A Real Options Approach for the Valuation of Highway Concessions

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The theory of real options offers an approach for the valuation of investments in real assets, based on the methodology developed for financial options. This approach is especially appropriate in the context of strategic decision making under conditions of uncertainty. This is the case for the valuation of highway concessions, where real options arise from certain clauses of the contracts, for example, a minimum traffic guarantee. The possible exercise of these kinds of rights means an added value for the project that cannot be easily captured using traditional procedures for investment valuation. In this paper, we develop a methodology to value these clauses of highway concessions and, for that purpose, we consider the traffic on the highway as the underlying asset in an option contract, taking into account that, when a nonfinancial asset is used, some adjustments have to be made to the options approach. This methodology is applied to the case of an already operating highway concession, using real data, and the authors obtain an estimate of the value of a minimum traffic guarantee, which depends on several parameters that are analyzed. The authors conclude that this methodology is an appropriate tool for the valuation of highway concessions that have operational flexibility.

Keywords: real options; highway; concession; traffic

1. Introduction

From an economic point of view, marginal costs for the use of uncongested transportation infrastructure are near zero, and therefore the latter can be treated as a public good. This is one of the reasons (together with other type of reasons: historical, political, etc.) that explains why the responsibility for planning, construction, and management of transportation infrastructure is generally assumed by government, while its financing relies on public budgets. However, the growing demand for transportation infrastructure in modern societies and the scarcity of public funds are increasing the level of private participation in the financing and management of transportation infrastructure. Moreover, when congestion or environmental costs appear in a determined transportation infrastructure, there are also efficiency reasons to introduce a market mechanism in the management of that infrastructure.

The traditional way to attract private financing for transportation infrastructure is the concession system. This type of concession normally implies huge investments, which are nearly always irreversible, long in duration, and ordinarily made under conditions of uncertainty. Besides, these projects usually involve a high level of financial leverage, so conditions set by financial institutions are very strict. However, and despite all of this, private participation in transportation infrastructure projects is becoming more common. Moreover, it may also happen that the government owns a certain transportation infrastructure, and it decides to transfer it to a private company, which undertakes the tasks of maintaining and operating the infrastructure, charging a tariff to users. In these cases, the private company will normally pay a price to the government to buy the concession business. In fact, this is becoming an alternative way for governments to obtain new funds, that can be used for the development of the transportation network. In this paper, we will use the term “investment” both for the creation of a new infrastructure and for the purchase of an existing asset.

Concessions for highways (new projects or existing highways), in particular, are increasing in numerous countries. Some European countries (France, Italy, and Spain, as well as others) have a long tradition in this area. Certain Latin American and Asian countries have also developed a system of concessions for their highways. Yet, perhaps the most outstanding developments in this area are to be found in the United States, where important highways have recently begun to be managed by the private sector through the system of concessions. The Indiana Toll Road, the Chicago Skyway and different stretches of new highway projects in Texas and Florida are recent
The theoretical analysis of the decision-making process in investment projects. In the case of transportation infrastructure investments, this approach has been used by Zhao, Sundararajan, and Tseng (2004) for a theoretical analysis of the decision-making process in highway development. Zhao's paper adopts the point of view of the public authority that is entrusted with the maintenance and expansion of the highway network, while in our paper, we use a real options model to calculate the value of some of the options embedded in a concession business. In addition, our research contributes some empirical results, like the testing of the hypothesis that traffic volume on toll highways follows a geometric Brownian motion (GBM), the measure of volatility of traffic on toll highways and the estimation of the $\beta$-parameter for highway concessionaire firms, which are quoted in the stock market.

One of the problems of real options is that, while financial options are usually well defined, in the case of real assets it is difficult to point out clearly the options involved. However, in the specific case of highway concessions, it is possible to determine the options, which are inherent to the business of the concession, starting from the terms of the contract of the concession, and considering the highway traffic as the underlying asset. It must be made clear that these options exist, and can be evaluated, for both parties involved in said contract: both the government and the concessionaire company. The following are some of the existing or possible real options in highway concessions: exchange rate guarantee, public participation loans, minimum traffic guarantees (traffic floors), maximum traffic limitations (traffic caps), extension of the concession, anticipated reversion, establishment of subsidies, etc. In this paper, we develop a methodological tool, based on real options, to evaluate highway concessions in which this kind of mechanisms exist. Therefore the methodology described in this paper permits the introduction of quantitative analysis to the strategic evaluation of a project, providing the decision-making process with a solid base.

It is important to notice that we can differentiate between the highway concession business and the infrastructure itself. Normally, the highway cannot be traded in the market, because it remains as a public asset, and it returns to the public authority at the end of the concession period. However, the concession business can normally be traded under certain conditions, which are established in the concession contract. These conditions generally imply the public authority’s agreement to accept the new concessionaire. Therefore, transactions of highway concessions do take place, but we cannot say that there is an open market for them. This raises the problem of how to provide a market price of risk for this kind of assets. As we will see in this paper, we have used, for that purpose, the correlation between the return of quoted highway concessionaire firms and the evolution of the whole stock market.
In our model, we use traffic on the highway as the underlying asset, as we have already pointed out. Real options that are embedded in the concession contract are thus calculated as a derivative of the traffic volume. This means that we consider traffic as the only source of uncertainty that affects the value of the options.

To illustrate the methodology, we have carried out a simulation exercise. We have calculated the value of a minimum traffic guarantee on a highway concession, applying different levels of the guarantee. We have used real data (for investment, duration of works, concession period, etc.) of a recently built highway, for which a concession agreement was established. The calculation was made ex post, i.e., once the highway entered into operation, and therefore this case can be considered as a concession on an existing highway.

2. The Real Options Approach
Real options represent a systematic and integrated approach for the evaluation of real assets, using the options theory. This approach is especially appropriate in the context of strategic decision making and evaluation of investment opportunities under conditions of uncertainty.

An option provides the opportunity of taking a decision after watching how events evolve. On the exercise date, if everything has happened as expected, the right given by the option will be exercised, but if an unforeseen, or unlikely event has taken place, another decision will be taken. This means that the return obtained is not strictly lineal, as it depends on the decision that is adopted. Nonlinear returns can be a tool to reduce the exposure to uncertainty, and this is extremely useful for managers.

From a traditional point of view, the higher the level of uncertainty, the lower the value of the project. Under the real options approach, a higher level of uncertainty can mean a higher value of the asset if managers identify and use their options to meet the future and sometimes unexpected evolution of events with flexibility. Options that are inherent to investments permit managers to reduce the exposure to bad results and increase the ability to exploit, when they occur, good results, and thus to increase the value of the investment project.

In a strict sense, the real options method is the extension of financial options theory to options on real assets. Financial options models use the close relationship between the underlying asset and the derivative asset (the option) to replicate the results of a riskless asset. The possibility of establishing a replicating portfolio permits the calculation of the value of any contingent financial claim, simply through observation of the capital markets. This evaluation methodology makes it possible to use the known price of an asset to estimate the relevant price of other assets whose value depends on the former.

Let us suppose that we have a derivative asset whose value \( Q_i \) depends on one state variable \( S \) and on time, i.e., \( Q_i = Q_i(S, t) \). \( S \) can be, for example, the price of a stock quoted in the capital markets. We can always build a riskless portfolio with an appropriate combination of the derivative asset and the underlying asset.

If we hold one unit of the derivative asset, for example, we can build a riskless portfolio by adding a short position of the underlying asset. In particular, we would need a short position of \( \frac{\partial Q_i}{\partial S} \) units of the underlying asset. Therefore the value of our portfolio would be (Hull 2006)

\[
H = Q_i - \frac{\partial Q_i}{\partial S} S. \tag{1}
\]

In a differential period of time \((dt)\), the change in the value of our portfolio would be

\[
dH = dQ_i - \frac{\partial Q_i}{\partial S} dS. \tag{2}
\]

This change in the value of our portfolio must be consistent with the return obtained by a riskless asset. Therefore, if \( r \) is the risk-free interest rate, we would obtain

\[
dQ_i - \frac{\partial Q_i}{\partial S} dS = r \left( Q_i - \frac{\partial Q_i}{\partial S} S \right) dt. \tag{3}
\]

If both the underlying asset and the derivative asset pay a dividend, the relevant equation would be

\[
\left( dQ_i + D_i dt \right) - \left( \frac{\partial Q_i}{\partial S} dS + \frac{\partial Q_i}{\partial S} S dt \right) = r \left( Q_i - \frac{\partial Q_i}{\partial S} S \right) dt. \tag{4}
\]

While the concept of replicating a portfolio offers a calculation method that is consistent with the prices and variables that determine the value of contingent claims, the concept of arbitrage guarantees that the estimate obtained is an equilibrium price. If the price resulting from the replicated portfolio and the derivative price were not the same, investors would detect arbitrage opportunities that would be exploited until the equilibrium were reached, and both prices coincide.

At first, it seems that the use of replicating portfolios and the arbitrage mechanism requires that the assets involved be negotiable both in the short and in the long term and also that the markets where these assets are traded be complete. When we are analyzing options on financial assets that are traded in capital markets, we can assume that the specified required conditions are met. However, it is not the same in the
case of real options, their underlying assets, and the markets where these assets are traded.

The literature on the application of the real options theory in complete markets is extensive. Dixit and Pindyck (1994); Trigeorgis (1995); Brennan and Schwartz (1985); and McDonald and Siegel (1986) are some of the most representative authors. The basic assumption in this case is that there exist a sufficient number of lineally independent assets in the market to allow for the construction of a replicating portfolio. This is one of the fundamental hypotheses in options theory, making a risk-neutral evaluation possible.

When it is not possible to build a portfolio of assets that span the stochastic variations of the project, or when the correlation between the project and the market portfolio is not perfect, it is said that the market is incomplete. If a market is not complete, the introduction of a new investment could extend the set of investment opportunities, modifying the existing equilibrium, and therefore the price of the rest of the assets. In these conditions, both the replicating model, and the application of that model to the evaluation of real options, are not feasible.

In this case, Copeland and Antikarov (2001) propose the adoption of the present value of the project without any options as the best market value for the project. This value is calculated through the discount of cash flows generated by the project, using the discount rate obtained with the well-known method of the capital asset pricing model (CAPM). This would allow for the use of the project itself as a basic asset in the replicating portfolio, because it is assumed that the project with no flexibility is, however, highly correlated with the value of the project with options. This hypothesis is known as “marketed asset disclaimer” (MAD). The use of the project itself as a part of the replicating portfolio makes the market complete for that project, and it guarantees a perfect correlation between the project and the spanning assets. Under this condition, the risk-neutral evaluation can be used.

In general, the value of real options evolves according to the price of another asset (like the value of a determined project) on which the exercise claim of the option is established. The price of this asset coincides with the present value of expected cash flows generated by the project. When there is not a market to trade this current of cash flows, the literature on real options recommends the use of the state variable on which the project cash flows depend. This variable, which induces the risk of the project cash flows, can be a financial asset, a physical asset, or any other variable, such as the temperature in a country or region, the market share of a company, or the traffic on a highway.

Harrison and Kreps (1979) and Cox, Ingersoll, and Ross (1985) prove that the hypotheses of replicating portfolio and arbitrage can be used, in general, to evaluate a specific claim that depends on a random variable whose nature is different from the claim itself. For evaluating any contingent claim whose price evolves in a random way, depending on changes over time to a number of other variables, it is enough to assume the existence of the same number of assets whose risk depend on, and only on, those variables. With \( n + 1 \) assets (including the contingent claim) that depend on \( n \) state variables, it is always possible to build a riskless portfolio and impose the equilibrium condition.

In summary, despite the analogies between financial and real claims, the latter present certain peculiarities that require an adjustment in the deductive reasoning on which the options theory is built. However, this does not invalidate the application of the theoretical foundations of the evaluation of financial options to the evaluation of real options.

In our paper, we use the traffic volume (a non-financial variable) as the underlying asset to evaluate a highway concession. Therefore we are assuming that traffic is the only source of uncertainty that affects the value of the options embedded in the concession agreement. This is a reasonable assumption in cases such as the existence of a minimum traffic guarantee. Then, we can consider the present value of the expected cash flows of the concession as a part of the replicating portfolio that allows for using the risk-neutral evaluation methodology, which has been described above. Therefore it is not necessary that the underlying asset be traded in the market to use the model.

3. Real Options on Highway Concessions
Real options arise in a natural way from the interpretation of the clauses established in the actual contracts of highway concessions. After all, the real options approach contributes a valuation tool, but the terms of the options are embedded in the contracts that regulate rights and obligations for both contracting parties. In the case of highways, some of the existing or possible real options in the projects have been mentioned already: exchange rate guarantee, public participation loans, minimum traffic guarantees (traffic floors), maximum traffic limitations (traffic caps), extension of the concession, anticipated reversion, establishment of subsidies, etc. These mechanisms reduce the cash flow volatility, add flexibility to the project, and allow a better management of the concession based on the occurrence of future events.

The possible exercise of this series of rights represents an added value for the project that is not captured by the traditional procedures of valuation. So,
the lack of an appropriate quantitative tool has prevented the effectiveness of these mechanisms from reaching their maximum potential. The habitual practice of calculating the NPV by means of the discount of cash flows, leads to erroneous results when the project incorporates a certain flexibility.

The options most commonly found in the terms of reference for highway concessions are European call and put options, although other possibilities exist. These are defined as “European” and not “American” put and call options because of the existence of a fixed and certain date of exercise of the right granted by the option.

To value these options, the theory of real options can be used, taking into consideration the following premises:

1. The present value of the project without any options is the best estimator of its market value, because it is assumed that the project without any flexibility is highly correlated with the project with options. This is the MAD hypothesis, already discussed in this paper.

2. The variations of the traffic volume, which is considered the state variable in the model, follow a random walk.

As in the case of the underlying assets in financial options, it is considered that the growth rate of traffic, \( \theta \), is normally distributed over a time interval \( dt \); so, absolute variations of \( \theta \), \( d\theta \), are lognormally distributed. Then, traffic can be modeled as a GBM, which can be described as

\[
d\theta = \alpha_\theta dt + \sigma_\theta dz,
\]

where \( \alpha_\theta \) is a drift term or growth parameter, \( \sigma_\theta \) is a measure of the traffic volatility, generally expressed by means of the standard deviation of the natural logarithm of \( \theta_t/d\theta_t \), and \( dz = \varepsilon(dt)^{1/2} \) is the increment of a standard Wiener process (Cabaña 2002), where \( \varepsilon \) is normally distributed with zero mean and unit standard deviation.

However, the GBM hypothesis for highway traffic is not evident. We have not found any empirical study on this matter. Therefore we have carried out a test for the GBM hypothesis for the evolution of traffic volume in toll highways, analyzing the series available for Spanish toll highways. The data set covers the annual average daily traffic (AADT) for 11 toll highway stretches. The number of observations has been 31 (that is, the period 1974–2004) for five of the highways, 29 for two of the highways, 27 for three of the highways, and 22 for the remaining highway (Sánchez Soliño and Lara Galera 2008). These highway stretches were located in different regions, and they had different features (interurban, urban, coastal highways with a touristic function, etc.). We have used the Dickey-Fuller (1979) approach, which is the most widely used methodology for testing if time series follow a random walk. It would have been desirable to have longer time series; Dixit and Pindyck (1994), for example, have run random walk tests for prices of raw materials, with time series that include 100 or more years of data. Unfortunately, there are no such long time series for highway traffic data. However, we ran the Dickey-Fuller (1979) test independently for each of the 11 highway stretches, and the main result of our research was that we could not reject the GBM hypothesis for traffic volume in any of them. Therefore we have assumed that hypothesis in this paper.

3. We can separate the market (or systematic) risk, for which the market is complete, and the private risks of the project (not correlated with the market), establishing a different treatment for these two sources of uncertainty.

Smith and Nau (1995) propose this separation, assuming that the investor has a diversified portfolio of investments and that the project does not represent an important parcel of its wealth. In this case, it can be assumed that the investor will be neutral to the private risk. This supposition is based on the fact that the market will remunerate the investor fundamentally for the systematic risk that he supports, because the nonsystematic risk can be diversified through its portfolio of investments. This is the hypothesis adopted in this paper.

Now, going back to the general theory of options pricing, we have considered that the value of any derivative asset can be calculated as the price of equilibrium of a contingent asset \( Q_t = Q_t(S_t, t) \), which depends on time and on one state variable \( S \). Now, we assume that \( S \) follows a Wiener process like the one indicated in Equation (5). Therefore

\[
dS = \alpha_S dt + \sigma_S dz.
\]

From Itô’s (1951) lemma, we obtain that the derivative asset also follows a Wiener process, which can be described as

\[
dQ_t = \left( \frac{\partial Q_t}{\partial S} \alpha_S S + \frac{\partial Q_t}{\partial t} + \frac{1}{2} \frac{\partial^2 Q_t}{\partial S^2} \sigma_S^2 S^2 \right) dt
\]

\[
+ \frac{\partial Q_t}{\partial S} \sigma_S dz.
\]

Now, if we apply the equilibrium condition, given by Equation 4, and we take into account Equations (6) and (7), we arrive at the general contingent claims asset pricing model for complete markets (Dixit and Pindyck 1994), given by

\[
\frac{1}{2} \frac{\partial^2 Q_t}{\partial S^2} \sigma_S^2 S^2 + \frac{\partial Q_t}{\partial t} (r - \delta) S + \frac{\partial Q_t}{\partial t} - r Q_t + D_t = 0.
\]
A key property of this model is that it does not involve any variables that depend on the risk preferences of the holders of the assets. In particular, the expected return of the underlying asset ($\alpha_s$, which depends on the level of risk aversion by investors) has disappeared in Equation (8). If risk preferences do not enter the equation, they cannot affect its solution. Therefore we will be able to calculate the value of the options under the assumption that the world is risk neutral. This result does considerably simplify the analysis of derivatives (Hull 2006).

In Equation (8), $r - \delta_S$ can be expressed as $\alpha_s - \lambda_S \sigma_S$, where $\lambda_S$ is the market price for risk of the state variable $S$, which can be obtained from the CAPM (see Dixit and Pindyck 1994 and Sharpe 1964):

$$\lambda_S = \frac{\mu_S - r}{\sigma_S} = \frac{E(R_m) - r}{\sigma_m} \rho_{Sm} = \frac{1}{\sigma_S} [E(R_m) - r] \beta_{Sm}. \quad (9)$$

In Equation (9), $\mu_S$ is the total expected return of asset $S$. Therefore it is assumed that the premium ($\mu_S - r$) for the systematic risk of the asset is equal to $\lambda_S \sigma_S$, i.e., the market price of risk ($\lambda_S$) multiplied by the quantity of risk (given by the volatility $\sigma_S$). In financial markets, this relationship is normally accepted, therefore a higher volatility implies a higher expected return of the asset. The parameter $\beta_{Sm}$ is widely used in financial economics to characterize a determined asset. It is defined as the covariance between the return of the asset $S$ and the return of the whole market portfolio, divided by the variance of the latter.

The partial differential Equation (8) must be fulfilled for any derivative asset $Q_i$ whose value is dependent on the state variable $S$ and the time variable $t$, allowing it to be valued through the imposition of the corresponding boundary conditions. The general contingent claims asset pricing model is generalizable for $n$ variables of state. In this case, it will be necessary to impose $2n+1$ boundary conditions.

Observe that $\alpha_s - \lambda_S \sigma_S$ acquires a special meaning because it is equivalent to considering a new drift $\alpha_{s, adjusted}$ adjusted by risk, eliminating from the initial drift $\alpha_s$ the portion that corresponds to the systematic risk of $S$, which is $\lambda_S \sigma_S$.

In the end, the neutral risk approach for the calculation of the value of an option that provides a payoff at a given time in the future is equivalent to the following procedure (Hull 2006):

1. Substitute a risk-adjusted drift $\alpha_{s, adjusted}$ (which is equal to $\alpha_s - \lambda_S \sigma_S$) for $\alpha_s$ in the stochastic process given by Equation (6).
2. Calculate the expected payoff from the option.
3. Then, discount the expected payoff at the risk-free interest rate ($r$).

The advantage of this procedure is that there is no need to make any assumptions on the risk preferences of the investor. However, there are some aspects of the procedure that are different for financial assets and real assets, as explained below.

1. **Financial Assets.** If the state variable is a stock or a financial asset that does not distribute any dividends, the expected return coincides with the expected price variation; that is, the return is equal to the expected rate of capital gains, being fulfilled such that

$$\mu_S = r + [E(R_m) - r] \beta_{Sm} = \alpha_S. \quad (10)$$

In this case, we can verify that $\alpha_{s, adjusted} = r$.

If the underlying asset is a stock or a financial asset that distributes a payout rate $\delta_S$, the only difference is that the total return would be the expected price variation of the asset plus the payout. Then, it would be fulfilled such that

- $\mu_S = \alpha_S + \delta_S$,
- $\alpha_{s, adjusted} = r - \delta_S$.

2. **Real Assets.** When the state variable is a non-financial variable or asset (such as the traffic on a highway), no payout is paid, but there is not, in principle, any relationship between the drift ($\delta_S$) and the volatility ($\sigma_S$) of the asset. This would mean that, in general, for nonfinancial variables: $\alpha_s \neq r \lambda_S \sigma_S$. Nevertheless, the model is still valid if we introduce a fictitious payout rate $\delta_{s, fictitious}$ so that

$$\mu_S - r = \alpha_s + \delta_{s, fictitious} - r = \lambda_S \sigma_S. \quad (11)$$

In this case, and taking into account Equation (9), the expected drift for the adjusted process will be

$$\alpha_{s, adjusted} = r - \delta_S = \alpha_s - [E(R_m) - r] \beta_{Sm}. \quad (12)$$

Observe that the use of this risk-adjusted growth rate allows for the application of the model given in Equation (8).

In Charoenpornpattana, Minato, and Nakahama (2003), the use of the fictitious payout rate $\delta_S$ is not mentioned in the description of the methodology used for the calculation of the value of real options in a highway concession. This means that, according to these authors, traffic volume would follow an adjusted process in which $\alpha_{s, adjusted} = r$. However, the use of this adjusted growth rate would be arbitrary, because we cannot assume in this case that $\lambda_S \sigma_S = \alpha_s - r$. In other words, we cannot assume that the market value of risk is implicit in the growth rate of traffic volume. From our point of view, this assumption would lead to wrong results.

The introduction of the fictitious payout rate $\delta_S$ (which can be calculated following an empirical method) allows us to calculate an adjusted growth rate ($\alpha_{s, adjusted}$, given in Equation (12) that substitutes...
for $\alpha_s$ in Equation (6)). This is equivalent to eliminate the systematic risk (that is, the risk that cannot be diversified in an investment portfolio) that affects the evolution of traffic volume, and it allows us to use the risk-neutral valuation procedure, as we have previously described. For the calculation of $\delta_s$, we have found first the $\beta$ parameter for the shares of quoted highway concessionaire firms, taking into account the correlation between the return of the whole market portfolio and the return of those shares (see §4.3). The correlation coefficient has been obtained starting from historical data in the Spanish stock market. In short, we consider that the use of the fictitious payout rate $\delta_s$ is necessary for the calculation, and its effects on the results are not negligible (see §4.4).

4. Application to the Pricing of a Minimum Traffic Guarantee

To apply our model, we have to make some assumptions on the institutional setting for concession agreements. In the first place, we assume that the level of tolls in a highway concession is fixed exogenously, stemming from the concession contract between the government and the concessionaire; so the latter has no tolling authority. The level of tolls can vary automatically (for example, applying an inflation index), but any other change has to be authorized by the government. In the second place, as for risk distribution between the public authority and the private concessionaire, we assume that normally the demand risk is assigned to the private operator. However, there may be certain clauses in the concession contract that limit the private operator’s risk. The purpose of our model is precisely the calculation of the value of this kind of clauses.

Demand risk is the one that normally deserves greater consideration in the case of a road concession, because the traffic level is hardly controllable by concession operators, and the risk of lower than forecasted levels of traffic undermines the project. This is especially true for traditional concessions, in which the income of operators is based on the collection of tolls, although it can also be true for shadow toll concessions, in which the income of the concession operator is proportional to the volume of traffic on the highway. In both cases, the deviations in traffic forecasts affect the expected cash flows of the project and therefore have an effect on the value of the concession. This risk is translated into more expensive and demanding financing conditions for the project.

The establishment of a level of minimum or guaranteed (by the public authority) traffic during some part of the concession period, or during its entire duration, works like a “floor” that reduces the variance of the future cash flows and makes the project more attractive to both promoters and lenders, limiting the possible adverse results. This kind of mechanism is commonly used in some countries (Vassallo and Sánchez Soliño 2006).

The theory of real options is useful for carrying out the valuation of this type of guarantee, because the establishment of a level of minimum or guaranteed traffic can be treated as an European put option, in which the strike price on the expiration date will be equal to the guaranteed minimum value. In this case, the value of the option is established as a derivative of the traffic, with the traffic considered to be the underlying asset. If the real traffic is higher than the guaranteed one at the expiration date, the value of the option will be null; in the contrary situation, the value of the option will be proportional to the difference between the strike traffic and the real one.

Let us suppose that we have a concession in which we made all the investments at the moment $t = 0$. The concession is operative during the years $1, 2, \ldots, i, \ldots, n$. The applicable toll rate in any year “$i$” will be $p_i$. If the expected traffic for year “$i$” is $\theta_{i}\pi_i$, then the income for this year will be $I_i = p_i \theta_{i}\pi_i$. In this context, we can use the AADT as the traffic variable.

Let us suppose that $C_i = C_i(\theta, t)$ is a European put, a real option whose value in $t = 0$ (initial moment) is obtained like a derivative of the volume of traffic $\theta$. This option grants the right of exercise in year “$i$” (date of expiration or maturity $T_i$) of a minimum guaranteed traffic $\theta_{gl,i}$ expressed as a percentage $\pi_i$ of the expected traffic $\theta_{i}\pi_i$:

$$\theta_{gl,i} = \pi_i \theta_{i}\pi_i.$$ (13)

The added value to the project (in $t = 0$), because of the $n$ options thus defined, will be

$$V_{options} = \sum_{i=1}^{i=n} C_i.$$ (14)

In addition, it is considered that the traffic variable follows a random walk process such as the one expressed in Equation (5), with $\theta_0$ as the initial expected traffic. In these conditions, applying Equation (8), the value of the option $C_i$ has to fulfill the equation

$$\frac{1}{2} \frac{\partial^2 C_i}{\partial \theta^2} \alpha_{\theta}^2 \theta^2 + \frac{\partial C_i}{\partial \theta} (r - \delta) \theta + \frac{\partial C_i}{\partial t} - r C_i = 0,$$ (15)

being $\alpha_{\theta}$ adjusted $= r - \delta = \alpha_0 - [E(R_m) - r] \beta_\theta$. The boundary conditions to apply are:

1. $C_i,T_i = p_i \cdot \max(\theta_{gl,i} - \theta_{Ti}, 0)$ (value of the derivative at the date of expiration).
2. $C_i = p_i \cdot \theta_{gl,i}$ when $\theta \to 0$ (limit price of the option when the underlying asset tends toward zero).
(3) \( C_i = 0 \) when \( \theta \to \infty \) (limit price of the option when the underlying asset tends toward infinity).

The previous partial differential equation is a parabolic type. Making the opportune changes of variables, this equation can be transformed into a diffusion equation, formally identical to the one-dimensional equation of heat transmission in physics. With the indicated boundary conditions, the equation has an analytical solution that can be expressed as

\[
C_i = p_i \left[ \theta_i e^{-\theta_i T_i} \Phi \left( -\frac{\ln(\theta_i/\theta_i) + (r - \delta - (\sigma^2/2)T_i)}{\sigma \sqrt{T_i}} \right) \right. \\
- \theta_i e^{-\theta_i T_i} \Phi \left( -\frac{\ln(\theta_i/\theta_i) + (r - \delta + (\sigma^2/2)T_i)}{\sigma \sqrt{T_i}} \right),
\]

where \( \Phi \) represents the function of accumulated probability of the standard normal distribution.

As a practical case for the application of this type of option, we have carried out a simulation exercise. We have calculated the value of an annual guarantee of minimum traffic (applying different levels of the guarantee) for the toll highway “Eje Aeropuerto,” which is a new access road to the international airport of Barajas, in Madrid. The calculation was made ex post, i.e., once the highway entered into operation, and therefore this case can be considered as a concession for an existing highway. Thus we can assume that traffic volume is the only source of uncertainty for the concession.

4.1. Static NPV of the Project

In the “Eje Aeropuerto” toll highway, a 25-year concession in which 327.82 million euros have been invested, the traditional methods of valuation yield the following results for the analysis of the investment (see Figure 1):

As may be observed in the previous graph, the static methods of valuation are limited to the calculation of the NPV, the internal rate of return (IRR) to the project and the IRR to the shareholder. As a complement, a sensitivity analysis has been made to evaluate the influence of the possible alterations of traffic on the expected results. For a 30% decrease of traffic in relation to the prediction, the NPV is negative and the IRR to the project and to the shareholder fall to very low values. In this case, the project would be unable to meet the debt service requirements during the first 10 years of the concession.

On the other hand, if a 30% increase of traffic (with regard to the expected values) takes place, the project could practically duplicate its predicted NPV. It must be observed that relatively narrow variations of traffic can make the project unfeasible or result in the exorbitant enrichment of the concessionaire. This circumstance is easily explained if we consider that these projects are strongly leveraged.

4.2. Traffic Characterization

The analyses of traffic made in the toll highway network in Spain, with historical annual data over a period of 30 years, show annual growth rates of traffic, which vary between 3.58% and 15.23% in the different highways, with an average value of 7.96% for the total network. Practically in every case, the growth of traffic presents a greater dispersion during the first 10 years of the concession, becoming stabilized afterwards.

For annual volatility, the measured values vary, with a single exception, between 5.33% and 9.65%, with 7.5% as the average volatility for the entire network. A greater dispersion of the measured volatility is also observed during the first years of the concession. After this initial period, volatilities remain almost constant, oscillating between 6% and 8% in practically all cases. With the habitual criterion of the classification of financial assets as a function of volatility, we should consider traffic as a moderate risk asset.

In the case of the Barajas airport access, we have taken the expected annual growth of traffic that was estimated in the traffic study for the concession, which was 3.5%. Regarding the volatility of traffic volume on the highway, we can take, in principle, the average value for the rest of the toll highway network in Spain.

4.3. Initial Parameters Used for Pricing the Options

As initial parameters for the accomplishment of the calculation relative to the options of the toll highway “Eje Aeropuerto,” the following have been taken into consideration:

(1) Risk-Free Interest Rate. We have used the yield of the Spanish treasury bonds with one year of maturity. In December 2006, it was 3.87%, in terms of the annual continuously compounded rate of return.
(2) Traffic Volatility. As pointed out above, we have calculated the average annual value for the Spanish toll highway network, which is 7.50%, and we have used it as the base case.

(3) Traffic Growth Rate $\alpha_t$ (Drift). We have taken a value of 3.50%, also in terms of annual continuously compounded rate. This value is coherent with the results of the traffic study realized for the “Eje Aeropuerto” highway.

(4) Market Yield. The average yield in the Spanish stock market (defined by the IBEX-35 index), during the last five years, has been 13.50%.

(5) $\beta$ of Traffic. The representative parameter of the systematic risk (the parameter $\lambda$) for the project (i.e., for the cash flows obtained by the concessionaire) and for the traffic must be equal. This can be expressed as

$$\lambda = \frac{\alpha_t + \delta - r}{\sigma_t} = \frac{\mu_{\text{project}} - r}{\sigma_{\text{project}}}.$$  

Replacing and operating in Equation (17), we arrive at the expression

$$\beta_t = \frac{\sigma_t}{\sigma_{\text{project}}} \beta_{\text{project}}.$$  

For the estimation of $\sigma_{\text{project}}$ and $\beta_{\text{project}}$, we have used, as a proxy, the parameters obtained for the series of the return of the shares of quoted highway concessionaire firms in the Spanish stock market. First, we calculated the volatility of that series, and the result was 22% for $\sigma_{\text{project}}$. For the calculation of the parameter $\beta_{\text{project}}$, we can use the following proxy:

$$\beta_{\text{project}} = \frac{\sigma_{\text{project}}}{\sigma_m} \rho_{qm},$$  

where $\rho_{qm}$ is the correlation coefficient between the series of the return of the whole market portfolio and the series of the return of the shares of quoted highway concessionaire firms, and $\sigma_m$ is the volatility of the return of the whole market portfolio. We obtained that $\rho_{qm} = 0.517$ and $\sigma_m = 0.25$. Then, the estimated value for $\beta_{\text{project}}$ is 0.45. Finally, if we operate in Equation (18), we obtain the result that the $\beta$ of traffic is equal to 0.15.

(6) Guarantee. A guarantee of 100% of the expected traffic as an initial value is assumed, although options are priced for different guarantee levels in the following section.

With these data, we obtain the following adjusted drift:

$$\alpha_{\text{adjusted}} = 3.5\% - [(13.5\% - 3.87\%) \cdot 0.15] = 2.06\%,$$

with $\delta = 3.87\% - 2.06\% = 1.81\%$ (see Equation (12)).

4.4. Value of the Guarantee
As has been previously mentioned, we can establish an analogy between the guarantee of minimum traffic and an European put real option. As the guarantee grows, the possibility of executing the option to protect the project from lower levels of traffic, makes the option more valuable. Also, the guarantee will be more valuable the higher the volatility.

Figure 2 Evolution of the Value of the Minimum Traffic Guarantee at $t = 0$
Figure 3  Evolution of the Accumulated Value of Minimum Traffic Guarantee at $t = 0$

Figure 2 shows the evolution of the value of the guarantee of minimum traffic, for the hypotheses set out in the previous section, and for different levels of guaranteed traffic. Figure 2 provides the value in $t = 0$ of the guarantee applied in each year, obtained applying Equation (16). It has been assumed that the level of guaranteed traffic (in percentage) is constant throughout the entire concession period, although it is possible to make the calculus assuming that the level of the guarantee changes every year.

As can be observed, as the percentage of guaranteed traffic increases, so does the value of the option in any given year.

Figure 4  Accumulated Value of the Minimum Traffic Guarantee for Different Volatilities and Guarantee Levels
The accumulated value at \( t = 0 \) of all the options, from any year to the end of the concession period, is indicated in Figure 3:

Figure 4 shows the accumulated value at \( t = 0 \) of all the options, depending on different levels of the minimum traffic guarantee, and on different values for the traffic volatility.

Volatility has a fundamental role in the value of the options, because the greater the volatility, the greater the probability of deriving profit from a high traffic, with the project protected against unfavorable values by exercising the option. Usually, the value of the options grows in a significant way as volatility increases. A limitation of the use of an analytical solution for the differential equations is that it does not allow for a variable volatility. Nevertheless, other numerical methods exist that make possible the use of a variable volatility, allowing for the assumption of greater value during the first years of the concession and lesser value for the rest of the concession period, when traffic becomes stabilized.

Some interesting conclusions can be obtained starting from the previous figures. If we divide all the values of the figures by the static NPV of the project, 91.81 million \( € \), it is possible to observe that a 100% guarantee of the expected traffic represents a value between 100% and 180% of the static NPV of the project, depending on the volatility.

With a volatility equal to 7.5% (calculated for the Spanish toll highways network), the value of the guarantee represents 124.24% of the static NPV; that is, 114.06 million euros. This means that the extended NPV moves from 91.81 million euros of the static model without options to 205.87 million euros, when we add the value of the options.

If we do not use the fictitious payout rate (\( \delta \)) to obtain the adjusted growth rate for the traffic, with the rest of the parameters unchanged, the value of the options would be 37.79 million euros. This means that a noncomplete adjusted growth rate clearly underestimates the value of this kind of guarantee.

Table 1 includes the values of the extended NPV (NPV*) for different guarantee levels, again with a risk-adjusted growth rate.

<table>
<thead>
<tr>
<th>Guarantee Levels</th>
<th>NPV* (( \pi = 70% ))</th>
<th>NPV* (( \pi = 80% ))</th>
<th>NPV* (( \pi = 90% ))</th>
<th>NPV* (( \pi = 100% ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>91.81</td>
<td>110.35</td>
<td>131.32</td>
<td>163.18</td>
<td>205.87</td>
</tr>
</tbody>
</table>

The methodology developed in this paper, based on the real options approach, provides a means for the calculation of the value that the different existing rights in contracts contribute to a highway concession project. The following are some of the existing or possible real options in highway concessions: exchange rate guarantee, public participation loans, minimum traffic guarantees (traffic floors), maximum traffic limitations (traffic caps), extension of the concession, anticipated reversion, establishment of subsidies, etc.

One of the advantages of the methodology followed in this paper is that it provides a risk-neutral procedure for the calculation of the options. On the contrary, if we use a calculation method based on a decision tree approach, it is necessary to specify an exogenous discount rate to obtain the value of the option. The problem (that cannot be solved in practice using the traditional approach) is that the existence of the option implies an effect on the relevant discount rate to be used. This problem is avoided by using a risk-neutral approach.

Furthermore, when there are several real options embedded in the contract, in some cases, we can calculate the value of each option independently (for example, if there is both a floor and a cap for traffic volume), because the existence of each option has no influence on the risk-free interest rate. Then, we can add the values of both options, taking into account that the value of a cap for traffic volume will be negative for the concessionaire, because it limits its revenues.

As an example in the application of the risk-neutral approach, the value of a minimum traffic guarantee in an already existing toll highway concession has been analyzed in this paper. The existence of this type of guarantee makes the concession considerably more attractive for the concessionaire and for lenders, because it limits the possible adverse results to the concessionaire if traffic does not reach the predicted levels.

Therefore, a minimum traffic guarantee benefits, in principle, the private partners of the highway concession, because their risks are reduced, while the public authority bears more risk. However, in a competitive environment, one could expect that these benefits for the concessionaire and for lenders are transferred (at least partially) to the users of the highway, through lower toll levels resulting from the tendering process, when the concession is awarded. From the point of view of the government, as the counterparty in a concession agreement, it is interesting to have an accurate estimation of the value of an option such as a minimum traffic guarantee. The government has ceded that option, and the methodology developed in this paper could be used as a basis for the accounting of the government’s liabilities stemming from the contract.

In any case, it is important for all the stakeholders in the concession agreement to have a clear understanding of the value of the risks involved. With respect to the traditional techniques of valuation, the method of real options is a powerful tool to suitably
quantify the value of the concession in such cases. In addition, it can also be a useful tool for policymakers to design “tailor-made” concessions.

The analysis developed in this paper shows that, by applying this methodology in a correct way, it can offer a feasible alternative to value projects with operational flexibility under uncertainty conditions, a situation that is common in highway concessions as well as in transportation infrastructure projects in general.

Notations
The following symbols are used in this paper:

- \( C_i \) = value of the option with maturity in year \( i \), with traffic as the state variable
- \( C_{i,T_i} \) = value of the derivative at the expiration date \( T_i \)
- \( D_i \) = payout of the \( Q_i \) asset
- \( dt \) = differential time interval
- \( dZ \) = increment of a Wiener process
- \( d\theta \) = differential change in traffic
- \( E(R_m) \) = market expected return
- \( H \) = value of a riskless portfolio
- \( I_i \) = concession income in year \( i \)
- \( p_i \) = toll rate in year \( i \)
- \( Q_i \) = value of the contingent claim asset \( i \)
- \( r \) = risk-free interest rate
- \( S \) = state variable
- \( t \) = time
- \( T_i \) = maturity time for the option \( C_i \)
- \( V_{\text{options}} \) = addition of the value of the \( n \) options \( C_i \)
- \( \alpha_S \) = growth rate of the state variable \( S \)
- \( \alpha_S^{\text{adj}} \) = adjusted growth rate of the state variable \( S \)
- \( \alpha_T \) = growth rate of traffic
- \( \alpha_T^{\text{adj}} \) = adjusted growth rate of traffic
- \( \beta_{\text{fin}} \) = beta of \( S \)
- \( \beta_{\text{project}} \) = beta of the cash flows for the concessionaire
- \( \beta_T \) = beta of traffic volume
- \( \delta \) = fictitious payout rate of traffic (state variable \( \theta \))
- \( \delta_S \) = payout rate of the state variable \( S \)
- \( \varepsilon_i \) = variable with normal distribution with mean 0 and standard deviation 1
- \( \Phi \) = function of accumulated probability of the standard normal distribution
- \( \lambda \) = price for the systematic risk of traffic
- \( \lambda_S \) = price for the systematic risk of the state variable \( S \)
- \( \mu_S \) = total expected return of \( S \)
- \( \mu_{\text{project}} \) = total expected return of the project
- \( \pi_i \) = level of guaranteed traffic in year \( i \), in percentage
- \( \rho_{\text{fin}} \) = correlation coefficient between the return of shares of quoted highway concessionaire firms and the market index
- \( \rho_{\text{sm}} \) = correlation coefficient between the state variable \( S \) and the market index
- \( \sigma_S \) = volatility of the state variable \( S \)
- \( \sigma_m \) = volatility of the market index
- \( \sigma_{\text{project}} \) = volatility of the cash flows for the concessionaire
- \( \sigma_T \) = traffic volatility
- \( \theta \) = traffic
- \( \theta_0 \) = initial expected traffic
- \( \theta_{\text{gi}} \) = guaranteed traffic in year \( i \)
- \( \theta_i^{\text{e}} \) = expected traffic in year \( i \)
- \( \theta_{T_i} \) = traffic at the expiration date \( T_i \)

References