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Tracing Causality and Co-movement between Pakistani and the Leading Foreign Stock Markets: A Graph Theoretic Approach

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Abstract

This paper developed and modify Peter and Clark (PC) causality algorithm to revisit the causal linkages between Pakistan and the leading foreign stock markets. Initially, the PC algorithm was conceived to determine causality in cross sectional data. Later on, (Swanson & Granger, 1997) for the first time used VAR residuals in PC algorithm to determine the causal ordering in time series. However, the weak point attached to VAR residuals are that it carries only contemporaneous causal information and remove all the past information. This study modify the PC algorithm based on recursive residuals proposed by (Rehman & Malik, 2014) and explore the causality among exchange rate, interest rate and stock market prices. The overall empirical results of modified PC algorithm indicate that causality is running from exchange rate, interest rate and stock market of India and Bangladesh to Pakistani stock market. The results observed from GARCH-GJR model show spill over effect from the leading foreign stock markets toward Pakistan stock market excluding Sir Lanka. The results of the study will guide the investors to be vigilant in decision making in diversified portfolio investment and hedging.

Keywords: *Financial Markets, PC algorithms, Causality, Graph theoretic Approach, GARCH, GJR.*

Introduction

Causality, risk and volatility across financial stock markets are of greater interest to academicians and policy makers. According to (Abou-Zaid, 2011) increase in the stock market volatility is due to the liberalization in the global financial market and fluctuations in the macroeconomic variables i.e. interest rate and exchange rate. Recently, (Ma, Yang, Zou, & Liu, 2020) also studied financial stock market liberalization, and causality transmission from one market to another market. They note that extreme risk is a part of this causality relationship and the directional co-movement between selected variables without due regard being given to the role of interest rate, exchange rate vis-à-vis stock market in this complex phenomenon. They also suggests that the financial stock market volatility has been influenced due to volatility in interest rates and exchange rates. Thus, economic policy makers and stock market analysts requires to conceive a policy that mitigate the effect of shock in the macroeconomic variables of one country on the other (domestic country). However, such an effective policy heavily relies upon the true results of stock markets causality studies. Numerous studies (D. K. Wong & Li, 2010) and (Chi, Li, & Bo, 2012) among others have attempted to determine the causal direction among stock markets and other macroeconomic variables using Granger type causality tests; however, these causality tests have some weak points as mentioned in the study of (Xing, Cambria, & Welsch, 2019) and the results cannot be fully relied. This study is, therefore, design to develop an advanced measuring method; modified PC algorithm of GTA to test the causality of financial stocks markets with high accuracy.

Several testing procedures have been applied over the last few decades for testing causality between financial stock markets; for instance (Granger, 1969) and PC causal algorithms of GTA developed by (Spirtes, Glymour, Scheines, & Heckerman, 2000) which is rarely applied in economics and finance. However, some week points are attached to these causality tests. Firstly, Granger causality does not have the ability to consider the structural causality and determines only the predictability not the causality ((Moneta, Chlaß, Entner, & Hoyer, 2011) and (Mazzarisi, Zaoli, Campajola, & Lillo, 2020). Granger-type causality check causal ordering between two variables or nodes. However, there may also be a chance of third node – Z which is the common cause of both A and B (Fazal, Rehman, Rehman, Bhatti, & Hussain, 2021). (Demiralp, Hoover, & Perez, 2008) argues that Granger causality fails to capture structural causality. Like, Granger causality, the original PC algorithms of GTA has also some weak flaws. Initially, PC algorithms were generally not designed for time series data. To make this approach applicable in economics, (Swanson & Granger, 1997) used the residuals series extracted through vector autoregressive (VAR) model and treat as original variables in the PC algorithms. They stated that VAR residuals contain the information about the causal ordering; and used these residuals series as original variables in the PC algorithm to find the causal direction. However, using VAR residuals carries some logical flaws as it carry only the current information about X_t and Y_t and purge the past causal information X_{t-i} and Y_{t-i}. Thus, causality cannot be correctly determined with a high chance to of being spurious. This paper is therefore set out to propose a new methodology that has overcome aforementioned problem and correctly identified the causality among exchange rate, stock market prices and interest rate for the four major SARCC countries i.e. Pakistan, India, Bangladesh and Sri Lanka (PIBS).

Research Objectives

The objectives of the study are:

1) To modify the original PC causality algorithm of graph theoretic approach by replacing VAR residuals with modified R recursive residuals presented by Rehman and Malik (2014) called modified PC algorithm.
2) To apply the novel modified PC algorithm and GARCH-GJR model to find the causality directions and spill over effect among the macroeconomic variables respectively i.e. exchange rate, stock market prices and interest rate for the four major SARCC countries.

The application of novel modified PC algorithm and research findings will make important contribution in the field of integrated financial markets. Likewise, methodologically speaking, the modified PC causality algorithm is equally applicable in environmental economics, health economics, econometrics and others as a standalone method that can be applied in other related research areas. The next sections of this paper contain related research overview (section 2), theoretical and methodological framework (section 3), results and discussion (section 4) and final section is conclusions provided in (section 5).

Related Research Overview

Planning and policy experts, economist and econometricians always have keen interest to investigate the financial market integration, causality and volatility spill over effect among different stock markets. Like other countries, the volatility spill over effect of stock markets has escalated, gradually in Pakistan and its partner's economies, because of the international financial liberalization. The effect exists, widely almost across all the financial markets in different regions. Many other researcher terms volatility in all financial markets an important aspect, which instigates the volatility and risk conduction procedure from one stock market to another. Most of the existing research on the risk transmission, volatility spill over effect and co-movement has been focussed around financial markets in the developed countries. There's been little focus on the developing countries. Some of the recent studies, as (Yousaf, Ali, & Wong, 2020) and (Kanas, 2000) conducted the study to investigate the co-movement between exchange rate volatility and stock prices for BRICS and Latin American countries using BEKK-GARCH as well as VAR model. They have concluded that the presence of return and volatility spill over within the particular BRICS countries stock market play relatively more important role than foreign exchange market. These studies did not find any significant relationship between exchange rates and stock prices. For Latin American countries the finding suggest that no feedback causality is found in case of Mexico and US stock market during global financial crises but found one sided return transmission from Mexico to US. The study also suggests that during chines crush the volatility spill over effect is bidirectional between the stock market of Brazil and china.

Many other studies used multivariate GRACH in mean (MGRACH) model to explore the internal and external spill over effect for the BRIC countries. (Xiong & Han, 2015) have tested the spill over effect between stock prices and exchange rate with the application of Granger causality test and error correction model. These studies presented the existence of stable and long run significant causal linkages between the exchange rate and financial stock markets. Similarly, (Panda & Deo, 2014) analyse different time periods like pre crises and post crises period for India using two variables, exchange rate and stock prices it is based on daily data and uses EGARCH model. By comparing different periods, it reveals that post crises period has more volatile as well as asymmetric distribution than the others. It also found that Nifty and USD were also significant and asymmetric spill over effect between these variables. An empirical study was conducted to explore the co-movement and causal connections between the exchange and stock markets with the application of wavelet analysis by (Chi et al., 2012) and found a one-sided volatility spill over from the stock market to the exchange market. (D. K. Wong & Li, 2010) has drawn parallel between the Asian financial crisis in 1997 and the 2007 financial crisis in United States and studied the importance of its external impacts for analysis. He has conclude that having the exchange rate remain relative stable during a financial crisis would be important for the normal function of the stock market.

As for as the research methods are concerned, studies financial stock markets have, mostly employed conventional methodologies. These studies were restricted to co-integration tests, Granger causality (GC) tests, VAR model, and impulse response function (IRF) for the investigation of causality and volatility spill over effect among various financial markets.

Study	Methodology	Results
(Alkan & Çiçek, 2020)	Multivariate GARCH Model	Fluctuations in the volatility in a domestic market transfer to other local or domestic markets.
(Kanas, 2000)	EGRACH Model	There was a co-movement between both the stock market to the exchange market.
(Panda & Deo, 2014)	EGRACH Model	Study found that post crises period has more volatile as well as asymmetric distribution as compare to other periods
(Y. S. Wong, Ho, & Dollery, 2012)	EGRACH Model	The results revealed that real exports are significantly affected by exchange rate volatility in most countries, but not in all South Asian countries.

(Christiansen, 2007)	GARCH model	The result suggests that with the initiation of euro the EMU bond markets have become much more assimilated.
(Yang & Doong, 2004)	EGRACH Model	Yang and Doong also investigated a strong causality and co-movement between these two markest i.e. stock market cause exchange market
(Shusong & Min, 2009)	EGRACH Model	Shusong and Min studied both the symmetric and asymmetric co-movement between the variables stock market (SM) and exchange market (EM). The finding shows that there is asymmetric co-movement from SM to EM but a symmetric from the EM to SM.
(Yousaf et al., 2020)	BEKK-GARCH	Unidirectional causality from Mexico to US stock market during GFC and during chines crush the feedback causality between the stock market of brazil and chin is found.

Some other studies like (Chi et al., 2012) and (Škrinjarić & Orlović, 2020) among others have attempted to look for the direction of causality and spill over effect using Granger type causality tests and GRARCH family models respectively.

However, recently integration in the financial markets of the growing/developing economies have revealed rapid progress with increased global integration. Therefore, application of conventional methods like Granger type causality tests to determine the causal directions among various financial stocks markets cannot be fully relied. This study is, therefore, design to develop an advanced measuring method; modified PC algorithm of GTA to test the causality of financial stocks markets with high reliability.

Theoretical and Methodological Framework

Modified PC Algorithm – Causality

The novel modified PC causality algorithm under the framework of graphical approach contains effective statistical tool to determine the causal relationship among the various financial stock markets. (Swanson & Granger, 1997) for the first time used it for time series data and assumed that information about causal ordering may be present in the covariance matrix of VAR error terms, therefore they treated VAR residuals as original variables in PC algorithm to find causal order.

The use of VAR residuals is likely to remove the non-stationarity problem from the time series data and correlation can be used to determine causality with a very low chance of being spurious. However, in the VAR model with variable X_t and Y_t, X_t is assumed to be a function of Y_{t-1} and others i.e. $X_t = f(Y_{t-1} \ldots.)$ and conversely, Y_t has assumed the function of X_{t-1} and others i.e. $Y_t = f(X_{t-1} \ldots.)$.
Consider a VAR model:

$$y_t = \alpha_1 + \beta_1 * x_{t-1} + \beta_2 * y_{t-1} + \varepsilon_{1t} \ldots (1)$$

$$x_t = \alpha_2 + \beta_3 * y_{t-1} + \beta_4 * x_{t-1} + \varepsilon_{2t} \ldots (2)$$

So, in the equation of Y_t only effect of X_t could be there, the effect of X_{t-1} and past values (X_{t-i} where i > 1) are removed. Thus, VAR residuals carries only contemporaneous information about the cross variable effect. This goes against the spirit of various definitions of causality such as Granger Causality. On the other hand, there are several univariate methods i.e. (Rehman & Malik, 2014) test residuals - modified R recursive residuals, which can eliminate the non-stationarity without removing the effect of cross variable feedback. In this study, we have developed modified PC algorithm of GTA by replacing VAR residuals with modified R recursive residuals and employed it in the current study to find the causality between Pakistan and the leading economies stock markets.

Graph Theoretic Approach: Notation and Terminology

Before starting the steps involved in modified PC causal algorithm; it is important to discuss the notations and statistical terms used in GTA.

Nodes and edge

A graph consists of a set *N* of nodes and a set of *E* edges. The nodes in the graph represent variables while edges show us relationship between pair of nodes or variables. These edges may have arrowheads showing the direction of causation.

Directed and undirected edge

Connection between two nodes through straight line is called *undirected edge* (A─B). While connection between two nodes through straight line having arrowhead is called *directed edge* (A→B). Edges having arrowheads indicating the direction of causation.

Skeleton Graph

The graph showing only the nodes and strip away all arrowheads from the edges is called skeleton.

Acyclic graph

The graph is acyclic when there is no feedback causal relationship. If there is arrow head on both ends of an edge, the relationship is called cyclical.

Causally sufficient graph

A graph is causally sufficient when all variables are observable i.e. that there are no latent variables

Condition of faithfulness

A graph and probability distribution is said to be faithful if and only if there is one to one correspondence between conditional independence relationships implied by causal Markov condition.

Mathematical notation of modified PC algorithm

Input: Dataset D with a set of variables V, and
significant level

Output: The undirected graph *G* with a set of edges *E*
Initially it is assumed that all variables or nodes are linked with each
other. Suppose depth *d = 0*

repeat
for each ordered pair of adjacent vertices *X* and *Y* in
G do
if *(|adj (X,G)\{Y}\|>=d)* **then**
for each subset *Z* ⊆ *adj (X,G) \{Y}* and
|*Z*|=d **do**
Test *I (X,Y | Z)*
If *I (X,Y | Z)* **then**
Eliminate linke/edge between *X* and *Y*
Store *Z* as the separating set of
(X,Y)
Update *G* and *E*
Break
end
end
end
end
Let *d = d + 1*
Until | *adj(X,G)\{Y}*| < *d*) for every pair of adjacent
vertices in *G;*

Steps involved in PC and Modified PC Algorithms of Graph Theoretic Approach

Modified PC algorithms has five steps to determine the causal ordering. In the first three steps, it learns from the data information and make a skeleton graph, while in the last two steps it orients the arrows head to construct the final causal graph. However, before explaining the steps involved in modified PC algorithms, it is important to discuss the assumptions and mathematical notation of modified PC algorithm. (Fazal et al., 2021) illustrates the following five steps.

Step 1. In the first step, it the algorithm **c**onstruct the general structure of graph in which all variables are connected through undirected links.
Step 2. The algorithm then test unconditional correlation between any two variables. If they are not unconditionally correlated then eliminate that connections
Step 3. Tests correlation between each two variables conditional on a third variable. If each pair of variables are conditionally uncorrelated then again eliminate their connections.
Step 4. This step is called orientation stage. In the previous step 3 if the pair is correlated conditional on the third variable, the third variable is said to be unshielded collider on that path, and arrows from the pair of variables are oriented toward the third variable. To explain the term unshielded collider, we have (Figure 2) which does not show any direct relationship between node *A* and B which mean that these two nodes are unconditionally uncorrelated, but conditionally correlated on node *D* (Hoover, 2020).

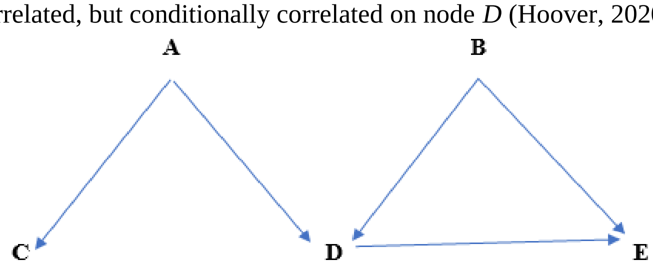

Figure 2: Directed Graph

Step 5. In this step, arrows are oriented on the basis of screening relationship. If two variables X and Y are not directly linked but are linked through a third variable Z as $X \rightarrow Z$--- Y so that one link points to the third variable say $X \rightarrow Z$ and the other link is undirected Z--- Y. So, orient the second link as $X \rightarrow Z \rightarrow Y$ because orienting the arrow toward Z shows that Z is unshielded collider and if it is true then this should be revealed in step 4. Thus, the intervening variable is a screen and not an unshielded collider, so the arrow cannot point toward it.

It is also important to note that the calculated values of conditional correlations will be compared with fisher's z test value. Fisher z test can be applied to test that whether the conditional correlations are significantly different from zero.

$$z(\rho(i,j|k) = \left[\frac{1}{2}\sqrt{n-|k|-3}\right] ln\left\{\frac{1+\rho(i,j|k|)}{1-\rho(i,j|k|)}\right\}$$

Where n is number of observations, (ρ(i,j|k) is the population correlation between i and j conditional on k. |k| is number of variables that we condition on.

Econometric Modelling for Volatility

The financial data is highly volatile in nature. To capture the effect of volatility in financial data series, (Engle, 1982) developed the Auto Regressive Conditional Heteroskedastic (ARCH) model. However, the problems attached to ARCH model is the extensive lag length and non-negativity restriction. For this (Bollerslev, 1986) proposed generalized auto regressive conditional heteroskedastic (GARCH) model by adding the lag term of conditional variance in the equation. Although the weak point attached to GRACH model is that it only considers positive shock and has not the power to capture the effect of negative shock. Finally, Glosten presented GJR model which is the modified and extended version of standard GARCH model and can analyse the asymmetric effect.

GARCH (p, q) Model

GARCH model contain the following two equations

Conditional Mean Equation (CME)

$$R_t = \alpha_0 + \beta * X_t + \varepsilon t \ldots (1), \qquad \text{Where } \varepsilon t \sim N\,(0, \sigma_t^2)$$

Conditional Variance Equation (CVE)

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{p} \varphi i * \sigma_{t-1}^2 + \sum_{i=1}^{q} \theta i * \varepsilon_{t-1}^2 \ldots (2), \qquad \text{Where } \theta_0 > 0,\ \theta_i \geq 0$$

The variable R_t and β represent the return series and vector of parameters in CME (1). In conditional variance equation (2), $\sigma_t{}^2$ shows conditional variance which depend on $\varepsilon_{t-1}{}^2$ and lag of conditional variance σ_{t-1}.

GJR (p q) Model

As discussed, that GJR model is the extended version of GARCH model which consist of the asymmetric term in the CVE (4) to capture effect of negative shock.

Conditional Mean Equation (CME)

$$R_t = \alpha_0 + \beta * X_t + \varepsilon t \ldots (3), \qquad \text{Where } \varepsilon t \sim N\,(0, \sigma_t^2)$$

Conditional Variance Equation (CVE)

$$\sigma_t^{2} = \theta_0 + \sum_{i=1}^{q} \theta i * \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \delta * \varepsilon_{t-i}^2 \; G_t + \sum_{i=1}^{p} \varphi i * \sigma_{t-1}^2 \ldots (4)$$

Where, θ_0 and $\theta_i > 0$

$0 \leq \delta i \geq 1$, is the range defined for the parameter of leverage effect.
$G_t = 1$ when $\varepsilon_{t-1} < 0$ and $G_t = 0$ when $\varepsilon_{t-1} \geq 0$

$G_t = 1$ for bad news, $G_t = 0$ for good news, if $\delta_I > 0$ then it shows that there is leverage effect, if $\delta_I = 0$ then both news has same impact.

Data and Variables

In this study we have used monthly data of various macroeconomic variables such as exchange rate, interest rate and stock market prices of Pakistan and the leading foreign markets i.e. India, Sir Lanka and Bangladesh. The data has been collected from International Financial Statistics (IFS), World development indicators (WDI) and Business recorder from the time period 2006 to 2018.

Results and Discussion

In the current study, the newly developed modified PC causality algorithm is employed to determine the causal directions among Pakistani and leading economies stock markets using monthly time series data of the past 144 months i.e. decemebr 2006 to decemebr 2018. The stock market indicies used for Pakistan, India, Banglades and Sir Lanka are KSE-100, BSE-SENSEX, CASPI and CSE all Share Index respectively. Before the application of modified PC causality algorithms and GARCH GJR models, we have graphically and discriptively analyse all the indicies to find the normality, ARCH effect and to explore that the series are symmetric or not symmetric displayed in Table 1.

Table 1. Discriptive Analysis

Variables	Mean	SD	Skew	JB	Kurtosis	Q-stat (10)	ARCH 1-10
KSE100	0.01	0.07	-1.90*	591*	9.7*	75.9*	32.3*
DLIST	0.01	0.06	-0.64*	42*	2.1*	71.7*	2.85*
DLSST	0.01	0.06	2.14*	4**	0.7**	76.1*	3.0*
DLBST	0.26	0.08	-0.60*	80*	3.5*	20.7*	9.4*
DLSER	0.00	0.01	0.72*	368*	7.9*	20.7*	4.9*
DLBER	0.00	0.01	-0.22*	616*	10.4*	45.4*	2.3**
DLPER	0.00	0.01	-0.11*	310*	7.3*	23.7*	2.8*
DLIER	0.00	0.02	-0.11*	9*	1.2*	63.5*	2.1**
DLIIR	0.00	0.04	-1.62*	917*	12.3*	22.4*	2.9*
DLPIR	0.01	0.05	1.41*	170*	4.7*	123.7*	123*
DLBIR	0.20	0.03	-1.31*	2438*	20.5*	11.2*	5.1*
DLSIR	0.18	0.05	-0.44*	72*	3.4*	64.5*	9.2**

The result displayed in (Table 1) is the summary statistic of the stock market indices and other control variables i.e. exchange rate and interest rate. The results show that the return series mean values for all countries variables are near to zero, which tell us the mean reversion behaviour of series. The standard deviation (SD) shows the deviation from the mean value. In case of small SD the deviation from mean value will be high. Similarly, the skewness, shows the shape of the distribution of the return series, the results shows that about all return series have negatively skewed.

The results also indicate that the series are not normal, which is confirmed from the Jarque-Bera (JB) test of normality. The column of Q-statics shows that there is autocorrelation problem because the null hypothesis of no autocorrelation is rejected for all data series. The Q-statistics squared series is also significant, and probability value is less than 5 %, so we are rejecting the null hypothesis and confirmed that there is serial autocorrelation in all return series. From the results of LM-ARCH test it is also confirmed that there is ARCH effect in the return series.

Estimated Results of PC Causality Algorithm

As discussed earlier that modified PC algorithm treat modified R recursive residuals as origional variables, therefore, first we have estimated the modified R residuals using the stock prices, exchange rate and interest rate variables of various countries through (Rehman & Malik, 2014) test procedure. These residuals series are then referred to the modified PC algorithm and by treating as original variables. The results of modified PC algorithms are provided in Figure 3 using TETRAD 4.9.1 software.

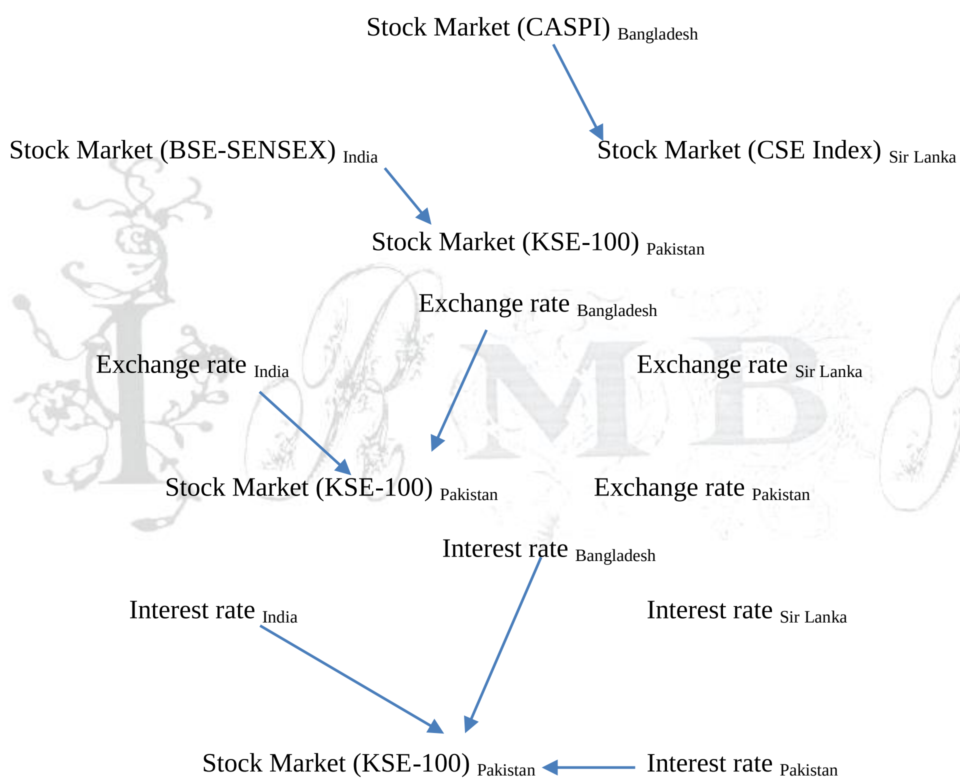

Figure 3. Causal directions indicated by modified PC algorithm

The results indicated by novel modified PC causal algorithm of GTA is displayed in the above Figure 3. In the first case (financial stock markets) the results shows us that causality is running from the stock market (CASPI) of Bangladesh and stock market (BSE-SENSEX) of India to the Pakistani stock market (KSE-100) while no causality has been found between Pakistani stock market (KSE-100) and Sir Lankan stock markets (CSE Index). In the second case there is a significant causal connection between the exchange rate of India and Bangladesh toward Pakistani stock market (KSE-100) are noted. This mean that the both countries exchange rate has significantly affected the stock market of Pakistan. These findings are interesting as conventional Granger type causality tests results indicate absence of causality relation between the exchange rate of Bangladesh and Pakistan stock market. These results are in line with

(Škrinjarić & Orlović, 2020) and (Kanas, 2000). Finally, in case third of Figure 3 it explores a significant existence of causal connections between interest rate of India and stock market of Pakistan. In all these three cases (exchange rate, stock market prices and interest rate) both India and Bangladesh significantly affect the stock market of Pakistan. This is because both countries are the key trading partner of Pakistan as compared to Sir Lanka. The market structure of India and Bangladesh are like the country Pakistan with strong association among their financial markets, and thus vulnerable to any financial market integration of these countries. Our results are in line with (Amarasinghe, 2015), (Gupta, Chevalier, & Sayekt, 2001) and (Ramsharan, 2019). They noted that a slight change in interest rate cause high change in stock prices.

The graph theoretic approach of modified PC algorithm determines the main causality channels affecting the Pakistani financial stock markets, however, to check the mean and volatility spill over effects among these stock markets, this study employed GARCH and GJR models. As discussed earlier that GRACH model consider only positive shocks, however to capture the effect of negative shock Glosten et al. (1993) introduce GJR which is the further extension of the GARCH model with the addition of asymmetric term in the conditional variance equation to capture the effect of both positive and negative shocks.

Estimated Results of GARCH and GJR Model

The estimated results of GRACH and GJR model are displayed in Tables 2, 3 and 4 as under:

Table 2. Stock Markets Spill over effect of Leading Economies on Pakistan (KSE-100)

Return Series Parameter	BAN (CASPI) ARMA(0,0) GJR (1)	SR (CSE Index) ARMA(1,1) GJR (1)	IND (BSE-SENSEX) ARMA(1,1) GJR (1)
Conditional Mean Equation (CME)			
Constant	-0.041	0.011	-0.137
α_0	(0.50)	(0.89)	(0.01)
AR(1)		-0.517	0.935
$\vartheta 1$		(0.119)	(0.00)
MA(1)		0.701	-0.99
$\emptyset 1$		(0.00)	(0.00)
Returns	-0.103	0.080	0.294
$\boldsymbol{\rho}$	(0.44)	(0.50)	(0.017)
Conditional Variance Equation (CVE)			
Constant	0.0043	0.0006	-9.305
θ0	(0.00)	(0.09)	(0.38)
ARCH (1)	-0.061	0.032	0.111
θ1	(0.79)	(0.75)	(0.31)
GARCH (1)	-0.167	0.833	0.88
$\varphi 1$	(0.32)	(0.08)	(0.00)
Variances	0.341	-0.05	0.124
$\boldsymbol{\gamma}$	(0.00)	(0.08)	(0.00)
GJR(1)	0.128	0.141	-0.149
$\delta 1$	(0.64)	(0.30)	(0.1907)
Residual Diagnostic Test			
Series	**Jarque Bera**	**Q^2-stat (5)**	**LM-ARCH (1-5)**
SRI_ST	112.80	0.99	0.916
	(0.00)	(0.96)	(0.96)
BAN_ST	71.28	5.59	4.656
	(0.00)	(0.34)	(0.45)
IND_ST	27.04	1.33	1.33
	(0.00)	(0.93)	(0.93)

The results displayed in (Table 2) shows mean spill over effect between Pakistan and India stock market because the return series of India in the conditional mean equation is significant at 5 % level of significance. We observe that return of Pakistan stock market may affect by Indian stock market return but there is no evidence of mean spill over effect of Bangladesh and Sri Lanka stock market on Pakistan stock market. In the variance equation, the results indicate that there is volatility spill over effect between Pakistan and all leading economies stock markets confirmed from the coefficient γ.

Table 3. Exchange rate Spill over effect of Leading Economies on Pakistan (KSE-100)

Return Series	**BAN ER to PAK (KSE-100)**	**SRI ER to PAK (KSE-100)**	**IND ER to PAK (KSE-100)**	**Pak EXC to (KSE-100)**
Parameter	**ARMA(1,1) GJR**	**ARMA(1,1) GJR**	**ARMA(1,1) GJR**	**ARMA(1,1)**
Conditional Mean Equation (CME)				
Constant α_0	0.0020 (0.98)	0.112 (0.18)	0.036 (0.27)	0.010 (0.86)
AR(1) $\vartheta 1$	-0.526 (0.27)	-0.241 (0.59)	-0.41 (0.38)	0.835 (0.00)
MA(1) $\emptyset 1$	0.65 0.09	0.376 (0.36)	0.564 (0.19)	-0.916 (0.00)
Returns $\boldsymbol{\rho}$	0.016 (0.96)	-0.088 (0.93)	-0.36 (0.15)	-1.632 (0.02)
Conditional Variance Equation (CVE)				
Constant θ0	0.0004 (0.21)	0.003 (0.00)	0.003 (0.05)	0.000 (0.25)
ARCH (1) θ1	0.077 (0.57)	-0.132 (0.45)	-0.122 (0.07)	0.121 (0.34)
GARCH (1) $\varphi 1$	0.399 (0.00)	0.065 (0.47)	0.399 (0.31)	0.765 (0.00)
Variances $\boldsymbol{\gamma}$	-0.44 (0.00)	11.88 (0.00)	-0.737 (0.00)	-0.122 (0.07)
GJR(1) $\delta 1$	0.15 (0.58)	0.142 (0.50)	0.142 (0.16)	2.578 (0.33)
Diagnostic Test Model				
Series	**Jarque Bera**	**Q^2-stat (5)**	**LM-ARCH (1-5)**	
SRI_ER	61.74 (0.00)	4.06 (0.54)	3.51 (0.62)	
BAN_ER	162.01 (0.00)	0.552 (0.99)	0.55 (0.98)	
IND_ER	578.4 (0.00)	1.64 (0.89)	1.48 (0.91)	
PAK_ER	21.71 (0.00)	1.9880 (0.851)	1.93 (0.85)	

Finally, it is concluded that Pakistan stock market volatility is affected by the volatility in all three countries stock markets. The term GJR is insignificant for each country stock market, and one can say that there is no leverage effect of India, Bangladesh and Sir Lanka stock market on Pakistan stock market. Through residuals analysis, it is also confirmed that the series for all countries are not normal as null hypothesis of JB test is rejected at 5% level of significance. The results also tell us that there is no auto correlation and no ARCH effect in the return series.

Table 4. Interest rate Spill over effect of Leading Economies on Pakistan (KSE-100)

Return Series	**BAN INT to PAK (KSE-100)**	**SRI INT to PAK (KSE-100)**	**IND INT ti PAK (KSE-100)**	**Pak INT to PAK (KSE-100)**
Parameter	**ARMA(1,1) GJR**	**ARMA(1,1) GJR**	**ARMA(1,1) GJR**	**ARMA(1,1)**
Conditional Mean Equation (CME)				
Constant α_0	-0.242 (0.99)	-0.075 (0.15)	-0.06 (0.0005)	0.036 (0.44)
AR(1) $\vartheta 1$	0.99 (0.00)			-0.705 (0.00)
MA(1) $\emptyset 1$	46923 (0.36)			0.824 (0.00)
Returns $\boldsymbol{\rho}$	-0.109 (0.95)	-0.004 (0.96)	0.300 (0.14)	0.1992 (0.17)
Conditional Variance Equation (CVE)				
Constant $\theta 0$	0.000 (0.89)	0.000 (0.00)	0.003 (0.07)	0.001 (0.00)
ARCH (1) $\theta 1$	0.042 (0.97)	-0.026 (0.32)	-0.037 (0.20)	-0.301 (0.00)
GARCH (1) $\varphi 1$	0.078 (0.99)	1.030 (0.00)	-0.180 (0.00)	0.785 (0.00)
Variances $\boldsymbol{\gamma}$	4.57 (0.90)	-0.095 (0.00)	2.217 (0.00)	0.138 (0.08)
GJR(1) $\delta 1$	0.43 (0.99)	-0.025 (0.501)	-0.146 (0.00)	0.355 (0.00)
Residual Diagnostic Test				
Series	**Jarque Bera**	**Q^2-stat (5)**	**LM-ARCH (1-5)**	
SRI_INT	259.6 (0.00)	4.66 (0.45)	4.430 (0.48)	
BAN_INT	210.3 (0.00)	0.45 (0.9)	0.44 (0.99)	
IND_INT	53.74 (0.0)	3.06 (0.34)	2.51 (0.54)	
PAK_INT	132.7 (0.00)	1.09 (0.954)	0.93 (0.96)	

In order to check the exchange rate volatility of Bangladesh, Sri Lanka, and India on Pakistan stock market, we have estimated GARCH and GJR model shown in Table 3. The results indicate that there is a mean spill over effect between Pakistan stock market prices and exchange rate of Pakistan because the return series of Pakistan exchange rate in conditional mean equation is significant at 5 % level of significance, showing that fluctuations in Pakistan exchange rate can also affect the return of the Pakistan stock market prices, but no evidence of mean spill over effect from Bangladesh, Sri Lanka and India are noticed. These results are in line with (Shusong & Min, 2009), (Yang & Doong, 2004) and (Škrinjarić & Orlović, 2020). In the variance equation, the results indicate that there is volatility spill over effect between Pakistan stock market and exchange rate of the leading economies. Finally, it is concluded that Pakistan stock market is affected by the volatility in all three countries exchange rate. The term GJR is insignificant, and we can say that there is no leverage effect of exchange rate on Pakistan stock market. Through residuals analysis, it is also confirmed that the series for all countries are not normal as null hypothesis of JB test is rejected at 5% level of significance. The results also reveal that there is no auto correlation and no ARCH effect in the return series.

Finally, (Table 4) indicate the mean and volatility spill over effects between stock market prices of Pakistan and volatility in the interest rate of the leading economies. The results show that there is no mean spill over effect between Pakistan stock market prices and interest rate of overall leading partner countries. The variance spill over effect between of stock market prices of Pakistan and interest rate of India, Sri Lanka are significant which indicate that shock in the interest rate of India and Sri Lanka can affect Pakistan stock market prices. The interest rate of Pakistan also affects that her own stock market prices.

Conclusion and Recommendation

The purpose of this study is to revisit the causal linkages between Pakistani and the leading foreign stock markets using the newly developed modified PC causality algorithm under the graph theoretical approach. This study developed and employed modified PC algorithm which successfully investigate the causal directions between Pakistan and the leading foreign stock markets i.e. India, Bangladesh and Sir Lanka based on 144 months' time series data (2006-2018) using causality discovery TETRAD V software. The study also applied GARCH and GJR model to check the mean and variance spill over effect among these stock markets.

The overall empirical results of modified PC algorithm indicate that causality is running from exchange rate, interest rate and stock market of India (BSE-SENSEX) and Bangladesh (CASPI) to Pakistani stock market (KSE-100). The results of GARCH and GJR model indicates that the return of Indian stock market (BSE-SENSEX) effect the return of Pakistan stock market (KSE-100) while the other two stock markets Bangladesh and Sir Lanka (CASPI) and (CSE Index) respectively have insignificant effect on Pakistan stock return (KSE-100). Furthermore, the volatility of Indian and Bangladesh stock market (BSE-SENSEX) and (CASPI) respectively have significant impact on Pakistani stock market (KSE-100). Similarly, uncertainty in currency markets and interest rate of leading countries have significant impact on Pakistan stock market. In short, the results observed from GARCH indicate a volatility spill over effect from India and Bangladesh stock markets, interest rate and exchange rate to Pakistani stock market. This is because both countries are the key trading partner of Pakistan, and financial market structure of these economies are alike having strong association. This study provides insight for policy makers and investors to be vigilant regarding decision making in portfolio allocation, currency risk and hedging in both domestic and international market.

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