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ESTIMATION OF UNDRAINED SETTLEMENT OF SHALLOW FOUNDATIONS ON LONDON CLAY

A. S. Osman, H.C. Yeow and M.D. Bolton
Department of Engineering, Cambridge University, UK, Arup Geotechnics, UK,
Department of Engineering, Cambridge University, UK
aseko2@cam.ac.uk , hoe-chian.yeow@arup.com, mdb@eng.cam.ac.uk

Abstract

The Mobilisable Strength Design (MSD) method is a new design approach conceived with the framework of plasticity theory. The objective behind the introduction of the MSD method is to achieve a simple unified design methodology, which could satisfy both safety and serviceability in a single step of calculation. In conventional terms, this offers a rational procedure for selecting safety factors according to the stress-strain behaviour of soil. The possible use of MSD in the design of shallow foundations is examined. The MSD method is used to back analyze the settlement performance of a structure founded on shallow foundations on London clay.

Keywords: Plasticity theory, Shallow foundations, Settlement.

Introduction

Engineers have basically two problems in considering design of shallow foundations: bearing capacity failure and excessive settlement. Bearing failure is checked using plasticity theory, whereas settlement is usually checked using elasticity. Conventionally, the calculations for settlement in saturated clay are divided into two components: immediate settlements due to deformation taking place at constant volume and the consolidation settlement accompanying the dissipation of pore water pressure (Skempton and Bjerrum, 1957). Excessive total or differential settlements are a main cause of unsatisfactory building performance. Although this is sometimes due to unexpected consolidation, the inadequacy of linear elasticity to describe the earlier phase of undrained settlement leads to significant uncertainties. This paper proposes a resolution of the latter problem.

The stress-strain behaviour of soil is highly non-linear from very small strains. Non-linear stress-strain characteristics can have a dominant influence on the form and scale of the displacement distribution of structures on soft clay. Therefore, there is a need for a simple design approach, which can relate successfully serviceability and collapse limits to the real nature of the soil.

A new design approach has been developed. The proposed design method treats a stress path in a representative soil zone as a curve of plastic soil strength mobilised as strains develop. Conventional bearing capacity factors are used to derive mobilised shear stresses from working loads. The working strain is then deduced from the mobilised shear stress using raw test data. Strains are entered into a simple plastic deformation mechanism to predict boundary displacements. Hence, the proposed Mobilisable Strength Design (MSD) method might satisfy both safety and serviceability in a single step of calculation.

Plastic deformation mechanism

Background

This solution uses the geometry of the well-known Prandtl mechanism (Fig. 1) for plane strain indentation to propose a plastic region of continuous deformation beneath a rigid circular punch. Outside this region, it is assumed that strain is negligible (Osman and Bolton, 2004a). The solution includes three zones of distributed shear. These zones are assumed to shear and deform compatibly and continuously with no relative sliding at their boundaries. Soil strains and compatible deformations are developed according to the shear stress that keeps the foundation in equilibrium.

The shear stresses in the soil are related to the external loading of the footing by the usual bearing capacity coefficient (N_c):

$$\sigma_{mob} = N_c c_{mob} \qquad (1)$$

where σ_{mob} is the applied bearing pressure, and c_{mob} is shear stress mobilized in the soil.

Compatibility conditions are satisfied through the specification of a kinematically admissible mechanism. Fig. 1 shows the selected deformation pattern in which there are no displacement discontinuities. Soil displacements vary quadratically with the position inside the plastic mechanism.

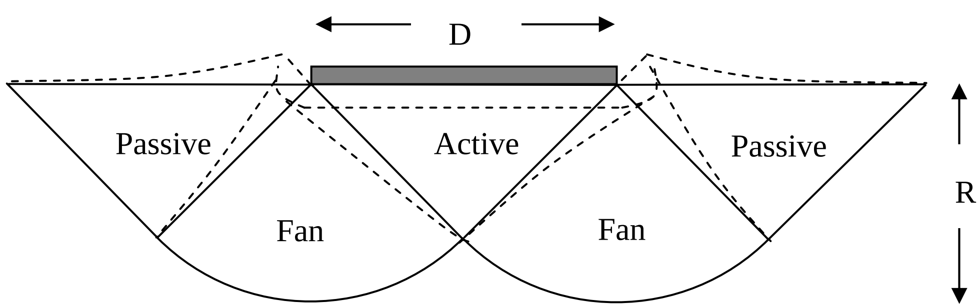


Figure 1 Plastic deformation mechanism

Since there is no volume change in undrained conditions; the following condition should be satisfied:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial v}{\partial z}=0 \qquad (2)$$

where u and v are the radial and the vertical displacement respectively, r is the radial distance from the centreline of the footing, and z is the depth below the ground surface.

The imposition of axial symmetry, the requirement for zero displacement at the outer boundary, together with Eq. (2), allow the parameters of the quadratic displacement field to be written down (Osman and Bolton 2004a). Each displacement component is proportional to the footing displacement δ. Strains can then be found from the first derivative of the displacements. Since the spatial scale is fixed by the footing diameter D, all strains components are proportional to δ/D.

The engineering strain γ, which is equal to 1.5 times the axial strain ε_a in an undrained triaxial test, can be defined as the difference between the maximum and the minimum principal strain. The average shear strain γ_{mob} mobilized in the deforming soil can be calculated from the spatial average of the shear strain in the whole volume of the deformation zone (Fig. 1):

$$\gamma_{mob}=\frac{\int_{vol}\gamma dvol}{\int_{vol}dvol}=1.33\frac{\delta}{D} \qquad (3)$$

A full mathematical derivation is given in Osman and Bolton (2004a).

A relation between applied bearing pressure and the displacement of footing can be established if the relation between shear stresses and shear strains can be obtained, such as from a carefully chosen undrained triaxial test. Fig. 2 summarizes the calculation procedure; mobilized shear stresses beneath shallow foundations are found from conventional bearing capacity factors. Strains required to mobilize these stresses are deduced from a triaxial test on a representative sample taken from a selected location in the plastic zone of influence. The footing settlements are then calculated from these strains using eq. (3).

The compromise of the new approach is therefore to couple together an equilibrium solution based on the mobilisation of a constant shear stress c_{mob}, with a kinematic solution based on the creation of an average mobilised shear strain γ_{mob}. Thus, the Mobilisable Strength Design MSD method can satisfy both safety and serviceability in a single step of calculation.

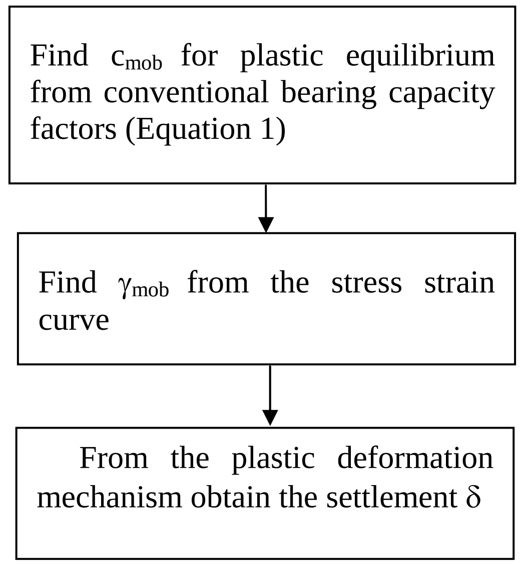


Figure 2 Calculation procedure

Osman and Bolton (2004a, 2004b) demonstrated the ability of this method to predict accurately and simply the settlements of shallow foundations by comparing its predictions with field studies and with comprehensive non-linear finite element analyses. Examples demonstrating the success of the MSD method are given for a variety soil conditions. Osman and Bolton (2004b) use the MSD method to back-analyse the well documented case of load tests of rigid square pads on Bothkennar soft clay (Jardine et al. 1995). In this paper, the performance a case history of shallow foundations on stiff London clay at Euston Road, London, UK is analyzed using the MSD method. Comparison is also made with non-linear finite element analysis in which the advanced BRICK model (Simpson 1992) is used.

Back analysis of a shallow foundation on London clay

The case history

The development consists of multi-storey concrete frame commercial development with a 4.5m basement. The site is located at 250 Euston Road, London, UK. The ground investigation showed that there is a surface layer of about 2m of fill consisting of a heterogeneous mixture of sand, gavel and construction debris followed by 2m layer of gravel. Beneath the gravel there is a 17m layer of London Clay. The undarined shear strength of London Clay increase with depth form 75-200 kPa. London Clay is overlaying Woolwich and Reading Beds. The loading on the pad foundations (size varies from 4.2x4.2m to 5.5x6.5m) varies from 50 to 170 kN/m^2, with majority of the pads loaded to 160kPa. The measured settlement was between 15 and 23 mm, with an average settlement at the centre of the foundations of 18.3mm.

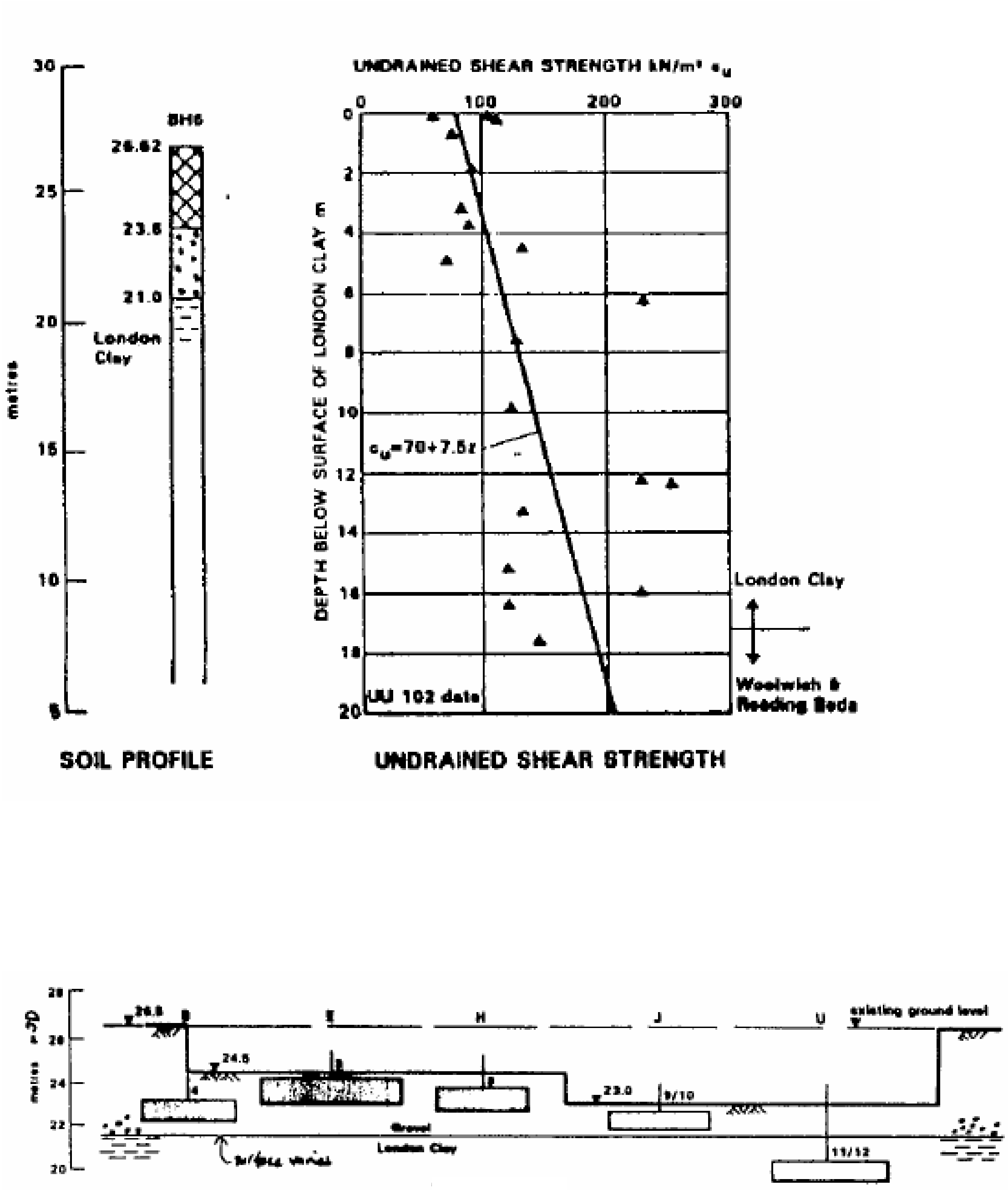


Figure 3 Euston site: soil properties and geometry

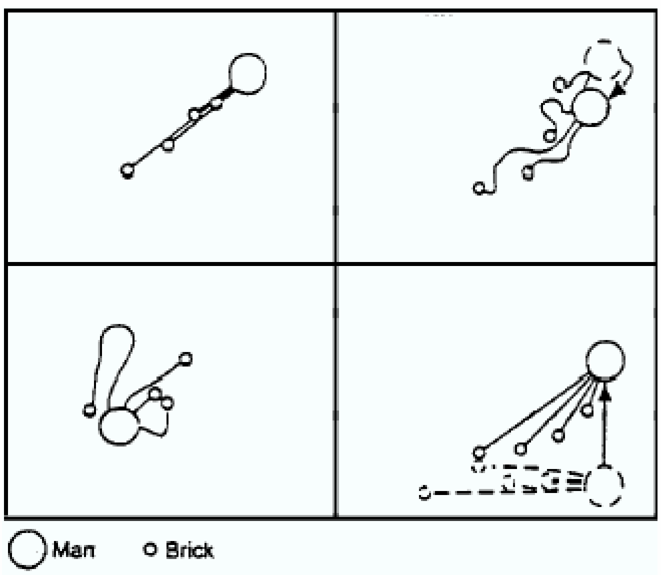
FE analysis

Finite element analysis was used to determine the footing settlement. In the FE analysis the BRICK model (Simpson 1992) was used. The BRICK model was formulated to model the dependence of the stiffness on the previous stress path direction. The model is essentially elastic-plastic with multiple kinematic yield surfaces. Simpson (1992) describes the model by analogy to a "man" walking around a room pulling behind him sets of bricks, each set on a separate string. Some possible paths for the man and the strings are shown in Fig 4a. The man represents the current strain state of an element of soil, and the bricks represent the plastic current strains in proportions of the soil within the element. When the man moves without movements of the bricks, the strains are entirely elastic; when all the bricks are moving by the same amount as the man, the strains are entirely plastic. The mobilised tangent stiffness against strain "S-curve" can be modelled in stepwise fashion as shown in Fig. 4b. At very small strain, the material is completely elastic; in the analogy none of the bricks is moving. As strain increases, one of the bricks start to move; plastic strains begin and there is a drop in the overall stiffness of soil. At larger strains, more bricks start to move; there is more drop in stiffness. The length of each step is the strain represented by the length of the string in the analogy and the height of the step of the step indicates the stiffness of the material. The BRICK model is expressed in strain space rather than in the conventional stress space. The model has six axes for the components of strain and these are the volumetric strain and five shear strains: ε_z-ε_x, $(2\varepsilon_y-\varepsilon_x-\varepsilon_z)/\sqrt{3}$, γ_{xy}, γ_{yz}, γ_{zx} with a compatible set of stress axes. The strings' lengths which govern the strain to failure and which are used to derive the angle of shearing resistance; are varied as a function of the relative proportion of the developing shear strains, giving a failure envelope relative to the five shear components. The model adopted the Drucker-Prager failure criterion expressed in terms of strains. This enables to maintain a continuous stiffness-strain curve without any discontinuity at failure. The elastic bulk modulus K_{max} is related to the elastic shear modulus G_{max} using the following function of small strain Poissons ratio (v)

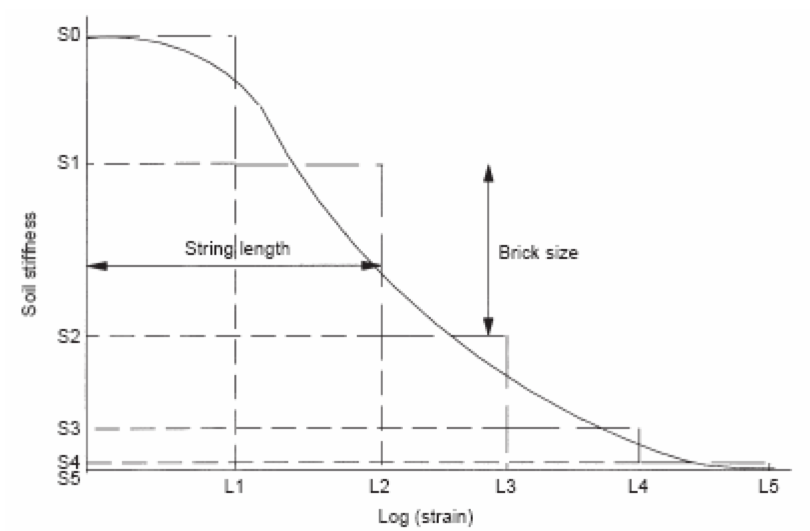
$$\frac{G_{\max}}{K_{\max}} = \frac{3(1-2v)}{2(1+v)} \tag{4}$$

K_{max} is assumed to be proportional to the elastic shear modulus G_{max}. The BRICK model has been successfully used to predict ground movements due to excavations, tunnelling and foundations (Simpson, 1992, Ng, 1992, Powrie et al. 1999, Lehane and Simpson 2000).

The footing is modelled in a FE axisymmetric analysis and adopted an equivalent diameter of 35m with an embedded depth of 4.5m. The soil was modelled using eight-noded quadrilateral consolidation elements. The mesh was sufficiently large to eliminate boundary effects so that the changes in stresses and displacements remote from the footing were negligible. Smaller elements were used near the footing where the changes of stresses and strains are significant. The bottom boundary was restrained from both horizontal and vertical movements while the left- and right-hand boundaries were restrained horizontally. Details of the finite element mesh are shown in Figure 5. The stress history of the soil was assumed to comprise one-dimensional consolidation followed by the removal of effective overburden pressure of 2000 kPa to create heavily overconsolidated clay. The FE analysis predicted 21 mm settlement assuming a bearing pressure of 150 kPa.



(a)



(b)

Figure 4 Brick model: (a) man and bricks analogy (b) stiffness discretization

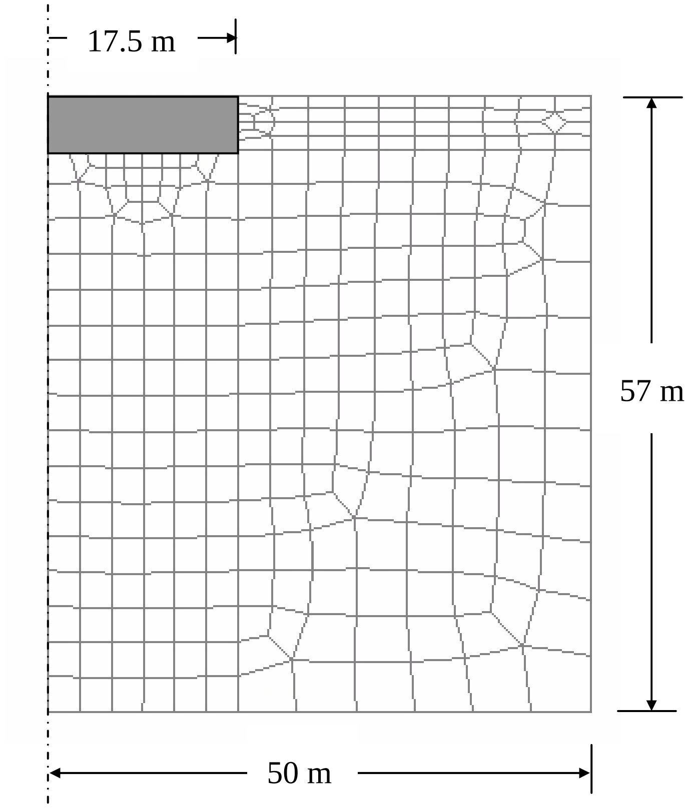


Figure 5 FE mesh

MSD calculations

In the MSD calculations, the geometry of the foundation (Fig. 3) is idealised and the analysis is carried out assuming a circular pad of equivalent area. The equivalent footing diameter is 35m. It is common in bearing capacity calculations to treat circles and squares as being equivalent (Skempton, 1951) although there is no theoretical justification for this assumption. Brinch Hansen's (1970) depth correction factor f_d was adopted to account for embedded depth (z) in the calculation of the mobilized strength in MSD of a foundation of width (D). The bearing capacity factor N_c should be increased by factor f_d.

$$f_d = 1 + 0.4z/D \tag{5}$$

However, no comparable adjustment was made to the plastic deformation mechanism. Although this approach is clearly approximate, it will be shown that the use of correction factor f_d can lead to acceptable predictions. The embedded depth of the raft is 4.5m which gives an overall bearing capacity factor N_c of 6.42.

In the absence of the actual stress-strain data, an FE simulation of undrained triaxial test was used to plot the representative stress-strain curve. The initial conditions of the simulated triaxial sample correspond to in-situ conditions of the representative soil element prior to the excavation. The soil is assumed to be controlled by the average stiffness in the zone of plastic deformation (Fig. 1); therefore the characteristic depth is selected at a depth of 0.3 times the width of the footing D measured from the footing level. This is equivalent to 9.1m below the top of London Clay which has a current vertical effective stress of about 187kPa, assuming ground water 1m above the Terrace Gravel. Erosion of the London Clay reduced the vertical effective stress to 91kPa before re-deposition to the current stress conditions. The BRICK model simulation allowed for deposition, erosion, and re-deposition, followed by a triaxial test to failure. The stress path for the representative soil element is shown in Figure 6a. The simulated stress-strain curve is shown in Figure 6b. Figure 6b also shows that the computed equivalent shear strength at this level is about 163kPa compared with 150kPa at 10.5m below the surface of London Clay as shown in Fig. 3.

The MSD calculations are as follows:

Equivalent diameter of the foundation (d) =35 m

Embedded depth (z) = 4.5 m

z/d=4.5/35=0.13

Bearing capacity factor (Equation 2) =6.1x (1+0.4x0.13) =6.42

Mobilised shear strength (c_{mob}) (Equation 1) = 150/6.42= 23.4 kPa

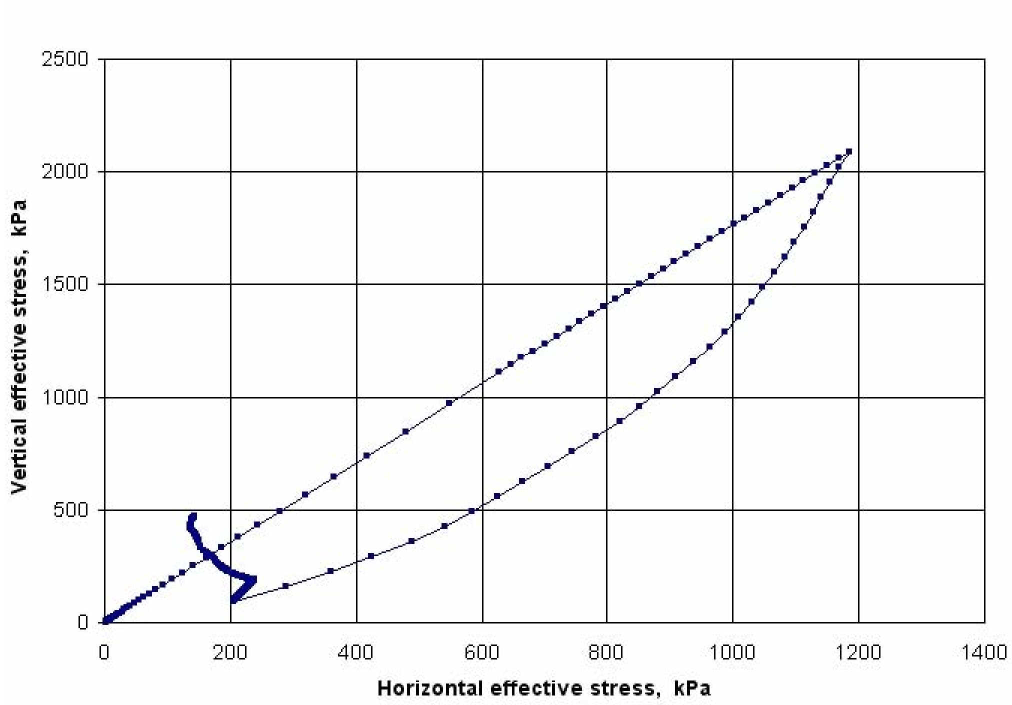
Deviatoric stress =2* c_{mob} =2x 23.4=46.7 kPa

Mobilised deviatoric strain (ε_q) (from Figure 4) =0.0475%

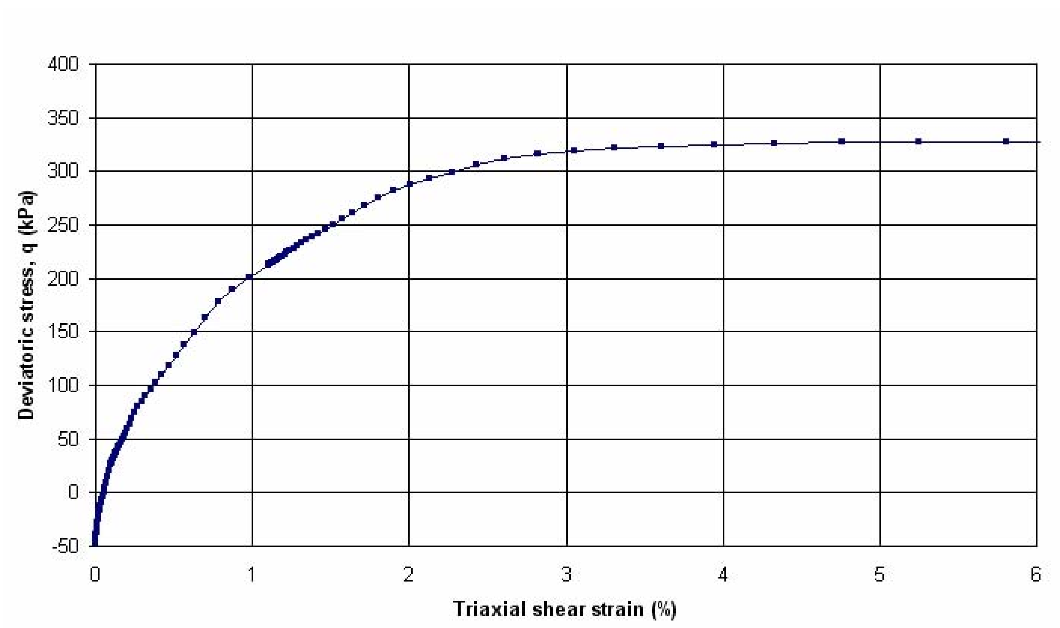
The engineering strain γ is equal to 1.5 times the axial strain ε_a in an undrained triaxial test =0.0475% *3/2=0.07125%.

Settlement is then from equation 3 is 0.07125%*35/1.33=19mm.

This value conforms well to the FE analysis (19mm compared with 21 mm). The key advantage of the MSD method is that it can predict accurately and simply the footing settlements directly from raw stress-strain data without the need for FE analyses or constitutive soil models.



(a)



(b)

Figure 6 representative stress-strain curve (a) stress-path for the representative sample (b) triaxial stress-strain data

Conclusion

Modelling soil stiffness properly in the analysis and design of shallow foundations is very important. The selection of soil parameters for design is sometimes difficult, since the properties of shallow and deep soil elements are quite different. The selection of a characteristic stress-strain curve is obviously necessary in design, but is difficult to decide. However, for design purposes of square or circular footings in homogenous soils, displacements can be assumed to be controlled by the average soil stiffness in the zone of deformation. Stress-strain data from an undisturbed soil sample taken at the mid-depth of the deformation mechanism can be used to deduce the average shear strength, which needs to be mobilised at the required shear strain in MSD calculations.

Serviceability and safety requirements should be based on the fundamental understanding of the stress-strain behaviour of the soil. The design strength that limits the deformations should be selected according to the actual stress-strain data from each site, and not derived using arbitrary factors.

An extension of bearing capacity theory to include plastic deformation mechanisms with distributed plastic strains can provide a unified solution for design problems. This application is different from the conventional applications of plasticity theory since it can satisfy approximately both safety and serviceability requirements and can predict stresses and displacements under working conditions. Also it provides simple hand calculations, which can give reasonable results compared with complex finite element analyses.

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References

Brinch Hansen, J. (1970). "A revised and extended formula for bearing capacity", *Danish Geotechnical Institute*, Bulletin 28, 5-11.

Jardine, R.J., Lehane, B.M., Smith, P.R. and Gileda, P.A. (1995). "Vertical loading experiments on rigid pad foundations at Bothkennar", *Geotechnique* **, 45**(4), 573-597.

Osman, A.S. and Bolton, M.D. (2004a). "A New Design method for retaining walls in clay", accepted for publication in *Canadian Geotechnical Journal*.

Osman, A. S., and Bolton, M. D. (2004b). "A new approach to the estimation of undrained settlement of shallow foundations on soft clay", *Proceedings of the IS-OSAKA 2004: Engineering practice and performance of soft deposit*, Osaka, Japan, 4-6 June 2004.

Ng, C. W.W (1992). "An evaluation of soil-structure interaction associated with a multi-propped excavation", *PhD thesis*, University of Bristol , UK.

Powrie, W., Chandler R. J., Carder D. R. and Watson G. V. (1999). "Back-analysis of an embedded retaining wall with a stabilization base slab". *Proceedings of the Institution of Civil Engineers: Geotechnical Engineering*, GT137, No 2, 75-86.

Simpson, B., (1992). "Retaining structures: displacement and design", *Geotechnique*, **42**(4), 541-576.

Skempton, A.W (1951). "The bearing capacity of the clay", *Proceedings of Building Research Congress*, vol. 1, 180-189.

Skempton, A.W, Bjerrum, L (1957). "Contribution to the settlement analysis of foundations on clay", *Geotechnique*, **7**(4), 168-178.