Fractal dimension of time-indexed paths

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Abstract

Measuring animal track or other trajectories (for example the track of a flowing particle) by divider method to obtain the fractal dimension without knowing the chronology of the points of the path, one can obtain erroneous results. The importance of time-indexing of the track’s point will be demonstrated by a simply geometrical example. Neglecting these indices, the error can be high enough to mask relevant differences between various movement paths.

Keywords: fractal dimension, tortuosity, divider method, trajectory, time, chronology

1. Introduction

Animals move in straight line only in very rare occasions; the path or trajectory of their movement is usually tortuous. The most common measure of the path tortuosity is the Hausdorff fractal dimension, $D_f \ [1-4]$. For straight (or nearly straight) paths embedded into two dimension $D_f \approx 1$, while for a very tortuous path $D_f \approx 2$. Habitats or parts of the habitats where the animal paths are very tortuous means that the animals
spend a lot of time in that area, i.e. the habitat is very favourable [5-9]. In this way the fractal dimension - as a measure of the tortuosity - can help us to quantify the "quality" of the habitats from the point of the animals (just like the interior-to-edge ratio or the Minkowski-Bouligand fractal dimension can quantify it from the point of the landscape geometry [10-12]).

Probably the simplest method to determine the fractal dimension of a tortuous line is the so-called divider method [2], when the length is measured by using straight yardsticks with various lengths. The fractal dimension can be calculated by using the following equation:

\[ L(\varepsilon) = A \varepsilon^{-D_f} \]  

where \( L \) is the length, measured by an \( \varepsilon \)-long yardstick, \( A \) is a constant and \( N \) is the number of yardsticks required to "cover" the whole line. Using a log-log plot for \( \varepsilon \) vs. \( L \) (this is the so-called Richardson-plot [13]) one might obtain a series of points located along a line. The slope of the line is \( 1 - D_f \). Obviously Eq.1 is true only for some mathematical objects; for a natural curve the linear fit of the points is only an approximation which is true only within some error and (which is often forgotten) only in a limited yardstick-range [2,14].

The divider-method is very simple and can be used for all kind of curves (unlike some other dimension-measuring methods, where the studied objects should satisfy some preliminary assumptions [15]). Unfortunately it has some disadvantage too. Probably the most serious one that the last divider step never falls exactly at the endpoint of the curve, therefore the measured \( L(\varepsilon) \) depends on the starting point of the measurement. For this
reason the path length will be truncated. Recently several successful methods has been proposed to avoid this problem [16,17].

In this short paper we would like to analyze a potential error caused by the often overlooked fact that animal paths are not only tortuous lines but also have chronology. Each point should have a time-index to show the direction; i.e. from neighbouring points one should be able to decide that the animal was first here, then there. Whenever the path has one or more loops, the improper use of the divider method can give an erroneous fractal dimension. In the following sections we are going to analyze this error by showing its extent to demonstrate that all patches should be handled as a time-indexed trajectory, instead of a simple curve.

2. Example and Results

On Fig. 1 a very simple path can be seen. It has a straight part and a perfectly circular loop touching the straight line just in one point (junction). Starting from the left side of the straight part and reaching the junction, the animal can choose from two different ways; it can take an extremely sharp turn, then follow the circle and after reaching the junction again, can take one more sharp turn and going to the end point, as it is shown on Fig. 1a. Alternatively, it can choose a smoother path by following the loop without turns, like the scenario represented on Fig. 1b. In the following analysis, the first scenario will be referred as "back" and the second one as "forth". It is probable that most animal would choose the "forth" one, but one can easily imagine a situation when repeated sharp turns would be also possible, like the appearance of a predator. Similarly,
the "forth" scenario is also more probable for any flowing particle, but they could be also forced to change their direction very suddenly after a collision.

On Fig. 1c and 1d it can be seen that using the divider method (and starting from the same point) the two paths will be covered in a different way. This difference will also cause difference in the measured $L$. For demonstration we measured a loop where the straight part is 300-unit long, the radius of the circle is 100-unit and the junction located at the first third of the straight part (at the 100$^{th}$ unit). The divider length ($\varepsilon$) varied between 10 and 100 units. To eliminate completely the error caused by the truncation (when the last divider step does not falls directly to the end of the path [16,17]) we used a "fractionated" last step; i.e. when the divider length was 10 units but the last full step ended 2.5 units before the path end, then we added $2.5/10=0.25$ unit into the number of the dividers covering the path. This can be done only in cases when the yardstick and the last part of the path are similar (straight line). The length ($L$) can be calculated by $L=Ne$ where $N$ is the number of yardsticks needed to cover the path with the given yardstick-length. The relative difference in the measured $L$ in the back and forth scenarios can be seen in Table 1 and on Fig. 2. The measured values are marked by squares while the solid line represents a Gaussian fitting. It can be seen that the error is size-dependent but not monotoneous (unlike the one caused by the truncation [16,17]), it has a maximum around $\varepsilon=60$ with a value around 20%. We cannot deduct any more general conclusion based on this one example (i.e. for other lines non-Gaussian fitting or some monotoneous dependence might be more accurate). Obviously this trajectory is very over-simplified but the analyzed effect can be demonstrated on it properly, while by using a real, very tortuous animal path for the demonstration would make the understanding more difficult.
The fact that the relative error is size dependent means that it can influence the estimated fractal dimension. On Fig. 3 an example can be seen when a set of data (solid squares) describing the $\log(\varepsilon)\text{-}\log(L)$ relationship for a fractal with $D_f=1.15$ (which is a quite average value for animal paths [16]) were modified in the range of $\varepsilon=(0.01-1)$ in the way that for the shortest yardsticks we introduced a +20% error while for the longest yardsticks the error was only 2% and between them the errors varied in linear way (which is a good approximation even in the reported Gaussian case between $\varepsilon=20$ to $\varepsilon=60$ or from $\varepsilon=60$ to $\varepsilon=90$). As it can be seen, the fractal dimension will be changed drastically in the short yardstick-range (from 1.150 to 1.227±0.002 in the 0.01-0.1 range), while remain almost unchanged at the long one. Scale-dependent fractal dimension values are not unknown [5,6] but we should keep in mind that the ranges for various fractal dimensions cannot be too small, because in that case the fractality would loose its meaning [14].

It can be seen that the difference between the two values (1.150 and 1.227) are quite high (0.077). We should keep in mind that one of the lowest reported value for real animal tracks is 1.06 (wood turtles) while the highest one is 1.16 (grasshoppers) [16], i.e. the whole meaningful range is hardly bigger that the error reported here. For this reason one has to know not only the path (for example as a bitmap picture) but also the order (first step, second step, third step, etc) of the point on the path to obtain any meaningful value for the fractal dimension in tortuosity studies.

The result suggest that the tortuosity of the curves (which is the same for the "back" and "forth" track) and the fractal dimension is not necessarily correlated whenever the fractal dimension is measured in time-indexed paths.
The direction of the movement can be determined from the observation of the position of individual footprints, although it is not possible in several cases, like snakes or snails [18]; in these cases only direct observation or radio tracking of the movement can give us the proper information. Sometimes it is very difficult but we do believe that this paper highlighted the importance of the "time indexing" by showing the error caused by neglecting it.

3. Conclusion

Several animal tracking methods record the path only as a line, without recording the chronology. Sometimes the chronological information will be lost during the digitalization. In this paper it has been shown that applying the divider method, the obtained fractal dimension on paths containing loops can be erroneous when the analyzed curve does not have time-indices. The extent of this error can be high enough to mask the difference between different tracks (or between different animals). The analysis of chronologic paths also can give us an example that tortuosity and fractal dimension is not always correlated.

Further research will be directed into the analysis of real animal paths as well as into the analysis of other fields where tortuous paths can be generated by various particles (like solid particles in a turbulent flow).

Appendix

It is obvious that the probability that an animal will make two sharp turn at the same point (like on Fig. 1a) is very small. But whenever one would like to analyze a
trajectory, it should be digitalized somehow. For example by making a bitmap picture, the digitalization would be done like it is represented on Fig. 4. As it can be seen, a "smoother" and more realistic trajectory (Fig. 4b) would give the same bitmap picture like the one with the sharp turns (Fig. 4a). The "digital" track is marked by the dark-grey pixels.

References


Table 1: The length of the sample loop with the “back” and “forth” scenario and the difference between them in percent.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$L_{back}$</th>
<th>$L_{forth}$</th>
<th>$(1 - \frac{L_{back}}{L_{forth}})$ in %</th>
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<td>10</td>
<td>870.2</td>
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<td>920</td>
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<td>901.4</td>
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<td>901</td>
<td>19.60044</td>
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<td>773.3</td>
<td>910.3</td>
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Figure legends

Figure 1. The "back" (a) and "forth" (b) scenarios to go along a geometrically similar path with different chronological order. The difference by measuring the length by the divider method in the "back" (c) and "forth" (d) scenarios are demonstrated too (grey lines represent the "yardstick"). Further description can be seen in the text.

Figure 2. The relative error of measuring the length by the divider method in the "back" and "forth" scenarios.

Figure 3. The effect of the observed error on the fractal dimension. Further information can be seen in the text.

Figure 4. Producing a "back"-type path from a smooth one by digitalization. The small arrows indicate the direction of the movement.
Figure 1

Figure 2.
Figure 3

Figure 4