Debt and Equity Valuation of IT companies: 
A Real Option Approach

Chung Baek*  
Brice Dupoyet**  
Arun J Prakash** 

Address for Correspondence 
Arun J Prakash  
Department of Finance  
CBA, Florida International University  
Miami, Florida 33199  
(305) 348-2680  
prakasha@fiu.edu

*University of Nebraska, Lincoln  
**Florida International University, Miami

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Abstract

We attempt in this study to estimate the fundamental equity value of a firm by combining two separate capital valuation techniques, namely the corporate debt valuation of Merton (1974) and the rational pricing technique of internet companies of Schwartz and Moon (2000). We use the Black Scholes (1973) approach proposed by Merton (1974) to infer an estimate of the value of the debt of the firm, and the pricing methodology of internet companies pioneered by Schwartz and Moon (2000) to estimate the total value of the firm. Making use of the fact that firm value is made up of debt and equity, we first derive a closed-form solution for the value of the debt and then back out implied fundamental equity values for three firms in the Information Technology sector, and show how in two cases out of three the share price is actually undervalued. We also provide inferences on the debt risk premium.
I. Introduction

The groundbreaking work of Black and Scholes (1973) in the field of option pricing has opened the doors to a myriad of extensions over a wide variety of areas. One of these extensions is the application of option theory to real assets for the purpose of valuing a firm’s capital. The pricing of real options isn’t without difficulties, however, as pointed out by Edward (2003). Real options can be mathematically challenging, may lack simplicity, are a relatively new instrument, may use non-tradable underlying assets, and finally may not accurately reflect the complex reality faced by managers. Moreover, the specific traits of the underlying real asset may be difficult to estimate and use as inputs to the model. Nevertheless, despite all of these issues, real options do come in very handy when traditional pricing tools fail to provide us with a clear answer to a valuation problem.

Real options are often associated with the pricing of enterprises considered having significant growth opportunities, and as such, tend to be involved with the valuation of high-tech firms such as electronics and pharmaceutical companies. Banerjee (2003) shows how valuation by real options of the R&D component of a pharmaceutical company can help account for a high market price otherwise unexplained by more traditional valuation tools. Lint and Pennings (2001) demonstrate how real options are able to capture the value of the flexibility present at each stage of a new product
development. For the purpose of implementing a real options valuation approach for the pricing of debt and equity of companies with uncertain cash flow, we thus select three Information Technology - IT from here on - firms likely to benefit from the techniques described in the next paragraph.

Merton (1974) shows how one can price risky discount bonds using an argument following that of Black and Scholes (1973). In a recent article, Schwartz and Moon (2000) propose a methodology to estimate the fundamental total value of a firm in cases where traditional valuation methods are difficult to implement, such as in the case of internet companies. We approach equity valuation under a new angle by combining these two techniques to compute an estimate of the fundamental equity value of firms in the IT sector as the difference between their fundamental total value and their debt value. Since equity market value is easily observable, it is then straightforward to compare whether a firm’s stock is priced accurately. Despite the common perception that IT firms are often not worth their share price, our results show that, valuing their equity using the models described above, two of the IT firms studied are actually undervalued while the third firm is overvalued.

We also study the debt characteristics of these firms, and since observing the current market value of the debt is a challenge, we are not able to estimate whether the debt itself is overvalued or undervalued. Instead, we focus on the risk premiums
associated with the bonds. Under the assumption that the firm’s debt can be expressed as a zero-coupon bond, we find that two of the firms have debt with fundamental risk premiums less than 1% while the third has a fundamental risk premium close to 3%. In other words, two are close to the riskless rate while the third displays moderate amounts of risk.

The paper is organized as follows. In section 2 we review how options relate to the value of a firm’s debt and equity. In section 3 we describe the valuation techniques used to estimate the fundamental value of the firms’ equity. We describe data collection and the choice or estimation of the parameters in section 4. We present empirical results in section 5 and conclude in section 6.

II. Real Option Theory with Debt and Equity

We begin this section with a brief review of how options relate to the valuation of a firm. Suppose that a company with total value X has outstanding issues of debt and equity with respective values D and E. Debt holders’ claims having priority over those of the equity holders, the equity holders are called residual claimants. If X is greater than D, the firm pays the debt holders, and the equity holders collect the difference X-D. If X is less than D, the debt holders claim the assets of the firm and the equity holders receive nothing. The payoff to the equity holders is thus identical to the payoff associated with a call option where the underlying asset is the value X of the firm and the exercise price is the amount of debt D. Note that the bondholders face a different payoff structure, albeit
again relating to options. As long as X is larger than D, the debt holders are guaranteed to receive D. But if X falls below D, debt holders lose the amount D-X. The payoff to the bondholders is thus similar to the payoff associated with being short on a put option where the underlying asset is the value X of the firm and the exercise price is the amount of debt D.

Now suppose that we are able to obtain the “true” total value of the firm. This number should be distinguished from the sum of the current debt and equity market values, as the true total firm value is not necessarily the same as the total market value. This fundamental value of the firm, once estimated, could then be compared against observed market values of the company – provided that both are observable – in an attempt to measure the levels of over(under) valuation. Alternatively, the total fundamental firm value could be estimated along with fundamental debt value, fundamental equity value could be inferred from the difference, and comparisons between theoretical and market share values could be made. This is the approach followed in the paper.

III. The Model

We combine two separate capital valuation techniques: the corporate debt valuation of Merton (1974) and the rational pricing technique of internet companies of Schwartz and Moon (2000). The Black Scholes (1973) approach used by Merton (1974) is used to infer an estimate of the fundamental value of the debt, and the valuation methodology of Schwartz and Moon (2000) is used to estimate the fundamental value of
the firm. We simplify the technique described in Schwartz and Moon (2000) somewhat in order to reduce the number of parameters needed for the simulation. We adapt the technique from Merton (1974) to our problem at hand, and assuming that the firm’s various obligations can be lumped into a representative single bond, construct a zero-return portfolio and solve the resulting partial differential equation in order to obtain a closed-form solution estimate of the fundamental debt value.

3.1 Fundamental firm value

Since the firm as a whole is not a financially traded asset itself, estimating the market value for the firm is challenging. However, the task may be tackled by implementing the methodology proposed by Schwartz and Moon (2000) to estimate the true value of the firm under a stochastic environment. By converting their continuous-time model to a discrete-time version with annual accounting data, we obtain an approximation of the true value of the firm. One drawback of the methodology, however, is the large number of parameters that must be estimated prior to the implementation of the Monte Carlo simulation. For tractability purposes, we thus make a few simplifying assumptions enabling us to reduce the number of parameters.

The key aspect of the internet company valuation method is to use risk-neutral discounted cash flow analysis to derive the value of the firm. The present value of the firm is the sum of the after-tax expected cash flows under the risk-neutral measure, discounted back to the present at the risk-free rate. The value today is therefore:

\[ X_v = E(TCF_t e^{-r}) \]  \hspace{1cm} (1)
where $X_0$ is the present value of the firm at time 0, $TCF_T$ is the total after-tax expected cash flow accumulated up to time $T$, $r$ is the risk-free rate and $E^*$ is the risk-adjusted expectation operator. Following Schwartz and Moon [2000], we assume that all cash flows generated remain as retained earnings, earn a rate of return equal to the risk-free rate and are distributed to the shareholders at a long-term horizon $T$. The liquidation value of the firm $TCF_T$ is calculated as the total expected cash flows accumulated up to time $T$ plus a multiple of EBITDA at time $T$. This implies that when the firm is liquidated at time $T$, the terminal value of the firm is assumed to be a multiple of EBITDA. We select the same multiple (10 times) as Schwartz and Moon (2000). Earnings are driven by revenues, and revenues are generated according to the following stochastic differential equation:

$$\frac{dR_t}{R_t} = (g_t - \bar{\lambda})dt + \nu dW_{t1},$$

(2)

where the drift $g_t$ is the expected growth rate on revenues, $\lambda$ is the price of risk already incorporating the volatility component, $\nu$ is a constant volatility of the percentage change on revenue, and $W_{t1}$ is a standard wiener process. Following Schwartz and Moon (2000), $g_t$ follows a mean-reverting process and has for dynamics:

$$dg_t = \kappa(\psi - g_t) dt + \xi_t dW_{t2},$$

(3)

where $\kappa$ is the speed of mean-reversion and $\psi$ is the long-term growth rate. The volatility $\xi_t$ of the change in the expected growth rate is assumed to equal $\varphi(\psi - g_t)$ where $\varphi$ is a constant. This implies that as the growth of the firm becomes stable, the volatility of the change in the growth rate decreases. We also assume that changes in growth rates are uncorrelated with aggregate wealth, implying that the market price of risk associated with $W_{t2}$ is null. Finally, we assume that $dW_{t1}$ and $dW_{t2}$ are uncorrelated with each other.
The net after-tax income $N_t$ can be written as follows.

$$N_t = (R_t - Exp_t - Dp_t) \quad \text{if} \quad LC_{t-1} > R_t - Exp_t - Dp_t$$

$$N_t = (R_t - Exp_t - Dp_t - LC_t) (1 - T_c) \quad \text{if} \quad R_t - Exp_t - Dp_t > LC_{t-1} \quad (4)$$

where $Exp_t$ consists of Cost Of Goods Sold (COGS), variable and fixed costs, and where $Dp$ is the depreciation cost. $LC_t$ is the loss carry-forward, and $T_c$ represents the corporate tax rate. We assume that the depreciation cost is constant over time. The dynamics of the loss carry-forward, $LC_t$, are described by

$$dLC_t = \text{Max}(-M_t dt, 0) \quad \text{if} \quad L_t = 0$$

$$dLC_t = -M_t dt \quad \text{if} \quad L_t > 0 \quad (5)$$

where $M_t = R_t - Exp_t - Dp_t$. In order to conduct the simulation, we transform the continuous-time processes of equations (2) and (3) into discrete-time ones. Since all the state variables are path-dependent, we obtain the risk-adjusted discrete-time versions of equation (2) through equation (4) as follows:

$$R_t = R_{t-\Delta t} \exp([(g_t - \lambda) - \frac{1}{2} \nu^2] \Delta t + \nu \Delta W_{\Delta t}) \quad (6)$$

$$g_t = g_{t-\Delta t} \exp(\pi_t) + \psi[1 - \exp(\pi_t)] \quad (7)$$

where $\pi_t = -(\kappa + 0.5 \varphi^2) \Delta t - \varphi \Delta W_{t2}$. Note that we need to estimate or have data for a total of 12 parameters in order to perform the Monte Carlo simulation. The number of parameters is however far smaller than that of Schwartz and Moon [2000]. Finally, the
true value of the firm is estimated by equation (1) using a Monte Carlo simulation and the fact that the total expected cash flows accumulated up to time $T$ are:

$$TCF_t = e^{r\Delta t}TCF_{t-\Delta t} + N_t \Delta p$$  

(8)

3.2 Fundamental debt and equity value (A closed-form solution)

We derive the fundamental value of the firm’s debt using a method proposed by Merton (1974). A zero-return portfolio is constructed and the resulting partial differential equation is solved in order to obtain a closed-form solution for the fundamental debt value. Then, since the fundamental firm value $X$ is the sum of the fundamental equity value $E$ and fundamental debt value $D$, the fundamental equity value can be recovered by taking the difference $X-D$. Assuming perfect market conditions and a known term structure, the continuous-time dynamics for the value of the firm are expressed by the following stochastic differential equation:

$$dX = (\mu X - \theta_i - \theta_e) dt + \sigma X dW,$$

(9)

where $X$ is the total value of the firm, $\mu$ is the instantaneous expected rate of return on the value of the firm per unit of time, $\theta_i$ and $\theta_d$ are the payouts per unit of time by the firm to equity-holders and debt-holders respectively, $\sigma$ is the volatility of the value of the firm per unit of time, and $dW_t$ is a standard Wiener process. Note that to keep the model tractable the volatility $\sigma$ is assumed constant over time.
We now let $H = B(X, t)$ where $B(X, t)$ is the market value of the debt and a function of the value of the firm and time. Since $H$ is assumed to be affected by a single factor, $X$, the dynamics of $H$ can be given by

$$dH = (\mu_X H - \theta_x)dt + \sigma_X H dW$$

where the drift $\mu_X$ is the instantaneous expected rate of return on $H$, $\sigma_X$ is the instantaneous standard deviation of $H$, and $dW$ is a standard wiener process.

Applying Ito’s lemma to equation (10) yields

$$dH = [\frac{1}{2} B_{xx} \sigma^2 X^2 + B_x (\mu X - \theta_x) + B_t]dt + B_x \sigma X dW$$

where $B_{xx}$ is the second-order partial derivative of $B$ with respect to $X$, $B_x$ is the first-order partial derivative of $B$ with respect to $X$, and $B_t$ is the first-order partial derivative of $B$ with respect to time $t$. Since equation (10) should be identical to equation (11), we obtain

$$\mu_X H = \frac{1}{2} B_{xx} \sigma^2 X^2 + B_x (\mu X - \theta_x) + B_t + \theta_x$$

and

$$B_x \sigma X dW = \sigma_H HdW$$

In order to obtain a partial differential equation for the market value of the debt function, we form a portfolio consisting of the firm, the debt, and the risk-free asset so that the net investment in this portfolio is zero. In other words, the sum of $w_1$, $w_2$, and $w_3$ is equal to zero where $w_1$ is the dollar amount invested in the firm, $w_2$ is the dollar amount invested in the debt, and $w_3$ is the dollar amount invested in the risk-free asset.
The instantaneous return of the portfolio $dr$, therefore is

$$dr = \frac{dX}{X} + \theta \frac{dH}{H} + \sigma \frac{dW}{W}$$

In the absence of arbitrage, the expected return on the portfolio should be zero because the net investment is null. From this condition and equation (13), the existence of a non-trivial solution implies the following singular matrix condition:

$$\left( \mu - r \right) \frac{B(X, H)}{H} = (\mu - r)$$

Combining equations (12) and (15) yields

$$\frac{1}{2} B \sigma^2 X^2 + B + rX - \theta X = 0$$

This partial differential equation must be satisfied by the market value function of the debt $B(X, t)$. Now, suppose that $F$ is the promised payment to the debt-holders at the maturity. As long as the market value of the firm, $X$, is larger than $F$, the firm will pay $F$ to the debt-holders at maturity. If $X$ is smaller than $F$, however, the firm will go bankrupt and the debt-holders will receive $X$ only. This can be summarized by:

$$\begin{align*}
\text{If } X & > F, \quad B = F \\
\text{If } X & < F, \quad B = X
\end{align*}$$

From these conditions, the boundary condition $B(X, T) = \min(X, F)$ can be set for the maturity date $T$. The other boundary condition is immediately obtained from the non-negativity of the debt. At any time $t$, $B(0, t) = 0$. By using these two conditions and under
the assumption that $\theta_s$ and $\theta_d$ are sufficiently small relative to $X$, if we let $\Delta = (\theta_s + \theta_d)/X$ we obtain the following closed form with details shown in the Appendix:

$$B(X, t) = F e^{-r(t - T)} - [F e^{-r(t - T)} \Phi(d_1) - X e^{-\Delta(t - T)} \Phi(d_2)] + (1 - e^{-\bar{r}(T - t)}) \frac{\theta_d}{r}$$

$$d_1 = \frac{-\ln\left(\frac{X}{F}\right) - \left(r - \Delta + \frac{1}{2} \sigma^2\right)(T - t)}{\sigma \sqrt{T - t}}$$

$$d_2 = d_1 + \sigma \sqrt{T - t}$$

where $\Phi(\cdot)$ is the standard cumulative normal distribution function and $\sigma$ is assumed to be close to the standard deviation of stock returns since the firm value is closely related to the stock value. In equation (17) the fundamental debt value consists of three components. On the right hand side, the first term is the present value of a zero-coupon bond, the next bracket term is the value of a put option, and the last term is associated with the present value of a coupon stream related to the remaining maturity. In other words, if the value of the firm $X$ goes below the face value of the debt, stockholders will turn over all of the assets to the debt-holders. The last term on the right hand side collapses to the present value of a perpetuity when coupons are made indefinitely. As, however, maturity approaches zero, the last term converges to zero. Since we assumed in section 3.1 that all cash flows remain as retained earnings until the firm is liquidated, the payout to the stockholders $\theta_s$ is zero. The fundamental debt value therefore depends on $\theta_d$: for example, if $\theta_d$ is equal to zero, $B(X, t)$ will be the fundamental debt value for a zero-coupon bond and if $\theta_d$ is not equal to zero, $B(X, t)$ will be the fundamental value for a coupon-bond.
We can now obtain the fundamental value of the equity by subtracting the fundamental debt value from equation (17) from the fundamental firm value. We get:

\[ E(X,t) = X - B(X,t) \]  

(18)

where \( E(X,t) \) is the fundamental value of equity at time \( t \) given the fundamental firm value \( X \).

### 3.3 Risk premium on risky debt

Our focus in this section is on the risk premium of the risky debt. Since in prior sections we assumed that the firm does not distribute its earnings but keeps them as retained earnings until liquidation occurs, \( \theta_0 \) must be zero. We are thus left with \( \theta_d \) as the only form of payout by the firm. With continuous compounding, if we denote the yield-to-maturity on the risky bond by \( R \) and a discount function of the coupons by \( K(R) \), the value of the debt can be expressed as:

\[ B(X,t) = F e^{-R(T-t)} + \theta_d K(R) \]  

(19)

Rearranging equation (19) in terms of the risk premium yields

\[ (R-r) = -\frac{1}{T-t} \ln\left[ \frac{Be^{r(T-t)} - \theta_d e^{r(T-t)} K(R)}{F} \right] \]  

(20)

Note that in the case of a zero-coupon bond, \( \theta_0 \) is equal to zero and the risk premium can be directly backed out from equation (19). When coupons are present, however, solving for the risk premium entails defining a function

\[ G(R) = (R-r) + \frac{1}{T-t} \ln\left[ \frac{Be^{r(T-t)} - \theta_d e^{r(T-t)} K(R)}{F} \right] \]  

(21)

and solving for the yield-to-maturity \( R \) numerically by finding its roots.
IV. The Data

We first collect a list of IT companies ranked by revenue levels from the Washington Post. Since not all data for those companies are available, only three companies are ultimately selected. Bearing Point, Inc. and American Management Systems, Inc. are chosen because while these two firms display the largest revenue in the list, their share price has seemed to reach a historical low. The third company, Costar Group, Inc displays significantly lower revenue levels but one of the highest share prices in the industry.

All data needed are obtained from CRSP and 10-K Annual Reports. Data for Bearing Point, Inc. are available from CRSP from 1998 to 2002, data for American Management Systems, Inc. from 1993 to 2002, and data for CoStar Group, Inc. from 1996 to 2002. In order to simulate fundamental firm value, starting values are obtained from 2003 10-K annual reports. We conduct our simulation on the basis of annual data and a future long-term horizon of 40 years. As mentioned earlier in this paper, we face some limitations when estimating the fundamental value of the debt. The debt characteristics not always being described in detail in the 10-K annual reports, we assume that all cash flows are the face values of risky bonds.

V. Empirical Results

5.1 Simulation for fundamental firm value

In order to simulate the fundamental firm value, we need to have parameters and starting values estimated and collected. Under the assumption that the past is a reasonable predictor of the future, we use statistical measures such as mean, variance, and
covariance from past data. Table 1 reports estimated parameters and starting values for the three firms. \( R_0 \), \( CF_0 \), and \( LC_0 \) are directly observable from the 2003 10-K annual reports. The starting growth rate, \( g_0 \) is estimated as an average of changes in past annual returns on revenues obtained from CRSP. We calculate \( v \) as the standard deviation of the changes in revenue returns and restrict these returns between +50% and -50% to avoid statistical bias due to isolated large changes in revenue returns. The price of risk \( \lambda \) is assumed to be a constant approximately equal to the beta of the return on revenue times the market risk premium. The mean-reversion speed coefficient \( k \) is calculated from past data as the reciprocal of the number of years needed to reach for the first time the estimated long-term growth rate. As the growth stabilizes over time, so does the volatility \( \varphi \). We thus take one-fourth of the standard deviation of past stock returns for \( \varphi \) and assume that \( r \) is the average yield on Treasury bills during the past decade. The depreciation cost, \( D_p \) is estimated as an average of historical depreciations. The constant percentage rate on revenue for total costs and expenses, \( \alpha \), is obtained from regressing total costs and expenses on revenue.

One problem is that Costar Group, Inc. has a regression-derived \( \alpha \) greater than one. Schwartz and Moon (2000) face the same issue with Amazon.com and opt to use a forecast instead. We similarly set the parameter to 0.98, a value considered large in this industry, to ensure that costs are not underestimated. Finally, the corporate tax rate is assumed to be 35%. With estimated values and starting values displayed in table 1, we simulate 10,000 paths and use equations (6) through (8) to estimate the fundamental values of the three firms. Table 2 reports the results. Since all annual data were obtained
as of the fiscal year of each company, the fundamental value approximates the true value of companies as of the most recent fiscal year.

5.2 Fundamental debt value

We now estimate the fundamental debt value for each company. This can be done based on sections 3.2 and 3.3. However, although the 10-K annual reports provide the face value of the various obligations to be paid at different maturities, they do not provide a detailed description of the debt characteristics. Moreover, equation (17) is on the basis of risky bonds only with one single maturity date. We therefore assume that all obligations can be expressed in terms of risky bonds and we calculate a face value-weighted duration to obtain an average maturity for all different debts. Then, based on the face value-weighted duration and the total face value, we estimate the fundamental debt value for a given coupon amount. Results are reported in table 3 for seven cases with coupon rates going from 0 to 6 percent.

Table 3 reports fundamental debt values for all seven cases and a risk premium estimate for the zero-coupon case. Note that the risk premium is independent of the coupon rate and thus for computational ease we use the zero-coupon case to infer the premium. Table 3 also shows that for a given face value, a higher coupon rate implies a higher fundamental debt value. While Bearing Point, Inc. has discount bonds across all seven cases; American Management Systems and the Costar Group have discount bonds for the first five cases and premium bonds for the last two. If current market debt values were observable directly, comparisons with their respective fundamental values could be made and used for financing and investment purposes. However, since debt values are
not directly observable, the main conclusion that we can draw is that with the assumption of a zero-coupon, American Management Systems, Inc. and CoStar Group, Inc. seem to have obligations whose characteristics are close to that of riskless bonds whereas the obligations of Bearing Point, Inc. display moderate amounts of risk.

5.3 Fundamental equity value

Since the equity value is the difference between firm value and debt value, we can now estimate the fundamental equity value of each company from the previous results. Table 4 reports fundamental equity value and corresponding share prices across all levels of coupon rates. Note that the fundamental stock price decreases with an increase in the coupon rate. The market price of a share of Bearing Point, Inc. on July 1, 2003 was $9.43 and the number of its outstanding shares was about 191 million. For all cases, its fundamental stock price falls between $13 and $14. This implies that its share price was undervalued by about 30%. The stock of American Management Systems Inc. traded for $15.51 on January 2, 2004 and about 42 million shares were outstanding. Since for all cases, the fundamental stock prices are between $34 and $36, the market price as of January 2, 2004 was under-valued by more than 50%. A possible explanation for this is that many IT stocks have been steadily decreasing during the past few years and that as a result some might have actually gone below fair levels. The share price of Costar Group, Inc, however, was overvalued, indicating that its persistently high share price may still be overestimating the value of the firm’s equity.
VI. Conclusion

We estimate in this paper the fundamental equity value of three firms in the IT sector by carefully combining two existing capital valuation techniques. We use the corporate debt valuation of Merton (1974) in connection with the rational pricing technique of Schwartz and Moon (2000) as a way to determine the fairness of the share prices. Merton (1974) proposes to use a Black and Scholes (1973) type of reasoning by building a zero-return portfolio in order to derive an estimate of the value of the firm’s debt. We adapt this method to the valuation of the debt of various IT companies and derive a closed-form solution as a reasonable approximation to the debt value. The methodology by Schwartz and Moon (2000) on how to value internet companies is then used to estimate the total value of the IT firms selected. We then retrieve fundamental equity values for the three firms by computing the difference X-D, and show how in two cases out of three the share price is actually undervalued. Risk premium on the various obligations are shown to be either very close to the riskless rate, or to remain at very moderate levels.
REFERENCES


This table provides estimated values for various parameters and starting values needed to implement the Schwartz and Moon (2000) total firm valuation Monte Carlo procedure. The notation is as follows: $g_0$ is the initial revenue growth rate, $\lambda$ is the price of risk, $\nu$ is the volatility of the percentage change in revenue, $R_0$ is the initial revenue level, $\kappa$ is the speed of mean-reversion in the growth rate process, $r$ is the risk-free rate, $Dp$ is the yearly depreciation cost, $\alpha$ is the constant percentage rate on revenue for total costs and expenses, $Tc$ is the assumed tax rate, $LC_0$ is the initial loss carry-forward and $CF_0$ is the initial cash flow of the firm.

<table>
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<tr>
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<td>$g_0$</td>
<td>0.20</td>
<td>0.12</td>
<td>0.60</td>
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<tr>
<td>$\lambda$</td>
<td>0.06</td>
<td>0.04</td>
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<tr>
<td>$\nu$</td>
<td>0.22</td>
<td>0.18</td>
<td>0.08</td>
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<tr>
<td>$R_0$ (million)</td>
<td>3139.3</td>
<td>961.6</td>
<td>95.1</td>
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<td>$\kappa$ (per year)</td>
<td>0.33</td>
<td>0.17</td>
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<td>$\varphi$</td>
<td>0.15</td>
<td>0.11</td>
<td>0.20</td>
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<tr>
<td>$r$ (risk-free rate)</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
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<tr>
<td>$Dp$ (million)</td>
<td>83.09</td>
<td>51.93</td>
<td>6.30</td>
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<tr>
<td>$\alpha$</td>
<td>0.96</td>
<td>0.93</td>
<td>0.98</td>
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<tr>
<td>$T_c$</td>
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<td>0.35</td>
<td>0.35</td>
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<tr>
<td>$LC_0$ (million)</td>
<td>0</td>
<td>3.6</td>
<td>0</td>
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<tr>
<td>$CF_0$ (million)</td>
<td>154</td>
<td>12.9</td>
<td>13.6</td>
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TABLE 2
Fundamental Firm Value

Fundamental values estimated by Monte Carlo simulation using 10,000 paths, as of the most recent fiscal year of each company.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Fundamental values (million)</td>
<td>3,221.99</td>
<td>1,754.15</td>
<td>628.28</td>
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</table>
TABLE 3
Fundamental Debt Value and Risk Premium

The fundamental Debt Value is calculated using Merton (1974) and the closed-form solution found in the appendix. The risk Premium is calculated for the 0%-coupon case using equation (20) in the paper.

<table>
<thead>
<tr>
<th>Case 1 (0% Coupon)</th>
<th>Company</th>
<th>Bearing Point Inc. (million)</th>
<th>American Management Systems Inc. (million)</th>
<th>CoStar Group Inc. (million)</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Face Value</td>
<td>741.506</td>
<td>267.7</td>
<td>32.166</td>
</tr>
<tr>
<td>Fundamental Debt Value (million)</td>
<td>553.54</td>
<td>231.38</td>
<td>27.57</td>
<td></td>
</tr>
<tr>
<td>Fundamental Risk Premium (%)</td>
<td>2.69</td>
<td>0.14</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>Case 2 (1% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>582.41</td>
<td>240.15</td>
<td>28.51</td>
</tr>
<tr>
<td>Case 3 (2% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>611.28</td>
<td>248.92</td>
<td>29.45</td>
</tr>
<tr>
<td>Case 4 (3% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>640.13</td>
<td>257.69</td>
<td>30.39</td>
</tr>
<tr>
<td>Case 5 (4% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>668.99</td>
<td>266.46</td>
<td>31.32</td>
</tr>
<tr>
<td>Case 6 (5% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>697.83</td>
<td>275.23</td>
<td>32.26</td>
</tr>
<tr>
<td>Case 7 (6% Coupon)</td>
<td>Fundamental Debt Value (million)</td>
<td>726.67</td>
<td>284.10</td>
<td>33.20</td>
</tr>
</tbody>
</table>
TABLE 4
Fundamental Equity Value and Stock Price
This table provides the fundamental equity value and fundamental stock price. These values are computed using the following equations:
Fundamental equity value = fundamental firm value − fundamental debt value.
Fundamental stock price = fundamental equity value / the number of shares.

<table>
<thead>
<tr>
<th>Company</th>
<th>Current Market</th>
<th>Case 1 (0% Coupon)</th>
<th>Case 2 (1% Coupon)</th>
<th>Case 3 (2% Coupon)</th>
<th>Case 4 (3% Coupon)</th>
<th>Case 5 (4% Coupon)</th>
<th>Case 6 (5% Coupon)</th>
<th>Case 7 (6% Coupon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Management Systems Inc. (as of 7/1/2003)</td>
<td>15.51</td>
<td>35.92</td>
<td>35.71</td>
<td>35.50</td>
<td>35.29</td>
<td>1,496.46</td>
<td>35.09</td>
<td>34.67</td>
</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
<td>42.15</td>
<td>600.71</td>
<td>599.77</td>
<td>598.83</td>
<td>597.89</td>
<td>596.96</td>
<td>34.00</td>
<td>33.90</td>
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<tr>
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<td>2,668.45</td>
<td>1,514</td>
<td>1,505.23</td>
<td>2,610.71</td>
<td>1,505.23</td>
<td>1,478.92</td>
<td>2,495.32</td>
</tr>
<tr>
<td>Case 6 (5% Coupon)</td>
<td>2,610.71</td>
<td>1,505.23</td>
<td>598.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
<td>2,495.32</td>
<td>1,478.92</td>
<td>596.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 7 (6% Coupon)</td>
<td>13.32</td>
<td>35.50</td>
<td>33.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
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<td>1,478.92</td>
<td>596.02</td>
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<tr>
<td>Case 8 (7% Coupon)</td>
<td>13.32</td>
<td>35.50</td>
<td>33.95</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
<td>2,495.32</td>
<td>1,478.92</td>
<td>596.02</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 9 (8% Coupon)</td>
<td>13.32</td>
<td>35.50</td>
<td>33.95</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
<td>2,495.32</td>
<td>1,478.92</td>
<td>596.02</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 10 (9% Coupon)</td>
<td>13.32</td>
<td>35.50</td>
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<td></td>
</tr>
<tr>
<td>CoStar Group Inc. (as of 1/2/2004)</td>
<td>2,495.32</td>
<td>1,478.92</td>
<td>596.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

24
APPENDIX

The partial differential equation to solve is:

\[
\frac{1}{2} B_s \sigma^2 X^2 + B_s (r X - \theta_s - \theta_d) + B_s + \theta_d - r \ B = 0
\]

with boundary conditions \(B(0,t) = 0\) and \(B(X,T) = \text{Min}(X,F)\)

With the assumption that \(\theta_s\) and \(\theta_d\) are sufficiently small relative to \(X\), let’s rewrite the PDE as follows:

\[
\frac{1}{2} B_s \sigma^2 X^2 + B_s X (r - \Delta) + B_s + \theta_d - r \ B = 0
\]

where \(\Delta = (\theta_s + \theta_d) / X\).

Let \(t = T - 2t/\sigma^2 \), \(X = Fe^x\), and \(B(X,t) = Fb(x, \tau)\).

Since \(B(X, T) = F\text{Min}(e^x, 1)\) and \(B(X, T) = Fb(x, \tau)\), we have \(b(x, \tau) = \text{Min}(e^x, 1)\)

Rewriting the above PDE with respect to \(b, x, \) and \(\tau\) yields

\[
\frac{\partial b}{\partial \tau} = \frac{\partial^2 b}{\partial x^2} + \frac{2(r - \Delta)}{\sigma^2} \frac{\partial b}{\partial x} - \frac{2r}{\sigma^2} b + \frac{2\theta_d}{\sigma^2 F}
\]

Let \(\alpha = 2(r - \Delta) / \sigma^2 - 1\), \(\beta = 2r/\sigma^2\), \(\delta = 2\theta_d/\sigma^2 F\), and \(b(x, \tau) = u(x, \tau) + \delta/\beta\).

Since \(\theta_s\) and \(\theta_d\) are sufficiently small relative to \(X\), \(\lambda\) is not much affected by the stochastic movement of \(X\). This is a slight approximation but of very little consequences on the actual solution.
The partial differential equation then becomes
\[ \nu_t = \nu_{xx} + \alpha \nu_x - \beta \nu \]

Now, let \( \nu(x, \tau) = e^{px+q\tau} u(x, \tau) \). Rearranging the above PDE with respect to \( u \),
\[ u_t = u_{xx} + (2p + \alpha) u_x + (p^2 + \alpha p - \beta - q) u \]

By letting \( p = -\alpha/2 \) and \( q = -\alpha^2/4 - \beta \), we can get the heat equation:
\[ u_t = u_{xx} \]

And since \( b(x, 0) = e^{px} u(x, 0) + \delta/\beta \) and \( b(x, 0) = \min(e^x, 1) \),
\[ u(x,0) = \min[e^{(x-x_0)^2/4\tau} \delta, e^x - \frac{\delta}{\beta}] \]

A well-known solution of the heat equation is
\[ u(x, \tau) = \frac{1}{2\sqrt{\pi \tau}} \int_{-\infty}^{\infty} u(s,0) e^{-\frac{(x-s)^2}{4\tau}} ds \]

Therefore, using \( u(x, 0) \), we can derive the function \( u(x, \tau) \) as follows.
\[ u(x, \tau) = \frac{1}{2\sqrt{\pi \tau}} \int_{-\infty}^{\infty} \left[ e^{(x-s)^2/\delta^2 \beta^2} - \frac{\delta}{\beta} e^{(x-s)^2/\beta^2} \right] ds + \frac{1}{2\sqrt{\pi \tau}} \int_{-\infty}^{\infty} \left[ e^{(x-s)^2/\beta^2} - \frac{\delta}{\beta} e^{(x-s)^2/\beta^2} \right] ds \]

Letting \( z = (s - x)/(2\tau)^{0.5} \),
\[ u(x, \tau) = \frac{1}{2\sqrt{\pi \tau}} \int_{-\infty}^{\infty} \left[ e^{(x-z)^2/\delta^2 \beta^2} - \frac{\delta}{\beta} e^{(x-z)^2/\beta^2} \right] \frac{1}{(2\tau)^{0.5}} dz + \frac{1}{2\sqrt{\pi \tau}} \int_{-\infty}^{\infty} \left[ e^{(x-z)^2/\beta^2} - \frac{\delta}{\beta} e^{(x-z)^2/\beta^2} \right] \frac{1}{(2\tau)^{0.5}} dz \]

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Rearranging each term in the right hand side,

\[ u(x, \tau) = e^{\frac{x^2}{2(1+\alpha/2)\sqrt{2\tau}}} \Phi(d_i) + \int e^{\frac{\alpha}{\beta} \frac{x^2}{2(1+\alpha/2)\sqrt{2\tau}}} \Phi(d_i) \, \frac{\delta}{\beta} e^{\frac{\alpha}{\beta} \frac{x^2}{2(1+\alpha/2)\sqrt{2\tau}}} \]

\[ d_i = -\frac{x}{\sqrt{2\tau}} - (1+\frac{\alpha}{2})\sqrt{2\tau} \]

\[ d_i = -\frac{x}{\sqrt{2\tau}} - \frac{\alpha}{2}\sqrt{2\tau} \]

Finally, repeating the change of variable technique, we obtain an approximate analytic solution based on the assumption that \( \Delta \) is sufficiently small,

\[ B(X, t) = e^{-\frac{(r-\Delta)}{2}\sigma^2(T-t)} \Phi(d_i) - X e^{-\frac{(r-\Delta)}{2}\sigma^2(T-t)} \Phi(d_i) + (1-e^{-\frac{(r-\Delta)}{2}\sigma^2(T-t)}) \frac{\theta}{r} \]

\[ d_i = \frac{-\ln\left(\frac{X}{F}\right) - \left( r - \Delta + \frac{1}{2} \sigma^2(T-t) \right)}{\sigma\sqrt{T-t}} \]

\[ d_i = d_i + \sigma\sqrt{T-t} \]