Electron-electron bound state in light of magnetic dipole-dipole interaction

2020

Arayik Emilievich Danghyan

E-mail: araik_d@hotmail.com

Introduction

There are many publications out there dealing with the problem of magnetic interaction between elementary particles with intrinsic dipole moments. Basically, the magnetic interaction becomes significant at sufficiently small distances; therefore, the problem is complicated by the need to take into account relativistic effects. The derived equations with the composite potential of the Coulomb and magnetic dipole-dipole interactions generally do not have a clear and simple analytical solution. In this paper, we propose an approach to study a particular case of electron-electron interaction by numerically solving the M2 equation [7] [9].

Keywords: electron-electron pair, magnetic dipole, quantum mechanics, relativistic equation.

Composite electromagnetic potential

The potential energy of magnetic dipole-dipole interaction between two electrons can determined using formula (1). $W = \frac{\mu_0}{4\pi}\left(\frac{\vec{\mu}_{e1}\vec{\mu}_{e2}r^2 - 3\left(\vec{r}\vec{\mu}_{e1}\right)\left(\vec{r}\vec{\mu}_{e2}\right)}{r^5}\right)$ (1) where $\mu_{e1} = \mu_{e2} = \mu_B$ (2) represents an intrinsic magnetic dipole moment, which, as we know, is equal to the Bohr magneton $\mu_B = \frac{e\hbar}{2m}$ (3), r – the distance between electrons, μ_0 – the magnetic constant.

Let us rewrite formula (1), taking into account scalar products and formula (2).

$W = \frac{\mu_0}{4\pi}\mu_B^2\left(\frac{\cos\theta_{12} - 3\cos\theta_1\cos\theta_2}{r^3}\right)$(4) where θ_{12} represents the angle between vectors $\vec{\mu}_{e1}$ and $\vec{\mu}_{e2}$, θ_1 and θ_2 – the angles between the magnetic moment vectors and the line joining particles.

It is clear that the potential energy (W) depends on the mutual arrangement of dipoles.

The following mutual arrangements of dipoles [1] are of interest to us (Fig. 1)

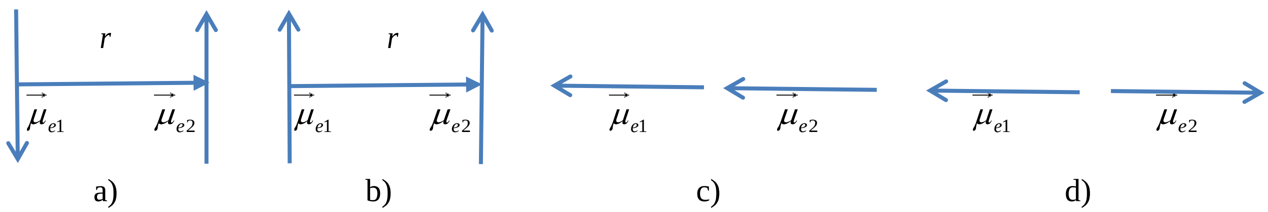

Fig. 1 Mutual arrangements of the intrinsic magnetic moments of electrons.

Cases a) and c) represent electron attraction; hence; the energy is negative. Cases b) and d), on the other hand, represent electron repulsion; therefore, the energy is positive.

It is clear that electron pairing requires attraction. Therefore, we can determine the energy (W) for cases a) and c) using formula (4), in which case we obtain:

a) $\theta_{12} = 180^0$, $\cos\theta_{12} = -1$, $\theta_1 = \theta_2 = 90^0$ $\cos\theta_1 = \cos\theta_2 = 0$, $W_a = -\dfrac{\mu_0\mu_B^2}{4\pi r^3}$

c) $\theta_{12} = 0^0$, $\cos\theta_{12} = 1$, $\theta_1 = \theta_2 = 0^0$, $\cos\theta_1 = \cos\theta_2 = 1$, $W_c = -\dfrac{2\mu_0\mu_B^2}{4\pi r^3}$

Therefore, the attractive potential energy for case c) is twice the value of the energy for case a). Note that case a) corresponds to oppositely directed electron spins. Therefore, the total spin is equal to 0. The electron spin direction coincides for case c); therefore, the total spin is equal to 1.

The Coulomb potential energy of interaction between two electrons is represented as follows: $W_{Coulomb} = \dfrac{e^2}{4\pi\varepsilon_0 r}$ The total potential energy of electromagnetic interaction for case c) can be calculated as follows:

$$U = W_c + W_{Coulomb} = -\frac{2\mu_0\mu_B^2}{4\pi r^3} + \frac{e^2}{4\pi\varepsilon_0 r} \quad (5)$$

Let us rewrite formula (5) to make it more convenient. To do this, we will use the well-known relation $\varepsilon_0\mu_0 = \dfrac{1}{c^2}$, which can be rewritten as follows: $\mu_0 = \dfrac{1}{\varepsilon_0 c^2}$ (6). Let us substitute values μ_0 (Formula 6) and μ_B (Formula 3) into the potential energy formula (5). We obtain: $U(r) = -\dfrac{2e^2\hbar^2}{4\pi\varepsilon_0 c^2 r^3 4m^2} + \dfrac{e^2}{4\pi\varepsilon_0 r}$ (7). Let us rewrite formula (7) using Hartree atomic units.

In the Hartree atomic unit system, the following values of physical constants are accepted: $a_0 = 1, m = 1, e = 1, \hbar = 1, c = 137.03599971, 4\pi\varepsilon_0 = 1$ Finally, we obtain the following equation for the composite potential energy of electromagnetic interaction: $U(r) = -\frac{1}{2c^2 r^3} + \frac{1}{r}$ (8). Let us graph the resulting equation (Fig. 2)

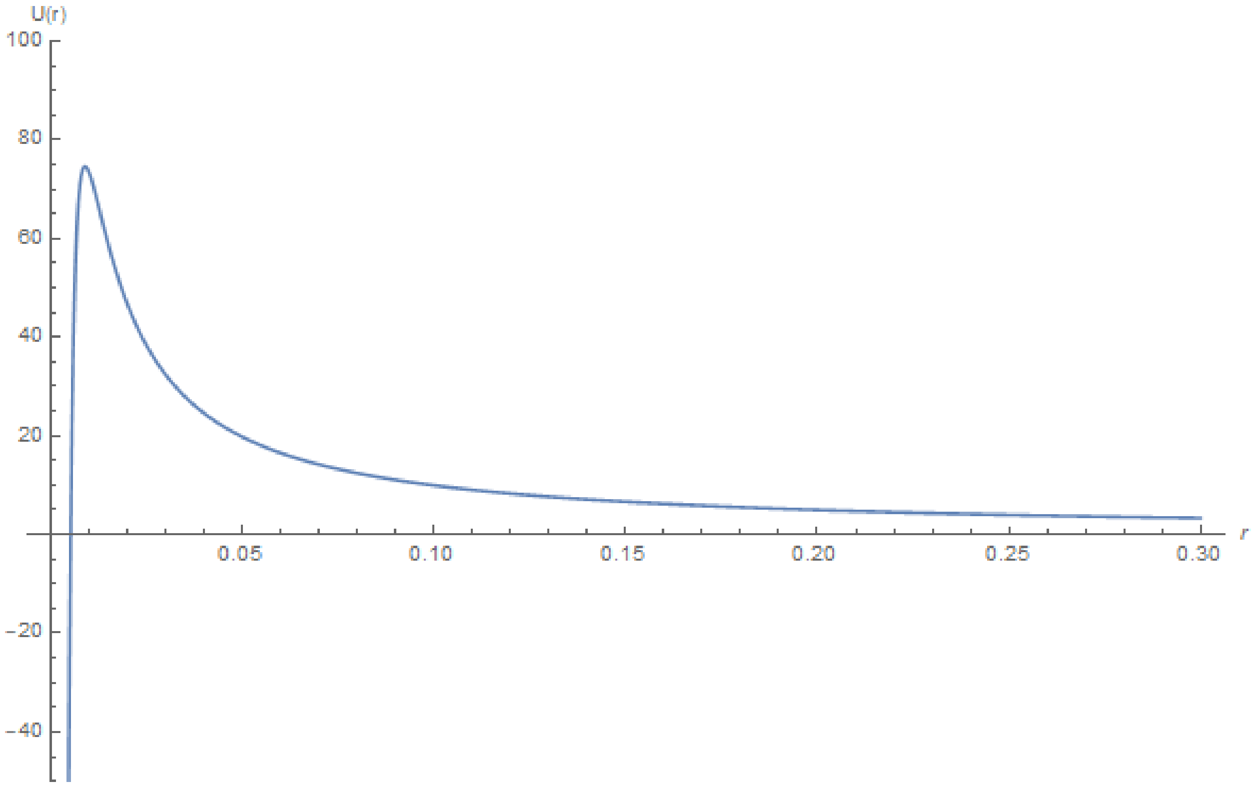

Fig. 2 Graph of the composite potential energy of electron-electron interaction.

As we can see, the obtained potential consists of a Coulomb barrier and a deep potential well and has a characteristic maximum.

Let us determine the coordinates of the maximum potential energy. $r_{Max} = 0.008937$ in Hartree atomic units. If we multiply this by the Bohr radius, we will obtain $r_{Max} = 0.008937 * 52.9 = 0.472788$ pm.

$U(r_{Max}) = 74.592950$ in Hartree atomic units. If we multiply this value by the Hartree energy, we will obtain: $U(r_{Max}) = 74.592950 * 27.2 = 2028.928244$ eV.

M2 radial equation for the electron-electron pair

Let us designate the electron-electron pair by the "*ee*" symbol. The *ee* pair is a hydrogen-like entity. And since the masses of these two electrons are equal, it is necessary to replace the mass (*m*) with the reduced mass ($\mu = \frac{m}{2}$) in the equation for the hydrogen atom [8]. Let us write the M2 radial equation for the *ee* pair:

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{l(l+1)}{r^2}R - \frac{1}{\hbar^2}\left[\frac{\mu^4 c^6}{\left(E - U(r)\right)^2} - \mu^2 c^2\right]R = 0 \quad (9)$$

It is difficult to solve the resulting equation using the analytical methods due to the complex dependence of the composite potential. Therefore, we will apply a numerical solution method using FlexPDE software package http://www.pdesolutions.com/.

The numerical solution of the radial equation in the spherical coordinate system

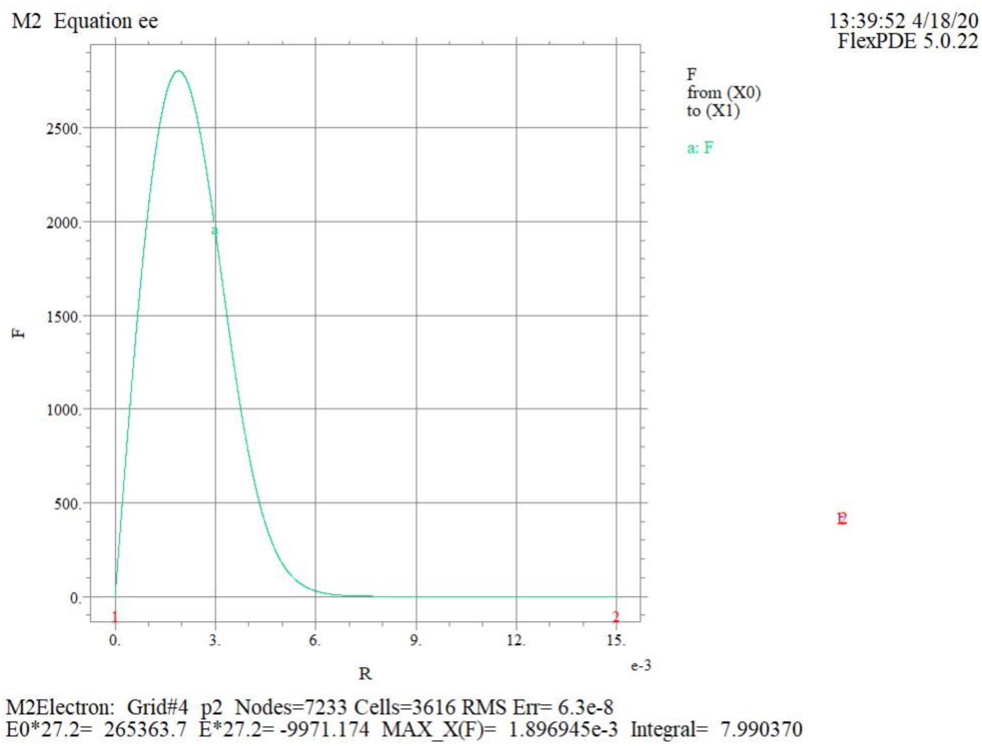

Fig. 3 The radial wave function of the electron-electron pair

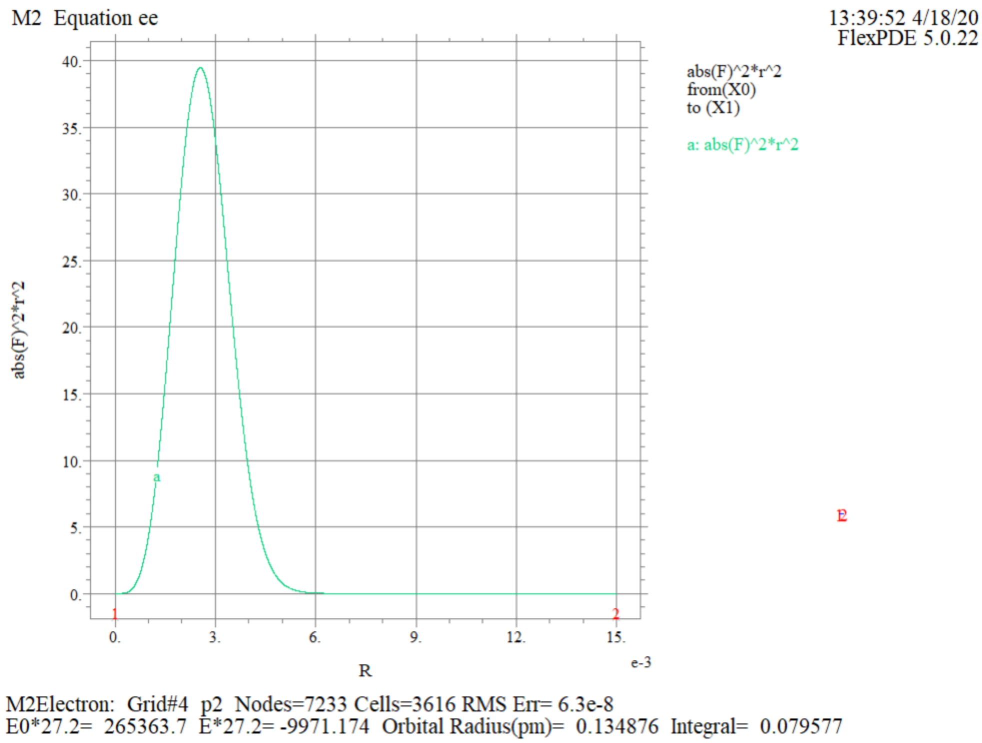

Fig. 4 The radial probability density of the electron-electron pair

As a result of the numerical solution of equation (9), the following parameters of the *ee* pair were obtained: the orbital radius is equal to 0.134876 pm. Note that this is the distance between the electrons, between the maxima of the probability density. The binding energy is equal to 265363.7 eV. As we can see, the convergence of the equation is very high, which means that there is a high probability that this bound state of electrons may actually exist.

The numerical solution of the M2 equation in the cylindrical coordinate system

As we can see, the radial equation (9) includes the angular momentum quantum number l. But, as we know, it doesn't include the magnetic quantum number *m*. Nor does it give us a complete picture of the spatial configuration of electron shells and the magnitude of the magnetic quantum number *m*. This data can be obtained by solving the equation in the cylindrical coordinate system.

The numerical solution of the equation in the cylindrical coordinate system showed us that the convergence is possible only if the value of the magnetic quantum number is $m=1$. We also note that further research is needed in order to obtain a solution for the orientation of magnetic moments a). Most importantly, the possibility of existence of a deep bound state of the electron-electron pair has been theoretically proven.

If this theory is experimentally confirmed, it will be further developed.

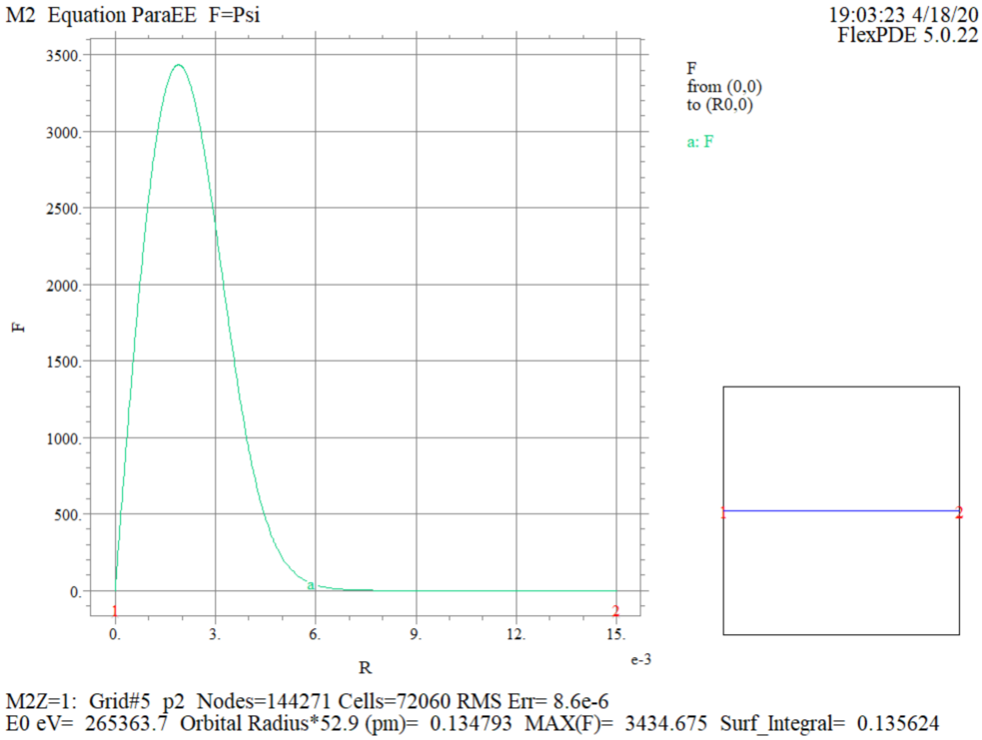

Fig. 5 Wave function of the *ee* pair in the cylindrical coordinate system

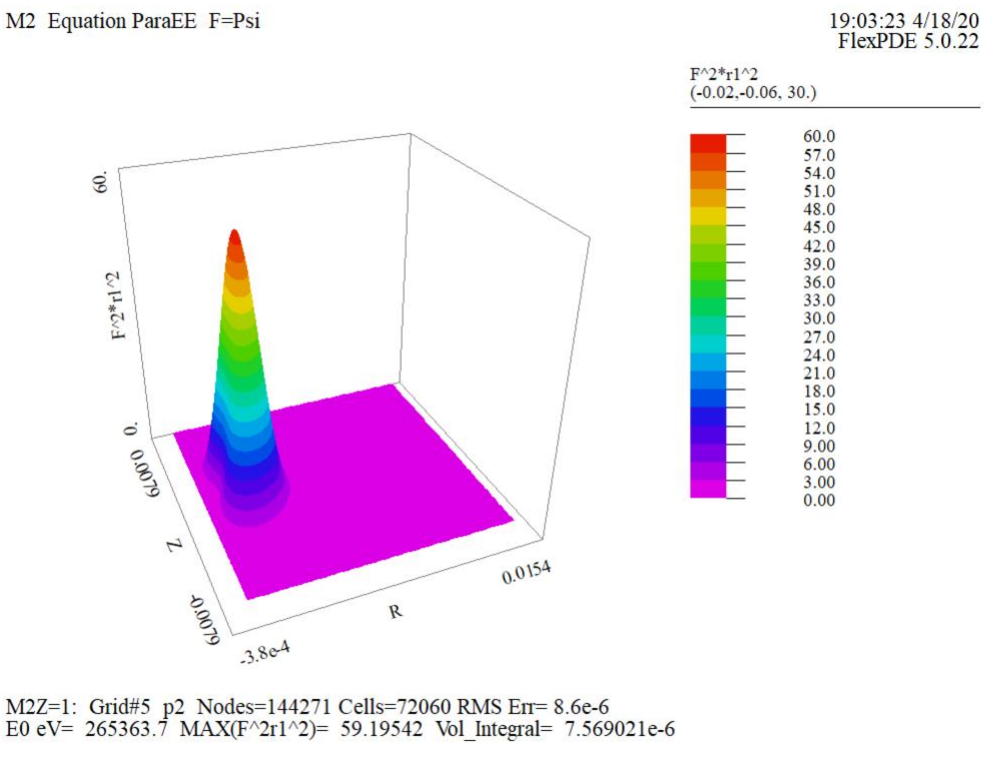

Fig. 6 Probability density in the cylindrical coordinate system

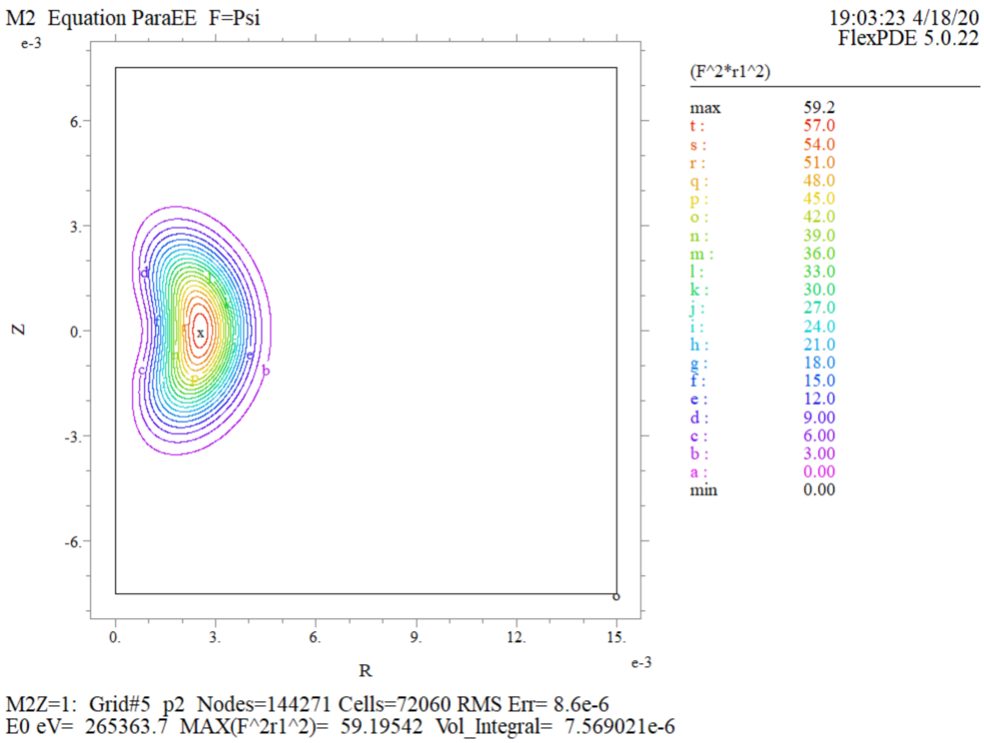

Fig. 7 Projection of probability density in the cylindrical coordinate system

Therefore, as a result of the numerical solution of the equation in the cylindrical coordinate system, the following results have been obtained: the orbital radius is equal to 0.134793 pm. Again, note that this is the distance between the maxima of the probability density, which is the diameter. The binding energy is equal to 265363.7 eV. And, as we said, the magnetic quantum number is $m=1$. In addition, we see that the spatial distribution of the electron density is represented by a toroid. However, if we take a look at the solutions of the radial equation (9) in the spherical coordinate system, we will see that it resembles a spherical distribution with an empty region inside.

Results and discussions

Simultaneous consideration of the Coulomb electrostatic repulsion and magnetic dipole-dipole attraction between two electrons gives us reason to believe that a stable bound state may exist when the electrons approach each other close enough. This is also mentioned by many other authors in their publications. However, it has not yet been possible to obtain an accurate calculation of all the parameters for such a state.

In this paper, the possibility of existence of a deep bound state of the electron-electron pair is proved based on the numerical solution of the M2 relativistic equation. The exact values of the ground state parameters have been obtained. The orbital radius is equal to 0.1348 picometers. In fact, this is the diameter, since the equation is set up taking into account the reduced mass. Therefore, the variable r

represents the distance between the electrons. The binding energy: 265363.7 electron volts. The total spin: $s=1$. The magnetic quantum number: $m=1$. The wave functions and probability density distributions have been obtained in the spherical and cylindrical coordinate systems. It has been shown that this distribution is represented by a toroid. This characteristic form of the probability density gives us reason to believe that the *ee* pair in its ground state is a neutral entity.

In addition to the fact that the above calculations and results are of theoretical interest, they may also be of great practical value. As shown, the formation of the *ee* pair leads to the release of energy at 265363.7 eV. This is of great practical value, which means that environmentally friendly power plants could be built based on this phenomenon.

We think there has been provided enough evidence to conduct an experimental search for an elusive particle.

List of publications:

1. TO THE POSSIBILITY OF BOUND STATES BETWEEN TWO ELECTRONS Alexander A. Mikhailichenko, Cornell University, LEPP, Ithaca, NY 14853, USA
2. Electromagnetic Theory of the Nuclear Interaction
   Bernard Schaeffer
3. On certain features of electron-proton interaction
   Popenko V.I. Scientific and Production Corporation "Kiev Institute of Automation"
4. F. Mayer, J. Reitz, "Electromagnetic Composites at Compton Scale", arXiv:1110.1134v1 [physics.genph] 10 Sep 2011
5. Solution of the Dirac equation with Coulomb and magnetic moment interactions
   Barut, A. O. Kraus, J.
   Journal of Mathematical Physics, Volume 17, Issue 4, pp. 506-508 (1976).
   April 1976
6. Two-body problems with magnetic interactions Hesham Mansour, Ahmed Gamal
   Journal of Nuclear and Particle Physics 2019; 9(2): 51-58
7. Danghyan A.E. "New equation of relativistic quantum mechanics"
8. Dangyan A.E. "Hydrogen Atom. Exotic states. Part one"
9. Dangyan A.E. "Hydrogen Atom. Exotic states. Part two"