

RESEARCH ARTICLE

Constraint Effect on Minimum Tracking Error Volatility: A Global Asset Approach

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Abstract

This paper studies the out of sample tracking error of minimum variance portfolios of global assets, equities and bonds. The methodology follows the one presented by Jagannathan and Ma (2003) regarding the risk reduction in US stock portfolios using weight constraints. The sample covariance matrix is used. Optimal minimum tracking error and minimum variance portfolios are derived using upper/lower and no restrictions. We show results assuming different revision frequencies and transaction costs assumed. The data used are monthly indices of stocks, bonds, gold oil and spreads from 1996 until 2013. Differences in relative risk, due to restrictions or rebalancing frequency, vary from 2 bps to 18 bps.

Keywords: *Portfolio Optimization, Minimum Tracking Error, Relative Risk Reduction, Constraints, Global Assets*

1. Introduction

Inspired by the report of Poullaouec (2008), our research is focused on the Minimum Tracking Error Volatility (TE) performance of global assets.

As per Chan et al (1999), the minimization of Tracking Error Volatility may be improved in forecasts if structure is imposed to the covariance matrix of stock return. However, the authors result suggest that, in terms of out of sample TE performance, the sample covariance matrix tracking error volatility vis-à-vis the benchmark delivers similar results like other models. The problem with TE portfolios, however, as per Roll (1992), is that as long as the benchmark towards which the minimization strategy is constructed, is not mean-variance efficient, TE portfolio is not efficient. Benchmarks, as widely proven in the finance literature, are passive indices that do not properly represent risk-return performance of the whole universe of investment opportunities, due to lack of information.

Based on Clarke et al (2006), however, the tracking error of factor constrained covariance of TE models result in lower relative volatility towards benchmark as compared with unconstrained, or sensitivity constraint models.

Pollayec (2008) finds that relative risk reduction of TE is, on average, 55 bps higher than style allocation strategies. Jorion (2003) shows that adding constraints to the portfolio can result in superior performance of actively managed portfolios.

Jagannathan and Ma (2003) study the effects of weight constraints on tracking error performance. As per the authors, the monthly sample covariance matrix estimator is among best performance regarding absolute risk reduction. The reasoning behind, according to the authors, is the fact that weight constraints on factor models are not supposed to have any positive effect on absolute/relative risk reduction vis-à-vis benchmarks. Zheng and Liang (2014) introduce a robustly optimized minimum TE portfolio, which,

considering the transaction costs, can be more efficient in portfolio selection

To the best of our knowledge, this is the first research works that studies the effect of considering global asset classes, weight constraints in the optimization input and transaction costs, revision frequencies in the out of sample empirical result of minimum TE volatility performance. Global assets are represented by regional indices, with the main assumption of dividing the world of investment opportunities in four parts, US, Europe, Asia Pacific and Developing Nations. Minimum Tracking Error variance portfolios are estimated using Chan et al (1999) sample covariance matrix of monthly indices return. The unconstrained, short-sale constrained, and upper bound constrained TEs are derived for the global asset portfolio, global bond portfolio and global equity portfolio, as per the methodology of Jagannathan and Ma (2003).

The relative and absolute risk reduction of unconstrained, short-sale constrained, and upper bound constrained minimum variance portfolios is compared between GMVPs and with the equally weighted benchmark of global and distinct asset classes.

Results are presented for one year and three year revision frequency for three global portfolios, where the covariance matrix is estimated using monthly returns. Transaction costs are included in the model, applying the methodology suggested by DeMiguel et al (2009a).

This paper is organized as follows: in the next section will be presented the research methodology of portfolio construction, optimization. In section 3 we present the data and a brief description of their statistical characteristics. Section four analyzes the empirical findings of the minimum variance portfolio of global assets, global equity and global bonds. Conclusions and recommendations of this study are considered in section 5.

2. Research Methodology

The methodology of this paper follows closely the one used from Chen et al (1999) and Jagannathan and Ma (2003). The returns are presented from a European Investor's perspective (EURO). This section describes the variance-covariance matrix estimation approach and optimization. Then, it is briefly explained the inclusion of transaction costs.

2.1. Portfolio optimization

Frequently mentioned in the finance literature, the estimation error of the variance covariance-matrix of returns is much less significant in optimization than expected return. The problem, however, still exists, and, as the work of Chan et al. (1999) points out, there is no simple answer for the model to be used in forecasting the variance-covariance matrix of returns.

There are three widely used models for dealing with estimation error of the covariance matrix of returns: the factor models, shrinkage estimators and portfolio weight constrains.

In line with Jagannathan and Ma (2003), Chan et al. (1999) and DeMiguel et al. (2009a), in this paper it is used the sample variance-covariance matrix estimation technique.

We follow the methodology of Chan et al (1999) in constructing minimum tracking error variance portfolio, who imply that the construction TE portfolios is the as construction global minimum variance portfolios (GMVPs), if returns are expressed in excess of benchmarks. We compute the return of every index at time t in excess to the benchmark:

$$R_{it} = r_{it} - r_{bt} \quad (1)$$

Where, R_{it} represent the excess return of index i on the benchmark at time t, r_{it} represent the return of index i at time t, and r_{bt} represents the return of the benchmark at time t.

We employ the minimum variance portfolio in line with Markowitz (1952) portfolio selection framework. Monthly returns are used. Following the work of Chan et al. (1999) and Jagannathan et al. (2003), the estimation window for sample covariance matrix is 60 months. The rebalancing frequencies applied are 12 months and 36 months.

The sample covariance matrix S is given by the formula:

$$S = \frac{1}{T-1} \sum_{t=1}^T (R_t - \bar{R})(R_t - \bar{R})' \quad (2)$$

where T is the sample size, R_t is the $N * 1$ vector of stock return at time t, and \bar{R} represents the sample mean of these returns. Given the estimated sample covariance matrix, the Minimum Tracking Error Volatility portfolio (TE) is the solution of the minimization exercise given by the formula:

$$\min_{W \in \mathbb{R}} W' \Sigma W \quad (3)$$

$$\text{s.t. } \sum_{i=1}^n W_i, t = 1 \quad (4)$$

where W_i, t denotes the weight of asset i at period t. The constraint in equation 4 implies that the total portfolio weight, including short positions, should sum up to 1. This is the classical unconstrained TE portfolio.

The work of Jagannathan and Ma (2003) introduces the study of the shrinkage like effect of the additional no short sale constrain and upper bound constraint, presented below in equation 5 and 6, of minimum tracking error volatility

since both of them have delivered accordingly promising out of sample performance results.

$$W_i, t \geq 0, \text{ for } i = 1 \dots n \quad (5)$$

$$W_i, t \leq W^{\max}, \text{ for } i = 1 \dots n \quad (6)$$

The three portfolios, namely unconstrained, no short sale constrained, no short sale and upper bound constrained for global assets, global equities, and global bond TE are studied in this paper. Moreover, the same process described in formulas (1)-(6) is applied to returns not in excess to benchmarks for portfolios constituents, for estimating global minimum variance portfolios (GMVPs).

2.2. Transaction costs

The transaction costs in the time series of portfolio returns are calculated in accordance with the methodology used in the work of DeMiguel et al. (2009a) for the three minimum variance portfolios and the benchmarks. Let R_p denote the portfolio return before the revision, which is given by the formula:

$$R_p = \sum_{i=1}^n R_{i, t+1} * W_{i, t} \quad (7)$$

The rebalance of the portfolio at time t+1 will upsurge a trade with a magnitude of $|W_{i, t+1} - W_{i, t}|$. Let c denote the proportional transaction cost¹. After each revision period the overall transaction cost will be:

$$c * \sum_{i=1}^n |W_{i, t+1} - W_{i, t}| \quad (8)$$

The Wealth net of transaction cost at time t+1 can be written as:

$$Wealth_{t+1} = Wealth_t * (1 + R_p) * [1 - c * \sum_{i=1}^n |W_{i, t+1} - W_{i, t}|] \quad (9)$$

The Return net of transaction cost is given:

$$R_n = \frac{Wealth_{t+1}}{Wealth_t} - 1 \quad (10)$$

The same approach is repeated in each revision period for all portfolios and benchmarks. The revision frequency for the GMVP will be one year and three years, while for the benchmarks will be five years.

2.3 Tracking Error measure

Following the explanation of tracking Error from Reilly and Brown book, Chapter 16 (2012), we estimate the annualized tracking error of TE portfolios as follows.

Let r_{pt} be the return of the TE portfolio resulted from the optimization input at time t, equal to:

$$r_{pt} = \sum_{i=1}^N w_{it} r_{it} \quad (11)$$

Where w_i is weight of asset i at time t and r_{it} is return of asset i at time t. Then, excess return of portfolio is calculated at time t is calculated:

$$R_{pt} = r_{pt} - r_{bt} \quad (12)$$

The variance of R_{pt} for a sample size T is estimated as:

$$\sigma_{R_p}^2 = \frac{\sum_{t=1}^T (R_{pt} - \bar{R})^2}{(T-1)} \quad (13)$$

Where R is the mean of R_{pt} .
Finally, the annualized tracking error can be estimated as:

$$TE = \sigma_{R_p} * \sqrt{T} \quad (14)$$

T is the sample size in months.

3. Data

The data used in this research are monthly returns of regional bond, equity indices and global oil, gold indices

chosen from DATASTREAM database software package available at ALBA Graduate Business School. The period under consideration is December 1996 – May 2013.

Table 1 lists the of global asset portfolio indices. For equity and bonds the world is divided in four regions, US, Europe, Asia Pacific, Developing Countries. The bond markets for Europe and US, since more indices with longer time series are available, include corporate and government fixed income indices separately.

Table 1: Global Asset Portfolio Constituents

| | |
|---------------------------------|----------------------------|
| CGBI WGBI EU ALL MATS | Europe Gov. Bond Index |
| REX GENERAL BOND | Europe Corp. Bond Index |
| CGBI USBIG OVERALL ASIA PACIFIC | Asia Pacific Bond Index |
| BARCLAYS EM WORLD ALL SERIES* | Emerging World Bond Index |
| ML CORP MASTER | US Corp. Bond Index |
| CGBI WGBI US ALL MATS | US Gov. Bond Index |
| MSCI EUROPE | European Stock Index |
| MSCI EM :I | Emerging World Stock Index |
| MSCI AC ASIA PACIFIC | Asia Pacific Stock Index |
| MSCI USA | US Stock Index |
| MLCX Gold | Gold Index |
| S P GSCI Crude Oil | Oil Index |

Referring to the global bond portfolios, addition to the previous indices, in portfolios are added two spreads supposed to add in the model risk premium of global long term government bonds in excess to short term government bonds and risk premium of global corporate bonds in excess to long term government bonds. This choice is made to observe how spread strategies reflecting differences in risk affect the optimization and performance of TE, GMVPs and benchmark.

The global oil index and global gold index are included in the equity portfolio due to the similarity of their risk-return profile with the equity indices. Two spread strategies, which proxy the global risk premium of small capitalized stocks in excess to large capitalized and global risk premium of value stocks in excess to growth stocks, are included in global equity portfolio.

The benchmark in all portfolio is the equally weighted portfolio of TE and GMVP constituents. The second benchmark is rebalanced every five years. Returns are expressed in EUROS.

4. Empirical Analysis

In this section we present the risk-return characteristics of Minimum Tracking Error Volatility (TE) and Global Minimum Variance Portfolios of global assets, global bonds, and global stocks. The first subsection focuses on the effect on weight constraints on the relative and total risk reduction differences of TE and GMVP portfolios rebalanced yearly. The second subsection analyses the effect of less frequent revision policy on the risk reduction of unconstrained/constrained global asset, equity and fixed income portfolios.

4.1 Risk Reduction

By comparing tables 2 and 3, global asset TE portfolio result in a total risk, as measured by the standard deviations of returns, more than three times higher than total risk of

GMVPs of global assets, when no constraint and short-sale constraint is applied. If upper/lower bound constraints are applied, total risk and portfolio return of global assets almost double when portfolio optimization is applied to minimize in sample relative deviations from the equally weighted benchmark. It seems, that, unless severe restrictions are imposed, TE global asset portfolios do not represent any benefit in terms of risk-return performance vis-à-vis GMVP of global assets, independently from the transaction costs assumed.

Although Global bond TE/GMVP total risk differences decrease as more severe constraints are imposed, returns decrease by more in relative terms. The most interesting TE portfolio is the unrestricted one in the global equity case. Return advantage (more than three times) seem to overcome the risk increase (2.6 times) as compared with unconstrained GMVP. Restrictions, however, seem not to influence the out of sample tracking error of all minimum TE volatility portfolios, with differences arising due to restrictions that vary from 2 basis points to 7 basis points, when portfolios are rebalanced yearly. The global bond TE portfolio shows the lowest out of sample annualized tracking error. It is, on average, almost 9.5 (6) lower than global equity (global asset) portfolios. Such phenomenon is also shown in the total risk reduction features of bond portfolios in table 2, although the worst global portfolio approximately experiences 4.5 higher risk than the global bond portfolios. The main result of this section is that, as restrictions generally increase the total risk of optimized portfolios, they seem to have no effect on relative risk reduction of global portfolios of different assets and specialized markets, such as bonds and stocks, independently from the assumed transaction costs.

Table 2: Summary Statistics of Annualized GMVP and Benchmarks (%) –
1 Year Revision Frequency

| | 50 bps transaction cost | | | | 25 bps transaction cost | | | |
|-----------------------|-------------------------|-----------|-------|---------|-------------------------|-----------|-------|---------|
| | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. |
| Global Assets | | | | | | | | |
| Unconstrained GMVP | 5.15 | 2.66 | 35.14 | -21.44 | 5.14 | 2.43 | 26.63 | -21.44 |
| Constrained GMVP | 5.25 | 2.55 | 29.10 | -21.05 | 5.25 | 2.53 | 25.91 | -21.05 |
| Max. Constrained GMVP | 5.48 | 4.97 | 50.51 | -39.72 | 5.48 | 4.96 | 50.51 | -39.72 |
| Benchmark | 5.59 | 8.34 | 73.75 | -92.73 | 5.59 | 8.34 | 73.75 | -92.73 |
| Global Bond | | | | | | | | |
| Unconstrained GMVP | 1.69 | 2.14 | 29.70 | -36.81 | 1.69 | 1.86 | 21.66 | -28.92 |
| Constrained GMVP | 2.91 | 2.34 | 19.96 | -21.75 | 2.90 | 2.30 | 18.66 | -18.79 |
| Max. Constrained GMVP | 3.55 | 2.73 | 25.26 | -18.00 | 3.55 | 2.71 | 25.26 | -18.00 |
| Benchmark | 3.49 | 5.17 | 47.08 | -33.63 | 3.49 | 5.17 | 47.08 | -33.63 |
| Global Equity | | | | | | | | |
| Unconstrained GMVP | 3.35 | 4.71 | 50.88 | -58.01 | 3.35 | 4.69 | 50.88 | -58.01 |
| Constrained GMVP | 3.82 | 4.91 | 47.93 | -51.29 | 3.82 | 4.91 | 47.93 | -51.29 |
| Max. Constrained GMVP | 5.81 | 7.22 | 57.06 | -88.67 | 5.81 | 7.22 | 57.06 | -88.67 |
| Benchmark | 5.97 | 10.51 | 82.95 | -145.89 | 5.97 | 10.51 | 82.95 | -145.89 |

Table 3: Summary Statistics of Annualized Minimum Tracking Error Volatility and Benchmarks(%) –
1 Year Revision Frequency

| | 50 bps transaction cost | | | | | 25 bps transaction cost | | | | |
|-----------------------|-------------------------|-----------|--------|---------|-------|-------------------------|-----------|--------|---------|-------|
| | Mean | Std. Dev. | Max. | Min. | TE | Mean | Std. Dev. | Max. | Min. | TE |
| Global Assets | | | | | | | | | | |
| Unconstrained GMVP | 8.96 | 9.80 | 88.11 | -136.18 | 13.29 | 8.96 | 9.81 | 88.11 | -136.18 | 13.28 |
| Constrained GMVP | 9.06 | 9.80 | 88.11 | -134.29 | 13.22 | 9.06 | 9.80 | 88.11 | -134.29 | 13.22 |
| Max. Constrained GMVP | 9.06 | 9.80 | 88.11 | -134.29 | 13.22 | 9.06 | 9.80 | 88.11 | -134.29 | 13.22 |
| Benchmark | 5.59 | 8.34 | 73.75 | -92.73 | 0.00 | 5.59 | 8.34 | 73.75 | -92.73 | 0.00 |
| Global Bond | | | | | | | | | | |
| Unconstrained GMVP | 4.60 | 5.03 | 58.88 | -38.27 | 7.19 | 4.60 | 5.03 | 58.88 | -38.27 | 7.19 |
| Constrained GMVP | 4.55 | 5.01 | 58.88 | -37.50 | 7.17 | 4.55 | 5.01 | 58.88 | -37.50 | 7.16 |
| Max. Constrained GMVP | 4.54 | 5.00 | 58.88 | -38.10 | 7.16 | 4.54 | 5.00 | 58.88 | -38.10 | 7.16 |
| Benchmark | 3.49 | 5.17 | 47.08 | -33.63 | 0.00 | 3.49 | 5.17 | 47.08 | -33.63 | 0.00 |
| Global Equity | | | | | | | | | | |
| Unconstrained GMVP | 10.32 | 12.05 | 102.74 | -164.41 | 16.62 | 10.32 | 12.05 | 102.74 | -164.41 | 16.63 |
| Constrained GMVP | 10.32 | 12.05 | 102.74 | -164.41 | 16.62 | 10.32 | 12.05 | 102.74 | -164.41 | 16.63 |
| Max. Constrained GMVP | 10.28 | 12.08 | 102.74 | -164.41 | 16.64 | 10.28 | 12.08 | 102.74 | -164.41 | 16.65 |
| Benchmark | 5.97 | 10.51 | 82.95 | -145.89 | 0.00 | 5.97 | 10.51 | 82.95 | -145.89 | 0.00 |

4.2 Special Case: Three Years Rebalancing Frequency

Based on tables 4 and 5, risk reduction/return differences between TE and GMVPs seem not to differ with revision frequencies, with the speed of decrease in difference of deviation of return higher than the speed of decrease of out of sample return for the global asset cases. The opposite, as in the case of one year rebalancing frequency, is observed for the specialized markets of equity and fixed income, according to this study.

No consistent improve/increase of tracking error, due to less frequent rebalancing policy, can be identified when

comparing results from table 4 and 5. Changes in relative risk, due to three year revision frequency, vary from improve of 5 bps in the global asset portfolio, to a deterioration of 18bps for the global bond portfolio. Less frequent rebalancing do not change the conclusions of the previous section, as global minimum tracking error portfolios revised every three years are not significantly influenced from weight restrictions in terms of out of sample relative risk reduction

Table 4: Summary Statistics of Annualized GMVP and Benchmarks (%) -
3 Year Revision Frequency

| | 50 bps transaction cost | | | | 25 bps transaction cost | | | |
|-----------------------|-------------------------|-----------|-------|---------|-------------------------|-----------|-------|---------|
| | Mean | Std. Dev. | Max. | Min. | Mean | Std. Dev. | Max. | Min. |
| Global Assets | | | | | | | | |
| Unconstrained GMVP | 5.01 | 2.59 | 31.61 | -28.39 | 5.00 | 2.36 | 26.63 | -23.49 |
| Constrained GMVP | 5.07 | 2.51 | 29.10 | -22.17 | 5.07 | 2.49 | 25.91 | -22.17 |
| Max. Constrained GMVP | 5.82 | 4.92 | 50.51 | -40.14 | 5.82 | 4.91 | 50.51 | -40.14 |
| Benchmark | 5.59 | 8.34 | 73.75 | -92.73 | 5.59 | 8.34 | 73.75 | -92.73 |
| Global Bond | | | | | | | | |
| Unconstrained GMVP | 1.42 | 2.11 | 27.01 | -36.81 | 1.42 | 1.83 | 18.75 | -28.92 |
| Constrained GMVP | 2.66 | 2.28 | 18.22 | -21.75 | 2.66 | 2.23 | 18.22 | -18.79 |
| Max. Constrained GMVP | 3.46 | 2.75 | 25.26 | -18.00 | 3.46 | 2.73 | 25.26 | -18.00 |
| Benchmark | 3.49 | 5.17 | 47.08 | -33.63 | 3.49 | 5.17 | 47.08 | -33.63 |
| Global Equity | | | | | | | | |
| Unconstrained GMVP | 3.38 | 5.06 | 49.79 | -78.33 | 3.38 | 5.06 | 49.79 | -78.33 |
| Constrained GMVP | 3.98 | 5.08 | 49.53 | -77.67 | 3.98 | 5.08 | 49.53 | -77.67 |
| Max. Constrained GMVP | 6.36 | 7.49 | 70.84 | -115.58 | 6.36 | 7.49 | 70.84 | -115.58 |
| Benchmark | 5.97 | 10.51 | 82.95 | -145.89 | 5.97 | 10.51 | 82.95 | -145.89 |

Table 5: Summary Statistics of Annualized the Minimum Tracking Error and Benchmarks (%) - 3 Year Revision Frequency

| | 50 bps transaction cost | | | | | 25 bps transaction cost | | | | |
|-----------------------|-------------------------|-----------|--------|---------|-------|-------------------------|-----------|--------|---------|-------|
| | Mean | Std. Dev. | Max. | Min. | TE | Mean | Std. Dev. | Max. | Min. | TE |
| Global Assets | | | | | | | | | | |
| Unconstrained GMVP | 8.80 | 9.85 | 88.11 | -136.18 | 13.25 | 8.80 | 9.81 | 88.11 | -136.18 | 13.23 |
| Constrained GMVP | 8.89 | 9.79 | 88.11 | -134.29 | 13.17 | 8.89 | 9.77 | 88.11 | -134.29 | 13.17 |
| Max. Constrained GMVP | 8.89 | 9.78 | 88.11 | -134.29 | 13.17 | 8.89 | 9.77 | 88.11 | -134.29 | 13.17 |
| Benchmark | 5.59 | 8.34 | 73.75 | -92.73 | 0.00 | 5.59 | 8.34 | 73.75 | -92.73 | 0.00 |
| Global Bond | | | | | | | | | | |
| Unconstrained GMVP | 4.36 | 5.22 | 58.88 | -38.40 | 7.37 | 4.36 | 5.18 | 58.88 | -38.40 | 7.33 |
| Constrained GMVP | 4.40 | 5.19 | 58.88 | -38.32 | 7.34 | 4.40 | 5.16 | 58.88 | -38.32 | 7.31 |
| Max. Constrained GMVP | 4.34 | 5.17 | 58.88 | -39.37 | 7.32 | 4.33 | 5.15 | 58.88 | -39.37 | 7.29 |
| Benchmark | 3.49 | 5.17 | 47.08 | -33.63 | 0.00 | 3.49 | 5.17 | 47.08 | -33.63 | 0.00 |
| Global Equity | | | | | | | | | | |
| Unconstrained GMVP | 9.91 | 12.26 | 105.50 | -164.41 | 16.70 | 9.91 | 12.27 | 105.50 | -164.41 | 16.71 |
| Constrained GMVP | 9.91 | 12.26 | 105.50 | -164.41 | 16.70 | 9.91 | 12.27 | 105.50 | -164.41 | 16.71 |
| Max. Constrained GMVP | 9.79 | 12.26 | 105.77 | -164.41 | 16.71 | 9.79 | 12.26 | 105.77 | -164.41 | 16.72 |
| Benchmark | 5.97 | 10.51 | 82.95 | -145.89 | 0.00 | 5.97 | 10.51 | 82.95 | -145.89 | 0.00 |

5. Conclusions

This paper studies the relative risk reduction of minimum tracking error portfolios for global assets, global stocks and global bonds. The world is divided in four regions: US, Europe, Asia – Pacific and Developing Countries. International indices of stocks, corporate/government bonds, spreads, oil and gold monthly returns are used as optimization inputs. TEs and GMVPs are estimated considering the restrictions introduced by Jagannathan and Ma (2003).

Weight constraints have no effect on out of sample tracking error reduction for the three global portfolios considered in this paper. The result is robust, considering different scenarios of transaction costs and revision frequency assumed. If severe constraints are included in the

optimization process, total risk and portfolio return of global assets experience similar growths if minimum TE strategy is applied.

One of the extensions of this paper may be the relative risk reduction effect from the application of more frequent data, such as daily return or weekly return, as input. Moreover, the comparison in performance through different performance metrics that consider both realized risk and return of TE strategy may attract the interest of academics and practitioners in the future.

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ⁱ In this research transaction costs will be assumed 50 basis point (bps) and 25 bps.