$ have been found for individual flocs ($1.20 < D < 1.36$) and floc populations ($1.21 < D < 1.37$) in streams (De Boer et al. 2000). Consequently, as the area of the objects becomes larger, the perimeter increases more rapidly than that for Euclidian objects, indicating that the boundary of the objects is becoming more convoluted.

A one-dimensional fractal dimension can be calculated for all objects in a particle distribution as

$$[2] \qquad P \propto l^{D_1}$$

where l is the maximum particle length. Values of $D_1 > 1$ indicate that, with increasing object size, the perimeter increases faster than the object length scale, so the object becomes more complex for larger objects. The fractal dimension of aggregates in two dimensions is

$$[3] \qquad A \propto l^{D_2}$$

Values of $D_2 < 2$ indicate that, as the object size increases, the projected area increases slower than the square of the length scale. In this case, the projected area of larger objects is less than that of Euclidean objects of the same scale because of elongation of the larger objects or because the larger objects surround or partially surround regions that are not part of the object. The lower limit of D_2 for aggregates dominated by differential sedimentation is 1.60 (Jiang and Logan 1991).

Description of rotating flume

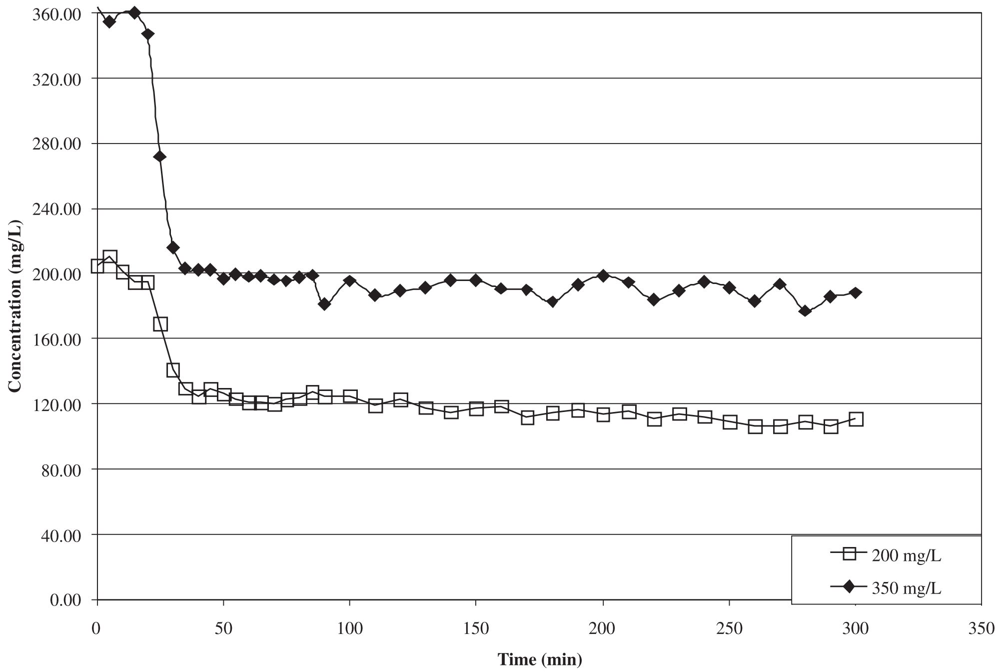
The settling experiments were conducted in a rotating circular flume 5.0 m in diameter located at the National Water Research Institute at Burlington, Ontario, Canada. The flume has a rectangular cross section measuring 0.30 m in width and 0.30 m in depth, and it rests on a rotating platform, which is 7.0 m in diameter. An annular cover plate (ring) fits inside the flume with a radial clearance of ~1.5 mm on either side. Complete details of the flume can be found in Krishnappan (1993). The characteristics of flows generated in this flume were studied both experimentally (Krishnappan 1993) and theoretically (Petersen and Krishnappan 1994). These studies showed that the flow field generated in the flume was two-dimensional with nearly constant bed shear stress across the width of the flume. The turbulence characteristics of the flows generated in the flume were quantified using the computational fluid dynamics model PHOENICS (Krishnappan et al. 1994; Krishnappan and Engel 2004).

The flume is equipped with a laser Doppler anemometer (LDA) that measures the flow field. Measurements made using this instrument were used to verify a three-dimensional numerical model of turbulent flows in rotating flume assemblies (Petersen and Krishnappan 1994; Krishnappan et al. 1994). A Malvern particle-size analyzer mounted on the flume measured the size distributions of the cohesive sediment directly in the water column. Sediment–water mixtures were drawn from a sampling port on the side of the flume, and suspended sediment concentrations were determined by the filtration method (Inland Waters Directorate 1988). The sampling point was at mid-depth in the centre of the flume. A single point measurement to represent the depth average concentration is justified because of the low settling velocity of the sediment. For such sediments, a near uniform concentration profile in the vertical direction was obtained theoretically by Dhamotharan et al. (1981) and experimentally by Fukuda and Lick (1980).

Experimental procedure

Suspended sediment and river water were collected from the Hay River, Northwest Territories, for the deposition tests. Filtered river water and sediment passing through a 70 µm mesh screen were added to the flume. Full mixing of the sediment was achieved by rotating the flume and the lid at high speeds, corresponding to a bed shear stress of 0.6 N/m^2. After 20 min, the speed was lowered to a bed shear stress of 0.121 N/m^2 and maintained for a period of 5 h. Two initial

Fig. 1. Sediment concentration as a function of time.



sediment concentrations (200 and 350 mg/L) were used for the deposition experiments. Sediment concentration was measured every 5 min, and the size distribution was determined every 2 min with the Malvern particle-size analyser.

Image analysis

Water samples were drawn from the sampling port of the flume at 30 min intervals, and a triplicate 5 mL aliquot was transferred by pipette into 50 mL settling columns containing prefiltered Hay River water and then passed through 0.45 µm Millipore HA filters at low vacuum. Digital images of particles deposited on the filters were collected using the method of De Boer and Stone (1999). In this procedure, filters are rendered (semi) transparent by applying droplets of Stephens Scientific low-viscosity immersion oil (n_D(23 °C) = 1.5150) to distinguish particles from the background (i.e., the filter). Light microscopy and an image-analysis system were used to determine the area, longest axis, and perimeter of particle populations. Particles are sized by image analysis to a lower resolution of 2 µm (20× objective) using a Zeiss Axiovert 100 microscope fitted with a Sony XC75 CCD camera connected to a Pentium computer running the Northern Exposure™ image-analysis software. Observation of particle morphology during and after oil immersion indicated that immersion oil did not resuspend particles or affect particle morphology. During image analysis, background subtraction was applied to minimize the impact of nonuniform light levels. All fractal dimensions were calculated from the slopes of regression lines of the relevant variables (eqs. [1]–[3]) on double-logarithm plots for a minimum of 4500 particles per sample.

Results and discussion

Sediment concentrations

During the two deposition experiments, the initial sediment concentrations (200 and 350 mg/L) decreased gradually to steady state concentrations of 110 and 190 mg/L, respectively. The ratio between the initial and steady state concentration is 0.54 for both experimental runs. Figure 1 shows that the steady state sediment concentrations for the two runs are different, indicating the steady state concentration is a function of bed shear stress and initial sediment concentration. Such behaviour is peculiar to cohesive sediments only. In the case of noncohesive materials such as sand, the steady state concentration is a function of bed shear stress only (Yalin 1972). Partheniades et al. (1969) argued that for cohesive sediments, only strong flocs capable of resisting high shear stress near the bed are deposited while more fragile flocs break up in the region of high shear and stay in suspension. Consequently, only a certain fraction of the sediment in suspension can form strong flocs, and the fraction remaining in suspension is a function of the initial sediment concentration (Lick 1982).

Representative photomicrographs of cohesive sediment at steady state (T = 300 min) show that primary particles, micro-flocs, and larger flocs are present in the water column for the two initial conditions (Fig. 2). Klimpel and Hogg (1986) proposed a multistage growth model of floc formation where the building units of larger flocs are micro-flocs. In this model, micro-flocs are formed at the initial stage of flocculation, and through random collisions due to fluid shear they combine to form larger flocs. Spicer and Pratsinis (1996) studied the effect of shear on coagulation and found that aggregates reach an equilibrium or steady state structure and floc size distribution. They argued that floc breakage was the main process responsible for maintaining a stable particle-size distribution, thus preventing further aggregate growth. Images of particles at steady state (Fig. 2) support the floc formation model proposed by Klimpel and Hogg (1986) and clearly show the presence of micro-flocs in suspension, and these micro-flocs are the building blocks of larger flocs.

Fig. 2. Photomicrographs of flocs at T = 300 min for initial sediment concentrations of (*a*) 200 mg/L and (*b*) 350 mg/L.

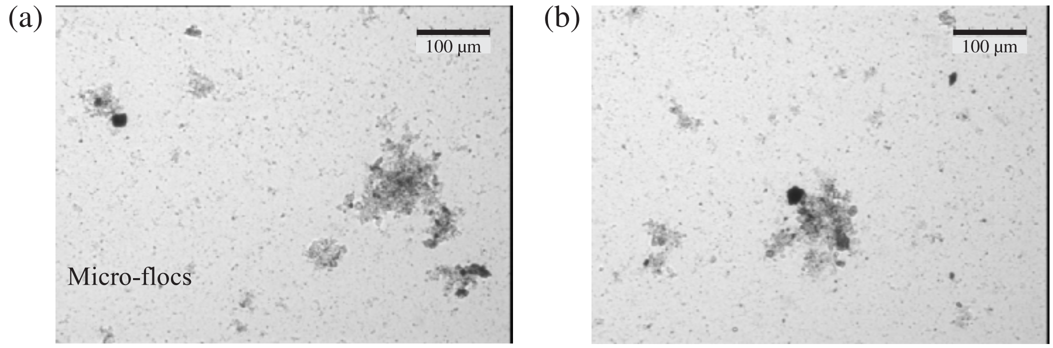
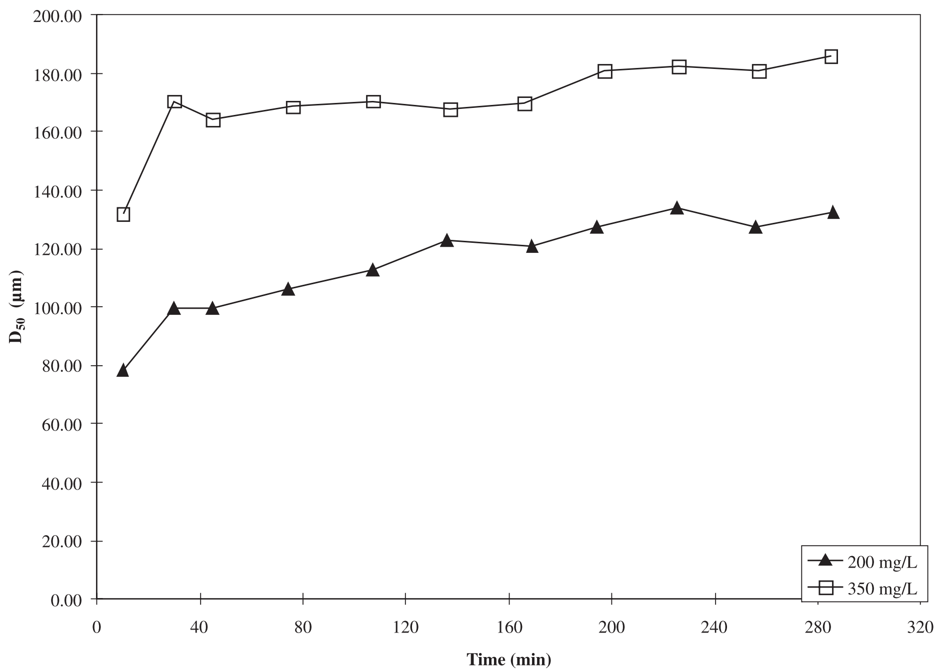


Fig. 3. D_{50} as a function of time.



Fractal dimensions as a function of time

The grain-size distributions of cohesive sediment measured as a function of time show that flocculation was a dominant process influencing the settling properties of sediment in the flume. Particle-size data for the two deposition experiments with different initial concentrations show that the median diameter (D_{50}) increased as a function of time (Fig. 3). As the flume speed was lowered from the initial condition to steady state, D_{50} continued to increase over time, indicating that larger flocs were being formed in suspension.

Fractal dimensions D, D_1, and D_2 changed slightly over the course of the two experiments, which suggests a change in the geometry of the particle populations (Figs. 4–6). The value of D changed from 1.32 at the beginning of the experiment to 1.36 at steady state, and D was generally higher for the experiment with an initial sediment concentration of 200 mg/L (Fig. 4). The increase in D represents a change in the particle shape over time and is interpreted as an increase in shape irregularity of larger particles compared to the smaller ones. The increase in shape irregularity is attributed to the formation of flocs in the flume. De Boer (1997) and De Boer and Stone (1999) reported similar D values for fluvial suspended sediment in Saskatchewan ($1.25 < D < 1.42$) and Ontario streams ($1.24 < D < 1.35$), respectively. They attributed changes in D to the effect of algal blooms in Saskatchewan and different sediment sources in Ontario streams, respectively.

D_1 and D_2 values were calculated as the slopes of the regression line for length and perimeter (eq. [2]) and length and area (eq. [3]), respectively. Values of D_1 ranged from 1.15 to 1.19, and values of D_2 ranged from 1.61 to 1.66 (Figs. 5, 6). The slight increases in D_1 and D_2 over time indicate a change in morphology towards more elongated particles. Similar ranges of D_2 values have been reported in the literature (Gorczyca and Ganczarczyk 1996; Namer and Ganczarczyk 1994; De Boer 1997).

Densely packed aggregates have higher fractal dimensions, whereas branched and loosely bound structures have lower fractal dimensions. Flocs become elongated as they grow, causing the collision frequency to increase and floc strength to decrease (Li and Logan 1997). Consequently, the trajectories of larger flocs become increasingly rectilinear and the dominant aggregation mechanism is differential sed-

Fig. 4. D as a function of time. Vertical bars denote ±1 standard deviation.

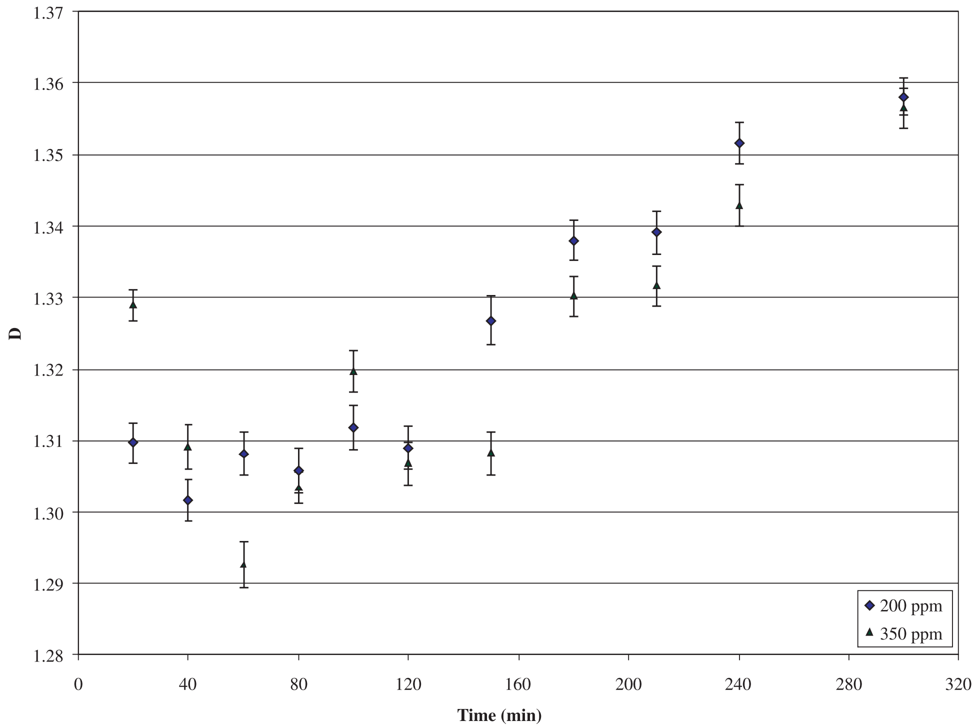
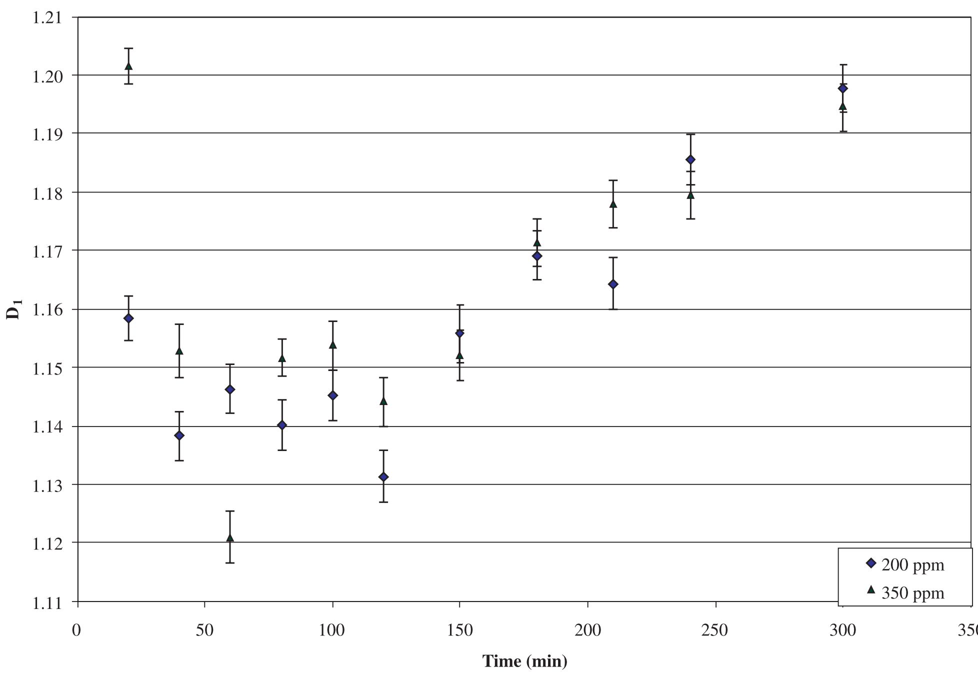


Fig. 5. D_1 as a function of time. Vertical bars denote ±1 standard deviation.



imentation (Thomas et al. 1999). The lower limit of D_2 for aggregates dominated by this mechanism is 1.60 (Jiang and Logan 1991). Based on the D_2 measurements in the present study, differential sedimentation is the predominant flocculation mechanism for cohesive sediment settling in the flume. The results are comparable with the range of modelled cluster–cluster aggregation structures that have fractal dimensions of $1.42 < D_2 < 1.61$ (Meakin 1989). The microflocs shown in Fig. 2 are building blocks of the larger flocs, and their stability at steady state is a function of the shear stress (Stone and Krishnappan 2003). The particle-size distributions at steady state for the two initial conditions are presented in Fig. 7. At steady state (T = 300 min), values of D, D_1, and D_2 were not significantly different (t test, p =

Fig. 6. D_2 as a function of time. Vertical bars denote ±1 standard deviation.

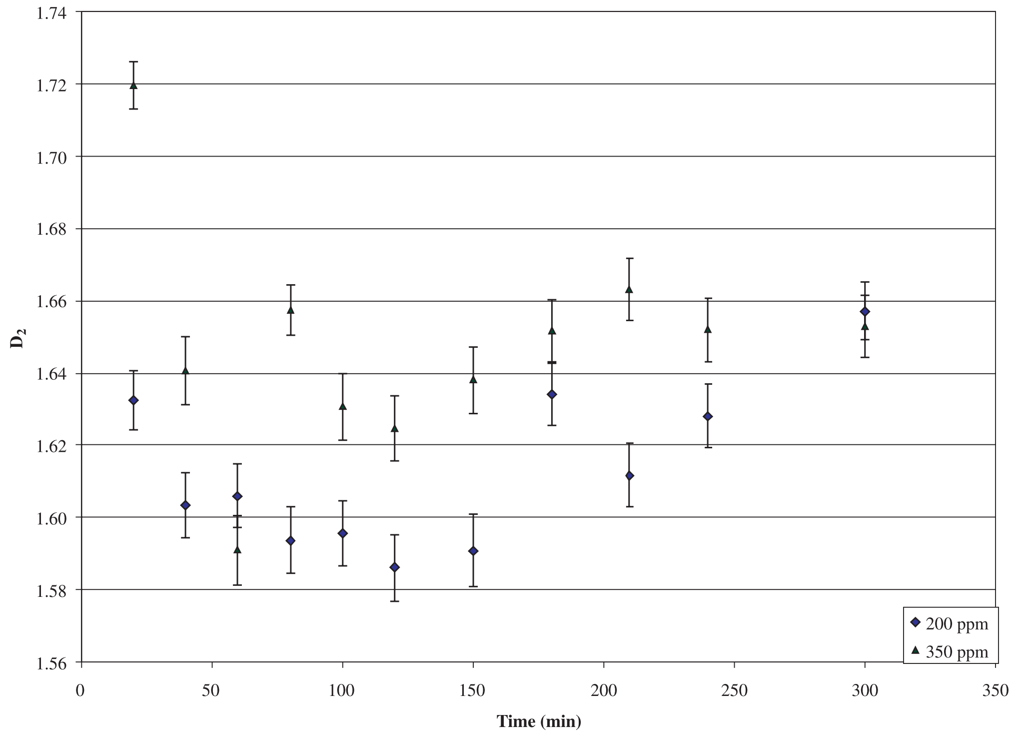
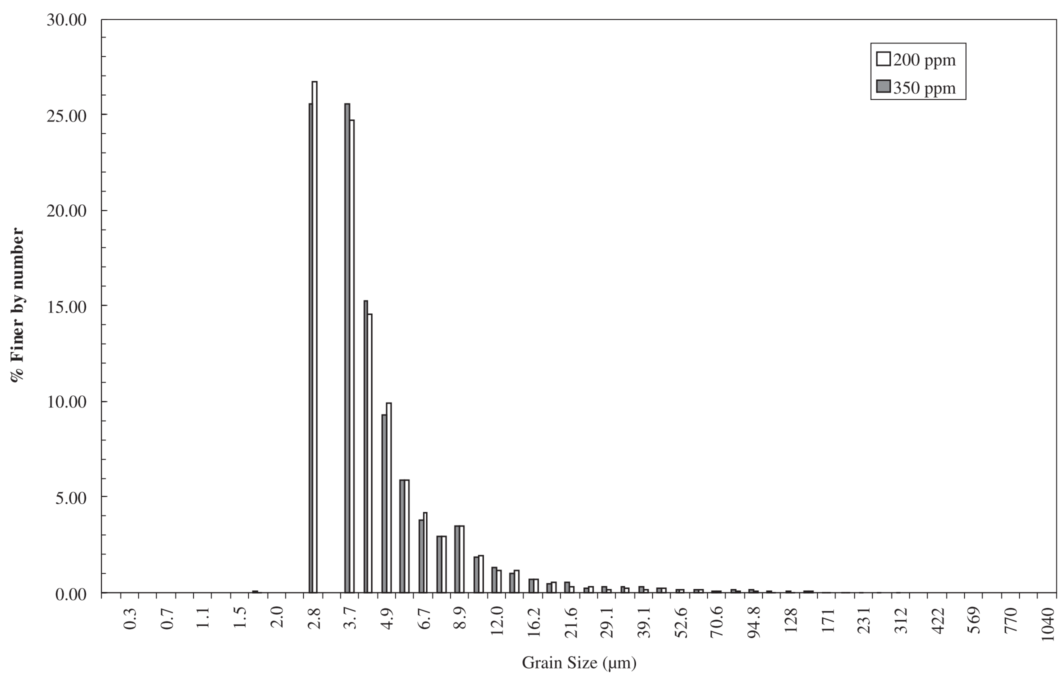


Fig. 7. Particle-size distribution (T = 300 min).



0.10), indicating that, although initial sediment concentrations influence the steady state sediment concentration, they have little effect on the morphology of particle populations.

Conclusions

The ratio between the initial and steady state concentrations for both experimental runs is 0.54, and the steady state sediment concentration is a function of the initial sediment concentration. At steady state, fractal dimensions D, D_1, and D_2 were not significantly different. Differential sedimentation is the predominant flocculation mechanism for cohesive sediment in the flume settling experiments. Photomicrographs of particles at steady state support the floc formation model proposed by Klimpel and Hogg (1986) and show the presence of micro-blocks in suspension, and these micro-blocks are the building blocks of larger flocs.

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