A Productivity-Oriented methodology for Local Area Network Design in Industrial Environments

Rafael Estepa, Antonio Estepa* Thiago Cupertino

Department of Telematics Engineering. University of Sevilla.
C/ Camino de los Descubrimientos s/n.
41092 Sevilla. Spain

Abstract

Industrial plants use conventional Local Area Networks (LANs) to access a growing number of Client/Server (C/S) applications such as Customer Relationship Management (CRM) or Enterprise Resource Planning (ERP) which have a direct impact on organization’s productivity. These LANs are typically extended throughout the plant which makes them exposed to occasional accidents such as fiber breakages or power failures. Reliable Network Design (RND) problems address the design of minimum-cost topologies resilient to link failures up to a certain degree. However, RND problems fail to capture some parameters of practical importance for organizations such as productivity losses due to network outages, the time period for which the network design is expected to be operating, or the fact that not all nodes are equally important for productivity.

We propose a new approach to LAN topological design named Productivity-Aware Reliable Network Design (PA-RND) that takes into account the productivity associated to each node of the network, minimizing not only the CAPEX but also the expected cost attributable to network downtimes over a certain period of network operation. Results show that our PA-RND problem optimizes the LAN topological design obtaining better results than current network design problems such as reliability constrained network design (RCND), minimum spanning tree (MST) or minimum cost ring (MCR).

Key words: Network Topology Design, LANs, Reliability.

1. Introduction

Computer Networks have become critical assets in today’s public and private corporations which rely more and more on the proper operation of their Local Area Networks (LANs) and Internet connection for running their business. If users can’t access intranet or Internet application-level services such as CRM, ERP or email due to network-service outages, the organization’s productivity is affected to some extent. Therefore, network reliability should be carefully addressed when designing a network topology since higher redundancy is likely to have a positive impact in productivity after a certain period of network operation.

Industrial Plants illustrate well the trade-off between reliability and productivity. In addition to control-specific networks, industrial plants use con-
tional LANs extended throughout the plant for running operational applications such as automated storage, customers’ orders, etc. which have a direct impact in productivity. Typically, such network consists of a centralized optical 1-10 Gbps Ethernet-switched network where terminals, located ubiquitously in the plant, transmit or receive operational information to servers connected to a root node. As an example, Fig. 1 represents three possible network designs for a plant with 4 nodes that connect dissimilarly-distributed users to the root node where servers and the Internet connection are located. There are two types of users: type A users are running a pallet storage and retrieval software; type B users are running both CMR and ERP applications as they are taking and tracking orders from customers. Users suffer a network outage if links or node fail and, after trying a self-restoration mechanism such as the Rapid Spanning Tree Protocol (RSTP) [1], no alternative paths are found to connect affected users with the root node. In such case, there is a penalty in terms of productivity which depends on the users affected (number and type) and network downtime.

Fig. 1. Topology designs with different reliability level: (a) MST (no redundancy), and (b), (c) with redundant links.

To further illustrate the dependency between productivity and topology, it is worthy to have a closer look to our three alternative network designs. Fig 1(a) shows the Minimum Spanning Tree (MST) topology which minimizes the CAPEX but does not have redundancy. If the link between node 1 and the root node fails, users from nodes 1 and 3 will suffer a network outage. This will have a penalty in terms of productivity. Figs 1(b) and 1(c) represent two alternative designs that exhibit the same reliability level (e.g. the same number of links and disjoint trees). Both topologies (b) and (c) withstand if a single link goes down, thereby keeping the productivity unaffected. Consequently, these topologies could be more profitable than (a) after a certain period of network operation due to saving in productivity penalties. Moreover, although (b) and (c) had the same reliability level (e.g. 0.99), one of them will be more profitable than the other after a certain period of operation (e.g. two years) due to differences in productivity savings as users are dissimilarly distributed.

In this paper we develop a new problem for network topological design termed Productivity-Aware Reliable Network Design (PA-RND) that addresses all the aspects described in the previous example. That is, to find the best topology in terms of expected profitability after a certain period of operation.

The remainder of this paper is organized as follows: Section 2 describes the network design problem faced. Section 3 provides a background on related works. Section 4 describes the cost model used in the objective function as well as its associated reliability model. Section 5 deals with the solution finding algorithm. Sections 6 present the results and its discussion and Section 7 concludes the paper.

2. Problem Description

The target scenario considered in this paper is (although not limited to) a medium-large private LAN where:

- The location and number of nodes are known.
- Delay and bandwidth within the LAN are not problematic for any productive application.
- It is possible to measure the probability of link breakage and node failure, which are considered independent events.
- There exists a centralized architecture (i.e., there is a central node through which servers are accessed). The reliability of the network is defined in terms of the availability of this central or root node to the rest of the nodes of the network. A network outage is defined as the situation in which

3. We assume that optical fiber is preferentially used in industrial environments due to large distances and electromagnetic noise. However, the models to be used in this paper can include a mix of different link types.

4. The main reasons to justify this assumptions are the high-speed of current 1-10Gb/s switched Ethernet with fiber links, the size of a LAN, the capacity of traffic prioritization of current Ethernet switches (IEEE 802.1p) and the fact that most productive TCP-based applications can tolerate delay and are elastic.

5. The root node is assumed to suffer no failures. This assumption is based on the fact that server rooms are typically well conditioned, secured and servers are clustered.
one or various nodes can’t find a valid physical path to reach the central node.

- When network outages occur due to nodes and/or links failure, the company’s activity is negatively influenced. The internal penalty due to productivity losses depends on the users affected at each node and the network downtime.

Our research has been conducted in an industrial plant where node and link accidents were somehow frequent due continuous works of expansion, and managers were concerned about significant productivity losses due to network outages. We think that this situation is of interest in any hazardous environment and therefore, generic enough to be applied in several other scenarios.

2.1. Objective Function

Let $G = (V, E)$ be a connected, undirected graph with $N = |V|$ nodes and $m = |E|$ edges which represent a network topology. We define the Overall Expected Cost (OEC) of a network $G$ as the cost of implementing the topological design (CAPEX) plus the expected cost attributable to network outages over a certain period of operation. Then our PA-RND problem consists of finding the best set of links $E$ that interconnects all nodes and minimizes the OEC of the network for a period of operation of $A$ years. This can be formulated as:

Minimize $OEC(G, A) = T(A) \cdot U(G) + C(G)$ \hspace{1cm} (1)

where $C(G)$ stands for the network CAPEX, $U(G)$ is the expected network downtime cost rate (i.e. cost rate of productivity losses due to network outages) and $T(A)$ is a factor that takes into account the expected period of network operation $A$ with an interest rate $f$ (i.e. $T(A) = A \cdot \sum_{i=1}^{A} [(1+f)^i]^{-1}$) to obtain the absolute cost due to network downtimes. Note that the time units of $A$, $f$ and $U$ should be the same.

Note that the two limits of the candidate topologies would be represented by the MST topology (minimum CAPEX but maximum expected productivity cost), and the full-mesh topology (maximum CAPEX and minimum expected network downtime cost).

3. Related Works

Topological Network Design (ND) problems have been profusely addressed on literature over the last two decades [2]. The generic problem addressed in ND is the selection of the subset of links that minimizes the capital expenditures CAPEX of a network subjected to one or more constraints, most notably, reliability [3–5].

A significant part of recent efforts have been focused on the design of carrier-level WAN optical-networks where multiple constraints such as capacity allocation [6], QoS [9] and availability at any layer [11] are expressed through a multi-layer model. Solutions to such complex problems are found using techniques such as protection cycles (p-cycles) [12,13] which are well suited to offer resiliency to one or dual link failures to the ring-like topologies of optical carrier-level WANs. However, carrier-level WAN problems are not necessarily practical for our target scenario. First, the layers of utility can be reduced to link-layer and physical layer (fiber) since current LANs speed (1-10Gbps) usually outperform classical C/S application requirements (e.g. bandwidth and delay) within the LAN for locally-delivered services. Second, the topology (i.e. centralized LAN with a root node) differs from those in carrier networks. Finally our reliability model and requirements do not pose any limitation to accidental failures of either links or nodes.

LAN design techniques have been traditionally focused in finding the Minimum-cost Spanning Tree (MST) and its related problems such as Degree-Constrained MST (CD-MST)[14,15] or Optimal Communication ST (OCST)[7]. However spanning trees solutions do not provide a reliable design but a minimum cost design. The Reliability Constrained Network Design (RCND) problem [4,16] seeks to minimize the CAPEX subjected to a reliability constraint. This can be formulated as [16]:

Minimize $C(G) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{ij} x_{ij}$ \hspace{1cm} (2)

subject to: $R(G) > R_0$

where $C(G)$ is the CAPEX of the network $G = (V, E)$, $c_{ij}$ is the cost of the edges connecting nodes $i$ and $j$, the $x_{ij}$ indicates whether the corresponding edge $e$ of $E$. $R(G)$ is the all-terminal reliability, that is the probability that the graph is still connected (even if some of the edges $e$ fail.) The estimation of reliability is typically done by either Monte Carlo simulations [4] or analytical models [19] which commonly consider a constant failure rate for network links [5]. The RCND problem has been solved us-
ing different techniques, most notably Genetic Algorithms (GAs) [5,28,16,18], and is the well-known network design problem that exhibits a closer similarity to our PA-RND problem. However, the RCND problem differs from our PA-RND problem in the following:

- **Objective Function.** The PA-RND do not include a reliability constraint in the objective function \(^6\). Moreover, the reliability concept is not used directly but indirectly as a sort of penalty function that rates the cost of unproductive network downtimes. Network failures involve operational costs (OPEX) such as repair \(^7\) and occasional penalties due to service interruption. These penalties can be due to external SLAs (in the case of operators [21]) or simply internal cost (in the case of a private LAN.) Nevertheless, the OPEX is not considered to be part of the RCND problem. OEC can be though of as the sum of the expected OPEX and CAPEX and hence, we believe that minimizing the OEC is more practical and intuitive than using a reliability constraint.

- **Expected time period of network operation.** LANs are assets expected to be working for a period of time. Any redundancy inserted in the topological design can be thought of as an investment expected to save network downtimes - and hence, money - over such operational period. Thus, in our PA-RND problem we have included this factor \((A)\) in the optimization problem.

- **Reliability Model.** RCND problems use all-terminal reliability. That is, their reliability constraint \((\text{e.g. } R(G) \geq 0.99)\) expresses the probability that all nodes can reach all others. The PA-RND problem do not use all-terminal reliability but all-to-one which is adequate for a centralized LAN. When workers can’t access to servers, they can’t work properly, reducing their productivity. Consequently for us, the reliability of a node \(n \in V\) measures the probability that it can reach the root node \(0 \in V\) independently whether it reaches the rest of the nodes or not.

- **Reliability Parameters.** We develop an analytical model where we do not only take into account the link failure rate but also node failure rates. This will let us consider practical aspects such as capt-

\[ C(G) = \sum_{i=0}^{N-1} \sum_{j=i+1}^{N-1} a_{ij} |l_{ij}| + b_{ij} \]  

where \(a_{ij}\) is the link-cost per length unit, \(|l_{ij}|\) is the link length and \(b_{ij}\) is a fixed cost per each link which can include items such as node connectors, ports, etc. If a link \(l_{ij}\) does not exist its cost is zero. Note that our cost model allows to independently adjust the cost of each link since different link types (e.g. fiber or STP) can be more appropriate for different parts of the network due to cost or technical reasons.

\[ \text{ for the purposes of our optimization problem, the topology cost will depend exclusively on the cost of the links in } E \]  

6 Constraints such as delay or bandwidth could be potentially added as extensions to the PA-RND problem. Consequently, topologies that do not met these restrictions should be discarded.

7 The repair cost have not been included in the scope of this paper.
The coefficients $a_{ij}$ and $b_{ij}$ also allow outweighing existing links in a topology re-design or extension.

4.2. Expected Network Downtime Cost Rate: $U(G)$

We define the Expected Network Downtime Cost Rate $U(G)$ as the cost rate due to productivity losses attributable to network-service outages. Note that $U$ represents a cost rate relative to the time units in which $A$ is expressed (e.g. hourly, daily or annual). $U$ can be broken down into the cost rate regarded to each node in the network as:

$$U(G) = \sum_{i=1}^{N-1} \Gamma_i(G) S_i$$  (4)

where $S_i$ represents the estimated productivity cost rate\(^9\) caused by the lost of direct or indirect connectivity between node $i$ and the root node (i.e. node 0). This cost can be estimated in several ways. For example it could be considered the number users of activity between node $i$ and the root node (i.e. service outage), due to link or node failures.

Note that $S_i$ allows to capture the impact of each node in productivity, outweighing those nodes more important for a specific organization. Or, equivalently, our model let us specify the relative economic importance of the reliability of each node in the topological design problem, which is a novelty in reliable LAN design problems.

4.3. Analytical Model for Reliability Calculation

With the previous definition of $\Gamma_i(G)$, we can express the all-to-one network reliability function as the average probability that nodes can reach the root node:

$$\bar{R}(G) = 1 - \frac{1}{N-1} \sum_{i=1}^{N-1} \Gamma_i(G)$$  (5)

To obtain an analytical expression for $\Gamma_i$, we will assume that a maximum of $L_F$ links and $N_F$ nodes can fail simultaneously and independently. Then, we can define the probability of service outage for node $i$ through the law of Total Probability as:

$$\Gamma_i(G) = \sum_{l=1}^{L_F} p_{il}^{(l)} \gamma_l + \sum_{n=1}^{N_F} q_{in}^{(n)} \gamma_n - \sum_{l=1}^{L_F} \sum_{n=1}^{N_F} p_{il}^{(l)} q_{in}^{(n)} \gamma_l \gamma_n$$  (6)

where $p_{il}^{(l)}$ is the conditional probability of service outage for node $i$ given that $l$ links have failed simultaneously, $\delta_l$ is the probability that $l$ links fail simultaneously, $q_{in}^{(n)}$ is the conditional probability of service outage for node $i$ given that $n$ nodes have failed simultaneously and $\gamma_n$ is the probability that $n$ nodes fail simultaneously.

4.3.1. Probability of simultaneous failures in Links and Nodes

Let the random variable $S_{ij}$ represent the state of a link $t_{ij} \in L$ in terms of being operative ($S_{ij} = 0$) or not ($S_{ij} = 1$). We consider the length as the risk factor of a link. Consequently, $P(S_{ij} = 1) = \lambda \cdot |l_{ij}|$.\(^{10}\) Then, we can compute $\delta_l$ by simply adding the probability of those link states of interest (i.e. those states in which $\sum_{n=1}^{N} S_{ij} = 1$). For example, in a topology with 3 links (e.g. $\{l_{01}, l_{02}, l_{12}\}$), the probability of 2 simultaneous failures $\delta_2$ would add up the probabilities of states $\{S_{01} = 1, S_{02} = 1, S_{12} = 0\}$, $\{S_{01} = 1, S_{02} = 0, S_{12} = 1\}$, and $\{S_{01} = 0, S_{02} = 1, S_{12} = 1\}$.

An estimation of $\gamma_n$ can be computed according to a simple model of operation in which a node is operating over a time period $\alpha$ (Mean Time Between Failures) followed by a repairing period $\beta$ (Mean Time To Repair). Assuming that both periods are exponentially distributed and that we have the average of both values from previous observation (measurement tests can be found in [22]), it can be used a Binomial Distribution for the estimated rate $\gamma_n$ of $n$ node simultaneous failure occurrences:

$$\gamma_n = \binom{N-1}{n} \rho^n (1 - \rho)^{N-1-n}$$  (7)

where $\rho = \frac{\beta}{\alpha + \beta}$. Note that we are assuming that all nodes (except for the root) exhibit the same failure probability.

4.3.2. Conditional Probabilities $p_{il}^{(l)}$ and $q_{in}^{(n)}$

These conditional probabilities can be computed algorithmically by finding all the paths from node

\(^9\) Both $S_i$ and $\Gamma_i$ represent rates rather than absolute values (i.e. relative to the time units in which $A$ is expressed)

\(^{10}\) Note that since link failures are independent events $P(S_{ij} = 1, S_{kl} = 0) = P(S_{ij} = 1) \cdot P(S_{kl} = 0) = \lambda \cdot |l_{ij}| \cdot (1 - \lambda \cdot |l_{kl}|)$
i to the root node after link or nodes failures. To expand on the procedure followed, let us define the following notation:

- Let $\mathbf{t} = \{t_1, t_2, \ldots, t_N\}$ be the set of all possible sub-topologies which result from removing $l$ links $\in E$ (see Fig. 3). For a topology with $m = |E|$ links, the number of possible sub-topologies would be $\binom{m}{l}$.
- Let $s_i(t_j)$ a variable that represents the state of node $i$ in terms of having connectivity with node 0 or not (i.e. values 0 and 1 respectively) for the sub-topology $t_j \in \mathbf{t}$. As shown in Fig. 3.
- Let $\tilde{p}_j^l$ be the probability of occurrence for the sub-topology $t_j \in \mathbf{t}$. Note that $\tilde{p}_j^l$ is readily obtained as the product of the relative weight of the length of each link removed in $t_j$.

\[ p_i^{(l)} = \sum_{j=1}^{\binom{n}{l}} s_i(t_j^l) \cdot \tilde{p}_j^l \quad (8) \]

The probability $q_i^{(n)}$ is calculated in a similar way to $p_i^{(l)}$ with the only difference that the probability for each sub-topology does not depend on the link length but on the number of nodes that has failed simultaneously. That is, we compute all the possible sub-topologies which would result from removing $n$ nodes $\in V$. When we remove a node, we also remove all its associated links $\in E$ as shown in Fig. 4.

Since all nodes have the same probability except for node 0, we can express $q_i^{(n)}$ as:

\[ q_i^{(n)} = \sum_{j=1}^{\binom{n}{n-l}} s_i(t_j^n) \cdot \frac{1}{(N-1)^n} \quad (9) \]

For example for a sub-topology in which links $l_0$ and $l_2$ have been removed, the conditional probability would be $|l_0| / |l_2| = L_T$, where $L_T$ is the sum of the length of all the links in the topology.

Let $s(1)$ be the probability of occurrence for the original topology, hence $s(1) = 1$. Note that the number of potential topologies increases as 2$^{N(N-1)/2}$.

5. Solution search method for the PA-RND problem

Due to the NP-hardness of our problem [27], exhaustive search is only feasible in small networks and a search method has to be provided as a general way to find the best solution.

Genetic Algorithms (GAs) were introduced by Holland in [23] and have demonstrated considerable success in providing good solutions to a wide variety of complex optimization problems including network design problems [14,28,5,2,8,20]. The main steps followed by these kind of evolutionary algorithms are depicted in Fig. 5. An initial population of individuals evolves according to the rules of some artificial genetic operators such as selection, crossover and mutation. The individuals selection is done as a function of their relative fitness (as evaluated by a fitness function) and, after the steps of crossover and mutation, the new generation is supposed to have better individuals.

It is a well-known fact that the performance of GAs can be improved by developing problem-specific heuristics in their operators [7,14]. In addition to some theoretical studies [25,26,24], literature provide us with a number of experimental works that show guidelines about the type of GA heuristics that provide best results for a specific problem such as Steiner-tree problem (ST) [2], Degree-Constrained Minimum Spanning Tree problem (CDMST) [14,15,10], Optimal Communication
Spanning Tree problem (OCST) [7], and most notably, the Reliable Communication Network Design problem (RCND) [17,18,16,5]. The results from these GA-centric works and our own experimental work have guided us in the development of the following GA heuristics to find the (sub)optimal solution for our PA-RND problem.

5.1. Initial Population and Population Size

Creating an initial population prone to be fit instead of randomly-generated individuals improves notably the performance of the GA [17,7]. Consequently, instead of a random initial population, we create initial individuals which exhibit properties valuables to minimize the OEC (i.e. low-cost topologies.) Our heuristic procedure is derived from Prim’s MST and creates individuals which are trees that interconnects our $N$ nodes with the $N-1$ cheaper links. The algorithm adheres to the following steps:

(i) Create an initial MST topology $G_1$;
(ii) For each link $l_{ij} \in G_1$ do:

2.1 Exclude the link $l_{ij}$ from $G_1$
2.2 Create another MST topology without the link $l_{ij}$;

As a result of our procedure we will have $N$ different individuals: the original one plus $N-1$ different MSTs derived from it (see Fig. 6).

The population size should be related to the network size $N$ [17]. In fact, all related works use a population size $\geq N$. We use $N$ as constant population size. Experimental results show that our heuristic provides diversity in individuals, avoiding the need for a large random population and, in turn, avoiding unnecessary computation resources.

5.2. Selection

This operator is responsible for selecting the best population individuals whose characteristics will be transferred to the descendants after a crossover process. There is no universal agreement on the best selection method [30] although most of related works use elitist selection schemes [31] that rate the relative goodness of the fit such as ranking selection [15] or proportional reproduction [18,28]. We use proportional reproduction but instead of ranking directly by the output of the objective function (Eq. 1), we use a control parameter $\eta$ that avoids preferential parents selection which could result on premature sub-optimal solutions. The selection operator is given as follows:

$$\text{Sel}(OEC(G,A)) = (OEC(G,A))^\eta$$ (10)

5.3. Crossover

The crossover operator combines the characteristics of two parents to create two new individuals. Thus, there is a maximum of $N/2$ possible crossover operations for each generation although the actual number of operations will be randomly determined since for each couple a crossover operation will happen with probability $P_c$. There are several types of crossover operators among which uniform-type exhibit better performance [14]. In fact, most works develop uniform-type crossover heuristics [17,15,28,7]. However, there is no agreement on the optimal crossover rate $P_c$ which ranges from 0.4 [28] to 1 [7,16].
5.4. Mutation

The mutation operator adds randomness on the population. Exchange-type mutation heuristics are dominant in literature and yield better results than insertion-type [14]. Hence, we develop a exchange-type mutation where for each pair of nodes \((i, j)\) on each created descendant, if the link \(l_{ij}\) does not exist it is created with a probability \(P_m\) and, if it exists, it is eliminated with the same probability.

The mutation rate \(P_m\) found in literature is usually small ranging between 0.01 [16] and 0.4 [28]. In [7] the authors set a mutation rate of \(1/N\), which is also aligned with the theoretical findings [24](pag. 5-26) that GA performance is increased if the mutation rate is reciprocal to the chromosome length. We also found experimentally that optimal results were achieved by relating \(P_m\) to the reciprocal of the chromosome length (i.e. number of potential edges) and some other variables from our PA-RND problem such as the expected time period of network operation. The mutation rate \(P_m\) is finally defined as:

\[
P_m = \frac{2bA}{N(N-1)} \tag{11}
\]

Where \(b\) is a coefficient which allows to modulate \(P_m\), \(N(N-1)/2\) is the number of potential mutations and \(A\) is the expected period of network operation being considered in \(OEC(G,A)\).

5.5. Fitness Evaluation and Population Replacement

The fitness evaluation is based on the objective function \(\min OEC(A)\) and the population replacement is done replacing the entire population by the \(N\) better individuals from the present generation, including the newly descendants.

5.6. Repair Function

Most GA-approaches intended for solving the RCND problem [11,18,16,5] use a repair heuristic to deal with those newly created individuals who do not meet the reliability constraint of the fitness function. The AP-RND problem however does not have such reliability constraint and hence, we do not use a repair function. Since \(G\) should be connected graphs by definition, we simply eliminate new individuals who are not connected after the crossover or mutation. We have checked that the percentage of new individuals born at each generation who does not meet this constraint is always below 1%. This can be traced back to the low rate applied to mutations and the own process of crossover. As suggested in [16], eliminating these defective individuals can be justified since their proportion is very small and improves time complexity in the calculation.

6. Computational Results

This section shows the computational results of the GA defined in previous Section for solving the AP-RND problem. We will start with a brief comment about the performance and complexity of the GA. Then, a comparison is made between our PA-RNC problem and other state-of-art problems for LAN topological design. Finally, an analysis will be made to show the sensitivity of the OEC to the parameters \(A\) and \(S\).

The general parameters for all simulations include the following. All the network scenarios are created over a square area of \(1Km^2\). On all simulations we have considered that the maximum number of simultaneous link and node failures is \(L_F = 4\) and \(N_F = 2\) respectively. The reliability parameters of nodes are set to: \(\alpha = 90\) days and \(\beta = 1\) day. The rate of link failures per time and length is set to 1 failure per 180 days for each 100m fiber link: \(\lambda = 0.00005556\). The cost per link length \(a_{ij}\) is set to 30 and a constant connectivity cost \(b_{ij}\) of 366 will be set for all links \(\in E\). The interest rate \(f\) is set to 0. The chosen parameters values are based on field-experience in an hazardous industrial plant. The general GA-related parameters used are selection \(\eta = 0.01\), crossover rate \(P_c = 0.6\), mutation coefficient \(b = 2\) for small networks (i.e. \(N \leq 8\)) and \(b = 1/4\) for \(N \geq 10\).
6.1. Expected performance of the GA

The performance of a GA includes both convergence (i.e. number of generations \( g \) until the improvement of the fitness function stalls) and the quality of the solutions (i.e. how far are the solution provided by the GA from the absolute optimum). The latter is shown in Table 1, where we evaluate the difference between the (sub)optimal solutions found by the GA after 200 generations and the absolute optimal found by exhaustive search, for different network sizes. Results show average values for 30 runs over 30 scenarios (i.e. randomly-located nodes), its standard derivation (\( \sigma \)), and the value of \( OEC(A) \) found for \( A \) values of 1, 2 and 3 years. CPU simulation time is also shown for each result.\(^{13}\)

Fig. 8 shows the convergence dynamic exhibited by our GA for different network sizes.

Our tests lead to expect a good performance from the heuristics defined in the GA. However, we are not showing a direct performance comparison with other GAs in related works since the problems solved (e.g. RCND) are different from AP-RND and our goal is not focused in GA performance.

The AP-RND problem is NP-hard [27]. In a first approach, the time-complexity [34] of the GA developed can be measured by the number of fitness function evaluations done during the course of a run [33]. The convergence shown in Fig. 8 let us set a maximum of \( g=60 \) generations for the rest of the results shown in this Section. The complexity exhibited by the fitness function (i.e. the complexity of the AP-RND problem) is \( O(m^2N^2 \log^2(m)) \), where \( m \) is the number of links and \( N \) is the number of nodes of

\(^{13}\)All simulations were computed on a Intel(R) Xeon(R) CPU X3210 @ 2.13GHz (4 GB cache) with four processors and 4 GB of RAM.

---

![Fig. 7. Illustrative crossover process.](image1)

![Fig. 8. Convergence of the GA.](image2)

---

**Table 1**

<table>
<thead>
<tr>
<th>Network Size (N)</th>
<th>A (years)</th>
<th>Difference From Optimal (%)</th>
<th>( \sigma ) (%)</th>
<th>OEC (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>3.76</td>
<td>3.60</td>
<td>36.919</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.94</td>
<td>1.75</td>
<td>71.441</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.85</td>
<td>0.68</td>
<td>10.9254</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3.67</td>
<td>3.19</td>
<td>41.709</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.10</td>
<td>0.94</td>
<td>79.654</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.69</td>
<td>0.48</td>
<td>117.164</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>3.27</td>
<td>2.30</td>
<td>47.430</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.52</td>
<td>0.76</td>
<td>91.292</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.65</td>
<td>0.41</td>
<td>141.110</td>
</tr>
</tbody>
</table>
the network. Therefore it exhibits lower complexity than the RCND problem (i.e. exponential) which is the network design problem more similar to our PA-RND. This can also be experimentally confirmed from the CPU time column in Table 2.

6.2. Goodness of the AP-RND problem

The AP-RND problem should be compared with other state-of-art network design problems to judge the goodness of our proposal. For this comparison we have chosen well-known LAN design problems such as Minimum-cost Spanning Tree (MST), Minimum Cost Ring (MCR) [35], and most notably, the RCDN problem [16]. The implementation done for for the MST follows the Prim’s algorithm while the other problems follow the cited references. The figure of merit used for the comparison is the \( OEC(G,A) \) which would result from the topologies obtained with the MST, MCR and RCND problems.

Table 2 shows the results obtained for different network sizes of 10, 15, 20, 25 and 30 nodes. For each network size, it is presented the CAPEX \( (C(G)) \) of the solution obtained, the value of the expected network downtime cost rate \( (U(G)) \) and the evaluation of the OEC for \( A \) values of 1, 2 and 3 years. The results represent average values obtained in 30 runs over 5 randomly-generated scenarios. The productivity cost rate \( S_i \) used for each node was \{5,000, 3,000, 1,500, 2,000, 500, 900, 1,000, 1,200, 2,000, 1,500, 600, 1,500, 800, 200, 1,000, 1,000, 500, 400, 1,200, 2,500, 300, 1,000, 1,500, 400, 500, 1,700, 2,000, 1,300, 400, 500\} for all \( i \in V \).

Results show that minimum CAPEX is obtained by the MST problem, followed by MCR. This is expected since MST and MCR topologies only have \( N − 1 \) and \( N \) links respectively. However, the topologies \( G \) obtained with those problems tend to exhibit the highest unavailability cost rate \( U(G) \) which can be traced back to the poor or null redundancy provided by those problems. The RCND problem have been used with two different reliability constraints: \( R(G) \geq 0.95 \) and \( R(G) \geq 0.99 \). In both cases, the solutions provided by the RCND problem exhibit the highest CAPEX but also expected unavailability cost rates lower than the MST and the MCR problems. Finally, the AP-RND problem is a good balance between CAPEX and \( U(G) \). It provides better CAPEX than RCND problems and expected unavailability cost lower than MST, MCR and RCND(0.95) problems. Taking the OEC of 1, 2 and 3 years as the figure of merit, the PA-RND problem exhibits an OEC average improvement of 15.5% for 1 year, 33.6% for 2 years and 41.9% for 3 years of network operation \( (A) \) with respect to the second best network design problem result. Note that CPU-time of the GA developed to solve PA-RND is in most cases better than the GA used to solve the RCDN [16], and that \( A \) increments have a negative effect in CPU-time since it produces more meshy topologies which affects negatively to the computation time of \( U(G) \).

6.3. Impact of the productivity losses \( (U(G)) \) and network operation period \( (A) \) in the network topology

To further illustrate how the AP-RND problem is sensitive to the productive use of each network node expressed in \( U(G) \), we have simulated two scenarios I and II with identical distance matrix (i.e. node distribution) but different downtime cost rates \( S_i \forall i \in V \) as specified in Table 3.

Table 3

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( S_i )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
<th>( 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>15.000</td>
<td>15.000</td>
<td>15.000</td>
<td>100</td>
<td>500</td>
<td>15.000</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>100</td>
<td>200</td>
<td>200</td>
<td>10.000</td>
<td>500</td>
<td>15.000</td>
<td>100</td>
</tr>
</tbody>
</table>

Fig. 9 shows the results obtained for scenarios I and II for 1, 2 and 3 years of network operation \( (A) \).

Table 4

<table>
<thead>
<tr>
<th>( A ) (years)</th>
<th>Scenario</th>
<th>OEC(G,A) (CAPEX)</th>
<th>C(G) (%)</th>
<th>OEC(G,A)/C(G) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>66.063</td>
<td>55.086</td>
<td>85.38%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>70.236</td>
<td>64.536</td>
<td>91.88%</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>70.512</td>
<td>68.262</td>
<td>96.8%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>78.694</td>
<td>72.072</td>
<td>91.58%</td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>78.960</td>
<td>78.558</td>
<td>99.49%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>80.082</td>
<td>75.378</td>
<td>94.12%</td>
</tr>
</tbody>
</table>

In the RCND problem we would only be able to specify the reliability constraint (i.e. 0.99), resulting in only one topology for both scenarios I and II. As shown in Fig. 9 and Table 4, the AP-RND problem yields two different solutions for scenarios I and II where those nodes with a higher downtime cost rate
<table>
<thead>
<tr>
<th>Network Size (N)</th>
<th>Problem</th>
<th>C(G)</th>
<th>CAPEX</th>
<th>$U(G)$</th>
<th>$OEC(A = 1)$</th>
<th>$OEC(A = 2)$</th>
<th>$OEC(A = 3)$</th>
<th>$\sigma$(%)</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>MST</td>
<td>44.004</td>
<td>270.867</td>
<td>314.871</td>
<td>585.738</td>
<td>856.606</td>
<td>9.37</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCR</td>
<td>53.118</td>
<td>11.431</td>
<td>64.549</td>
<td>75.980</td>
<td>87.411</td>
<td>8.94</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.95)</td>
<td>70.515</td>
<td>20.053</td>
<td>90.569</td>
<td>110.623</td>
<td>130.677</td>
<td>13.25</td>
<td>7.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.99)</td>
<td>82.270</td>
<td>5.121</td>
<td>87.391</td>
<td>92.512</td>
<td>97.633</td>
<td>14.15</td>
<td>7.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=1)</td>
<td>57.344</td>
<td>6.948</td>
<td>64.292</td>
<td>-</td>
<td>-</td>
<td>12.61</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=2)</td>
<td>63.640</td>
<td>3.418</td>
<td>-</td>
<td>70.476</td>
<td>-</td>
<td>12.25</td>
<td>5.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=3)</td>
<td>67.172</td>
<td>2.386</td>
<td>-</td>
<td>-</td>
<td>74.330</td>
<td>11.38</td>
<td>9.08</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>MST</td>
<td>47.160</td>
<td>377.354</td>
<td>424.514</td>
<td>801.868</td>
<td>1.179.223</td>
<td>6.22</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCR</td>
<td>55.920</td>
<td>25.090</td>
<td>81.829</td>
<td>107.739</td>
<td>133.649</td>
<td>6.13</td>
<td>10.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.95)</td>
<td>116.595</td>
<td>9.414</td>
<td>126.009</td>
<td>144.838</td>
<td>149.933</td>
<td>11.32</td>
<td>489.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.99)</td>
<td>110.798</td>
<td>134.777</td>
<td>138.855</td>
<td>142.933</td>
<td>149.325</td>
<td>11.32</td>
<td>489.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=1)</td>
<td>74.852</td>
<td>6.854</td>
<td>81.706</td>
<td>-</td>
<td>-</td>
<td>7.2</td>
<td>34.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=2)</td>
<td>83.797</td>
<td>3.065</td>
<td>-</td>
<td>89.927</td>
<td>-</td>
<td>12.25</td>
<td>244.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=3)</td>
<td>93.161</td>
<td>2.386</td>
<td>-</td>
<td>-</td>
<td>98.984</td>
<td>11.38</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>MST</td>
<td>67.722</td>
<td>709.745</td>
<td>777.467</td>
<td>1.487.213</td>
<td>2.146.958</td>
<td>6.57</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCR</td>
<td>77.718</td>
<td>54.969</td>
<td>132.687</td>
<td>187.656</td>
<td>242.626</td>
<td>6.13</td>
<td>28.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.95)</td>
<td>219.778</td>
<td>13.712</td>
<td>233.490</td>
<td>247.202</td>
<td>260.915</td>
<td>8.89</td>
<td>467.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.99)</td>
<td>233.284</td>
<td>3.350</td>
<td>256.635</td>
<td>263.336</td>
<td>282.75</td>
<td>9.98</td>
<td>788.32</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=1)</td>
<td>119.380</td>
<td>12.795</td>
<td>132.175</td>
<td>-</td>
<td>-</td>
<td>5.96</td>
<td>85.91</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=2)</td>
<td>130.957</td>
<td>5.742</td>
<td>-</td>
<td>142.441</td>
<td>-</td>
<td>6.36</td>
<td>285.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=3)</td>
<td>134.360</td>
<td>3.906</td>
<td>-</td>
<td>-</td>
<td>146.078</td>
<td>6.12</td>
<td>775.69</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>MST</td>
<td>86.529</td>
<td>695.357</td>
<td>781.886</td>
<td>1.477.244</td>
<td>2.172.602</td>
<td>6.57</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCR</td>
<td>98.514</td>
<td>107.119</td>
<td>205.633</td>
<td>312.753</td>
<td>419.872</td>
<td>5.79</td>
<td>49.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.95)</td>
<td>359.896</td>
<td>15.084</td>
<td>374.981</td>
<td>390.065</td>
<td>405.149</td>
<td>7.79</td>
<td>2.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.99)</td>
<td>377.878</td>
<td>2.295</td>
<td>380.173</td>
<td>382.468</td>
<td>384.763</td>
<td>5.27</td>
<td>5.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=1)</td>
<td>133.724</td>
<td>26.709</td>
<td>160.433</td>
<td>-</td>
<td>-</td>
<td>7.07</td>
<td>88.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=2)</td>
<td>147.066</td>
<td>14.834</td>
<td>-</td>
<td>176.734</td>
<td>-</td>
<td>7.31</td>
<td>371.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=3)</td>
<td>155.876</td>
<td>9.739</td>
<td>-</td>
<td>-</td>
<td>185.093</td>
<td>7.51</td>
<td>1.088</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>MST</td>
<td>112.407</td>
<td>1.340.501</td>
<td>1.452.908</td>
<td>2.793.409</td>
<td>4.133.910</td>
<td>3.61</td>
<td>3.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCR</td>
<td>132.123</td>
<td>193.504</td>
<td>325.627</td>
<td>519.130</td>
<td>712.634</td>
<td>6.05</td>
<td>103.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.95)</td>
<td>547.280</td>
<td>21.105</td>
<td>568.385</td>
<td>589.491</td>
<td>610.597</td>
<td>6.05</td>
<td>13.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RCND (0.99)</td>
<td>597.553</td>
<td>2.215</td>
<td>599.768</td>
<td>601.983</td>
<td>604.198</td>
<td>6.65</td>
<td>23.765</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=1)</td>
<td>178.438</td>
<td>41.125</td>
<td>219.563</td>
<td>-</td>
<td>-</td>
<td>6.50</td>
<td>284.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=2)</td>
<td>198.939</td>
<td>20.311</td>
<td>-</td>
<td>239.561</td>
<td>-</td>
<td>8.01</td>
<td>1.474</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PA-RND(A=3)</td>
<td>203.878</td>
<td>15.933</td>
<td>-</td>
<td>-</td>
<td>251.667</td>
<td>7.28</td>
<td>4.616</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Results for Medium-Large Networks (sizes from 10 to 30)

The reliability obtained as calculated from Eq. 5 ranged from 6% to 11%, with a trend to exhibit a higher redundancy. Moreover, although the two topologies from the AP-RND problem exhibited a similar network reliability. 

15 The reliability obtained as calculated from Eq. 5 ranged from 6% to 11%, with a trend to exhibit a higher redundancy. Moreover, although the two topologies from the AP-RND problem exhibited a similar network reliability.
solutions offered by the AP-RND are different since the topology adapts to the productive use of each node of the network, which is specific for each organization.

To conclude this section, we illustrate the influence of $A$ in the topologies through the results shown in Fig. 10 which shows topologies obtained for different network sizes and operation periods ($A$). It can be noticed that as $A$ becomes larger the topologies vary from a simple ring-like topology to a more meshy topology. The level of redundancy (and thereby, the CAPEX) increases with the expected period of network operation ($A$). This can be justified because $A$ weights the relative value of network downtimes (productivity penalties) over the CAPEX. This fact also reflects the trade-off between the CAPEX and the money savings due to the services outages avoided, which are more frequent on larger periods.

7. Conclusions and further work

This work addresses a comprehensive and practical proposal for reliable LAN design. Its main novelty is the definition of a new network design problem (AP-RND) which translates the impact of reliability into its economical effect within an organization through the concept of Overall Expected Cost. This allows the design of topologies that take into consideration the productive use of each node of the network, optimizing the expected profitability for the time period of network operation in which we want that the extra investment done in redundancy is paid off (a maximum R.O.I. of 3 years has been considered in this paper). We believe that considering the productivity-level impact in the network design is a significant step forward respect to current state-of-art LAN design problems.

Future research includes the consideration of repair cost in the $U(G)$ model and detailed study of optical-fiber networks, such as by-pass links and optical cross-connects, which are responsible for changes on the network routes. We also plan on doing further research on the applicability of our AP-RND problem to Cloud Computing applications.

References

Fig. 10. Examples of Topologies obtained for different network sizes (N=5 (a,b,c), N=10 (d,e,f) and N=15 (g,h,i)) and network operation periods of 1(a,d,g), 2(b,e,h) and 3 years(c,f,i).


