OSTBC Transmission in MIMO AF Relay Systems with Keyhole and Spatial Correlation Effects

Trung Q. Duong, Himal A. Suraweera, Theodoros A. Tsiftsis, Hans-Jürgen Zepernick, and A. Nallanathan

† Blekinge Institute of Technology, Sweden (E-mail: {dqt,hjz}@bth.se)
‡ National University of Singapore, Singapore (E-mail: elesaha@nus.edu.sg)
‡ Technological Educational Institute of Lamia, Greece (Email: tsiftsis@teilam.gr)
¶ King’s College London, United Kingdom (E-mail: arumugam.nallanathan@kcl.ac.uk)

Abstract—In this paper, we investigate the degenerative effects of antenna correlation and keyhole on the performance of multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay networks. In particular, we considered a downlink MIMO AF relay system consisting of an \( n_S \)-antenna base station \( S \), an \( n_R \)-antenna relay station \( R \), and an \( n_D \)-antenna mobile station \( D \), in which the signal propagation originated from the mobile station suffers from keyhole and spatial correlation effects. We have derived an exact expression for the moment generating function (MGF) of the instantaneous signal-to-noise ratio which allows us to analyze the symbol error probability and outage probability of the considered system. We have shown that although the mobile station is in a poor scattering environment, i.e., keyhole, the relay channel \((S \rightarrow R \rightarrow D)\) still achieves a cooperative diversity order of \( \min(n_R, n_D) \) provided that the channel from the source \((S \rightarrow R)\) is keyhole-free. This result is important to radio system designers since under such a severe scenario it is unnecessary to deploy a large number of antennas on the relay station \((n_R)\) but only requires \( n_R = n_D \) to obtain the maximum achievable diversity gain.

I. INTRODUCTION

The performance of multiple-input multiple-output (MIMO) systems heavily relies on the rich scattering propagation environment of wireless links. Specifically, rank deficiency of the channel matrix due to the keyhole effect of multiple antenna channels causes a decrease of the spatial multiplexing and diversity gains in MIMO systems [1], [2]. In a rich-scattering environment, i.e., the channel matrix is of full rank, a MIMO channel between an \( n_S \)-antenna source and an \( n_R \)-antenna destination can provide the maximum multiplexing gain of \( \min(n_S, n_D) \) and diversity gain of \( n_S n_D \). On the other hand, the keyhole effect causes the channel matrix to be rank deficient. As a consequence, the channel capacity reduces to that of single-input single-output (SISO) systems and the diversity gain is of \( \min(n_S, n_D) \) order.

So far, there has been a limited number of research works considering such keyhole effects on the performance of MIMO relay networks. To the best of the authors’ knowledge, only Souihli and Ohtsuki have very recently attempted to investigate the effect of keyhole for MIMO relay systems [3], [4]. In particular, by considering a downlink cellular network in which the channel between the source and relay enjoys a rich-scattering environment while the source-to-destination and relay-to-destination channels suffer from keyhole phenomenon, they have shown that using the relay can mitigate the effect of keyhole in terms of channel capacity.

However, [3], [4] have only focused on the multiplexing gain. How the keyhole channel affects the diversity gain is still unknown. Hence, in this paper, we investigate the cooperative diversity for a downlink keyhole MIMO system. Besides the keyhole effect, antenna correlation is another key factor degrading the performance of a MIMO system. In the downlink it is reasonable to assume that source and relay nodes are fixed stations and have enough space to deploy multiple antennas without incurring any spatial correlation. However, the destination, which is a mobile station, can be a portable device resulting in spatial correlation of co-located multiple antennas.

In this work, we apply orthogonal space-time block code (OSTBC) at the source to exploit the maximum achievable diversity. The performance of OSTBC transmissions over MIMO amplify-and-forward (AF) relay networks has been extensively studied: i) Symbol error probability (SEP) for semi-blind AF relay in dual-hop Rayleigh/Rician and Rayleigh/Rayleigh fading channel [5], [6], ii) Bit error probability (BEP) for channel state information (CSI)-assisted AF relay with and without direct link [7], [8], iii) Performance of CSI-assisted AF relay in Nakagami-\( m \) fading channels [9]. Very recently, the antenna correlation effect has been investigated in [10] and [11] for Rayleigh and Nakagami-\( m \) fading channels, respectively. None of these works has considered the keyhole effect.

In this paper, we therefore analyze the performance of downlink MIMO AF relay networks taking into account both keyhole and spatial correlation effects. Our contributions can be summarized as follows: We have derived an exact expression for the moment generating function (MGF) of the instantaneous signal-to-noise ratio (SNR) which allows us to assess the SEP and the outage probability. We have shown that the relaying channel, i.e. the link between source, relay, and destination, offers a cooperative diversity order of \( \min(n_R, n_D) \) with the condition that \( n_S > \min(n_R, n_D) \). The obtained diversity gain implies that under such an unfavorable channel condition, deployment of more than \( n_D \) antennas at
the relay yields no diversity gain.

Notation: A vector and a matrix are written as bold lower case and upper case letters, respectively. \( I_n \) represents the \( n \times n \) identity matrix, \( \mathbf{0} \) denotes the all zero vector, and \( \| \mathbf{A} \|_F \) defines Frobenius norm of the matrix \( \mathbf{A} \). \( \mathbb{E} \{ \cdot \} \) is the expectation operator. \( \Re \) is real part and \( j^2 = -1 \). The complex circularly symmetric Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \) is denoted by \( CN (\mu, \sigma^2) \). Let \( \mathbf{x} \in \mathbb{C}^m \) be the vector-variate complex Gaussian distribution with mean \( \mathbf{u} \) and variance \( \mathbf{\Sigma} \) defined as \( \mathbf{x} \sim \mathcal{N}_m (\mathbf{u}, \mathbf{\Sigma}) \). Finally, let \( \mathbf{X} \in \mathbb{C}^{m \times n} \) be the matrix-variate complex Gaussian distribution defined as \( \mathbf{X} \sim \mathcal{N}_{m,n} (\mathbf{M}, \mathbf{\Sigma}, \mathbf{\Phi}) \) if \( \text{vec}(\mathbf{X}) \) is \( mn \)-variate complex Gaussian distributed with mean \( \text{vec}(\mathbf{M}) \) and covariance \( \mathbf{\Sigma}^T \otimes \mathbf{\Phi} \). The PDF of \( \mathbf{X} \) is given by

\[
p_{\mathbf{X}} (\mathbf{X}) = \pi^{-mn} \det (\mathbf{\Sigma})^{-n} \det (\mathbf{\Phi})^{-m} \times \exp \left[ - \text{tr} \{ \mathbf{\Sigma}^{-1} (\mathbf{X} - \mathbf{M}) \mathbf{\Phi}^{-1} (\mathbf{X} - \mathbf{M})^\dagger \} \right].
\]

II. SYSTEM AND CHANNEL MODEL

A. Protocol Description

We consider a MIMO fading relay channel consisting of a source terminal \( \mathcal{S} \), a destination terminal \( \mathcal{D} \), and a relay terminal \( \mathcal{R} \). Let us denote \( \mathbf{H}_0 \), \( \mathbf{H}_1 \), and \( \mathbf{H}_2 \), as the random channel matrices with the channel mean power \( \Omega_0 \), \( \Omega_1 \), and \( \Omega_2 \) for the links \( \mathcal{S} \rightarrow \mathcal{D} \), \( \mathcal{S} \rightarrow \mathcal{R} \), and \( \mathcal{R} \rightarrow \mathcal{D} \), respectively.

In this paper, similarly as in [3], [4], we consider a downlink scenario of cellular networks where \( \mathcal{S} \) and \( \mathcal{R} \) terminals are fixed base stations (BSs) and can be installed at strategic locations by network operators. Hence, we assume that the \( \mathcal{S} \rightarrow \mathcal{R} \) link enjoys a rich-scattering environment leading the channel gain matrix \( \mathbf{H}_1 \) to be of full rank. On the other hand, in downlink systems, \( \mathcal{D} \) is considered as the mobile station (MS) and applicable to be in a poor scattering environment. In order to model this practical scenario of interest, we make an assumption on the poor environment of \( \mathcal{S} \rightarrow \mathcal{D} \) and \( \mathcal{R} \rightarrow \mathcal{D} \) links as keyhole channels, i.e., \( \mathbf{H}_0 \) and \( \mathbf{H}_2 \) are unit rank. Due to space constraints, it is challenging to deploy multiple antennas. The lack of spatial separation causes the correlation effects on MS to be inevitable. In contrast, there is sufficient space to deploy multiple antennas at BSs without causing spatial correlation. Under this practical consideration, we assume that only the destination has the antenna correlation effect represented by the correlation matrix \( \mathbf{\Phi} \in \mathbb{C}^{n_{\mathcal{S}} \times n_{\mathcal{S}}} > 0 \).

We next describe the OSTBC transmission over a dual-hop AF relay network in detail. During a \( T_c \)-symbol interval, \( N \log_2 M \) information bits are mapped to a sequence of symbols \( x_1, x_2, \ldots, x_N \) selected from \( M \)-PSK or \( M \)-QAM signal constellation \( \mathcal{S} \) using Gray mapping with average transmit power per symbol \( P_s \). These symbols are then encoded into an OSTBC codeword denoted by an \( T_c \times n_\mathcal{S} \) transmission matrix \( \mathbf{G} \) whose elements are linear combinations of \( x_1, x_2, \ldots, x_N \) and their conjugate with the property that columns of \( \mathbf{G} \) are orthogonal [12]. The transmission rate of the OSTBC is \( R_c = N/T_c \).

We assume that the channel is subject to frequency-flat fading and is perfectly known at the receiver but unknown at the transmitter. The signal received at \( \mathcal{D} \) and \( \mathcal{R} \) in the first time slot is given by

\[
\begin{align*}
Y_0 &= H_0 \mathbf{X} + W_0 \\
Y_1 &= H_1 \mathbf{X} + W_1
\end{align*}
\]

where \( \mathbf{X} = \mathbf{G}^T \) is the OSTBC matrix. We denote \( H_0 \sim \mathcal{N}_{n_{\mathcal{S}},n_{\mathcal{S}}} (0, \Omega_0 \mathbf{I}_{n_{\mathcal{S}}}) \), \( H_1 \sim \mathcal{N}_{n_{\mathcal{R}},n_{\mathcal{S}}} (0, \Omega_1 \mathbf{I}_{n_{\mathcal{R}}}, \mathbf{I}_{n_{\mathcal{S}}}) \) as random channel matrices and \( W_0 \sim \mathcal{N}_{n_{\mathcal{S}},n_{\mathcal{S}}} (0, N_0 \mathbf{I}_{n_{\mathcal{S}}}) \), \( W_1 \sim \mathcal{N}_{n_{\mathcal{R}},n_{\mathcal{S}}} (0, N_0 \mathbf{I}_{n_{\mathcal{R}}}, \mathbf{I}_{n_{\mathcal{S}}}) \) as the additive white Gaussian noise (AWGN) matrices at \( \mathcal{D} \) and \( \mathcal{R} \), respectively.

The received signal at \( \mathcal{R} \), \( Y_1 \), is then multiplied by an amplifying gain \( G \), and retransmitted to the destination terminal. In this paper, we assume that the relay terminal operates in semi-blind mode and consumes the same amount of power as the source leading to the amplifying gain \( G^2 = 1/(\gamma_1 \Omega_1 + 1/\gamma_2) \), where \( \gamma = P_s/N_0 \) is the average SNR. The signal received at the destination in the second time slot is given by

\[
Y_2 = H_2 G Y_1 + W_2
\]

where \( H_2 \sim \mathcal{N}_{n_{\mathcal{R}},n_{\mathcal{S}}} (0, \Omega_2 \mathbf{I}_{n_{\mathcal{R}}}, \mathbf{I}_{n_{\mathcal{S}}}) \) and \( W_2 \) is the AWGN matrix at \( \mathcal{D} \), e.g., \( W_2 \sim \mathcal{N}_{n_{\mathcal{R}},n_{\mathcal{S}}} (0, N_0 \mathbf{I}_{n_{\mathcal{R}}}, \mathbf{I}_{n_{\mathcal{S}}}) \).

B. Maximum-Likelihood Detection

In this section, we first derive the maximum-likelihood (ML) detection of OSTBC over the dual-hop \( \mathcal{S} \rightarrow \mathcal{R} \rightarrow \mathcal{D} \). The ML metric decomposes into a sum of \( N \) terms, where each term exactly depends on one complex symbol \( x_n, n = 1,2,\ldots,N \). Consequently, the detection of \( x_n \) is decoupled from the detection of \( x_p \) for \( n \neq p \), and is given as follows [5], [6]

\[
\hat{x}_n = \arg \min_{\tilde{x}_n \in \mathcal{S}} \left( \left\| \mathbf{\Psi}^{-1/2} \mathbf{H}_1 \right\|_F^2 x_n + \eta \left( \left\| \mathbf{\Psi}^{-1/2} \mathbf{H}_2 \right\|_F^2 \right) \tilde{x}_n^2 \right)
\]

where \( \eta \in \mathcal{CN} \left( 0, N_0 \left\| \mathbf{\Psi}^{-1/2} \mathbf{H}_2 \right\|_F^2 \right) \). As shown in (5), OSTBC encoding and decoding convert a MIMO relay fading channel into \( N \) equivalent SISO Gaussian subchannels with a path gain of \( \left\| \mathbf{\Psi}^{-1/2} \mathbf{H}_2 \right\|_F^2 \), where \( \mathbf{H} = \mathbf{G} \mathbf{H}_1 \mathbf{H}_2 \mathbf{G} \mathbf{H}_2^* \mathbf{H}_1^* \).

The signals from \( \mathcal{S} \) and \( \mathcal{R} \) are combined together using the maximal-ratio combining (MRC) technique. The instantaneous SNR at \( \mathcal{D} \) is shown as \( \gamma_D = \gammaSD + \gammaSDRD \), where \( \gammaSD \) and \( \gammaSDRD \) are the instantaneous SNRs over the single-hop \( \mathcal{S} \rightarrow \mathcal{D} \) and dual-hop \( \mathcal{S} \rightarrow \mathcal{R} \rightarrow \mathcal{D} \), respectively, and can be written as [5]

\[
\begin{align*}
\gammaSD &= \alpha \left\| \mathbf{H}_0 \right\|_F^2 \\
\gammaSDRD &= \alpha \left( \left\| \mathbf{I}_{n_{\mathcal{R}}} + G^2 \mathbf{H}_2 \mathbf{H}_2^* \right\|_F^{-1/2} \mathbf{H}_1 \right)_F^2
\end{align*}
\]

where \( \alpha = \gamma/(R_{c,n_{\mathcal{S}}}) \).
III. PERFORMANCE ANALYSIS

In this section, we derive the SEP and the outage probability of OSTBC over MIMO AF relay networks taking the keyhole and spatial correlation effects into account. We begin our analysis by calculating the MGF of $\gamma_D$. The following lemma will be helpful for this calculation.

**Lemma 1:** Let the two independent random vectors $\mathbf{z} \in \mathbb{C}^m$ and $\mathbf{y} \in \mathbb{C}^n$ be complex Gaussian distributed, i.e., $\mathbf{z} \sim N_0(m, I_m)$ and $\mathbf{y} \sim N_0((0, I_n))$ and assume there exists a positive definite matrix $\mathbf{M} \in \mathbb{C}^{m\times m}$ and a positive constant $\Omega$ so that $\mathbf{Z} = \sqrt{\Omega} \mathbf{M}^{1/2} \mathbf{y}$. Then the probability density function (PDF) of $\|\mathbf{Z}\|^2_2$, $p_{\|\mathbf{Z}\|^2_2}(z)$, is given by

$$p_{\|\mathbf{Z}\|^2_2}(z) = \sum_{i=1}^{\mu} \sum_{j=1}^{n} \frac{2 \Xi_{i,j} z^{j-1}}{(\Gamma(j)^{(n)}(\Omega \lambda_i)^{j-1})} \mathcal{K}_{n-j} \left( \frac{2 \sqrt{z}}{\Omega \lambda_i} \right)$$

(8)

where $\mathcal{K}_n(\cdot)$ is the modified Bessel function of the second kind [13, eq. (8.432.3)]. For the case of independent fading, i.e., $\mathbf{M} = I_m$, (8) can be simplified as

$$p_{\|\mathbf{Z}\|^2_2}(z) = \frac{2 \Xi_{i,j} z^{j-1}}{(\Gamma(n)\Omega^{j-1})} \mathcal{K}_{n-j} \left( \frac{2 \sqrt{z}}{\Omega} \right).$$

(9)

**Proof:** Let us denote $\mathbf{w} = \mathbf{M}^{1/2} \mathbf{y}$. We then can express $\|\mathbf{Z}\|^2_2 = \Omega \|\mathbf{w}\|^2_2 \|\mathbf{y}\|^2_2$. Since $\|\mathbf{w}\|^2_2$ can be seen as the Hermitian quadratic form in complex normal variables, the MGF of $\|\mathbf{w}\|^2_2$ can be shown as [14]

$$\phi_{\|\mathbf{w}\|^2_2}(s) = \mathbb{E}_{\|\mathbf{w}\|^2_2} \left( e^{-s\|\mathbf{w}\|^2_2} \right) = \prod_{i=1}^{\mu} \left( 1 + s \lambda_i \right)^{-\mu_i}$$

(10)

where $\lambda_1, \lambda_2, \ldots, \lambda_\mu$ are $P$ distinct non-zero eigenvalues of the matrix $\mathbf{M}$ with multiplicities $\mu_1, \mu_2, \ldots, \mu_\mu$, respectively. Further, by expanding the partial fraction for the product in (10) and applying the inverse Laplace transform of $\phi_{\|\mathbf{w}\|^2_2}(s)$, the PDF of $\|\mathbf{w}\|^2_2$ can be obtained as

$$p_{\|\mathbf{w}\|^2_2}(w) = \sum_{i=1}^{\mu} \sum_{j=1}^{n} \Xi_{i,j} w^{j-1} \frac{1}{\Gamma(j)^{(n)}(\Omega \lambda_i)^{j-1}} \exp \left( -\frac{w}{\lambda_i} \right)$$

(11)

where the expansion coefficients $\Xi_{i,j}$ can be expressed as

$$\Xi_{i,j} = \frac{\lambda_i^{-j}\mu_i}{(\mu_i-j)! \partial^{\mu_i-j}(\Omega \lambda_i)^{-\mu_i}} \left. \left( \prod_{i=1,i \neq i}^{\mu} (1 + s \lambda_i)^{-\mu_i} \right) \right|_{s = -1/\lambda_i}.$$

Since $\mathbf{w}$ and $\mathbf{y}$ are statistically independent, we have

$$p_{\|\mathbf{Z}\|^2_2}(z) = \int_{0}^{\infty} p_{\|\mathbf{w}\|^2_2}(w) p_{\|\mathbf{y}\|^2_2} (\frac{z}{\Omega w}) \frac{1}{\Omega w} dw$$

$$= \sum_{i=1}^{\mu} \sum_{j=1}^{n} \Xi_{i,j} z^{j-1} \Gamma(j)^{(n)}(\Omega \lambda_i)^{j-1} \int_{0}^{\infty} w^{j-1} \frac{1}{\Omega w} \exp \left( -\frac{w}{\lambda_i} - \frac{z}{\Omega w} \right) dw$$

(12)

where (12) follows from the fact that $\|\mathbf{y}\|^2_2$ is a chi-square random variable (RV) with $2n$ degrees of freedom and mean $n$. Finally, by making use of the result provided in [13, eq. (3.471.9)], we obtain (8) which completes the proof.

A. MGF of Instantaneous SNR after OSTBC Decoding

Clearly, the MGF of $\gamma_D$, $\phi_D(s)$, is the product of the MGF over the single-hop $S \rightarrow D$, $\phi_{SD}(s)$, and the MGF over the dual-hop $S \rightarrow R \rightarrow D$, $\phi_{SRD}(s)$. In the following, we first consider $\phi_{SD}(s)$ and then $\phi_{SRD}(s)$.

For the direct communication, i.e., $S \rightarrow D$ link, by utilizing the Lemma 1 and making use of [13, eq. (6.643.3)], the MGF of $\gamma_{SD}$, $\phi_{SD}(s)$, can be expressed as

$$\phi_{SD}(s) = \sum_{i=1}^{\mu} \sum_{j=1}^{n} \Xi_{i,j} \left( \frac{1}{\Omega_0 \lambda_{2i} \alpha_S} \right)^{j+n-1} \exp \left( \frac{1}{\Omega_0 \lambda_{2i} \alpha_S} \right)$$

$$\times \mathcal{W}_{\frac{n}{s} + j - 1, \frac{n}{s} - j - 1} \left( \frac{1}{\Omega_0 \lambda_{2i} \alpha_S} \right)$$

(13)

where $\mathcal{W}_{\alpha,\beta}(z)$ is the Whittaker $\mathcal{W}$ function [13, eq. (9.222)].

Next, applying [13, eq. (9.220.2)] reduces (13) to

$$\phi_{SD}(s) = \sum_{i=1}^{\mu} \sum_{j=1}^{n} \Xi_{i,j} F_0(n, j; -\Omega_0 \lambda_{2i} \alpha_S).$$

(15)

It is important to note that the result given in (15) is new and does not appear in the literature.

For the dual-hop, the MGF $\phi_{SRD}(s)$ can be obtained by evaluating

$$\phi_{SRD}(s) = \mathbb{E}_{H_2} \left\{ \det \left( I_{nSR} + \sigma_0 \Omega_{1} G^2 H_2^2 \right)^{-1} \right\}$$

(16)

$$= \mathbb{E}_{H_2} \left\{ \det \left( I_{nSR} + \sigma_0 \Omega_{1} G^2 H_2^2 \right)^{-1} \right\}$$

$$\times \mathbb{E}_{H_2} \left\{ \det \left( I_{nSR} + \sigma_0 \Omega_{1} G^2 H_2^2 \right)^{-1} \right\}$$

(17)

$$= \mathbb{E}_{H_2} \left\{ \left( \frac{\det \left( I_{nSR} + \sigma_0 \Omega_{1} G^2 H_2^2 \right)}{\det \left( I_{nSR} + \sigma_0 \Omega_{1} G^2 (1 + \sigma_0 \Omega_{1}) H_2^2 \right)} \right)^{n_S} \right\}$$

(18)

where (16) can be obtained from (1) and (7) by utilizing the fact that $\phi_{SRD}(s) = \mathbb{E}_{H_2} \{ e^{-s \gamma_{SRD}} \}$. Let us denote $\chi$ as a non-zero eigenvalue of $H_2^2$. Since $H_2$ is of unit rank, it is easy to see that $\chi = \|H_2\|^2_2$. Using this property, we can rewrite (18) as

$$\phi_{SRD}(s) = \int_{0}^{\infty} \left[ \frac{1 + G^2 \chi}{\chi} \right]^{n_S} p_{\chi}(\chi) d\chi.$$
Substituting (8) in (19) and applying the binomial theorem results in

\[ \phi_{\text{SRD}}(s) = \sum_{i=1}^{P} \sum_{j=1}^{\mu_i} \sum_{k=0}^{\infty} \Theta_{i,j,k} (1 + \beta \chi)^{-n_s} \]

\[ \times \frac{n_s + 1}{2} \chi^{k-1} K_{n_s-j} \left( 2 \sqrt{\chi \lambda_{2i}} \right) d\chi \]  

where

\[ \Theta_{i,j,k} = \frac{2 \Xi_{i,j} G^{2k}(n_s)}{\Gamma(j) \Gamma(n_R) \Omega_{2i}^{n_s-j}} \], \quad \beta = G^2(1 + s \alpha \Omega_i) \]

**B. Symbol Error Probability**

Using the MGF approach, the SEP of considered systems including the single-hop (S → D) and the dual-hop (S → R → D) for M-PSK modulation can be given by [16]

\[ P_e^{(SD+SRD)} = \frac{1}{\pi} \int_{0}^{\pi - \frac{\pi}{2}} \phi_{\gamma_0}(g) \left( \frac{g}{\sin^2 \theta} \right) d\theta \]

where \( g = \sin \left( \frac{\pi \chi}{M} \right)^2 \). In our case, the finite-range integral is intractable for the closed-form derivation of the exact SEP although it enables us to numerically evaluate the SEP performance. The SEP given in (21) can be tightly lower bounded as [17]

\[ P_e^{(SD+SRD)} \approx \left( \frac{M-1}{2M} - \frac{1}{6} \right) \phi_{\gamma_0}(g) + \frac{1}{4} \phi_{\gamma_0}(\frac{4g}{3}) + \left( \frac{M-1}{2M} - \frac{1}{4} \right) \phi_{\gamma_0}(\frac{g}{\sin^2[\pi(M-1)/M]}). \]

By multiplying \( \phi_{\text{SRD}}(s) \) with \( \phi_{SD}(s) \), respectively, given in (20) and (15) and substituting in either (21) or (22), we can obtain the SEP of keyhole MIMO AF relay systems.

**C. Outage Probability Analysis**

Another standard performance metric characterizing the diversity systems operating over fading channels is the so-called outage probability and is defined as the probability that the instantaneous SNR \( \gamma_D \) falls below a given threshold \( \sigma \), i.e.,

\[ P_p^{(SD+SRD)} = \Pr (\gamma_D \leq \sigma) = F_{\gamma_0}(\sigma) \]

where \( F_\gamma(\cdot) \) is the cumulative distribution function (CDF) of \( \gamma_D \). The CDF can be obtained from the inverse Laplace transform of the MGF, i.e., \( F_{\gamma_0}(\cdot) = \mathcal{L}^{-1}\{\phi_{\gamma_0}(s)/s\} \), where \( \mathcal{L}^{-1}\{\cdot\} \) is the inverse Laplace transform operator. By utilizing the MGF function of \( \gamma_D \) given in Section III-A, we can obtain \( F_{\gamma_0}(\cdot) \) using simple numerical techniques. Specifically, we obtain a tight approximation for the outage probability performance as [18]

\[ P_p^{(SD+SRD)} \approx \frac{A}{\sigma} \sum_{p=0}^{Q} \left( \frac{Q}{p} \right) \sum_{q=0}^{p+q} \left( \begin{array}{c} p+q \\ q \end{array} \right) \frac{(-1)^p}{\delta_p} \left( \frac{A+2\epsilon x_p}{2x} \right)^p \]

where \( \delta_p \) is a constant defined as \( \delta_p = 2 \) if \( p = 0 \) and \( \delta_p = 1 \) if \( p = 1, 2, \ldots, P \). Here, \( A, P, \) and \( Q \) are predefined integers.

When only considering the relaying channel, i.e., \( S \rightarrow R \rightarrow D \) link, we can utilize another approach to obtain a more tractable outage probability, which will be described in the sequel. By definition, the PDF of \( \gamma_{SRD} \) can be obtained from (19) as

\[ p_{\gamma_{SRD}}(\gamma) = \int_{0}^{\infty} L^{-1}\left\{ \left( 1 + G^2 \frac{\lambda}{1+G^2 \lambda} \right)^{-n_s} \right\} p_{\chi}(x) dx \]  

where \( b = \frac{\beta}{(R, n_s)}. \) Then, using the result derived in Lemma 1 for (25) and applying the binomial theorem for the term \( (1 + G^2 \lambda)^{-n_s} \), the PDF of \( \gamma_{SRD} \) can be rewritten

\[ p_{\gamma_{SRD}}(\gamma) = \sum_{i=1}^{P} \sum_{j=1}^{\mu_i} \sum_{k=0}^{\infty} \Lambda_{i,j,k} \gamma^{n_s-1} e^{-\frac{\gamma}{2}} \]

\[ \times \int_{0}^{\infty} \frac{x^{n_s+j+k-n_s-1} e^{-\frac{\gamma x}{2}}}{\gamma^{n_s}} K_{n_s-j} \left( 2 \sqrt{\chi \lambda_{2i}} \right) dx \]

where \( \Theta_{i,j,k} = \frac{2 \Xi_{i,j} G^{2k}(n_s)}{\Gamma(j) \Gamma(n_R) \Omega_{2i}^{n_s-j}} \). Then, integrating (26) results in the CDF of \( \gamma_{SRD} \) as

\[ F_{\gamma_{SRD}}(\gamma) = \sum_{i=1}^{P} \sum_{j=1}^{\mu_i} \sum_{k=0}^{\infty} \Lambda_{i,j,k} \gamma^{n_s-1} e^{-\frac{\gamma}{2}} \]

\[ \times \int_{0}^{\infty} x^{n_s+j+k-n_s-1} e^{-\frac{\gamma x}{2}} K_{n_s-j} \left( 2 \sqrt{\chi \lambda_{2i}} \right) dx \]

which readily enables us to get the outage probability for the case when we neglect the direct link. This result will be helpful in the next section when we investigate the effect of keyhole and correlation on the diversity gain supported by the relay.

**D. Achievable Diversity Gain**

The diversity order obtained by the single hop, i.e., \( S \rightarrow D \) link, is \( d_{SD} = \min(n_S, n_D) \). The achievable diversity order over the dual-hop can be expressed as

\[ d_{SRD} = \lim_{\gamma \rightarrow \infty} \frac{-\log P_e^{(SD+SRD)}}{\log \gamma} = \lim_{\gamma \rightarrow \infty} \frac{-\log \phi_{\gamma_{SD}}(g)}{\log \gamma} \]

For independent keyhole fading, by substituting (12) in (19) and exchanging the variable \( \gamma = t \), we have

\[ \phi_{\gamma_{SD}}(g) = \int_{0}^{\infty} \xi_1 \left[ \frac{1 + G^2 \gamma^{-1} t}{1 + G^2 \gamma^{-1} t + at} \right]^{(n_s+n_0)/2-1} \]

\[ \times \gamma^{-(n_s+n_0)/2} K_{n_s-n_0} \left( 2 \sqrt{\chi \lambda_{2i}} \right) dt \]

where \( \xi_1 = 2 \Omega_2^{-1}(n_s+n_0)/[\Gamma(n_R) \Gamma(n_D)] \) and \( a = G^2 \Omega_1/(R, n_s). \) For assessing the high SNR asymptotic of SEP, we make use of fact that for small values of \( x, K_n(x) \) can be approximated as [19]

\[ K_n(x) \approx x^{(n_s)} \left\{ \begin{array}{cl} -\ln \left( \frac{x}{2} \right) & \text{for } n = 0 \\ \frac{\Gamma(n+1)}{2} \left( \frac{x}{2} \right)^n & \text{for } n \neq 0 \end{array} \right. \]

Applying (30) in (29) results in the high SNR asymptotic of \( \phi_{\gamma_{SRD}}(g) \) which then allows us to derive the cooperative
diversity gain. To do so, we have to distinguish two different cases as follows:

- **Case I**: \( n_R \neq n_D \). In this case, we have \( n = n_D - n_R \neq 0 \).

  Replacing \( \mathcal{K}_{n_D-n_R} (\cdot) \) in (29) by its approximation in (30) and neglecting the small terms in the integrand of (29) yields

  \[
  \phi_{\text{SRD}} (g) \left( \frac{\text{large} \gamma}{\text{large} \Gamma} \right) \approx \frac{1}{\min(n_R, n_D)} \gamma - \min(n_R, n_D) \times \int_0^\infty \xi_2 (1 + at)^{-ns} t^{\min(n_R, n_D) - 1} dt \tag{31}
  \]

  where \( \xi_2 = \frac{\Gamma(|n_D - n_R|)(\Omega_2)^{n_D-n_R}}{2} \). Note that the integral in (31) converges when \( n_S > \min(n_R, n_D) \), which then yields

  \[
  \phi_{\text{SRD}} (g) \left( \frac{\text{large} \gamma}{\text{large} \Gamma} \right) \approx \frac{\xi_2 \Gamma(|n_R, n_D|) \Gamma(n_S - \min(n_R, n_D))}{a^{\min(n_R, n_D)} \Gamma(n_S)} \tag{32}
  \]

  From (32), we have \( d_{\text{SRD}} = \min(n_R, n_D) \).

- **Case II**: \( n_R = n_D \). In this case, \( n = n_D - n_R = 0 \).

  Similarly as in Case I, if \( n_S > n_R \) then we get

  \[
  \phi_{\text{SRD}} (g) \left( \frac{\text{large} \gamma}{\text{large} \Gamma} \right) \approx \frac{\ln \gamma}{n_R} \times \frac{\xi_1 \Gamma(n_R) \Gamma(n_S - n_R)}{2a^n \Gamma(n_S)} \tag{33}
  \]

  From (33), applying the l’Hôpital’s rule for \( \lim_{\gamma \to \infty} \frac{\ln \phi_{\text{SRD}} (g)}{\ln \gamma} \) results in \( d_{\text{SRD}} = n_R \).

By combining the two cases, the cooperative diversity gain provided by the relay channel, i.e., \( S \to R \to D \) link, is of order

\[
  d_{\text{SRD}} = \min(n_R, n_D) \tag{34}
\]

with the condition that \( n_S > \min(n_R, n_D) \). Since our considered system is the downlink communication, i.e., \( n_S > n_D \), the aforementioned condition is always true. Hence, we consequently can state that, the relay link exhibits a cooperative diversity gain of \( \min(n_R, n_D) \).

The above-obtained cooperative diversity order for MIMO AF relay networks with keyhole does not depend on the first hop but the second hop. It is actually in line with a previous result for single-antenna AF relay systems, i.e., among two hops, the more severe one solely determines the system performance. More importantly, it provides insights into the system design. In fact, when \( n_R \geq n_D \), increasing the number of antennas at the relay station results in no spatial diversity boost.

### IV. Numerical Results

In this section, we present numerical results for MIMO AF relay networks with keyhole and antenna correlation using OSTBCs in selected scenarios. The analytical results of SEP and outage probability corresponding to the closed-form expression are generated based on (22) and (24), respectively. The channel mean powers for all links are assumed to be identity, i.e., \( \Omega_\ell = 1 \) for \( \ell = 0, 1, 2 \). For the correlation matrix at the destination \( \Phi \), we use the exponential correlation model from equispaced diversity antennas, i.e., the correlation coefficient \( \rho_{i,j} \) among the \( i \)-th and \( j \)-th input channel \( \rho_{i,j} = \rho^{|i-j|} \) with \( i, j = 1, 2, \ldots, n_D \). We apply 8-PSK modulation for all considered examples and the threshold \( \sigma = 10 \) dB.

Fig. 1 and Fig. 2 illustrate the SEP and outage performance of MIMO AF relay systems equipped with 3, 2 and 2 antennas at S, R, and D, respectively, i.e., an (3, 2, 2) system, under the keyhole and antenna correlation effects. The correlation coefficient is varied as \( \rho = 0.3, 0.5, 0.7, 0.9 \). As can clearly be observed from the two figures, the SEP and outage performance are degraded with the increase of antenna correlation coefficients. The excellent agreement between analytical and simulation results verifies the correctness of our analysis.

To demonstrate the cooperative diversity obtained by the relay channel, we only plot the SEP of the \( S \to R \to D \) link.
destination suffers from a keyhole effect.

\[
\min(\ldots)
\]

as the maximum achievable cooperative diversity gain is of

\[
(\ldots)
\]

be made for the example in Fig. 4. This result is expected

\[
(\ldots)
\]

from 3 to 6 or fix

\[
(\ldots)
\]

(omitting the direct link) for \((6, n_R, 3)\) and \((5, 2, n_D)\) in Fig. 3 and Fig. 4, respectively. We either fix \(n_D = 3\) and vary \(n_R\) from 3 to 6 or fix \(n_R = 2\) and vary \(n_D\) from 2 to 5. As shown in Fig. 3, increasing \(n_D\) while keeping \(n_D\) remains unchanged results in no diversity enhancement. The same observation can be made for the example in Fig. 4. This result is expected as the maximum achievable cooperative diversity gain is of \(\min(n_R, n_D)\) when the signal propagation originated from the destination suffers from a keyhole effect.

V. CONCLUSIONS

In this paper, we have studied effects of the most severe factors, i.e., keyhole and antenna correlation, on the performance of MIMO wireless relay systems. We have derived a new exact expression for the MGF of the instantaneous SNR and using this result the system’s error performance was investigated. We show that under such environment the relay channel still provides a cooperative diversity gain of \(\min(n_R, n_D)\). This result is important to radio system designers to achieve the maximum diversity gain while keeping the low system complexity in terms of the number of antennas deploying on the relay station. Our analysis is verified by Monte-Carlo simulations.

REFERENCES


