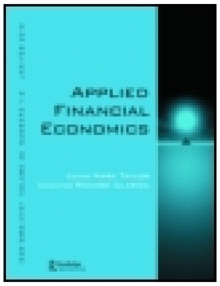

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A simple approach to valuing a multinational firm's tax shields

A. Pierru [a] & T. Atallah [a]
[a] King Abdullah Petroleum Studies and Research Center (KAPSARC) , P.O. Box 88550, Riyadh , 11672 , Kingdom of Saudi Arabia
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A simple approach to valuing a multinational firm's tax shields

A. Pierru* and T. Atallah

King Abdullah Petroleum Studies and Research Center (KAPSARC), P.O. Box 88550, Riyadh, 11672, Kingdom of Saudi Arabia

We consider a multinational firm that seeks to maximize its total amount of interest tax shield while following a constant debt ratio policy on a global level. The firm's total interest tax shield can then be considered as a piecewise-linear increasing function that is concave with respect to the firm's value. As a result, the expected interest tax shield can be much safer than the firm's free cash flow, depending on the firm's current value. With a simple no-arbitrage model, we derive the discount factor to apply to the total interest tax shield expected by the multinational firm. We show that this formula generalizes standard results of the literature on interest tax shields valuation.

Keywords: tax shields; multinational firm; financial leverage; corporate tax rate; APV

JEL Classification: G30; G32

I. Introduction

This article shows that a multinational firm following a constant debt-to-value ratio policy and seeking to maximize its expected total amount of interest tax shields should discount this amount at a rate that is a function of its current value. This rate can be much lower than the return expected on its unlevered equity. This result is new since the literature on tax shields valuation – including recent contributions by Fernandez (2004), Arzac and Glosten (2005), Cooper and Nyborg (2006), Grinblatt and Liu (2008), Liu (2009), Qi (2011) and Barbi (2012) – has never specifically considered the case of multinational firms.

We thus consider a multinational firm that faces multiple tax jurisdictions while maintaining constant its debt-to-value ratio[1] on a global basis. The firm rebalances debt in order to satisfy a target debt ratio computed with respect to the total value of all the firm's assets, with the possibility of raising debt capital via the foreign affiliates that operate these assets. We assume that all affiliates' debts are guaranteed by the parent, or are originally external debt contracted by the parent and subsequently lent to the affiliates (as internal debt). In brief, every dollar of the loans considered here is assumed to be ultimately financed by external debt and included in the calculation of the debt ratio targeted by the firm.

In most countries, interest expenses are deductible for corporate tax purposes. In determining their financial structure, purely domestic firms only have to deal with the domestic tax system. Multinational firms, however, face the more complicated choice of determining the allocation of their debts to the parent firm and the subsidiaries across all countries in which the multinational operates, taking account of the dispersion in statutory corporate tax rates.

*Corresponding author: E-mail: axel.pierru@kapsarc.org

[1] Graham and Harvey (2001) report in their survey that 81% of firms have some form of target debt-to-value ratio, and that the range around the target is tighter for larger firms. By using a partial-adjustment model of firm leverage, Flannery and Rangan (2006) find strong evidence that firms do have target capital structures.

In this article, we assume that the multinational firm maximizes the total value of its interest tax shields by first borrowing via affiliates located in high-tax countries. In other words, the firm first associates loans with assets susceptible to producing interest tax shields higher than those that can be obtained elsewhere. Nevertheless, real-world frictions, such as the role of collateral, restrict feasible debt-to-asset ratios across tax jurisdictions. For instance, the firm may not easily issue much debt in high-tax jurisdiction A if most of its tangible assets are in low-tax jurisdictions B. In addition, for each asset, the amount of debt for which interest payments are deductible from the taxable income may not exceed a certain limit imposed by the local revenue service in charge of tax collection. This limit can be a function[2] of the amounts invested in local projects by the affiliate. More generally, many countries impose rules[3] that limit the tax deductibility of interest paid by firms. All these considerations result in constraints that the firm has to take into account when it seeks to maximize its total interest tax shield. In this sense, the firm optimally allocates its debt capacity between its various affiliates. Empirical evidence supports this assumption. For instance, Newberry and Dhaliwal (2001) find some evidence that US multinationals are more likely to issue bonds through a foreign subsidiary if this subsidiary is located in a country with generally high corporate tax rates compared to the United States. Desai *et al.* (2004) find strong evidence that affiliates of US multinational firms alter the overall level of debt in response to tax incentives. Buettner *et al.* (2009) provide similar results for German multinationals. Mills and Newberry (2004) find that non-US multinationals with relatively low average foreign tax rates use more debt in their US subsidiaries than those with relatively high average foreign tax rates. In Graham and Harvey's (2001) survey, 45% of respondents indicate that tax implications are important or very important determinants of leverage, especially for larger public firms and for decisions concerning the financing of subsidiaries.

As a crucial consequence, the multinational firm's total interest tax shield can be considered as an increasing piecewise-linear function that is concave with respect to the firm's total interest payment (and, therefore, to the stochastic firm's value). Note that in the usual case of a firm facing a single corporate tax rate, the firm's interest tax shield is simply a linear function of the interest paid by the firm.

Intuitively, this concavity should have a 'sheltering effect' on the value of the firm's portfolio of interest tax shields, since the loans that produce interest tax shields at the lowest tax rate will serve as 'adjustment variables' when the firm's value stochastically fluctuates. In other words, because it is concave with respect to the firm's value, the total amount of interest tax shields is less sensitive to variations in this value. As a result, being less risky than the firm's free cash flow, the firm's total interest tax shield should be discounted at a rate lower than the firm's unlevered cost of equity.

In this article, we elaborate a simple no-arbitrage model that captures the effect of concavity through the expectation of the firm's total interest tax shield under the risk-neutral measure. The next section presents the assumptions made about the dynamics of the firm's debt, free cash flow and unlevered value. Section III introduces the assumptions made on the evolution of the constraints faced by the firm in its optimal debt allocation process, and models the concavity of the firm's total interest tax shield with respect to its interest payment. Sections IV and V derive the valuation formula, and the corresponding discount factor, of the firm's expected interest tax shield. This formula is interpreted in Section VI and illustrated with a numerical example in Section VII. Section VIII concludes.

II. Dynamics of the Firm's Debt, Free Cash Flow and Unlevered Value

In this simple model, we assume that all the firm's assets (located in various affiliates) have the same[4] operating risk and therefore the same expected unlevered return on equity. This allows us to directly consider the firm's total unlevered value (i.e. the sum of all the assets' unlevered value), denoted as A_t on each date t, which is here the only source of uncertainty. In the Markovian setting of our analysis, the firm's current unlevered value A_0 is known, and we let the Brownian motion B drive the uncertainty relating to changes in this value, according to the following geometric Brownian process:

$$dA_t = gA_t dt + \sigma A_t dB \qquad (1)$$

[2] A practical example is given, from a capital-budgeting viewpoint, by Pierru and Babusiaux (2008) who considers the petroleum exploration and production sector where debt financing giving rise to tax shields in a given country usually depends on the investments made in that country. In Norway, for example, interest paid to lenders is fully deductible from taxable income, provided the amounts borrowed do not exceed 80% of the investment.

[3] The 'thin-capitalization' rules, for instance, typically permit deductions of interest payments on loans made by the parent or another affiliate up to a given debt-to-equity ratio (i.e. interest payments from an excess leverage cannot be deducted from the tax base). Buettner *et al.* (2008) note that two-thirds of OECD countries imposed such rules in 2005.

[4] The reason may be that the firm targets a specific debt ratio for each set of projects with the same operating risk.

On each date t, the firm's total free cash flow,[5] denoted as F_t, is assumed to be a fixed (positive) percentage b of the firm's unlevered value:

$$F_t = bA_t \quad (2)$$

The expected free cash flow is therefore a growing perpetuity and, from Equations 1 and 2, the return ρ expected on the firm's unlevered equity is equal to $b + g$. The firm's debt is assumed to be contracted at the instantaneous risk-free interest rate r (taken as constant throughout time) and the debt ratio w targeted by the firm is defined with respect to the firm's unlevered value.

We consider that the firm pays interests and taxes on periodic dates, let us say at the end of every year. In addition, similar to Miles and Ezzell (1985), we assume that the amount of interest paid on each date only[6] depends on the firm's debt at the previous date (i.e. 1 year earlier). The total interest paid by the firm at date t is therefore $(e^r - 1)wA_{t-1}$.

III. Firm's Interest Tax Shield with Respect to Interest Payment

For a given year t, let us rank the multinational firm's z affiliates in decreasing order of the interest tax shields they are susceptible to produce. Each affiliate j (with $j \in \{1,...,z\}$) is assumed to be subject to the corporate tax rate θ_j (assumed constant throughout time). Let $m_j(t, A_{t-1})$ be the maximum attainable amount of interest payment producing interest tax shields at this rate θ_j in year t. In practice, $m_j(t, A_{t-1})$ should depend on both the maximum feasible debt-to-asset ratio and the fiscal rule defining the maximum amount of deductible interest payments for affiliate j.

Without any loss of generality, we hypothesize the following ranking:

$$\theta_1 \geq \theta_2 \geq \; ... \; \geq \theta_z \geq 0$$

We assume that in year $t - 1$ the firm rationally allocates loans to its affiliates according to this ranking, i.e. so as to maximize its total interest tax shield in year t. Let $l_j(t, A_{t-1})$ be the maximum amount of interest payment producing tax shields at rates equal to or greater than θ_j attainable in year t:

$$l_j(t, A_{t-1}) = \sum_{k=1}^{j} m_k(t, A_{t-1}) \quad (3)$$

with the following construction: $0 < l_1(t, A_{t-1}) < l_2(t, A_{t-1}) < \; ... \; < l_z(t, A_{t-1})$.

The portion of the firm's interest payment that is in excess to $l_z(t, A_{t-1})$ is assumed to produce tax shield at a rate θ_{z+1} (with $\theta_{z+1} \leq \theta_z$ and, possibly, $\theta_{z+1} = 0$).

A difficult question is now to define these limits to deductible interest payments, since these limits are susceptible to vary over time along with the constraints faced by the firm in its optimal debt allocation process. These variations may be partially deterministic and, for instance, due to foreseeable changes in the book value of assets or in local fiscal policies. To some extent, these variations may be uncertain: unexpected changes in the firm's value may presumably affect the subsidiaries' taxable incomes in the short run, and, in a longer run, are susceptible to modify the cash flows available for investments in new or existing assets.

To illustrate this, let us consider a multinational firm whose affiliate a is subject to the (highest) tax rate θ_1 and affiliate b to the (next highest) tax rate θ_2. Let us assume that affiliate a is located in a country where deductibility of interest payments is, every year, possible for a debt amount that cannot exceed the accounting value of the affiliate's assets in the previous year. Let us further assume that for some business reasons the firm has decided to stop investing in affiliate a, whose total book value decays at the annual rate 10%. We therefore have

$$l_1(t, A_{t-1}) = m_1(t, A_{t-1}) = l_1(1, A_0)e^{-0.1(t-1)}$$

Let us now hypothesize that, for strategic reasons, the firm's management sets the objective[7] of locating (in every future year) less than 20% of the firm's total operational value in the country where affiliate b operates, with a feasible debt-to-asset ratio that cannot exceed 80% of affiliate b's value. By assuming that the local IRS does not impose any limit on the amount of deductible interest payments, we then have

$$m_2(t, A_{t-1}) = 0.16A_{t-1}$$

And from Equation 3

$$\begin{aligned} l_2(t, A_{t-1}) &= m_1(t, A_{t-1}) + m_2(t, A_{t-1}) \\ &= l_1(1, A_0)e^{-0.1(t-1)} + 0.16A_{t-1} \end{aligned}$$

[5] The free cash flow is an after-tax operating cash flow that includes no interest tax shields.
[6] A – more complex – alternative would be to consider that the interest paid in period t depends on the path followed by the firm's unlevered value during the 1-year time interval $[t - 1, t]$, the firm's debt ratio being continuously maintained constant.
[7] This objective could be achieved with local disinvestments whenever the corresponding constraint is violated.

This illustration suggests the following simplified – but convenient – way of modelling the limit $l_j(t, A_{t-1})$ ($j \in \{1, 2, ..., z\}$):

$$l_j(t, A_{t-1}) = w(e^r - 1)(\alpha_j(t) + \beta_j(t)A_{t-1}) \quad (4)$$

where $\alpha_j(.)$ and $\beta_j(.)$ are non-negative known functions of time, with $\alpha_j(.) \leq \alpha_{j+1}(.)$ and $\beta_j(.) \leq \beta_{j+1}(.)$ for all $j \in \{1, 2, ..., z-1\}$.

To remain consistent with the debt allocation problem considered here, we also assume[8] that $\beta_j(t)$ ($j \in \{1, 2, ..., z\}$) is smaller than unity.

According to Equation 4, the maximum (attainable) amount of interest payments deductible at a rate equal to or greater than θ_j in year t is an affine function of the total interest paid by the firm $(e^r - 1)wA_{t-1}$. The firm's total interest tax shield generated in year t can therefore be conveniently written as follows:

$$\theta_1(e^r - 1)wA_{t-1} - \sum_{j=1}^{z} (\theta_j - \theta_{j+1}) \max\left[0, (e^r - 1)wA_{t-1} - l_j(t, A_t)\right] \quad (5)$$

The meaning of the two terms in Equation 5 is straightforward: first we consider that the full interest payment $(e^r - 1)wA_{t-1}$ generates interest tax shields at the highest tax rate θ_1, then we subtract the 'losses' in interest tax shields caused by the successive decreases in tax rate. By combining Equations 4 and 5, the firm's total interest tax shield generated in year t is equal to

$$w(e^r - 1)\left(\theta_1 A_{t-1} - \sum_{j=1}^{z} (\theta_j - \theta_{j+1}) \max\left[0, \left(1 - \beta_j(t)\right)A_{t-1} - \alpha_j(t)\right]\right) \quad (6)$$

Let us assume that in year $t-1$ the realized value of A_{t-1} is such that the last dollar of the firm's debt capacity generates a tax shield at the rate θ_u. From Equation 6, the firm's total interest tax shield is then

$$w(e^r - 1)\left(\theta_1 A_{t-1} - \sum_{j=1}^{u-1}(\theta_j - \theta_{j+1})((1 - \beta_j(t)) A_{t-1} - \alpha_j(t))\right) \quad (7)$$

The derivative of Equation 7 with respect to A_{t-1} is therefore

$$w(e^r - 1)\left(\theta_1 - \sum_{j=1}^{u-1}(\theta_j - \theta_{j+1})(1 - \beta_j(t))\right) \quad (8)$$

Equation 8 can be rewritten as follows:

$$w(e^r - 1)\left(\theta_u + \sum_{j=1}^{u-1} (\theta_j - \theta_{j+1})\beta_j(t)\right) \quad (9)$$

This (nonnegative) derivative remains constant as long as the last allocated dollar of the firm's debt capacity yields an interest tax shield at the same rate θ_u. Let us now assume that the realized value of A_{t-1} is higher and such that the last dollar of debt allocated among the firm's subsidiaries yields an interest tax shield at the (next lower) rate θ_{u+1}. The derivative of the firm's total interest tax shield is now

$$w(e^r - 1)\left(\theta_{u+1} + \sum_{j=1}^{u} (\theta_j - \theta_{j+1})\beta_j(t)\right) \quad (10)$$

By subtracting Equation 9 from Equation 10, we obtain:

$$-w(e^r - 1)(\theta_u - \theta_{u+1})(1 - \beta_u(t)) \quad (11)$$

The term in Equation 11 is negative, which proves that the (nonnegative) derivative of the firm's total interest tax shield is a decreasing function of A_{t-1}. The firm's total interest tax shield is therefore piecewise concave with respect to the firm's unlevered value, as illustrated by Fig. 1. This concavity results from the optimal debt allocation process inside the multinational firm.

From a pragmatic viewpoint, when considering a real-life multinational firm, a curve similar to that in Fig. 1 can always be drawn. It suffices to rank all its affiliates' debts by decreasing order of the interest tax shield produced[9] on a per-dollar-of-interest basis. The concave function is then obtained by plotting the cumulative interest tax shield with respect to the cumulative affiliates' interest payment. One could consider that this curve implicitly results from an optimal debt allocation that would take into account all real-world constraints faced by the firm.

A stochastic change in the firm's value results in a proportionate change in its debt capacity, and, therefore, in its total interest payment 1 year later. Consequently, when its value increases (decreases) between years $t-1$ and t, the firm is expected to optimally adjust its affiliates' debts in a straightforward manner, by moving to the right

[8] If for a given j we had $\beta_j(t) \geq 1$, the limit $l_j(t, A_{t-1})$ would never be binding and no firm's debt would generate tax shields at a rate lower than θ_j; to revert to the debt allocation problem considered here, we would then have to set z equal to $j-1$.
[9] If necessary, the tax rate used for the computation of the interest tax shield can be the composite of both affiliate and parent tax rates with suitable adjustments.

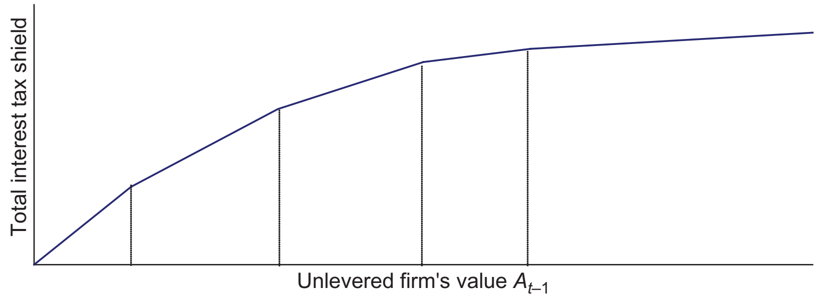

Fig. 1. Firm's total interest tax shield with respect to the firm's unlevered value

(left) on the curve of year $t+1$, up to the point corresponding to its new debt capacity in year t. The total amount of interest tax shield produced in any year is thus the value given by the corresponding curve for the total interest paid by the firm during this year.

IV. Valuation of the Firm's Expected Interest Tax Shield

Under the risk-neutral measure, the firm's unlevered value has an expected rate of growth equal to the difference between the risk-free rate and the dividend yield (from payouts of the free cash flows):

$$dA_t = (r-b)A_t dt + \sigma A_t dB' \quad (12)$$

The total interest tax shield received by the firm in each year t is here valued as a derivative in a dynamic complete market.

From Equation 6, the firm's total interest tax shield generated in year t can be written as follows:

$$w(e^r-1)\left(\theta_1 A_{t-1} - \sum_{j=1}^{z}(\theta_j - \theta_{j+1})(1-\beta_j(t)) \max\left[0, A_{t-1} - \frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right) \quad (13)$$

Interestingly, Equation 13 can be interpreted as if, with a 1-year delay, the firm were receiving a fraction of A_{t-1} and paying fractional values of z expiring call options written on A_{t-1}.

By using Equation 13, in the absence of arbitrage, the present value of the interest tax shield generated in year t is:

$$w(e^r-1)e^{-rt}E'_{(0,A_0)}\left\{\theta_1 A_{t-1} - \sum_{j=1}^{z}(\theta_j - \theta_{j+1}) (1-\beta_j(t)) \max\left[0, A_{t-1} - \frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\} \quad (14)$$

where $E'_{(0,A_0)}$ denotes the expectation operator under the risk-neutral measure, knowing A_0 in $t=0$.

Since Equation 12 gives $E'_{(0,A_0)}\{A_{t-1}\} = e^{(r-b)(t-1)}A_0$, the term in Equation 14 becomes

$$w(e^r-1)\left(\theta_1 A_0 e^{-r-b(t-1)} - e^{-r}\sum_{j=1}^{z}\left((\theta_j - \theta_{j+1}) (1-\beta_j(t))e^{-r(t-1)}E'_{(0,A_0)}\left\{\max\left[0, A_{t-1} - \frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\}\right)\right) \quad (15)$$

Let $C_j(t-1)$ denote the current market price (in $t=0$) of a European call option, which gives the right to buy the firm's unlevered value at the exercise price $\left(\frac{\alpha_j(t)}{1-\beta_j(t)}\right)$ at the expiration date $t-1$. We have

$$e^{-r(t-1)}E'_{(0,A_0)}\left\{\max\left[0, A_{t-1} - \frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\} = C_j(t-1) \quad (16)$$

From Equations 15 and 16, the present value of the firm's interest tax shield generated in year t is

$$w(e^r-1)\left(\theta_1 A_0 e^{-r-b(t-1)} - e^{-r}\sum_{j=1}^{z}(\theta_j - \theta_{j+1}) (1-\beta_j(t))C_j(t-1)\right) \quad (17)$$

By using the standard formula giving the value of a European call option written on an asset following a geometric Brownian process and paying a continuous dividend yield, we have

$$C_j(t-1) = e^{-b(t-1)}A_0 N\left(d_{1,j}(t)\right) - e^{-r(t-1)}\frac{\alpha_j(t)}{1-\beta_j(t)} N\left(d_{2,j}(t)\right) \quad (18)$$

where $d_{1,j}(t)=\frac{\ln\left(\frac{(1-\beta_j(t))A_0}{\alpha_j(t)}\right)+\left(r-b+\frac{1}{2}\sigma^2\right)(t-1)}{\sigma\sqrt{t-1}}$,

$$d_{2,j}(t)=d_{1,j}(t)-\sigma\sqrt{t-1}$$

$N(.)$ is the standard normal distribution function.

By combining Equations 17 and 18, the present value of the firm's total interest tax shield generated in year t is therefore

$$w(e^r-1)\left(\frac{\left(\theta_1-\sum_{j=1}^{z}(\theta_j-\theta_{j+1})(1-\beta_j(t))N(d_{1,j}(t))\right)A_0}{e^{r+b(t-1)}}+\frac{\sum_{j=1}^{z}(\theta_j-\theta_{j+1})\alpha_j(t)N\left(d_{2,j}(t)\right)}{e^{rt}}\right) \quad (19)$$

Formula 19 is the most important result of this article, since it gives the present value of the interest tax shields expected by a multinational firm whose free cash flow is a growing perpetuity.

Let us consider the standard case where the firm faces one tax rate only: $\theta_j=\theta_1\ \forall j\in\{2,\ldots,z+1\}$. Formula 19 then simplifies to

$$\frac{\theta_1(e^r-1)wA_0}{e^{r+b(t-1)}}=\left(\theta_1(e^r-1)we^{g(t-1)}A_0\right)e^{-r-\rho(t-1)} \quad (20)$$

According to Equation 1, $e^{g(t-1)}A_0$ is the firm's unlevered value expected in year $t-1$. The right-hand side of Equation 20 is therefore equal to the interest tax shield expected by the firm in year t multiplied by $e^{-r-\rho(t-1)}$. This latter term is precisely the discount factor derived by Miles and Ezzell (1985) under consistent assumptions.

V. Discount Factor for the Expected Interest Tax Shield

From Equation 13, the interest tax shield expected in year t is

$$w(e^r-1)E_{(0,A_0)}\left\{\theta_1A_{t-1}-\sum_{j=1}^{z}(\theta_j-\theta_{j+1})(1-\beta_j(t))\max\left[0,A_{t-1}-\frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\} \quad (21)$$

where $E_{(0,A_0)}$ denotes the expectation operator under the historical probability, knowing A_0 in $t=0$.

Since from Equation 1 we have $E_{(0,A_0)}\{A_{t-1}\}=e^{g(t-1)}A_0$, the term in Equation 21 is equal to

$$w(e^r-1)\left(\theta_1A_0e^{g(t-1)}-\sum_{j=1}^{z}\left((\theta_j-\theta_{j+1})(1-\beta_j(t))E_{(0,A_0)}\left\{\max\left[0,A_{t-1}-\frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\}\right)\right) \quad (22)$$

The analytical expression of the term in Equation 22 can be easily obtained by exploiting similarities with relationship (Equation 16). By multiplying the right-hand side of Equation 18 by $e^{r(t-1)}$ and by replacing $r-b$ by ρ in the resulting expression, we deduce that

$$E_{(0,A_0)}\left\{\max\left[0,A_{t-1}-\frac{\alpha_j(t)}{1-\beta_j(t)}\right]\right\}=e^{g(t-1)}A_0N\left(d_{3,j}(t)\right)-\frac{\alpha_j(t)}{1-\beta_j(t)}N\left(d_{4,j}(t)\right) \quad (23)$$

Where $d_{3,j}(t)=\frac{\ln\left(\frac{(1-\beta_j(t))A_0}{\alpha_j(t)}\right)+\left(g+\frac{1}{2}\sigma^2\right)(t-1)}{\sigma\sqrt{t-1}}$,

$$=d_{1,j}(t)+\frac{(\rho-r)}{\sigma}\sqrt{t-1}$$

$$d_{4,j}(t)=d_{2,j}(t)+\frac{(\rho-r)}{\sigma}\sqrt{t-1}$$

By combining Equations 22 and 23, the firm therefore expects to receive on date t a total interest tax shield equal to

$$w(e^r-1)\left(e^{g(t-1)}\left(\theta_1-\sum_{j=1}^{z}(\theta_j-\theta_{j+1})(1-\beta_j(t))N\left(d_{3,j}(t)\right)\right)A_0+\sum_{j=1}^{z}(\theta_j-\theta_{j+1})\alpha_j(t)N\left(d_{4,j}(t)\right)\right) \quad (24)$$

The discount factor to apply to the interest tax shield expected in year t is equal to the ratio of the term in Equation 19 and to the term in Equation 24 (in which $b+g$ is replaced by ρ):

$$\frac{\left(\theta_1 - \sum_{j=1}^{z}(\theta_j - \theta_{j+1})(1-\beta_j(t))N(d_{1,j}(t))\right)A_0 e^{-r-\rho(t-1)} + \left(\sum_{j=1}^{z}(\theta_j - \theta_{j+1})\alpha_j(t)N\left(d_{2,j}(t)\right)\right)e^{-r-(r+g)(t-1)}}{\left(\theta_1 - \sum_{j=1}^{z}(\theta_j - \theta_{j+1})(1-\beta_j(t))N(d_{3,j}(t))\right)A_0 + e^{-g(t-1)}\sum_{j=1}^{z}(\theta_j - \theta_{j+1})\alpha_j(t)N\left(d_{4,j}(t)\right)} \tag{25}$$

VI. Interpretation of the Discount Factor and Special Cases

The discount factor given by formula 25 is a weighted sum of the discount factors $e^{-r-\rho(t-1)}$ and $e^{-r-(r+g)(t-1)}$, although the sum of the two weights is in general not equal to unity. It is derived by discounting – at the risk-free interest rate – the interest tax shield expected under the risk-neutral probability. Various special cases of formula 25 are worth to be considered.

First consider the standard case where the firm faces one tax rate only: $\theta_j = \theta_1 \ \forall j \in \{2,...,z+1\}$. Formula 25 then immediately simplifies to $e^{-r-\rho(t-1)}$, which is the factor derived by Miles and Ezzell (1985) for discounting the interest tax shield generated in year t.

Let us now turn to the case $t = 1$. Formula (25) gives a discount factor of e^{-r}, which is consistent with the fact that the firm's total amount of interest tax shields in year 1 is assumed to be known with certainty. More generally, in our Miles-Ezzell's world, the firm's total interest tax shield generated in every period is always certain over one period (since formula 25 is equal to e^{-r} times a function of $t-1$).

Another interesting case occurs when $\alpha_j(t) = 0$ for all $j \in \{1,...,z\}$; $d_{1,j}(t-1)$ and $d_{3,j}(t-1)$ are then equal to $+\infty$ for every j and formula 25 gives $e^{-r-\rho(t-1)}$. In other words, when every limit on deductible interest payment simply remains proportional to the unlevered firm's value, the discount factor collapses to the Miles-Ezzell factor. This result can be easily explained: when the firm's total interest payment and the limits on deductible interest payments are all proportional to the firm's unlevered value, formula 13 is equal to A_{t-1} times a (known) function of time. The firm's total interest tax shield therefore bears the same risk as the firm's unlevered value, except in the last year. Similarly, for an infinite t, formula 25 tends towards the Miles-Ezzell factor when one reasonably hypothesizes that $\lim_{t\to\infty}\alpha_j(t) = 0$ for all j (as, in the very long run, the uncertainty on the firm's value and investment policy should prevail).

More generally, when A_0 tends towards zero, the firm's interest tax shield is generated in a 'local' Miles-Ezzell world – in the sense that it tends to depend only on the first linear piece – and, consequently, formula 25 tends towards $e^{-r-\rho(t-1)}$.

In the special case where $\theta_{z+1} = 0$ and $\beta_j(t) = 0$ for all j, formula 25 tends towards e^{-rt} for an infinite A_0, since then the exposure of the (finite) total interest tax shield to the operating risk tends towards zero. In all other cases, formula 25 tends towards $e^{-r-\rho(t-1)}$ for an infinite A_0, as the 'infinite part' of the interest tax shield remains proportional to the firm's unlevered value.

VII. Numerical Illustration

In this example, we value the interest tax shield expected by a multinational firm in year 2. The free cash flow expected by the firm is assumed to be a perpetuity (i.e. $g = 0$ and $\rho = b$). The firm owns two affiliates A and B that are respectively subject to the (high) tax rate θ_1 and (low) tax rate θ_2. Owing to local fiscal rules, the amount of interest payments that can be deducted from affiliate A's taxable income in year 2 is limited to a fixed amount m_1 (in other words, $\beta_1(2) = 0$). We assume that there is no ceiling on interest deductibility in affiliate B, which amounts to, take here, $z = 1$.

As the interest tax shield expected by the firm in year 2 is uncertain over only 1 year, we introduce γ such that $e^{-r-\gamma}$ equates the discount factor given by formula 25:

$$e^{-r-\gamma} = \frac{\left(\theta_1 - (\theta_1-\theta_2)N\left(d_{1,1}(2)\right)\right)A_0 e^{-r-\rho} + \left((\theta_1-\theta_2)\alpha_1(2)N\left(d_{2,1}(2)\right)\right)e^{-2r}}{\left(\theta_1 - (\theta_1-\theta_2)N\left(d_{3,1}(2)\right)\right)A_0 + \left((\theta_1-\theta_2)\alpha_1(2)N\left(d_{4,1}(2)\right)\right)} \tag{26}$$

with:

$$d_{1,1}(2) = \frac{\ln\left(\frac{A_0}{\alpha_1(2)}\right) + r - \rho + \frac{1}{2}\sigma^2}{\sigma}, \quad d_{2,1}(2) = d_{1,1}(2) - \sigma,$$

$$d_{3,1}(2) = \frac{\ln\left(\frac{A_0}{\alpha_1(2)}\right) + \frac{1}{2}\sigma^2}{\sigma}, \; d_{4,1}(2) = d_{3,1}(2) - \sigma$$

From (26) we have

$$\gamma = \ln\left(\frac{(\theta_1 - (\theta_1 - \theta_2)N(d_{3,1}(2)))A_0 + ((\theta_1 - \theta_2)\alpha_1(2)N(d_{4,1}(2)))}{(\theta_1 - (\theta_1 - \theta_2)N(d_{1,1}(2)))A_0 e^{-\rho} + ((\theta_1 - \theta_2)\alpha_1(2)N(d_{2,1}(2)))e^{-r}}\right) \tag{27}$$

From now on, γ is called the 'equivalent discount rate', with the understanding that the firm's interest tax shield is less risky than the free cash flow if $\gamma < \rho$.

Let us set $r = 0.05$, $\rho = b = 0.15$, $\sigma = 0.2$, $\theta_1 = 0.5$, $w = 0.4$, $l_1(2, A_1) \equiv \$2$ million. For the sake of illustration, for θ_2 we consider the following three alternative values: $\theta_2 = 0, \theta_2 = 0.1$ and $\theta_2 = 0.3$. From Equation 4, we have $\alpha_1(2) = \frac{2}{0.4(e^{.05} - 1)} \cong 97.5$ and $\beta_1(2) = 0$.

For the three different values of θ_2, Fig. 2 gives γ in function of the firm's current value A_0. When θ_2 is higher, the firm's expected interest tax shield is greater but riskier (since discounted at a higher rate). As already explained, when the firm's current value tends towards infinite, γ tends towards the return expected on the firm's unlevered equity (risk-free interest rate) when $\theta_2 = 0.1$ or $\theta_2 = 0.3$ ($\theta_2 = 0$).

VIII. Conclusions

A multinational firm that follows a constant leverage-ratio policy and seeks to maximize its total amount of interest tax shields has the option of optimally reallocating its debt capacity in the future. By acting as a natural hedge, the future revisions of the optimal firm's debt allocation reduce the return currently expected on the firm's portfolio of interest tax shields. As a result, the interest tax shields expected by the firm should be discounted at a rate lower than the expected return on its unlevered equity, even if the firm continuously maintains a constant debt-to-value ratio.

Intuitively, the interest tax shields produced at the highest tax rate are almost certain, since they would be 'the last to vanish'. They would not realize only in the (unlikely) case of a dramatic decrease in the firm's value. They should therefore be discounted at a rate only slightly greater than the risk-free rate. On the contrary, the loans that produce interest tax shields at the firm's marginal tax rate will serve as 'adjustment variables' when the firm's value stochastically fluctuates. Their interest tax shields have consequently to be discounted at a rate close to the expected return on unlevered equity. The firm's various

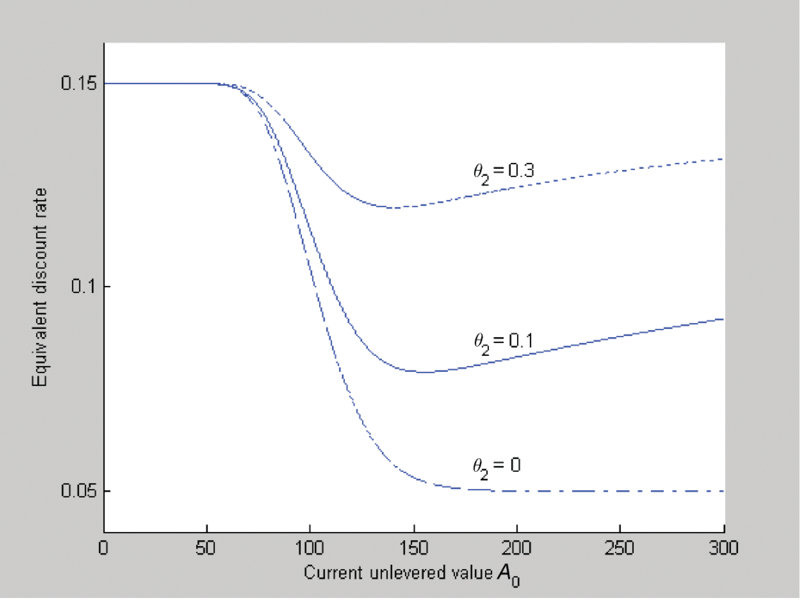

Fig. 2. Equivalent discount rate of interest tax shield with respect to the firm's current value

interest tax shields have therefore distinct exposures to the operating risk. In average, the firm's portfolio of interest tax shields has an expected return lower than the return expected on unlevered equity.

Even if the formulas in 19 and 25 are derived under simplified assumptions, they have interesting implications for the discounting of multinational firms' capital cash flows. More generally, they provide useful insights for those carrying out corporate valuation and interested in the value of multi-nationality.

References

Arzac, E. R. and Glosten, L. R. (2005) A reconsideration of tax shield valuation, *European Financial Management*, **11**, 453–61.

Barbi, M. (2012) On the risk-neutral value of debt tax shields, *Applied Financial Economics*, **22**, 251–8.

Buettner, T., Overesch, M., Schreiber, U. *et al*. (2008). The impact of thin-capitalization rules on multinationals' financing and investment decisions, Discussion paper, Economic Studies, No. 03/2008, Deutsche Bundesbank, Frankfurt am Main.

Buettner, T., Overesch, M., Schreiber, U. *et al*. (2009) Taxation and capital structure choice – evidence from a panel of German multinationals, *Economic Letters*, **105**, 309–11.

Cooper, I. A. and Nyborg, K. G. (2006) The value of tax shields IS equal to the present value of tax shields, *Journal of Financial Economics*, **81**, 215–25.

Desai, M. A., Foley, F. and Hines, J. R. (2004) A multinational perspective on capital structure choice and internal capital markets, *Journal of Finance*, **59**, 2451–87.

Fernandez, P. (2004) The value of tax shields is NOT equal to the present value of tax shields, *Journal of Financial Economics*, **73**, 145–65.

Flannery, M. J. and Rangan, K. P. (2006) Partial adjustment toward target capital structures, *Journal of Financial Economics*, **79**, 469–506.

Graham, J. R. and Harvey, C. R. (2001) The theory and practice of corporate finance: evidence from the field, *Journal of Financial Economics*, **60**, 187–243.

Grinblatt, M. and Liu, J. (2008) Debt policy, corporate taxes, and discount rates, *Journal of Economic Theory*, **141**, 225–54.

Liu, Y. C. (2009) The slicing approach to valuing tax shields, *Journal of Banking and Finance*, **33**, 1069–78.

Miles, J. and Ezzell, J. R. (1985) Reformulating tax shield valuation: a note, *Journal of Finance*, **40**, 1485–92.

Mills, L. F. and Newberry, K. J. (2004) Do foreign multinationals' tax incentives influence their US income reporting and debt policy?, *National Tax Journal*, **57**, 89–107.

Newberry, K. J. and Dhaliwal, D. S. (2001) Cross-jurisdictional income shifting by US multinationals: evidence from international bond offerings, *Journal of Accounting Research*, **39**, 643–62.

Pierru, A. and Babusiaux, D. (2008) Valuation of investment projects by an international oil company: a new proof of a straightforward, rigorous method, *OPEC Energy Review*, **32**, 197–214.

Qi, H. (2011) Value and capacity of tax shields: an analysis of the slicing approach, *Journal of Banking and Finance*, **35**, 166–73.