Modelling in Modelica and position control of a 1-DoF set-up powered by pneumatic muscles

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ABSTRACT

The characteristics of pneumatic artificial muscles – or McKibben muscles – make them of great interest for the development of robotic applications such as orthoses or certain wearable robots. In order to research the applicability of these actuators in marketable applications, an experimental one-degree-of-freedom set-up based on pneumatic muscles manufactured by Festo was built at Ikerlan. After the detailed description of the experimental set-up, the paper presents the modelling of a pneumatic muscle in Modelica as a new component totally compatible with objects from commercial libraries, thus enabling any mechatronic device that contains pneumatic muscles to be modelled. It then offers a description of the experiments performed to identify the model in the case of real pneumatic muscles. With a view to adjusting the static model to the experimental tests, the inclusion of a new polynomial term depending on muscle contraction is proposed by the authors. The paper then shows the complete model of the experimental set-up in Dymola/Modelica. The part related to the modelling ends with a validation of the model with experimental data. The experimental set-up is very non-linear and very difficult to control properly. As a reference, an enhanced PID controller was designed, and at the same time, a robust controller $H_\infty$ and a sliding-mode controller based on an observer were designed and implemented. After this, a position controller based on an internal pressure loop for each pneumatic muscle was tuned up. The paper goes into detail regarding each of the four position controllers designed, and finally, a comparison is made by means of experimental results.

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1. Introduction

At present, most robots use actuators and control systems featuring high mechanical impedance. However, there is evidence in the natural world of the fact that natural impedances in animals are quite low. Many robotic applications require low impedance in terms of interaction with the environment. Protheses, orthoses and exoskeletons or wearable robots are some such cases. Observation of nature and the use of biologically inspired components (sensors and actuators) open up new options for the design of biomimetic robotic devices.

Although electrical actuators are not biomimetic, they are still the most frequently used in biomimetic robots owing to the fact that they integrate easily, are high-powered, low-cost and have a straightforward interface. However, major advances are being made in the development of artificial muscles in order to replicate the biological qualities of animal muscle. Today’s new technologies include electroactive polymers (EAPs), although their applicability is still far removed from marketable applications. Pneumatic artificial muscles – or McKibben muscles – are a very interesting alternative.

Pneumatic muscles have high-performance actuating features and a good power/weight ratio and lightness, enabling the easy design of low impedance robotic applications in terms of interaction with the environment. Unfortunately, like animal muscles, they reveal certain very non-linear force–length characteristics, and it is not easy to control them and obtain the performance features required by some applications [1,2]. The modelling of pneumatic muscles and controller design have been active research issues in recent years [3–7].

Although pneumatic muscles were invented in the mid-twentieth century, it is only in the last two decades that any development has been made in the knowledge, control and applicability of the same. Groups of researchers from different universities such as the University of Washington (USA), Arizona State University (USA), Salford University (UK), the University of Ulsan (South Korea), Vrije University Brussel (Belgium), Vanderbilt University (USA) or Wright State University (USA) have been studying the behaviour
of pneumatic muscles for years, developing mechatronic applications based on this type of actuators.

Several examples of the successful inclusion of pneumatic muscles in mechatronic devices for rehabilitation purposes can be found in scientific literature.

Applications in the form of an exoskeleton exist for both upper and lower limbs. The upper limb applications include the device called RUPERT, developed at Arizona State University, the latest version of which now has 5 degrees of freedom actuated by pneumatic muscles [8,9]. The research group at Salford University has developed solutions for both upper and lower limbs. The upper limb solution is noteworthy for its 7 degrees of freedom, all actuated by pneumatic muscles [10,11]. The lower limb solution is applicable from the waist to the feet and was designed with 10 degrees of freedom [12]. Another interesting device can be seen in [13], where a prototype of an exoskeleton for gait rehabilitation is presented. Exoskeletons with a more simple design also exist, with one single joint, for either the hand [14], the elbow [15] or the ankle [16].

Other researchers have opted for pneumatic muscle-based solutions but of the end-effector type, i.e. non-wearable devices. At L’Aquila University in Italy a device with 2 degrees of freedom was developed for functional recovery therapy [17]. At Ulsan University in the Korean Republic, researchers have been working on similar devices with 1 and 2 degrees of freedom for years, with the aim of obtaining valid solutions for rehabilitation [18,19].

There are also prostheses actuated by pneumatic muscles which replace amputated limbs. [20] shows a hand and forearm prosthesis where the movement of four fingers is controlled by means of pneumatic muscles, while in [21,22] the usability of this type of actuators in lower limb prostheses is studied in depth.

On a very different note from the rehabilitation and function recovery applications are the pneumatic muscle-based mechatronic devices such as biorobots [23,24].

Ikerlan has designed and built IKO (Ikerlan’s Orthosis), a 5-DoF upper-limb exoskeleton. It is a biomechatronic device aimed at helping the user carry out daily tasks that require a certain effort [25,26]. Before designing it, in order to research the applicability of pneumatic muscles in marketable applications, and, particularly, in the development of orthoses and wearable robots, an experimental 1-DoF arm powered by pneumatic muscles manufactured by Festo was designed and constructed (see Fig. 1). This experimental set-up is the core of this article.

The experimental set-up is very non-linear and very difficult to control properly. Owing to the fact that the results obtained with a classical PI controller were not good, the authors decided to apply other advanced control techniques. Firstly, a PID-based controller was enhanced with linear and non-linear internal loops. However, good performance requires the use of robust or non-linear control techniques and, in this context, the application of different control techniques is found in the literature.

Many attempts have been made to control muscle position using robust control techniques such as sliding-mode [27,28] and $H_\infty$ [29]. Ref. [30] shows an adaptive robust control where a class of saturated adaptive robust control (SARC) laws using the discontinuous projection method are applied. Predictive control is also proposed in [31] for control of the position and pressure by means of non-linear model predictive trajectory control, and in [32] with the adaptive fuzzy sliding-mode control (AFSMC). To obtain the desired tracking performance, an inversion based control (IBC) concept was used and presented in [33].

Consequently, a robust linear control technique $H_\infty$ [34] and a robust non-linear technique, sliding-mode [35], were firstly applied to control the experimental set-up presented in this work. Despite the fact that the results achieved with these controllers are fairly good, owing to the non-linearity of the set-up and air pneumatic muscles themselves the performance level is not identical throughout the displacement range.

Subsequently, following an idea applied in work by Caldwell and Tsagarakis [36], the authors developed another position controller based on an internal pressure loop for each muscle. This position algorithm requires the use of one servo-valve for each pneumatic muscle instead of one single valve for each DoF, as used with the algorithms that were designed and implemented previously in the experimental set-up described in this paper.

![Fig. 1. Picture and schematic diagram of the experimental set-up and pneumatic circuit.](image-url)
After the introduction the paper gives a detailed description of the experimental 1-DoF mechanism based on a pair of antagonistic pneumatic muscles.

In the part related to modelling issues, the paper first presents a brief review of the pneumatic muscle models that feature in the literature. The analytical model chosen is then described, followed by the experimental trials performed to identify the model parameters. It then goes on to explain the implementation of the model in Dymola/Modelica as a new component compatible with the PneuLib library. The paper then shows the complete modelling of the experimental device in the aforementioned package. Lastly, validation of the model with experimental data is shown.

The main aim of the last part of this paper is to describe each of the four position controllers that have been designed: the enhanced PID controller, the $H_\infty$ controller, the sliding-mode controller and the controller based on internal pressure loops. Finally, the paper concludes by comparing the performance of the different control algorithms by means of experimental results.

2. Description of the 1-DoF experimental set-up

A human arm orthosis-type application has been taken into consideration when designing the set-up. To this end and albeit with a single degree of freedom, it was considered that it should allow for the greatest angular displacement possible, and that it should be able to transport the greatest possible mass at the tip (emulating a weight borne by the hand). On the other hand, however, it needed to be confined to the length of the pneumatic muscles. In seeking a compromise between all the specifications, a displacement of around 60° and a maximum mass to be moved at the tip of 8 kg were set. By attempting to minimise the length of the muscle required, the design focused on the mechanism that would enable the arm and inertias to rotate with good dynamics by means of the two muscles.

The pneumatic muscle chosen was the DMSP-20-200N manufactured by Festo, and the resultant mechanism is shown in Fig. 2. The parameter values defining the mechanism are:

- $a = 5 \text{ mm}$; $b = 85 \text{ mm}$; $c = 491 \text{ mm}$; $d = 40.6 \text{ mm}$
- $e = 129.4 \text{ mm}$; $\alpha = 0$–60°; $\beta = 120$–180°; $r = 32 \text{ mm}$

From these values the distance $L$ (mm) between the ends (or joining points of the mechanism) of the pneumatic muscles is:

$$L = \sqrt{175059 + 2841.6 \cdot \cos \alpha - 26624 \cdot \sin \beta}$$

When there is no pressure on the muscles, the distance $L$ is 423 mm, with the length of the muscle being 200 mm. The centre of the arm mass with regard to the centre of rotation is at a height of 17.6 mm and at a horizontal distance of 205 mm, considering that the arm is in the horizontal position. The arm mass is 0.987 kg. The centre of the additional masses placed on the end of the arm (up to a maximum of 8 kg) would be at a height of 24 mm and at a horizontal length of 367 mm with regard to the centre of rotation, always bearing in mind that the arm is in the horizontal position. Fig. 1 shows a picture of the prototype constructed. The set-up may be rotated so that the arm moves in a horizontal plane and the effects of gravity are therefore cancelled out.

![Fig. 2. Geometric model of the 1-DoF experimental set-up.](image-url)
One Festo MPYE-5-1/8HF pneumatic servo-valve is initially used for actuation, with a working pressure of 6 bar. Festo SDE-D10 pressure sensors are used to establish the entry pressure of each muscle. The model includes a FAGOR S-D90 encoder which supplies 180,000 pulses per turn, for accurate measurement of the rotation angle of the arm. A load cell is also included on the lower stop of the model, to allow the force exerted by the arm against this stop to be measured. The stop may be fastened at different angles, so that the force exerted by the muscles may be calculated at different lengths and with different supply pressures.

The schematic diagram of the set-up, which includes the control hardware, sensors and pneumatic circuit, is also shown in Fig. 1. As the figure shows, a Festo MPYE-5-1/8HF pneumatic servo-valve is used as a standard for actuation. This servo-valve is able to operate the two pneumatic muscles. The controller hardware is PIP8, an industrial PC made by the company MPL, which is very similar to The MathWorks’ xpCTargetBox. A PC104 card (Sensoray model 526) was incorporated into the PIP8 in order to read and write all the system signals. Control algorithms were implemented in Simulink and code was generated and downloaded in the aforementioned hardware by means of two The MathWorks tools: RTW and xpCTarget.

3. Pneumatic muscle modelling

3.1. Pneumatic muscle static model selection

Many authors have worked on the idea of obtaining a model that represents the behaviour of these devices, the most important of which are given below. The purpose of the model is to discover a relationship between the pressure and length of the pneumatic muscle with the force it exerts along its entire axis. Fig. 3 shows the well-known outline of the constitution of the pneumatic muscle. L is the length of the cylinder and D the diameter. Assuming the inextensibility of the mesh material, the geometric constants of the system are the thread length c and the number of turns n for a single thread. α is the angle between the thread and the long axis of the cylinder. The angle changes with the length of the muscle.

On the basis of this, Tondu and Lopez [3] carried out a mathematical development based on the application of a virtual work theorem, which led them to obtain the following equation:

\[
F(ε, P) = (\pi \cdot r_0^2)P\alpha(1 - k \cdot ε)^2 - b
\]

with:

\[
b = \frac{1}{\sin^2(α_0)}
\]

where:

\[
ε = \frac{l_0 - l}{l_0}, \quad 0 \leq ε \leq ε_{\text{max}}, \quad α = \frac{3}{\tan^2(α_0)}
\]

\[
l = l_0 \cdot \sin(α)
\]

\[
D \leftrightarrow \text{n turns} \leftrightarrow \text{c} = \frac{l}{n \pi D}
\]

Right: Geometric model of a pneumatic muscle.

Thus, in [4,5,37], formulas equivalent to (1) are put forward based on the same principles as Tondu and Lopez [3] and may be readily deduced from each other. All of them show that the force exerted by the muscle has a linear relation with the internal pressure and a non-linear relation with the contraction.

As it is very difficult to know the exact value of some of the internal design and construction parameters of muscles, a model was chosen in which the parameters are set through experimental testing. Consequently, for the model put forward in this work, Eq. (2) proposed by Petrovic [5], which is equivalent to Eq. (1), was initially chosen due to the simplicity of the second order polynomial adjustment

\[
F = (D_1 + D_2 \cdot q + D_3 \cdot q^2)P
\]

with q being the displacement \( q = l_0 - l \). In the process for obtaining this equation, the most important simplification made is the elimination of the friction.

However, Eq. (2) predicts the same contraction for various pressures at force \( F = 0 \). To reflect the correlation between pressure and contraction when \( F = 0 \), some authors [1–3] add two correction functions that depend on the pressure: one for pressures higher than 2 bars and other for lower pressures. The authors propose adding a new term that depends on the contraction, as it is easier to identify from the experiments carried out. Thus, Eq. (2) becomes:

\[
F = (D_1 + D_2 \cdot q + D_3 \cdot q^2)P + \varphi(q)
\]

3.2. Static pneumatic muscle model identification experiments

The selected static analytical model (3), which relates the force exerted by the muscle with its internal pressure and contraction, comprises a set of parameters. To find the values of these parameters, the test platform shown in Fig. 4 was designed.

As shown in the figure, the upper part of the muscle was fixed mechanically to the platform and a hook was added to the lower end on which different weights can be placed. The top right section of the figure shows the valve that controls the pneumatic pressure from the source. Finally, a linear sensor was added to provide the muscle contraction with proportional tension. The data on the muscular contraction and pneumatic pressure were sent to and received by a PIP8, an industrial PC made by the company MPL. The above platform was used for two different types of tests, although they are redundant for calculating the model parameters.

The first of the experiments consists of measuring the contraction of the pneumatic muscle submitted to a constant load at variable pneumatic pressure. This gives a graph of the muscular contraction in accordance with a variable pneumatic pressure of between 1 and 6 bar. As shown in the graph in Fig. 5, there are as many curves as there are different loads used in the experiments. In addition, the hysteresis behaviour of the pneumatic muscles is shown, where very different contraction values may be obtained for the same pressure, depending on how this pressure value was obtained.

The second experiment consists of measuring the contraction of the pneumatic muscle under constant pressure and with a variable load, i.e. the muscle was loaded with a variable weight of 0–120 kg at a constant pneumatic pressure and the static contraction
Fig. 4. Platform built for model identification trials.

Fig. 5. Contraction-pressure results for different loads.
obtained in each case was measured. In this case, the muscle hysteresis is also notable.

In theory, the results obtained in both experiments are equally valid for obtaining the relation to be characterized between the force, pressure and contraction. However, in practice, it is preferable to use the data from the first experiment owing to the large quantity of sample data and the fact that they have been obtained without altering the system, unlike the second experiment, in which each addition and removal of weight involves vibrations. Thus, the curves in Fig. 6 are obtained from the curves of the first experiment and provide a linear relation between the force and the internal pressure for a given contraction.

Owing to the hysteresis of the pneumatic muscle, there are two curves for each contraction level, and the chosen approach was to show the behaviour of the pneumatic muscle by means of a reference curve positioned between the two. In accordance with the lines shown in Fig. 6, the extra term in equation (3) has been adjusted as a fourth-order polynomial, increasing the number of parameters to be identified to eight. Therefore, the pneumatic model proposed is:

$$F = (D_1 + D_2 \cdot q + D_3 \cdot q^2)P + (a_1 + a_2 \cdot q + a_3 \cdot q^2 + a_4 \cdot q^3 + a_5 \cdot q^4)$$

(4)

Accordingly, the following values for the parameters are obtained from successive linear approximations of the set of curves:

$$D_1 = 242.261; \quad D_2 = -3.269; \quad D_3 = -0.005$$
$$a_1 = 84.057; \quad a_2 = -44.37; \quad a_3 = 2.952;$$
$$a_4 = -0.101; \quad a_5 = 0.001$$

As has been mentioned above, the set of curves formed by the average value for each level of contraction has been taken into account on calculating the values (Fig. 6). The model will therefore not be able to reflect the hysteresis of the actuator, which is due to the friction not considered by the model.

3.3. Pneumatic muscle model implementation in Modelica

The model described above was implemented by using the good qualities of the physical system modelling language known as Modelica. Modelica is a freely available object-oriented language for the modelling of large, complex, and heterogeneous physical systems. It is suited for multi-domain modelling, for example, and in this case, mechatronic models in robotics involving pneumatic, mechanical and control subsystems. Modelica also allows hybrid systems to be modelled as it is based on synchronous differential, algebraic and discrete equations, enabling unified mathematical description of continuous time and discrete event parts of a model [38].

The muscle model was developed ensuring its connectivity with other objects described in Modelica and compatible with the commercial pneumatic library PneuLib. The model therefore contains two mechanical interfaces corresponding to the muscle anchorage points and a pneumatic interface allowing it to be connected to an external pneumatic circuit.

Consideration has been given to the different main parameters that characterize the physical properties of this type of muscle: nominal length; nominal diameter; the initial angle between the membrane fibres and the muscle axis; and the set of parameters of equation (4). Other magnitudes that may be considered as parameters are the heat transfer coefficient, for instance, or the maximum pressure beyond which the system delivers a warning message.

In order to characterize the physical behaviour of the muscle, a set of equations was implemented in the Modelica code. The first corresponds to the previously analyzed Eq. (4) that describes the static mechanical force exerted by the muscle. To encode the fact...
that the muscle only works in contraction, according to Otter et al. [38], a discrete event was added to the continuous equation. The event was implemented by means of a conditional sentence. A similar solution has been used for the internal temperature and air mass, giving the model a hybrid character.

There are other important equations for determining the pressure on the muscle, due to its importance for the dynamic behaviour of this actuator. It may be calculated from the ideal gas equation:

$$P \cdot V_{\text{eff}} = m \cdot R \cdot T_{\text{muscle}}$$  \hspace{1cm} (5)

where the air mass ($m$) and the internal temperature ($T_{\text{muscle}}$) are considered as state variables of the model. $V_{\text{eff}}$ is the effective volume of the muscle, where, following the model proposed in [39], even the flattening experienced on the ends of the muscle when it is contracted has been taken into consideration for calculation purposes.

The last of the equations considered corresponds to the thermodynamic equation:

$$m \cdot \frac{dT_{\text{muscle}}}{dt} + T_{\text{muscle}} \cdot \dot{m} = \gamma \cdot T_{\text{inout}} \cdot \dot{m} + \frac{dV_{\text{eff}}}{dt} \cdot \frac{P}{C_v} - h \cdot s \cdot \frac{dT_{\text{muscle}} - T_{\text{sou}}}{C_p}$$ \hspace{1cm} (6)

with $\dot{m}$ being air mass flow rate, $\gamma$ the ratio of specific heat capacities, $T_{\text{inout}}$ the temperature of air entering or leaving the muscle, $c_v$ the specific heat capacity at constant volume, $s$ the heat transfer surface area, $h$ the heat transfer coefficient and $T_{\text{sou}}$ the temperature of the surroundings.

As a result, the muscle model implemented in Modelica is static, modelled as a gas spring with non-linear characteristics, and which does not contemplate friction (friction inside the rubber tube material, friction inside the braided shell structure, friction between the rubber tube and the braided shell and friction of pressurized gas) or other non-linear phenomena, i.e. those appearing in the form of hysteresis.

A dynamic model would be obtained on introducing the friction and inertial terms of the moving parts of the actuator. However, as in this case the new pneumatic muscle element defined in Modelica is to be used as one of several elements in conjunction with other mechanical/pneumatic elements, the inertia moving the actuator will be determined by the structure and the mechanism, and the friction – which would include both the internal friction of the muscle and that of the structure – could be reflected as an element external to the pneumatic muscle in the complete model, as described in the following section. Obviously, the friction – both Coulomb and dynamic – must be suitably estimated so that the complete model reflects the real situation.

A more complete description of the Modelica code implemented can be found in [40].

4. Modelling of the 1-DoF experimental set-up

4.1. Non-linear modelling of the experimental set-up

For the composition of the prototype dynamic model, the inter-object connection properties offered by the graphic interface of the Modelica-based modelling and simulation tool called Dymola have been used. Dymola supports hierarchical model composition, and libraries of reusable components are available in many engineering domains.

Fig. 7 is a graphic representation of the prototype model designed in Dymola. The following paragraphs give a detailed explanation of its most significant parts. The construction of the model includes elements developed expressly for this application (such as the pneumatic muscle model described in Section 3), as well as parts that belong to commercial libraries (Multi-body and Pneu-Lib). Given the multi-disciplinary nature of the prototype, the model contains parts from different domains that are related by special objects providing the connection between the mechanical and pneumatic parts.

The element that represents the metal arm is an object called bodyBox, which models a rigid rectangular solid. Its mass is determined by the data for density, length, thickness and height. An actuated revolute joint object is used to define the rotation axis of the body. A damper and a non-linear element with Coulomb friction that model all the friction forces present in the set-up are attached to the joint. The stops against which the arm impacts when it reaches the limits have also been modelled using bodyBox elements.

In order to model the impact between the arm and the stops, the Collision class was developed and two instances were included in the model, one for each point of contact. The force of the impact is calculated on the basis of a spring and shock absorber model. The objects enabling interaction of the pneumatic parts with the mechanical parts are connected to the anchoring points of each
muscle. These objects (*lineForce*) form a line of force between the two mechanical ports. The magnitude of this force depends on the actuator connected to the ports. In this case, the actuators are of course two objects of the type that model the pneumatic muscle as described in detail in Section 3.

The behaviour of the Festo MPYE-5-1/8HF servo-valve used was modelled on the basis of the *PneuLib* library’s propValveNoStates class, which describes a proportional valve, with second order spool dynamics and no lumped volumes at the ports. The pneumatic interfaces of the muscles are connected to two of the outputs on the proportional valve.

### 4.2. Experimental validation of the non-linear model

In order to validate the full model implemented in Dymola/Modelica, open-loop tests were first of all performed on the prototype. The available control element was the servo-valve that accepts an input signal in the range of 0–10 V. A control signal consisting of a ±0.9 V impulse train on the central value of +5 V was applied to the servo-valve with a period of 4 s. Tests were performed with different weights placed on the end of the arm. With the open-loop signal, the arm moved along its entire sweep from one end to the other, touching the stops lightly. This test verified and adjusted the dynamics of the model. Fig. 8 shows the experimental results and those obtained in the simulation with a weight of 3 kg on the end of the arm. The angle shown in the figures corresponds to the angle measured from the vertical position. The dynamic model was tuned up using the damper values and those of the component that models the Coulomb static friction.

As shown in the Fig. 8 both the model and the prototype involve a vibration that originates from displacements in both directions in the intermediate area of movement, changing the dynamics of the system and making it slower. These vibrations are due to the prototype’s own configuration: when the arm moves, two opposite phenomena take place in the performing muscle: (i) it tightens and diminishes its strength and (ii) the pressure increases, as does the strength. Both effects are also inversely reflected in the other muscle and thus there comes a moment when the strength generated by both muscles is evened out. As a result, the arm vibrates. This behaviour is also shown in the results of the model. This vibration, although much more damped, is also reflected in the results of other researchers. In the case of the prototype presented in this paper, the vibration is more accentuated due to the short lever and the high movement speed.

After the open-loop tests, closed-loop trials were performed and a basic PI controller was used. The parameters were tuned up with a position leap of 10° and a slope of 20°/s performed in the central part of the arm displacement area. In any case, the objective at this stage was not to find the optimum value of the parameters. The prototype does not behave linearly throughout its movement range and there are notable differences depending on the area in which the position leap is performed. For the same PI controller the vibration is more obvious the nearer the arm is to the bottom stop. Both the moment when the vibration appears and its intensity depend on the displacement area.

Fig. 9 shows the results obtained when the position step is performed in the upper part of the arm displacement area. As mentioned above, the vibration appears clearly in the transient response and is shown by the simulation result, which follows the experimental result very closely.

### 4.3. Linear models of the experimental set-up

One of the main objectives of the prototype is to emulate the movement of the forearm orthosis, analysing the adaptation of the pneumatic muscles in such applications. Assuming that the hand with the orthosis should be able to transport a determined weight, for purposes of controller design the nominal conditions were considered to be when the robotic arm has a load of 3 kg at the tip. Furthermore, the load may be reduced or increased by up
to 6 kg. The specifications therefore presume that the system must be robust for this load interval.

The non-linear model in Dymola/Modelica can be integrated as an $S$-function in a non-linear model in Simulink. On the basis of this model, reduced linear models for use in designing controllers were deduced. In the standard case, using only one servo-valve (Fig. 1), the system is considered as a SISO system, where the input is the input voltage to the servo-valve and the output is the angle position of the robotic arm.

The linear models were obtained by linearising the non-linear model in Simulink, after a jump in the voltage input of 0.5 V and a duration of 1 s, occurring in the middle displacement area. The models were simplified and reduced in Matlab. The linearizations were made considering different masses on the end of the arm. The nominal model is taken as having a weight of 3 kg at the tip of the arm.

The resulting transfer function of the nominal model is as follows:

$$G_{3N}(s) = \frac{0.25s^2 + 31s + 175}{0.075s^3 + 4.26s^2 + 9.36s + 0.003}$$ (7)

As extreme cases in the considered working range, a load of 6 kg is considered at the tip of the arm, with the following reduced linear model:

$$G_6(s) = \frac{20.6s + 63.5}{0.008s^3 + 0.09s^2 + s^2 + 9.26s}$$ (8)

and the case of no weight at the tip, where the linear model is:

$$G_0(s) = \frac{0.33s^2 + 61.74s + 142.5}{0.00003s^5 + 0.00153s^4 + 0.058s^3 + 1.96s^2 + 5.98s}$$ (9)

Fig. 10 shows these plants in the frequency domain. As can be seen, the models show significant resonance. When the load is increased, the resonance frequency is reduced, together with the gain. The differences between the linear models are considered as nominal model uncertainties. Furthermore, these linear models do not reflect the significant influence of friction, which is very different throughout the robotic arm’s range of displacement. These linear models were used to design the robust position controllers for the standard configuration with a servo-valve. For the control algorithm based on internal pressure loop design, which requires two servo-valves, analytical models were not used.

5. Position controllers

5.1. Classic PI and enhanced PID controllers

As a previous step, to serve as a reference for comparing the results obtained with different advanced controllers to be designed and implemented on the prototype, a classic PI position controller was developed. Also, on the basis of this PI but taking into account the results obtained in the different zones, it was completed with other internal linear and non-linear loops. Fig. 11 shows the structure of the most elaborate controller constructed from a position PID and with which the best results were obtained.

The position PID is complemented with a speed loop and an acceleration loop. A speed feedforward was also added to improve the dynamic response. A more internal non-linear loop partly compensated the effects of gravity. For the case of the basic PI controller the tuned gains were $K_p = 0.062$ and $K_i = 0.029$. For the case of the improved PID (Fig. 11), the tuned gains were $K_p = 8.5$, $K_i = 0.009$, $K_d = 0.8$, $K_v = 0.02$, $K_v = 0.005$, $K_n = 0.0025$ and $K_a = 0.0002$.

5.2. $H_\infty$ controller

The $H_\infty$ problem can be defined as that of finding a controller, $K$, such that the value of the norm $\infty$ of a characteristic vector of the system controlled is minimised below the unit:

$$\left\| \begin{bmatrix} W_1 \cdot S & W_2 \cdot R & W_3 \cdot T \end{bmatrix} \right\|_\infty \leq 1$$ (10)
The sensitivity functions establish the effect of the disturbance signals at the output (S-function), the noise on signal measurement (T) and the noise on the control (R) at the plant output. The nominal performance can be established using the weight function $W_1$ applied to the sensitivity function output $S = (1 + G \cdot K)^{-1}$, so that for low and medium frequencies it is verified that $||W_1 \cdot S||_\infty < 1$. To specify the robustness of the controller's stability, suitable transfer functions are used to describe the permissibility levels. Thus, the inverse of the complementary sensitivity function $T$, or closed-loop transfer function, represents a bound of the maximum multiplicative disturbances permitted for the stability of the closed-loop system. In the same way, the inverse of the control sensitivity $R = K \cdot S$ represents a bound to the permitted additive disturbances. Thus, the robustness of the stability can be specified by the appropriate
weight functions, and so \( |W_2 \cdot R|_\infty < 1 \) \( |W_3 \cdot T|_\infty < 1 \). Depending on the type of problem to be solved, the sensitivity vector may not need to contain all three of these components, and minimizations are habitual for vectors \([S, T]\) or \([S, R]\).

The design method focuses on the specification of the weight functions that set the characteristics of robustness and system performance, i.e. those which model the frequency response of the sensitivity functions.

The selected weight functions are:

\[
W_1(s) = \frac{s + 0.23}{s(2.4s + 0.001)}
\]

\[
W_3(s) = \frac{2.8s^2 + 27s + 76}{25s + 85}
\]

\[
W_2(s) = 1
\]

A pole at the origin was added to the weight \( W_1 \) to increase the integral action of the controller and reduce the steady-state error. The multiplicative uncertainties were estimated on the basis of the linear transfer functions \( G_0 \) and \( G_2 \). Therefore, the weight \( W_3 \) was chosen as a bound of the multiplicative uncertainties. Fig. 13 shows the multiplicative uncertainties and the weight functions \( W_1 \) and \( W_2 \).

Using the Matlab “Robust Control Toolbox” and minimising the mixed sensitivity vector (10), the following controller \( K(s) \) was obtained:

\[
K(s) = \frac{15.51s^7 + 782.7s^6 + 3.001e4s^5 + 0.21e6s^4 + 4.511e6s^3 + 6.01e6s^2 + 4.425e6s + 170.1}{s^8 + 124.3s^7 + 8.263s^6 + 3.363e5s^5 + 6.246e6s^4 + 2.041e7s^3 + 2.43e7s^2 + 1.574e7s + 0.006295}
\]

5.3. Sliding-mode controller

To develop the sliding-mode controller, work by Edward and Spurgeon [42] was taken into account. A tracking requirement was incorporated using a comprehensive action approach. From the initially considered system,

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

\[
y(t) = Cx(t)
\]

which is assumed to be square, where \( A \) and \( B \) are the matrices representing the nominal system and \( x(t) \) is the state vector, an integral action is introduced by means of an additional state:

\[
\dot{x}_t(t) = r(t) + y(t)
\]

where \( r(t) \) represents the set-point value and \( y(t) \) is the system output.

The sliding-mode control is based on taking the system to a surface where the closed-loop dynamics will be governed by the equations that are established, free from any unmodelled disturbances. In this way, bearing in mind the incorporation of a tracking requirement, the sliding surface is defined by:

\[
S = \{ x \in \mathbb{R}^{n+p} : SX = S; r \}; \quad [S; S_2]
\]

where \( S \) and \( S_2 \) are design parameters governing the movement dynamics. \( S_2 \) will be of dimension \( n \) and \( S_2 \) of dimension \( p \).

To establish the sliding surface \( S \), the system's uncertainties must be taken into account, estimated through the nominal models that are presented. Once \( S \) has been set, the control signal for designing a sliding-mode controller including an integral action approach and tracking requirement is:

\[
u = u_i(\dot{x}, r) + u_n
\]

where the linear component is:

\[
u_i(\dot{x}, r) = L\dot{x} + L_1r + L_2r
\]

\[
L = -SB^{-1}(SA - \phi S); \quad L_1 = -SB^{-1}(\psi S + S_2 M);
\]

\[
L_2 = SB^{-1}S_2
\]

and where \( S \), and \( \phi \) are design matrices.

The non-linear component of the control law (18) will be a function of the surface \( S \) multiplied by a gain \( K \) established in the design process.

\[
u_n = -K \cdot \text{sgn}(S\dot{x})
\]

To design the controller described above the state vector should be measured, which is not possible in the experimental set-up under study, and an observer was therefore designed for its estimation. In a linear system such as (15), the observer can be expressed by the formula:

\[
\dot{x}(t) = \bar{A}x(t) + Bu(t) - G(Cx(t)) - y(t) + FB
\]

where \( \bar{x} \) represents the state vector estimated. In this way, the observer takes the form of a system model, which is impelled by the misalignment between the plant output and the observer output. The linear gain \( G \) is calculated as follows:

\[
G = T_0^{-1} \begin{bmatrix} A_{12} \\ A_{22} - A_{12} \end{bmatrix}
\]

where \( T_0 \) is the matrix representing the change of coordinates between the given system and its canonical form and \( A_{12} \) is a diagonal matrix established in the design. \( v \) is the sign function of \( Cx(t) - y(t) \) multiplied by a design constant \( K \). The matrix \( F \) is calculated by means of \( F = BP \), where \( P \) is the solution to the Lyapunov equation:

\[
A_{12}^T P + PA_{12}^T + I = 0
\]
As a summary, the controller design and implementation process was as follows: on the basis of the nominal model $G_{NN}$, the first step was to convert the system given by a transfer function into its expression by means of the state space, obtaining the $A$, $B$ and $C$ matrices required for the design (see Appendix A). Once the matrices that defined the system were obtained, the next step was to design the observer to obtain the matrices $G$ and $F$. To do this, the value $A_{22}^T$ had to be established. This was done by trial and error, setting the desired output estimation error pole. After designing the observer, the sliding-mode controller was designed in accordance with the equations described. The desired $\lambda$ poles had to be set for the closed-loop system, which provides the system robustness. The rest of the design matrices (see Appendix A) were set experimentally to obtain good response performance. The design matrices used in the controller appear in Appendix A.

Despite the fact that this new solution initially doubles the variables that have to be controlled for each degree of freedom, it can be considered as a single-variable approach for each joint. Based on the symmetrical co-contraction of the opposing muscles, an asymmetrical variation is set for the pressure of each muscle. Thus, based on an initial pressure ($P_0$) the setting is increased for one of the muscles and reduced by the same amount ($\Delta P$) for the other.

$$P_{1\text{ref}} = P_0 - \Delta P; \quad P_{2\text{ref}} = P_0 - \Delta P$$

Accordingly, from the control point of view, the system is still SISO with the angular position of the joint ($\theta$) as the output and the pressure variation ($\Delta P$) as the input.

Special mention must be made of the relevance of the parameter $P_0$ in the rigidity or impedance of the joint. Indeed, it could be designed as a variable parameter whose value changes depending on certain environmental conditions.

5.4. Internal pressure loop based controller

One different approach when controlling an actuated joint using a pair of opposing muscles consists of independently controlling the pressure of each muscle [36]. Of course, this requires the incorporation of one valve for each muscle on the device, assuming the cost this involves.

$$P_{1\text{ref}} = P_0 - \Delta P; \quad P_{2\text{ref}} = P_0 - \Delta P$$

Accordingly, from the control point of view, the system is still SISO with the angular position of the joint ($\theta$) as the output and the pressure variation ($\Delta P$) as the input.

Fig. 14 shows the full control schematic based on the internal loops that control the pressure in each muscle, implemented by means of PI algorithms. As it has been already mentioned, the pressure set-point for each controller is set on the basis of an initial value ($P_0$), adding and subtracting the same quantity ($\Delta P$). The value of this increase/reduction is the output of the most external loop of the controller (the position loop). This loop has also been implemented by means of a PID algorithm. The gains of both pressure loops were adjusted to the values $K_p = 4$, $K_i = 4$, and the gains of the position loop to $K_p = 0.21$, $K_i = 1.2$, $K_d = 0.04$, being $P_0 = 3$ bar.

6. Experimental results

In a human arm orthosis (an end application considered for pneumatic muscles), the set-point position is generated by the user's intention of movement; it is not a path pre-set by a controller. On tuning the controllers, small jumps in position of $10^{-5}$ have therefore been used. The performance specifications are that the response to these jumps must be as quick as possible with very little overshoot and no vibration.
Initially, all the controllers were tuned and tested in simulation, using the non-linear model developed in Modelica and the resulting linear models. The controllers, discretized with a sampling time of 2 ms, were then included in the actual set-up. The controllers were tuned up experimentally in the first displacement zone with a load of 3 kg and evaluated in the overall displacement range with different weights. The values of the parameters obtained experimentally varied between 5% and 10% with respect to those obtained on simulation. In order to compare the performance levels of the various control algorithms Figs. 15–17 show the experimental responses to a ramp input of 10° and a slope of 20°/s applied in three different areas of the displacement range, the mass at the tip being the nominal of 3 kg.

Analysing the system response in the first displacement area (shown in Fig. 15), with the PI (PI) and enhanced PID (Enhanced PID) controllers, there is no overshoot; however, the settling time is quite high. Of the rest, the sliding-mode is the algorithm with the biggest overshoot and the controller \( H_\infty (H_{\infty}) \) takes a long time to eliminate the steady-state error. The algorithm based on the pressure loops (Press Loops) takes the arm to 25° before any other.

Fig. 16 corresponds to a jump in the intermediate zone and shows that the PI controller involves an overshoot and an excessive settling time, whereas the response of the enhanced PID follows the set-point until a small vibration appears. The controller \( H_\infty \) has an initial behaviour that is similar to the PI, even though it has no overshoot and the steady-state error is compensated. The response of the sliding-mode controller increases quickly and, despite a slight overshoot, it is capable of eliminating the steady-state error. It can be concluded that the response of the controller based on the pressure loops is clearly the best, since despite an overpass equal to the case of the sliding-mode, its settling time is lower.

In the lower zone (Fig. 17), besides increasing the overshoot and the slowness of the previous zones, the position controlled by the PI begins to oscillate. The performance levels of the enhanced PID

---

**Fig. 14.** Control diagram of the internal pressure loops based algorithm.

**Fig. 15.** Experimental results in the upper displacement zone.

**Fig. 16.** Experimental results in the intermediate displacement zone.

**Fig. 17.** Experimental results in the lower displacement zone.
also fall because its settling time is increased. The response of the algorithm $H_\infty$ also worsens and becomes much slower; it needs more than 20 s to reach the steady-state. The sliding-mode controller shows an overshoot somewhat greater and there is a certain amount of oscillation. Once again, the controller based on the pressure loops offers the best performance levels, obtaining a faster system, with smaller overshoot and without steady-state error. The control signals corresponding to the jump of Fig. 17 are plotted in Fig. 18. The input range of the valves is $[-5, 5]$ V, and in the case of the internal pressure loops based controller only the control

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**Fig. 16.** Experimental results in the middle displacement zone.

**Fig. 17.** Experimental results in the lower displacement zone.
signal of one of the valves is shown. The sliding-mode controller’s control signal presents the typical chattering effect due to the nonlinear part of the control law.

Figs. 19 and 21 show the angular position of the arm with different weights on the tip in response to a ramp input of 40° amplitude and 2 s duration for the two controllers with the best dynamic performance.
performance levels: the sliding-mode controller and the controller based on internal pressure loops. Figs. 20 and 22 show the control signals that provide the angular displacements plotted in the previously mentioned figures. In the case of the controller based on pressure loops (Fig. 22) only the signal for one of the valves is shown. Despite the fact that the transient response of the system

![Fig. 20. Control signals for the sliding-mode controller with different tip masses.](image1)

![Fig. 21. Experimental results with different tip masses for internal pressure loops based controller.](image2)
controlled by the sliding-mode algorithm (Fig. 19) is similar for all the weights, there are small differences at the overshoot level. It is sufficient to look at Fig. 21 to see that the controller based on the pressure loops is so robust with regard to the load variation that the figure shows hardly any differences between the results of the different weights.

7. Conclusions

Pneumatic artificial muscles, or McKibben muscles, are of great interest for the development of robotic applications that feature low impedance in terms of their interaction with the environment, as may be the case with orthoses or certain wearable robots. In this paper, the modelling of a pneumatic muscle in Dymola/Modelica is shown. The model has been implemented ensuring connectivity with the other Modelica libraries, and it can therefore be used as a basis for constructing a model of any device powered by this type of actuators. Furthermore, a new polynomial term depending on the muscle contraction has been added to the well-known model of the static force exerted by the muscle. The component obtained has been applied to the modelling of a 1-DoF mechatronic device actuated by an opposing pair of pneumatic muscles. The non-linear model has been validated by open-loop and closed-loop experimental trials, which underline the fact that it is very difficult to obtain a unique non-linear model that is valid for all displacement zones.

An analysis of the position control of the aforementioned experimental set-up has also been made. The non-linear nature of the pneumatic muscles and of the actual set-up built means that it is very difficult to control, since the system response varies across the movement range. Furthermore, the possibility of positioning additional weight on the tip of the arm forces us to consider the controller robustness for a wide range of weights.

The paper presents various control solutions and compares the results obtained. Four of the controllers implemented (PI, enhanced PID, $H_\infty$, and sliding-mode) work with one single servo-valve. The results obtained with the classic PI and enhanced PID controllers vary greatly from one zone to another in the displacement range, as well as with the variation of the mass at the tip. The $H_\infty$ controller was designed to achieve a certain level of robustness with regard to the weight of the load. However, its response is not the same across the movement range. In order to achieve a level of control capable of working correctly across the range the sliding-mode technique was implemented. The results presented illustrate this decision. Furthermore, the change of the load does not seem to affect the response to any great extent. This does not mean that control algorithms such as the enhanced PID or the $H_\infty$ are not valid, but they would have to be designed or tuned in different operating zones and a gain-scheduling strategy would have to be applied.

Finally, despite the change in the pneumatic circuit, a control algorithm was designed and implemented based on the independent control of the pressure of each muscle. The results obtained are the best of the comparison with regard to performance levels and for compensating the non-linearity of the prototype. In addition, the study of the robustness in comparison with the load also offers very good results.

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Appendix A

The design values used in Eqs. (15), (17), (19), (20) and (22) to design the sliding-mode controller are as follows:
\[
A = \begin{bmatrix}
-5.716 & -125.6904 & -0.0044 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}; \quad B = \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]
\[
C = 2.9 \quad 415.3 \quad 2350
\]
\[
S = \begin{bmatrix}
0 & -1 & -9.28 & -33.64
\end{bmatrix}
\]
\[
L = -23.5624 \quad 93.5496 \quad -672.7956
\]
\[
L_r = 0.2863; \quad L = 0.0143
\]
\[
S_r = -0.0143; \quad P = 0.025
\]
\[
\phi = -0.2; \quad K = 0.02
\]
\[
G = \begin{bmatrix}
-1.19609 \\
0.3491 \\
0
\end{bmatrix}
\]
\[
F = 14.3243
\]
\[
K_{obs} = -2
\]