Acceptance Sampling Based on Truncated Life Tests in the Birnbaum Saunders Model

Ayman Baklizi* and Abed El Qader El Masri

We develop acceptance sampling plans assuming that the life test is truncated at a preassigned time. The lifetimes of the test units are assumed to follow the Birnbaum Saunders distribution. The minimum sample size necessary to ensure the specified average life is obtained and the operating characteristic values of the sampling plans and producer's risk are presented. An illustrative example is given.

KEY WORDS: Acceptance sampling; Birnbaum Saunders model; life tests

1. INTRODUCTION

The Birnbaum Saunders distribution has many applications in a variety of contexts. For example, it can model the failure time of the fatigue process; failure is the event that a crack length exceeds a threshold value. Some other applications and motivations for this model can be found in Johnson et al. (1995).

Inferential procedures in this distribution have been discussed by many authors, see Engelhardt et al. (1981), Ahmed (1988), and Achcar (1993) among many others. However, little attention has been paid to acceptance sampling based on life tests for this distribution. Acceptance sampling is one of the major fields in statistical quality control. It is used by the consumer to decide whether to accept or reject lots of products shipped from the producer. Acceptance sampling is often used when the inspection of the product is costly or destructive; more details are given by Duncan (1974).

The problem we are considering is that of finding the minimum sample size necessary to ensure a certain average life when the life test is terminated at a preassigned time ($t$) and when the number of failures does not exceed a given acceptance number ($c$). The lot is accepted if the specified average life can be established with a preassigned probability ($P^*$) specified by the consumer. This kind of life tests is discussed by Sobel and Tischendorf (1959) for the exponential model. The Weibull model is considered by Goode and Kao (1961), while Gupta and Groll (1961) considered the Gamma model. Kantam and Rosaiah (1998) and Kantam et al. (2001) considered the half-logistic and the log-logistic distributions. Baklizi (2003) considered the Pareto model of the second kind. In Section 2, we shall give the proposed acceptance sampling plans and the operating characteristic. The results and a descriptive example are given in Section 3.

2. THE SAMPLING PLANS

The cumulative distribution function and the probability density function of the Birnbaum Saunders distribution are given, respectively, by

\[ F(t, \sigma, \alpha) = \Phi\left(\frac{1}{\alpha} \left(\frac{t}{\sigma}\right)\right), \quad t > 0, \sigma, \alpha > 0, \quad (1) \]

\[ f(t, \sigma, \alpha) = \frac{\exp\left(\alpha^{-2}\right)}{2\alpha\sqrt{2\pi}\sigma} t^{-3/2}(t + \sigma) \]

\[ \times \exp\left(-\frac{1}{2\alpha^2}\left(\frac{t}{\sigma} + \frac{\sigma}{t}\right)\right), \quad t > 0, \sigma, \alpha > 0. \quad (2) \]
A sampling plan consists of

\( n \): The number of units on test.

\( c \): An acceptance number such that if \( c \) or fewer failures occur during the test time \( t \), the lot is accepted.

\( tl/\sigma_0 \): where \( \sigma_0 \) is the specified average life.

The consumer risk is fixed such that it does not exceed \( 1 - P^* \). Assume that the lot is large enough to be considered infinite so that the theory of the binomial distribution is applied. What we want is the minimum sample size \( (n) \) such that

\[
\sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i} \leq 1 - P^*,
\]

where \( p = F(t, \sigma_0) \) and we took \( \alpha = 1 \). Notice that \( p \) is a function of \( t/\sigma_0 \). Thus, the experiment needs only to specify this ratio. The minimum values of \( n \) satisfying Equation (3) were obtained for \( P^* = 0.75, 0.9, 0.95, 0.99, \) and \( t/\sigma_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 \). This choice is consistent with that of Gupta and Groll (1961), Kantam and Rosaiah (1998), and Kantam et al. (2001).

The operating characteristic of the sampling plan \((n, c, t/\sigma_0)\) gives the probability of accepting the lot. This probability is given by

\[
L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i},
\]

where \( p = F(t, \sigma) \), \( \alpha = 1 \), considered as a function of \( \sigma \). Here, also \( \alpha = 1 \). Values of the operating characteristic as a function of \( \sigma/\sigma_0 \) for some selected sampling plans are given in Table I.

The producer’s risk is the probability of rejecting the lot when \( \sigma \geq \sigma_0 \). Under the sampling plan under consideration, and given a value for the producer’s risk, say 0.05, one may be interested in knowing what value of \( \sigma/\sigma_0 \) will ensure the producer’s risk less than...
or equal to 0.05. This value of $\sigma/\sigma_0$ is the smallest number $\sigma/\sigma_0$ for which $F((t/\sigma_0)(\sigma/\sigma))$ satisfies the inequality

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i} \geq 0.95. \quad (5)$$

For a given sampling plan $(n, c, t/\sigma_0)$ at specified confidence level $P^*$, the minimum values of $\sigma/\sigma_0$ satisfying Equation (5) are computed and presented in Table II with $\alpha = 1$.

Tables I–III can be generated for any other value of $\alpha$. A Mathematica program that does this is available from the author upon request.
The results are given in Tables I–III. Assume that an experimenter wants to establish that the true unknown average life is at least 1,000 hours with confidence \( P^* = 0.95 \). The experiment will be stopped at \( t = 942 \) hours. Let the acceptance number be \( c = 2 \), then the required \( n \) from Table III is 11. If during 942 hours no more than 2 failures out of 11 are observed then the experimenter can assert with confidence 0.95 that the average life is at least 1,000 hours.

For the sampling plan \((n = 11, c = 2, t/\sigma_0 = 0.942)\), the operating characteristic values from Table I are as follows.

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3. THE RESULTS AND A DESCRIPTIVE EXAMPLE

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Table III. Minimum Values of \( n \) Necessary to Ensure an Average Life Exceeding \( \sigma_0 \) with Probability \( P^* \) and an Acceptance Number \( c \)
This means that if the true mean life is twice the specified mean life, then the producer’s risk is about 0.45 while it is about zero when the true mean life is 10 times the specified mean life.

Table II can be used to get the value of $\sigma / \sigma_0$ for various choices of $c$, $t / \sigma_0$ in order that the producer’s risk may not exceed 0.05. For example, the value of $\sigma / \sigma_0$ is 3.52 for $c = 2$, $t / \sigma_0 = 0.942$, $P^* = 0.95$. This means that the product should have an average life of 3.52 times the specified average life of 1,000 hours in order that the product be accepted with probability 0.95.

REFERENCES


