15th Triennial World Congress, Barcelona, Spain

A FUZZY LOGIC METHOD FOR AUTOTUNING A PID CONTROLLER: SISO AND MIMO SYSTEMS

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Abstract: In this paper a new method for autotuning SISO and MIMO PID fuzzy logic controllers (FLC) is proposed. The fuzzy autotune procedure adjusts on-line the parameters of a conventional PID controller located in the forward loop of the process. Fuzzy rules are based on the representation of human expertise on how can be the behaviour of gain and phase margins of a control system to efficiently compensating the system errors. Performance and robust stability aspects are assessed by practical and simulated examples.

Keywords: fuzzy control, PID control, multivariable feedback control, nonlinear systems, stability.

1. INTRODUCTION

For years the fuzzy logic control has proved its broad potential in industrial applications (Altrock and Gebhardt, 1996; Qin, *et al*., 1998). The fuzzy control theory has been applied to a number of systems with single-input and single-output (SISO) structures, mainly to overcome uncertain parameters and unknown models (Hu *et al*, 1999). Generally, fuzzy control shows good performance for controlling nonlinear and uncertain systems that could not be controlled satisfactorily by using conventional controller, for example, a conventional PID controller (Ying, *et al*., 1990). Also, in applications when there are multiple-input and multiple-output (MIMO) systems with strong loop interactions, conventional controllers do not work well and advanced control conceptions are required. Control literature of MIMO fuzzy logic controllers (MIMO FLC) shows limited results and a great effort has been used by researchers to derive stable control strategies. Usually, a MIMO FLC is tuned by trial-and-error that means a tedious and time-consuming task, and design techniques for systematic tuning must be obtained. Also, MIMO FLC applications are frequently solved by using the conventional decoupling theory and with single FLCs, resulting in high-dimensional rule-bases that may not be implemented in practical systems, due to required processing time (Nie, 1997).

In this paper a method for autotuning SISO and MIMO FLC is proposed. The autotune procedure adjusts on-line the parameters of a conventional PID controller located in the forward loop of the process.
In order to give an autotuning capacity to SISO and MIMO cases, a scheme of identification and sequential multivariable identification are implemented by using relay feedback (Wang and Shao, 1999; Luyben, 1990; Shen and Yu, 1994; Shiu and Hwang, 1998). Since the transfer function in each step of the sequential design has a mix of underdamping and overdamping behavior, a second-order plus dead-time structure is adopted as plant model. Also, fuzzy rules, employed to determine the set of PID gains, are based on the representation of human expertise on how must be the behavior of gain and phase margins of a control system to efficiently compensating the system errors. In both SISO and MIMO cases, gain and phase margins are determined by a set of Mandani rules and the membership function of the fuzzy sets are based on the system error and its difference.

Performance and robust stability aspects are assessed by practical and simulated examples of SISO and MIMO systems. Simulation results for other conventional control algorithms are also included for comparison purpose. The proposed control scheme offers advantages over the conventional fuzzy controller such as: *i*) a systematic design is attained in both SISO and MIMO cases; *ii*) it is necessary only one rule base for all loops; *iii*) the tuning mechanism is simple and control operators can easily understand how it works; and *iv*) it is completely autotuned, requiring only one relay feedback experiment per loop.

2. AUTOTUNING OF FUZZY PID CONTROLLER: SISO CASE - FPID-SISO

Since the proposed controller uses a nonlinear fuzzification algorithm and output membership functions, the controller can be considered as a nonlinear PID where parameters are tuned on-line based on error *e(t)* and change of error *de(t)* about a setpoint *r(t)*, as shown in Fig. 1. The system error is compensated by a set of fuzzy linguistic rules which are derived from the experience and knowledge of a control designer on how can be the behaviour of gain and phase margins for efficiently compensating the system error. In this sense the FPID-SISO can be interpreted as a fuzzy gain scheduling PID controller.

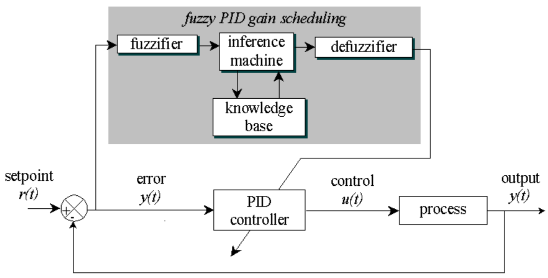


Fig. 1. Fuzzy logic controller system.

In order to implement the FPID-SISO, it is necessary to identify a process model and to design a conventional PID control as a starting point. After that, the fuzzy engine is designed.

2.1 Tuning phase: identification and initial PID parameters

An approximated model for the process is considered. A second-order transfer function with time-delay is usually enough for practical systems and is given by

$$G_p(s) = \frac{e^{-sL}}{as^2 + bs + c} \tag{1}$$

where *a, b*, *c* and L_d are unknown parameters and they need to be determined by a feedback relay experiment. Depending on *a*, *b* and *c*, the model may have real or complex poles and it is representing both monotonic and oscillatory processes. Under relay experiments, the parameters *a*, *b* and *c* are given by the following equations

$$c = \frac{1}{K_p}; \quad b = \frac{sin(\omega_u L)}{\omega_u K_u}; \quad a = \frac{c + \frac{\cos(\omega_u L)}{K_u}}{\omega_u^2} \tag{2}$$

$$\omega_u = \frac{2\pi}{T_u}; \quad G_p(j\omega_u) = -\frac{1}{K_u} = -\frac{\int_0^{t_u} y_r(t)e^{-j\omega_u t}dt}{\int_0^{t_u} u_r(t)e^{-j\omega_u t}dt} \tag{3}$$

where, $y_r(t)$ and $u_r(t)$ are process and relay output, respectively. K_u and ω_u are process critical gain and frequency, respectively.
Literature shows many methods for tuning PID control. In this paper, the initial PID parameters are determined by considering the transfer function of the form (Wang and Shao, 1999)

$$G_c(s) = k\left(\frac{As^2 + Bs + C}{s}\right) \tag{4}$$

where $A=K_d/k$, $B=K_c/k$, $C=K_i/k$ and (K_c, K_i, K_d) are the PID gains.

Zeros of the controller are chosen to cancel the poles of the process model, *A*=*a*, *B*=*b* and *C*=*c*. Then, the following relationship holds

$$G_p(s)G_c(s) = \frac{ke^{-sL}}{s} \tag{5}$$

where *k* is obtained by considering the gain (A_m) and phase margin (Φ_m) and the gain crossover frequency (ω_g) and phase crossover frequency (ω_p). So, the following relation can be derived

$$\Phi_m = \frac{\pi}{2}\left(1 - \frac{1}{A_m}\right) \tag{6}$$

and PID parameters are

$$\begin{bmatrix} K_c \\ K_i \\ K_d \end{bmatrix} = \frac{\pi}{2A_m L}\begin{bmatrix} b \\ c \\ a \end{bmatrix} \tag{7}$$

A typical value for A_m, taking into accounting a conventional PID controller, is A_m=3, so that Φ_m=60°.

2.2 Autotuning fuzzy logic controller engine

The gain margin A_m and the phase margin Φ_m, Eq.(6) and (7), are considered linguistic variables which values are defined with respect to the same universe of discourse specified by human expertise about the operational knowledge of the process. It is assumed that the feedback system gain and phase margins are in prescribed ranges [$A_{m,min}$, , $A_{m,max}$] and [$\Phi_{m,min}$, $\Phi_{m,max}$], respectively. For convenience, values of A_m are normalized into a range between zero and one by the following linear transformation

$$A_m^{'} = (A_m - A_{m,\min})/(A_{m,\max} - A_{m,\min}) \tag{8}$$

$$\Phi_m^{'} = (\Phi_m - \Phi_{m,\min})/(\Phi_{m,\max} - \Phi_{m,\min}) \tag{9}$$

where $A_m^{'}$ and $\Phi_m^{'}$ are normalized gain and phase margins, respectively.
Values of $A_m^{'}$ are determined by a set of fuzzy rules of the form

If *e(t)* is A_i and *de(t)* is B_i then $A_{m,i}^{'}$ is C_i; i=1...n (10)

where $A_{m,i}$ is the gain margin for *i* rule, A_i, B_i and C_i are fuzzy sets on the corresponding supporting sets. The membership functions of these fuzzy sets for *e(t)* and *de(t)* are shown in Fig.2c. Fuzzy rule base sets are obtained from operator's expertise by using the step response of the process. Figs. 2a and 2b show an example of a desired time response and a fuzzy rules base.

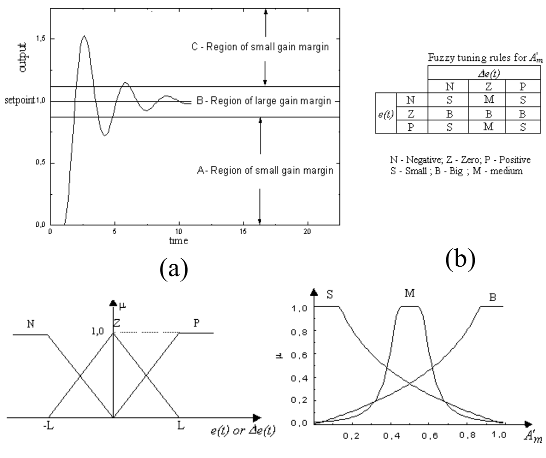


Fig. 2. (a) Plant response and fuzzy rule base sets; (b) rules base; (c) membership functions.

The fuzzy set C_i may be either Big or Small and it is characterised by logarithmic membership functions. The grade of the membership μ and the variable $A_m^{'}$ has the following relation

$$\mu_B(A_m^{'}) = -\tfrac{1}{\eta}\ln(1 - A_m^{'}) \tag{11}$$

$$\mu_M(A_m^{'}) = 1 - e^{-\left(\delta / |0.5 - A_m^{'}|\right)^{2.5}} \tag{12}$$

$$\mu_S(A_m^{'}) = -\tfrac{1}{\eta}\ln(A_m^{'}) \tag{13}$$

where, η and δ are adjustable parameters, in Fig. 2c η=4 and δ=0.1.

The truth value of the ith rule in Eq. (10) μ_i is obtained by the product of the membership function values in the antecedent part of the rule (Nie, 1997).

$$\mu_i = \mu_{Ai}[e(t)] \cdot \mu_{Bi}[\Delta e(t)] \tag{14}$$

Based on μ_i, values of $A_m^{'}$ for each rule are determined from their correspondent membership function. The implication procedure is shown in Fig. 3.

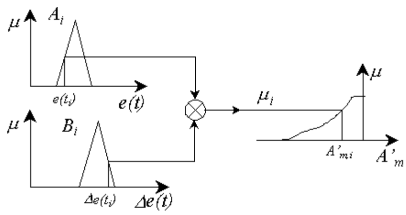


Fig. 3. Implication procedure of a fuzzy rule.

By using the membership functions in Fig.2c, the following condition holds

$$\sum_{i=1}^{n} \mu_i = 1 \tag{15}$$

Then, the defuzzification process, Fig. 3, yields

$$A_m^{'} = \sum_{i=1}^{n} \mu_i A_{m,i}^{'} \tag{16}$$

Once $A_m^{'}$ is obtained, A_m is calculated from Eq. (8) and PID parameters are derived from Eq. (7).

2.3 Simulation and experimental results for the FPID-SISO

The new fuzzy PID controller is now assessed for its ability to control nonlinear and time-varying plants, and to evaluate its performance in comparison with the corresponding PID control tuned without the fuzzy part of the algorithm. Four experiments are shown: three simulations and one practical system.
The first system to be tested is a second-order plus time-delay with the following transfer function

$$G_p(s) = \frac{1}{(2.5s+1)(3.75s+1)} e^{-2s} \tag{17}$$

Since it is known that the PID controller can deal with low-order linear systems with short dead-time and time-invariant, an error of 50% with variance of 0.01 is applied to the time-delay estimation for assessing the performance of the controllers. As shown in Fig. 4, FPID performs better than PID controller.

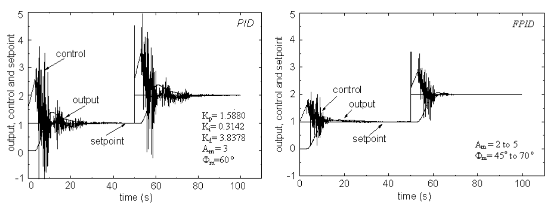


Fig. 4. Responses for (a) PID (left) ; (b) FPID (right).

Now, considering an error of 50% with variance of 0.01 in the time-delay of the system and a model rupture of 50% in the parameter *b*, the PID controller was not able to provide stability while the FPID still tracking the setpoint (there is a excessive overshoot in the setpoint response). Closed-loop results are shown in Fig. 5.

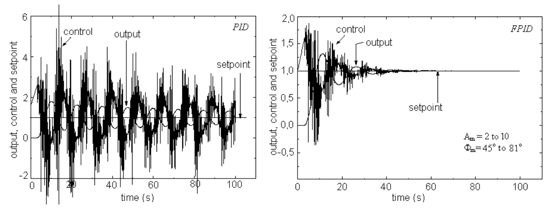


Fig. 5. Responses for (a) PID (left) ; (b) FPID (right).

Next, a nonlinear process given by Eq. (17) is simulated by using trapezoidal integration. The relay feedback experiment is used to tune the PID parameters around an operational point. Output, control and reference signals of the process when controlled by PID and FPID controllers are shown in Fig. 6. It is evident that the PID presents poor performance while a well damped setpoint response is achieved by FPID controller.

$$\dot{y}(t) = -y(t) + sin^2(\sqrt{|y(t)|}) + u(t) \tag{18}$$

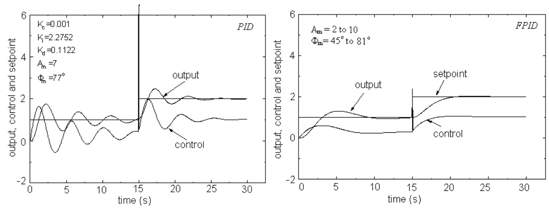


Fig. 6. Responses for (a) PID (left) ; (b) FPID (right).

In order to conclude monovariable performance tests, PID and FPID control approaches are assessed in a heating tunnel process implemented in the Department of Automation and Systems at the University of Santa Catarina. Details of the process are available in http://www.lcmi.ufsc.br/lcp/. The heating tunnel control system, shown in Fig. 7, is composed by a fan, a *DC* motor, a 50 cm long air duct with uniform transverse area, having on its right extremity an electrical heating resistance. The electrical heating part is driven by a power actuator circuit whose input is compatible with a *D/A* card. The temperature is measured by two sensors placed on the duct extremity. The hot air inside the duct is spread by the fan and the control problem is to regulate the temperature inside the duct (controlled variable) by actuating on the current through the electrical resistance (manipulated variable).

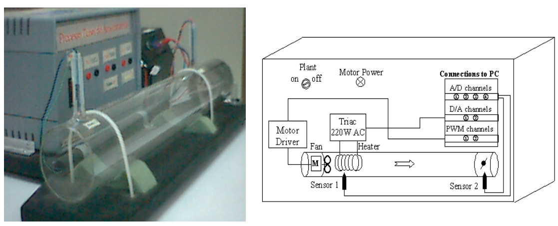


Fig. 7. Experiment Heating tunnel plant.

Step responses of FPID–SISO and PID controllers for different setpoints are given in Fig. 8.

Although both FPID and PID controllers give good control, FPID results is superior with overshoot, rise time and control variance minimum characteristics.

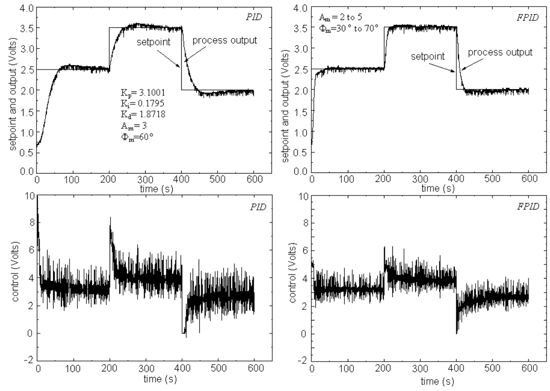


Fig. 8. Tunnel heating responses for PID and FPID controllers.

According to the set of locus of Fig. 9, $G_p(j\omega)$ does not enclose $-1/N(A,\omega)$ within $[\omega_1, \omega_2]$ so, by the circle stability criterion (Kin et al., 2000), the system is asymptotically stable.

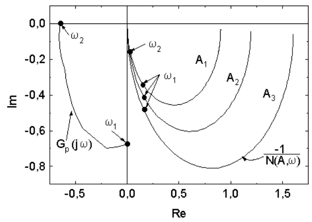


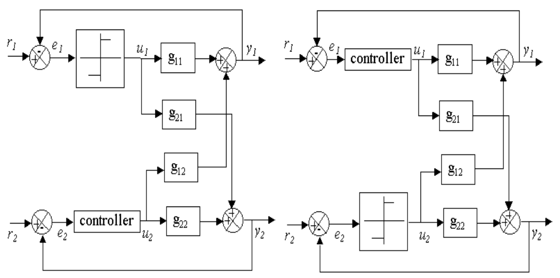
Fig. 9. Circle stability criterion, $G_p(j\omega)$ – heating tunnel Nyquist diagram; $N(A,\omega)$ – describing function of FPID controller; ω_1= 0.0305 rd/s; ω_2= 0.0528 rd/s.

3. AUTOTUNING OF FUZZY PID CONTROLLER: MIMO CASE - FPID-MIMO

With additional conditions, the proposed FPID-SISO controller, can be generalized for the MIMO case. As in the SISO case, the FPID-MIMO can be divided into two stages: identification and controller design phases. The identification phase copes with the auto-tuning capacity of the controller and to do so, a scheme of sequential multivariable identification is implemented. In the controller design phase, the main ideas applied to the FPID-SISO case is extended to the MIMO case. The reduced dimension of the rule base as well its simplicity correspond to the most important features of the proposed fuzzy scheme. In order to asses the FPID-MIMO performance the Wood-Berry distillation column (Luyben, 1990) is evaluated. Simulations have shown that the controller is capable of providing good overall system performance.

3.1 Identification phase

In this paper a sequential identification is implemented by utilizing a sequential relay scheme that treat the MIMO system as a series of SISO systems as shown in Fig. 10 for a 2x2 system (Luyben, 1990; Shen and Yu, 1994; Shiu and Hwang, 1998).



(a) (b)

Fig. 10. Identification under sequential relay.

In this kind of identification the transfer function $g_{ii,CL}$ (i=1,2) generally, has underdamped poles. So, the second-order plus dead-time model given by

$$G_{p,i}(s) = \frac{e^{-sL_i}}{a_i s^2 + b_i s + c_i} \quad , \qquad i = 1,\ldots,n \tag{19}$$

where a_i, b_i, c_i and L_i are unknown parameters to be determined, is used. Depending on a_i, b_i and c_i, the model may have real or complex poles. Hence, it is suitable for representing both monotonic and oscillatory processes like that in the Eq. (19).

The model parameters are determined from sequential relay experiment (Shen and Yu, 1994) and given by

$$c_i = 1/k_{p,i} \tag{20}$$

$$a_i = \frac{c_i - real[e^{-j\omega_{u,i}L_i} \big/ G_{p,i}(j\omega_{u,i})]}{\omega_{u,i}^2} \tag{21}$$

$$b_i = \frac{imag[e^{-j\omega_{u,i}L_i} \big/ G_{p,i}(j\omega_{u,i})]}{\omega_{u,i}} \tag{22}$$

where the index i is the loop number and ω_{ui} is the crossover frequency

3.2 *Autotuning multivariable fuzzy logic controller engine*

The multivariable fuzzy logic controller proposed in this paper utilizes fuzzy rules to determine the set of PID parameters. As in the SISO case, control signals in the MIMO case are generated by PID controllers, Fig. 11.

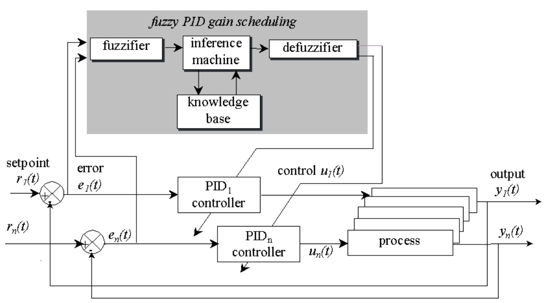


Fig. 11. Fuzzy gain scheduling PID controller: MIMO case.

MIMO fuzzy rules are the representation of human expertise on how must be the behavior of the gain and phase margins of a MIMO control system to efficiently compensating the system errors.

If $e_i(k)$ is A_v and $\Delta e_i(k)$ is B_v then $A_{m,j}$ is C_v (23)
v=1...p and i=1...n

where $A_{m,i}$, is gain margin for i loop, A_v, B_v and C_v are fuzzy sets on the corresponding supporting sets, p is the number of fuzzy sets and n is the number of loops in the multivariable system. The membership function of the fuzzy sets $e_i(k)$ and $de_i(k)$ as well the rule base are generalization of the SISO case. MIMO PID parameters are determined to ensure adequate gain and phase margins to the system.

Considering that sequential design is addressed, the MIMO PID parameters are tuned as in a SISO case. Considering a two-input/two-output system, as example of a MIMO transfer function, PID MIMO controller can be written as

$$G_{c,i}(s) = k_i \left(\frac{A_i s^2 + B_i s + C_i}{s} \right) \qquad i = 1, 2 \tag{24}$$

where A_i, B_i, C_i and k_i are determined by the same SISO case strategy. MIMO PID parameters for a TITO system is given by

$$\begin{bmatrix} K_{c,i} \\ K_{i,i} \\ K_{d,i} \end{bmatrix} = \frac{\pi}{2LA_{m,i}} \begin{bmatrix} b_i \\ c_i \\ a_i \end{bmatrix} \qquad i = 1, 2 \tag{25}$$

where the parameters $A_{m,i}$ is a nonlinear variable determined by the fuzzy engine.

Design steps for the auto-tuning MIMO fuzzy controller are: *i*) tune a Ziegler-Nichols PID controller considering individuals loops; *ii*) if the control system is unstable, one step of a sequential design should be done to tune a PID controller for the MIMO process; *iii*) identify the MIMO system by performing a relay experiment in each loop while the other loops are under PID controllers designed in step one. A transfer function like that of Eq. (19) is obtained for each loop; *iv*) define the discourse universe to the fuzzy variables; *v*) specify the maximum and minimum values to A'_m. Typical values for A'_m ranges from 2 to 5 and is corresponding to phase margin between 30 to 45; *vi*) apply the MIMO fuzzy controller engine.

3.3 *Simulation results for the FPID-MIMO*

In this section the distillation column of Wood and Barry, for separating methanol and water is used for simulation purposes. The Wood and Berry (WB) column presents strong loop interactions and the transfer function model is given by

$$\begin{bmatrix} X_t(s) \\ X_b(s) \end{bmatrix} = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \tag{26}$$

Manipulated variables are reflux, *R(s)*, and steam flow, *S(s)*. Controlled variables are distillate and bottom compositions, $X_t(s)$ and $X_b(s)$, respectively. The interactor matrix is diagonal with a strong couple between the loops. According to step one of the proposed algorithm, Ziegler-Nichols PID (ZN-PID) (Luyben, 1990) parameters must be used as a pre-tuning and are given in Table 1. Parameters of the identified transfer function under relay experiments are shown in Table 2.

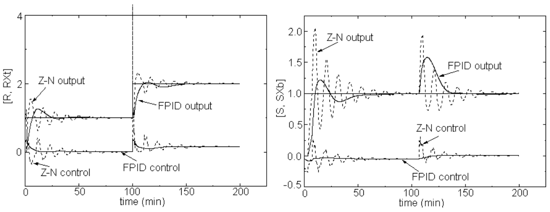
Table 1 – ZN pre-tune parameters.

	Parameters		
Loops	K_c	K_i	K_d
1	0.9450	0.2898	0.5545
2	-0.1960	-0.2177	-0.3175

Table 2 – Loop parameters for each transfer function.

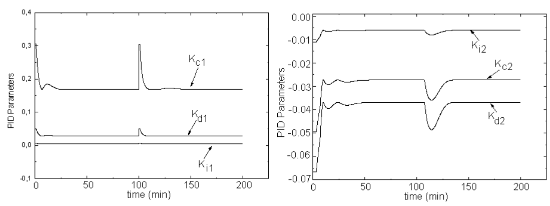
	Parameters			
Loops	a	b	c	L
1	0.0231	0.9487	0.1565	1.01
2	0.6222	0.4609	0.1032	2.98

Figs. 12 and 13 compare output, setpoint and control signals for WB column under ZN and FPID-MIMO control methods. Simulation results show that the FPID-MIMO is better than ZN method. For setpoint responses, the overshoot, the level of interactions and the settling time are improved with more stable responses.



(a) (b)

Fig. 12. Control, output and setpoint for WB column (a) pair R-Xt (b) pair S-Xb.



(a) (b)

Fig. 13. FPID-MIMO parameters.

4. CONCLUSION

A systematic method has been developed to design a fuzzy PID controller for SISO and MIMO cases. The method is based on gain and phase margin specifications and needs system identification under relay experiments. In the SISO case the gain is considered a fuzzy variable and in the MIMO case the sequential design is addressed in order to be possible the fuzziness of the gain and phase margins. The fuzzy PID controller derived successfully demonstrated better performance than the conventional PID controller for many case studies, particularly for nonlinear plants. The fuzzy PID controller is also able to tolerate many poor selections or inadequate implementation of the controller gains, for example, bad tuning for initial parameters of PID controller, which would make most conventional controller unstable.

The main goal of this paper is to provide to the plant operators with easy-to-understand fuzzy PID method for quickly achieving satisfactory control over unknown monovariable and multivariable systems. Despite its simplicity, the proposed method yielded monovariable and multivariable designs and a superior behavior to that resulting from empirical method based on trial-and-error procedure. The method has shown be adequated for practical applications, both in SISO and MIMO cases.

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