A MIXED TRAFFIC MODEL FOR PERFORMANCE EVALUATION OF WIRELESS NETWORKS

K. Aggelis, S. Louvros, A.C. Iossifides, A. Baltagiannis, V. Triantafyllou

1Physics Department, University of Patras, Greece,
2,5Dept. of Telecommunication Systems and Networks, Technological Educational Institute (TEI) of Messologi, Greece,
3Electronics Department, Alexander Technological Educational Institute (TEI) of Thessaloniki, Greece
4Department of Engineering Sciences and Applied Mathematics, University of Patras, Greece

1kosangel@gmail.com, 2splouvros@gmail.com, 3aiosifidis@el.teithe.gr, 4abalt@upatras.gr, 5triantaf@teimes.gr

Keywords: Mixed traffic, multi-state transitions, multilayer network, blocking probability.

Abstract: In contemporary and future mobile communication networks, variable services with different characteristics need to be served simultaneously. This paper proposes a novel mathematical model for the analysis of such cases based on a two-dimensional Markov approach. This model comprises higher order resource allocation and resource sharing with no packet traffic pre-emption priority and no buffers. The state probabilities of the model have been mathematically derived and can be used as a basis for the evaluation of performance metrics of modern high speed mobile networks supporting two different service classes. Based on the model, the blocking probabilities of two resource allocation strategies are evaluated for two simultaneous services of different characteristics.

1 INTRODUCTION

Nowadays mixed services multilayer cellular networks (HSPA, LTE, WiMAX) are the dominant solutions for wireless communication. In these networks, an ongoing set of services could be handed between different technology networks by allocating the necessary resources for the guaranteed bandwidth, based on proper capacity management, admission control and congestion control algorithms. Towards this end, several approaches have been proposed in international literature. Among them, Ermel et al. in [1], raised the question of the best strategy to partition the available cell capacity by describing three different strategies, i.e. complete partitioning, partial sharing and complete sharing. Wei-Yeh Chen et al. in [2] proposed an analytical model using two dimensional queues and employing a buffer for one of the two services proving that other service blocking probability can be reduced. Bynat et al. in [3], introduced a partial partitioning scheme using the extended Erlang-B law in order to simulate more accurately the performance of pre-emption strategy on mixed traffic cellular scenarios. Belzenara et al. in [4], introduced a more advanced approach with different operational scenarios and evaluated the performance of mixed traffic. Pylarinos et al. in [5], proposed a simple two dimensional queue model without any queue and considered only the case of lost pre-empted GPRS packets. They proved that for high load in the system, the GPRS blocking probability and pre-emption probability increases linearly. Islam et al. [6] proposed a three dimensional queue model to evaluate QoS, however the mathematical approach was mostly based on analytical calculations rather than on traditional Markov chain solution due to its complexity. Ramirez et al. [7] proposed an innovative resource allocation algorithm that combines allocation of resources to both voice and data packet traffic services and a combinatorial mathematical solution, trying to predict the available resources to be allocated for data traffic. Balint et al. [8] proposed a partial partitioning resource allocation scheme where three mixed traffic models have been studied and validated based on several performance parameters. A very good analysis was proposed by Carvalho et al. in [9], where the authors introduced a buffer management together with call admission control (CAC) algorithms.

All the aforementioned scientific works did not consider multi-resource allocation and resource
sharing. These two approaches, which are potential in contemporary wireless and mobile networks resource allocation algorithms, are incorporated in the proposed model. Additionally, following modern packet network design specifications, traffic pre-emption priority is excluded from the analysis. Based on these facts, our model provides a standard mathematical basis for resource allocation, capacity analysis and design requirements of contemporary mobile and wireless systems, such as HSPA, LTE and WiMax. Towards this end, the proposed model is used to predict the blocking probability of two different resource allocation strategies. A hard resource allocation strategy that respects subscribers’ resource requests (blocking requests that cannot be fully served) and a soft resource allocation strategy that allocates resources based on the availability of the system, probably leading to less allocated resources than requested by the subscriber. These scenarios are used as an example in order to prove the validity of the proposed model for analyzing various resource allocation algorithms of contemporary mobile systems.

2 MATHEMATICAL MODEL

The model developed assumes two services, namely, service $\mathcal{P}$, and service $\mathcal{Q}$, attempting to allocate resources from a common resource pool (e.g. carrier, slots, etc.). Each service follows a Poisson process with mean arrival rate equal to $\lambda_p$ and $\lambda_q$, respectively, and the corresponding service times are exponentially distributed with mean service times equal to $1/\mu_p$ and $1/\mu_q$, respectively. Moreover, for each service request the system could allocate more than one resource units with a specific probability. In this context, $p_i$ corresponds to the probability of a $\mathcal{P}$ service connection to be given one, two or more resource units, and $q_i$ corresponds to the probability of a $\mathcal{Q}$ service connection to be given one, two or more resource units, respectively. The system consists of a total number of $n$ resource units in a cell. Resource sharing is taken into account by allowing each resource unit to serve up to $v$ connections simultaneously. The proposed analytical model is based on a two-dimensional Markov chain where the state $E_{i,j}$ denotes that there are $i$ allocated resources for service $\mathcal{P}$ and $j$ allocated resources for service $\mathcal{Q}$ in the system. $P_{i,j}$ denotes the probability that the system is in state $E_{i,j}$. When the system is in state $E_{0,0}$, a total of $n$ resource units can be allocated either to $\mathcal{P}$ or $\mathcal{Q}$ service or both, satisfying $i + j = n$. In the same sense when the system is in state $E_{1,0}$ a maximum of $n-1$ resource units can be reserved. In the general case, from state $E_{i,j}$, for service $\mathcal{P}$, transition to states $E_{i+1,j}$ up

![Figure 1. The state diagram of the two-dimensional Markov model.](image-url)
to $E_{n-j}$ may take place with probabilities $p_1$ up to $p_{n-(i+j)}$, and, for service $\mathcal{Q}$, transition to states $E_{i,j+1}$ up to $E_{n-i,j}$ with probabilities from $q_1$ up to $q_{n-(i+j)}$, correspondingly. Figure 1 presents the state transition diagram of the corresponding two-dimensional Markov chain, where, $\pi_i = p_i \lambda_p^i \nu$ and $\kappa_j = q_j \lambda_q^j \nu$ are the probabilities of moving from state $E_{i,j}$ to $E_{i,j}$ and $E_{i,j}$ to $E_{i,j}$, respectively, and $\mu_p$, $\mu_q$, $\mu_v$ are the probabilities of moving from state $E_{i,j}$ to state $E_{i,j}$ and from state $E_{i,j}$ to state $E_{i,j}$, respectively (that may be different from the initial service rate of the services due to multiple resource allocation). The state probabilities of the model are given by equation (1) above, subjected to the following conditions

$$
P_i = q_i = p_i = q_i = 0$$

$$
P_{i,j} = 0 \quad \forall (i < 0) \cup (j < 0) \cup (i + j < n)$$

$$
\sum_{i=0}^{n} \sum_{j=0}^{n} P_{i,j} = 1, \quad \sum_{i=1}^{n-i} r_i = 1, \quad \sum_{i=1}^{n-j} q_i = 1
$$

so that to ensure that the arrival rate between any system state is stable.

2 BLOCKING PROBABILITY EVALUATION

Two different resource allocation strategies are considered in the following for the calculation of the blocking probability.

2.1 Hard resource allocation

In this case the subscriber requests a certain number of resource units, however the radio resource management (RRM) unit of the base station (BS) will either allocate the exact number of resource units requested or will block the request if the requested number of resource units is greater than the available resource units of the system at the moment of the request. Thus, a subscriber request of one resource unit from either service $\mathcal{Q}$ or $\mathcal{Q}$ will be blocked if the system is in any one of the diagonal states, that is, the states $E_{i,j}$ satisfying $i + j = n$. This is equal to the probability that the system has occupied all $n$ resource units, times, the probability $p_1$ or $q_1$ – based on what service is requested. Therefore, for service $\mathcal{Q}$, this blocking probability can be expressed as

$$
P_{B,\mathcal{Q}}^{(1)} = p_1 \sum_{i=0}^{n} P_{i,n-i}
$$

Correspondingly, a request of two resources will be blocked if the system is in any of the last two diagonals, that is, the states $E_{i,j}$ satisfying $i + j = n$ or $i + j = n-1$. Accordingly, the blocking probability of service $\mathcal{Q}$, for this case, can be expressed as

$$
P_{B,\mathcal{Q}}^{(2)} = p_1 \sum_{i=0}^{n} P_{i,n-i} + p_2 \sum_{i=0}^{n} P_{i,n-i-1}
$$

Generalizing for a $k$ resource units request the blocking probability may be calculated by taking into account the states $E_{i,j}$ satisfying $i + j = n$ or $i + j = n-1$ or ... or $i + j = n-k+1$, which can be expressed as

$$
P_{B,\mathcal{Q}}^{(k)} = \frac{1}{v} \left( p_1 \sum_{i=0}^{n} P_{i,n-i} + p_2 \sum_{i=0}^{n} P_{i,n-i-1} + \ldots + p_k \sum_{i=0}^{n} P_{i,n-k+1} \right)
$$

Finally, the overall blocking probability for service $\mathcal{Q}$ can be calculated by gathering all the relevant cases, providing

$$
P_{B,\mathcal{Q}} = \frac{1}{v} \sum_{m=1}^{n} m \sum_{i=0}^{n} P_{m,n-i-k+1}
$$

Likewise, the total blocking probability for service $\mathcal{Q}$ can be calculated by

$$
P_{B,\mathcal{Q}} = \frac{1}{v} \sum_{m=1}^{n} m \sum_{i=0}^{n} P_{n-i-k+1, j}
$$

while the mean number of occupied resource units is given by
\[ N_R = \sum_{i=1}^{n} \sum_{j=0}^{n-i} (i+j)P_{i,j} \] (8)

2.2 Soft resource allocation

In this case the subscribers may be allocated less resource units than the ones requested depending on the available resource units of the system at the moment of the request. This looks like an up-switching or down-switching functionality due to a congestion resolution algorithm. A subscriber request will be blocked, for either \( P \) and \( Q \) services, if the system lacks of resources, that is, when the system is in any one of the diagonal states, e.g. the states \( E_{ij} \) satisfying \( i + j = n \). Therefore, the blocking probabilities are

\[ P_{B,P} = \sum_{i=0}^{n} P_{n-i} \] (9)
\[ P_{B,Q} = \sum_{i=0}^{n} P_{n-i,j} \] (10)

for services \( P \), and \( Q \), respectively.

3 RESULTS

Using equations (1) and (6-10), the blocking probabilities for the hard and soft allocation strategies described, the average number of used resources in the system together with other system performance parameters could be evaluated. Thereby, the proposed model aims to be the basis for analyzing resource allocation strategies, given the values of mean arrival and service rate of mobile communication system services. As an example we have considered a realistic small scale scenario with the following characteristics:

- Number of resources in a cell: 20
- Service time for \( P \) service: 1/180
- Service time for \( Q \) service: 0.5
- For \( P \) service one allocated resource with probability \( p_1 = 0.6 \), two allocated resources with probability \( p_2 = 0.4 \)
- For \( Q \) service one allocated resource with probability \( q_1 = 0.5 \), two allocated resources with probability \( q_2 = 0.3 \), three allocated resources with probability \( q_3 = 0.2 \)
- Arrival rate of both \( P \) and \( Q \) services is considered to be in Erlang units.
- The simultaneous sharing of users per resource unit equals \( \nu = 4 \).

Figure 2 presents blocking probability for service \( P \) with hard resource allocation strategy, considering both services traffic. In this graph it is obvious that blocking probability increases fast when reaching the total capacity resource allocation. This result was expected since resource sharing is also considered and when reaching the total capacity most resource requests for high resources are rejected.

![Figure 2](image-url)

Accordingly, figure 3 presents the blocking probability for \( Q \) service in the hard allocation resource strategy case. In this case, the blocking probability increases faster than \( P \) service (Figure 2), since on the analysis and simulation results the probability of resource allocation has three resources. In such a case increasing the demand for resources in the hard resource allocation scenario rejects faster requests thus contributing to higher blocking probability.

![Figure 3](image-url)
Figure 4 presents blocking probability for \( P \) or \( Q \) service in the second case of soft resource allocation strategy. In this scenario both services present the same blocking probability since blocking takes place when the system is on the diagonal states. This means that blocking is declared when one only resource is requested but the system is using all resources. This is the reason that blocking probability increases slower than the hard scenario.

Finally, figure 5 presents the mean used resources in the system, based on equation (8). From this equation it is obvious that the dependence on the resources is linear, hence the graph is a standard plane.

![Figure 4](image4.png)

Figure 4. Blocking probability of \( P \) or \( Q \) service for soft resource allocation strategy with respect to the arrival rate of both services.

![Figure 5](image5.png)

Figure 5. Mean number of used resource units with respect to the arrival rate of both services.

4 CONCLUSIONS

This paper presented a new two-dimensional Markov model that incorporates multiple resource allocation and resource sharing cases. The model was mathematically analyzed and applied to a cellular system scenario comprising two services with different characteristics so that to predict the blocking probabilities and the mean number of used resource units. The application proves that the model suits very well the requirements posed by contemporary wireless systems and can be adapted to different resource allocation strategies in order to predict their performance.

REFERENCES


