Optical wave breaking of pulses in nonlinear optical fibers

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A new effect appears in femtosecond optical pulse compression, using single-mode fibers, that we describe as optical wave breaking. In the fiber, frequency-shifted light in the leading and trailing edges of a pulse overtakes unshifted light in the pulse tails. Mixing of these overlapping frequency components generates sidelobes on the pulse spectrum. The effect often leads to computational instabilities, but careful numerical simulations, including fiber loss, give results in excellent agreement with experiment.

The remarkable successes of numerical solutions of the nonlinear Schrödinger equation in describing experiments on the propagation of high-intensity pulses in single-mode optical fibers, and the compression of those pulses in grating-pair compressors, have led various investigators (including some of the present authors) to gloss over certain curious features of the numerical results. The computational instabilities that we encountered in simulating some recent experiments with very high compression ratios have encouraged us to reexamine these features and have led to an increased understanding of the phenomena involved in the nonlinear propagation of pulses in single-mode fibers. We find that the combined effects of self-phase modulation (SPM) and group-velocity dispersion (GVD) can lead to effects that we describe as optical wave breaking. We show that the wave breaking is responsible for some previously unexplained features in experimental spectra. Wave breaking produces fine structure in the pulse tails, such that accurate simulation requires a much finer numerical grid spacing than is essential in its absence. We show that by using a grid spacing adequate to describe the wave breaking, and by including the fiber loss, our numerical simulations predict both frequency spectra and compressed pulse widths that are in excellent agreement with experimental results for the largest compression reported to date.

This agreement further extends the range for which the nonlinear Schrödinger equation has been shown to describe the nonlinear propagation of optical pulses in single-mode fibers.

The technique of using SPM in a single-mode optical fiber to produce a frequency-chirped pulse, which is then compressed in a grating-pair compressor, has been shown to be a useful tool for the production of ultrashort optical pulses and has been responsible for a succession of "shortest optical pulses." On the leading edge of the pulse, the SPM, which is a result of the intensity-dependent refractive index, gives a frequency decrease or red shift; the trailing edge of the pulse experiences a corresponding blue shift. When GVD and SPM act together (in this Letter we consider only the case of positive or normal GVD), the GVD acts on the increased spectral width caused by the SPM to increase further the temporal width of the pulse. This results in a quite rectangular temporal pulse shape and in a linear frequency chirp over most of the pulse. It has been shown that this is advantageous for pulse compression with a grating pair.

A previous numerical study determined the achievable compression and optimum fiber length for normalized pulse amplitudes up to $A = 20$ (see below for descriptions of the normalized parameters) and provided extrapolated estimates of these quantities for higher-amplitude pulses. The recent experiments by Johnson et al. used $\approx 35$-ps pulses (from a frequency-doubled Nd:YAG laser) with normalized amplitudes of $A = 170$. The observed compression of $80 \times$ was somewhat smaller than the $109 \times$ predicted by the extrapolations, although the difference did not seem surprising considering that the amplitude is almost an order of magnitude greater than the highest value analyzed in the previous study. Of greater surprise was the spectrum (see Fig. 1), which displays well-developed sidelobes. The present study was undertaken to try to identify the cause of these sidelobes.

For the present study we model the pulse propagation in the fiber by the nonlinear Schrödinger equation but with a linear loss term:

$$\frac{\partial u}{\partial (z/z_0)} = -i \frac{\pi}{4} \left[ \frac{\partial^2 u}{\partial (t/t_0)^2} - 2|u|^2 u \right] - \gamma u. \quad (1)$$

In Eq. (1) $u$ is the (complex) optical field amplitude variable, $z$ is distance along the fiber, and $t$ is time in a coordinate system moving at the average group velocity, such that at any position $z$ the center of the pulse is at $t = 0$. The normalizing length, $z_0$, is defined in Ref. 1. The three terms on the right-hand side of Eq. (1) represent (respectively) GVD, SPM, and linear loss. The input pulse was of the form $u(z = 0, t) = A \operatorname{sech}(t/t_0)$,
Fig. 1. Experimental spectra of the input and output pulses from Ref. 3.

and the normalized pulse amplitude, $A$, is defined in Ref. 1.

The numerical procedures used to solve Eq. (1) for the output pulse from the fiber, and to model the action of a grating-pair compressor on that pulse, were identical to those described in Ref. 1. For the experiments of Ref. 3, the normalized input pulse amplitude was $A = 173$, and the normalizing fiber length was $z_0 = 8.88$ km, so for the 93.5-m fiber used the normalized length was $z/20 = 0.0105$. The fiber had a loss of 16 dB/km (at 0.53 μm), which corresponds to a loss coefficient $\gamma = 16.36$. (The 34.4-ps width of the input pulses corresponds to a normalizing time $T_0 = 19.5$ psec.) Except where noted, our calculations used these parameter values.

Our initial efforts to simulate the propagation of $A = 173$ pulses were unsuccessful, because the numerical integrations became unstable. Examination of the results showed that the instability began at a fiber length of about $z/z_0 = 0.0055$. In Fig. 2 we present calculated results for various fiber lengths. In Figs. 2(a)-2(c) the middle curve shows the intensity of the pulse as a function of time, the upper curve gives the instantaneous frequency (the time derivative of the phase of the pulse) as a function of time, and the lower part shows the frequency spectrum. In Fig. 2(a), at $z/z_0 = 0.0020$, the temporal shape of the pulse is only slightly broadened (the input pulse has a FWHM of 1.76 nanoseconds), and the instantaneous frequency function shows that there is now a nearly linear frequency chirp over most of the pulse width, and that steep transition regions have developed at the leading and trailing edges. In Fig. 2(c), at $z/z_0 = 0.0060$, we see the beginning of the structure that leads to the instability.

The steepening of the frequency function occurs because the red-shifted light near the leading edge of the pulse is traveling faster than, and overtaking, the unshifted light in the forward tails of the pulse (and vice versa on the trailing edge). In the absence of a dissipative mechanism, which could stabilize an optical shock, the shifted light will overrun the pulse tails, and thus in the leading and trailing regions the pulse will contain light at two different frequencies, which will interfere and generate new frequencies. We suggest that this phenomenon is somewhat analogous to the breaking of water waves and that it be described as optical wave breaking.

On the basis of the above analysis, we concluded that the instabilities in our initial simulations occurred because the wave-breaking phenomenon resulted in a structure on the pulse that was too fine to be described adequately by the mesh spacing used in those simulations. New simulations with a mesh spacing of $\Delta z/\lambda_0 = 3.125 \times 10^{-9}$ (about 4X smaller than used previously) gave the results in Fig. 3, which shows the instantaneous frequency function, the temporal pulse shape, and its frequency spectrum, for a length $z/z_0 = 0.01$, for a lossless fiber and for a fiber with a loss corresponding to the fiber used in Ref. 3. (For each fiber the simulation required about 24 min of processor time on a Cray-1 computer.) For both cases the simulation has remained stable, and the resulting pulse shape shows well-developed interference fringes on the leading and trailing edges. The frequency spectra clearly display the sidelobes that we were attempting to explain! More detailed examination of the results for shorter fiber lengths showed that the sidelobes in the spectrum are indeed associated with the occurrence of the interference effects. A convenient way to understand the generation of the sidelobes is to recognize that SPM can be described as a four-wave mixing phenomenon and that in the interference region the simultaneous pres-
We set out to extrapolate expressions of Ref. 1, and thus the major lossless fiber is very close to that predicted by the experiment. Note that the calculated compression for the small increase in the background on the compressed pulse is in excellent agreement with the experimental results (80X compression) reported for the interference effect, and including the fiber loss, numerical solutions of Eq. (1) are in excellent quantitative agreement with the experimental result is the loss in the fiber.

Simulations of the effect of a grating-pair compressor structure displayed in the calculated spectrum.) Since the experimental spectrum is an average over many pulses, we do not expect to see the fine structure displayed in the calculated spectrum.)

The spectrum for the fiber with loss [Fig. 3(b)] is in excellent agreement with the experimental spectrum of Fig. 1. (Since the experimental spectrum is an average over many pulses, we do not expect to see the fine structure displayed in the calculated spectrum.) Simulations of the effect of a grating-pair compressor on these pulses gave a compression of 98 for the lossless fiber and of 84 for the fiber with loss. Thus, by using a mesh spacing adequate to model the wave-breaking effect, and including the fiber loss, numerical solutions of Eq. (1) are in excellent quantitative agreement with the experimental results (80X compression) reported in Ref. 3. (The sidelobes on the spectrum result in a small increase in the background on the compressed pulse.) Note that the calculated compression for the lossless fiber is very close to that predicted by the extrapolated expressions of Ref. 1, and thus the major cause of the difference between that prediction and the experimental result is the loss in the fiber.

While we believe that we are presenting the first detailed description of the optical wave-breaking phenomenon, considerable evidence of it has already been published, including both the experimental results that we set out to explain and several theoretical (numerical) results. For \( A < 10 \) the wave breaking becomes significant only for fiber lengths greater than the optimum for pulse compression, and thus it is easy to ignore, but the interference fringes are quite evident in the data for \( A = 5 \) presented in Fig. 2 of Ref. 1 and more so in Fig. 1 of Ref. 10 for \( A = 20 \). The interference fringes are quite prominent in the numerical results of Lassen et al. for \( A = 39.06 \), despite the fact that they presented smoothed curves. (Using our numerical procedures, we have precisely reproduced the results of Ref. 9, but without smoothing.)

The interference fringes are prominent in the calculated temporal pulse shapes, but they have not yet been directly observed. Since the spectral bandwidth that contributes to the compressed pulse is approximately twice the frequency difference involved in the wave-breaking interference, the period of the interference fringes is approximately twice the width of the compressed pulse. This suggests that a cross correlation of the compressed and uncompressed pulses might have just enough resolution to show the interference fringes. Such experiments are in progress.

An important aspect of the present study is the demonstration of the enormous range of applicability of the nonlinear Schrödinger equation [Eq. (1)] for describing nonlinear pulse propagation in single-mode fibers. It is well known that each of the various terms in Eq. (1) represents the lowest-order approximation to the phenomenon that it is describing, and it is commonly assumed that higher-order terms will be significant for very high compression ratios and/or very short input pulses. Small discrepancies between experiment and theory have been carefully investigated in the hope of identifying higher-order terms. The present results show that the highest-compression-ratio experiments can be accurately described by Eq. (1) without invoking any higher-order terms. There is also some evidence that even in experiments resulting in the shortest optical pulses, the essential features can be described by Eq. (1) without invoking higher-order terms.

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References