Information Sharing in a Channel with Partially Informed Retailers

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While retailers have sales data to forecast demand, manufacturers have a broad understanding of the market and the coming trends. It is well known that pooling such demand information within a distribution channel improves supply chain logistics. However, little is known about how information-sharing affects wholesale pricing incentives. In this paper, we investigate a channel structure where a manufacturer and two retailers have private signals of the state of the demand. Our model identifies the presence of a pricing distortion, which we term the inference effect, when a manufacturer sets price to an uninformed retailer. Because of this inference effect, the manufacturer would like to set a low wholesale price to signal to the retailer that the demand is low. On the other hand, the manufacturer would like to set a high wholesale price so that he earns the optimal margin on each unit sold. Vertical information sharing benefits the manufacturer by eliminating the distortion caused by the inference effect, which is more profound in a channel whose retailer has a noisier signal. This result implies that when there is a cost associated with transmitting information, the manufacturer may choose to share information with only the less-informed retailer rather than with both.

Key words: channels of distribution; information sharing; retailing; game theory

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1. Introduction

There has been a great deal of attention in supply chain management on information-sharing initiatives. Initiatives, such as Collaborative Planning, Forecasting, and Replenishment (CPFR), involve the exchange of information among members of the distribution channel to improve supply chain efficiency. Much of the data that is shared concerns the state of consumer demand and is used by suppliers and retailers to improve forecasts and distribution planning. While much has been written about the logistic benefits for vertical information sharing, little attention has been paid to the strategic benefits. In this paper, we examine how pricing strategies are affected by the sharing of demand information within a distribution channel.

In an early application of CPFR, Warner-Lambert, makers of Listerine mouthwash, and Wal-Mart agreed to share demand forecasts six months in advance of the expected retail sale date with the intent to improve order accuracy and production planning (Seifert 2003, Småros 2003). The benefit commonly attributed to this and other similar initiatives\(^1\) is a more streamlined supply chain, which reduces inventories and lowers operating costs for all members of the supply chain.\(^2\) In this case, Listerine’s production scheduling was considerably smoother while Wal-Mart improved its in-stock position from 85% to 98% (Seifert 2003).

Such logistic benefits are now well known for pooling demand forecasts in a supply chain (Lee et al. 1997). In this research, we acknowledge these motivations and ask the next question: How does sharing information within a distribution channel affect a manufacturer’s pricing incentives? We take as given that a smoother supply chain can lower operational costs and allow a manufacturer to price more competitively. However, when controlling for these cost savings, our research shows that by sharing demand information with a retailer, a manufacturer may benefit additionally from higher wholesale prices.

The magnitude of this benefit for the manufacturer depends greatly on the characteristics of the distribution channel. Given that Warner-Lambert entered

\(^{1}\) Others include Continuous Replenishment Program and Efficient Consumer Response.

\(^{2}\) Manufacturers, for instance, enjoy smoother production schedules while retailers reduce inventories and avoid stockouts. See, for example, the 2002 CPFR Baseline Study from the Grocery Manufacturers of America.
into a sharing arrangement with the largest retailer in the world, it may also be tempting to reason that this potential benefit is largest with large retailers. Our model, however, indicates that this reasoning may not always hold. To illustrate, consider the following.

Apparel manufacturer VF, snack producer Nabisco, and Delta Faucet are other examples of suppliers who have benefited by implementing information exchange systems with the retailers who distribute their products. But, contrary to the example above, these suppliers did not initiate their information-sharing arrangements with dominant, power retailers like Wal-Mart, Target or Home Depot. Rather, they started exclusively with the smaller specialty retailers ShopKo, Wegmans, and TrueValue Hardware, respectively. This research investigates reasons why a manufacturer may, in some cases, find it optimal to exchange demand information with its low-volume retailer instead of sharing with a large-scale big-box retailer.

Our results rely on the notion that manufacturers and retailers all have different kinds of demand information. Retailers, for instance, have point-of-sale (POS) data, knowledge of their own merchandising efforts, and aggregate store measures. Moreover, the precision of this information may differ across retailers depending on the retailers’ size and location. Manufacturers, on the other hand, often have broader information about the market, consumer motivations, demographic patterns (Desrochers et al. 2003), and study the factors that affect sales of their own products (Blattberg and Fox 1995).

We interpret these data sources as providing the respective channel member a private signal about the state of demand. When information-sharing arrangements have been implemented, these signals can be pooled, yielding more accurate demand forecasts. But even when information-sharing arrangements have not been established, strategic choices permit inferences about other channel members’ signals. We illustrate, specifically, that a retailer may infer the manufacturer’s knowledge about demand through wholesale prices. A high wholesale price, for instance, tells the retailer that the manufacturer expects high demand. The retailer’s ability to use wholesale price to infer the manufacturer’s signal complicates the pricing problem facing the manufacturer. In particular, the retailer is more sensitive to (wholesale) price in the absence of information sharing. Thus the manufacturer, faced with the familiar channel inefficiencies of double marginalization, is induced to distort its wholesale price downward relative to the case with no demand uncertainty. We refer to the phenomenon that induces this distortion as the inference effect. The distortion is alleviated, however, when the manufacturer can directly communicate its signal to the retailer. We conclude that alleviating the adverse consequences of the inference effect is an incentive for a manufacturer to implement information-sharing arrangements with a retailer.

The magnitude of the inference effect, and, consequently, the benefit of vertical information sharing, depends on several factors. The degree of retail competition, the relative precision of a retailer’s stochastic signal, and whether competing retailers are already sharing demand signals are three important factors determining the benefits of vertical information sharing, which are explored in this research.

First, we investigate how the general competitive environment in which a retailer operates affects the impact of the inference effect. We might expect, for example, the inference effect to differ between a competitive discounter and a specialty niche retailer. As mentioned above, the inference effect stems from channel inefficiencies associated with double marginalization, and such inefficiencies are known to be less severe when retailers face strong competition. Consequently, our results suggest that a manufacturer may benefit more by sharing its information with retailers in less competitive environments.

Second, the relative precision of retailers’ demand signal directly affects the size of the inference effect. Volume of sales plays an important role in the accuracy of retailer information. Some retailers, because of their larger market share, have access to larger amounts of POS data than the others. In addition, a retailer’s investment in information technologies (ITs) is an important factor in determining the precision of that information. This paper shows that the adverse consequences of the inference effect are more pronounced for a channel in which the retailer has less precise information because the retailer is likely to rely more heavily on wholesale price for inferences about the state of demand. Thus, sharing information in these channels offers relatively more benefits than in channels with more accurate information.

Finally, we evaluate how the information possessed by one retailer affects the magnitude of the inference effect in a competing retail channel. We ask, for

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3 See Hill (2000), JDA Enterprise (2002), and VICS (1999) for accounts of these information-sharing initiatives.

4 A high wholesale price could imply many other things in addition to high demand, including competitive positioning, differences in costs, or differences in profit targets. However, when the retailer is updating its belief about demand, a high wholesale price increases the likelihood in a probabilistic manner that demand is higher.

5 For example, Wal-Mart holds 500 terabytes of data in its data warehouse (Beckham 2002). This database is second only to that of the U.S. government in size (Nelson 1998).
example, what is the manufacturer’s benefit of sharing information with Wal-Mart given that it is already sharing information with Wal-Mart’s competitor. Our model suggests that there may be diminishing returns to developing information-sharing arrangements with additional retailers. The implication of this result is that even if the cost of developing an information-sharing network stays constant, the payoff from this investment may depend on the number of channels with such a network already installed.

There are significant costs associated with implementing information-sharing arrangements in a given distribution channel. Despite the fact that the marginal cost of information transmission is virtually zero, there may also be large setup costs in building and developing communication technologies, solving incompatibility of IT systems between partners, buying forecast software, and training personnel. This implies that, despite the noted benefits of vertical information sharing, implementing a sharing arrangement in a given channel is not optimal unless the benefits exceed costs. While the investment in one channel may have spillover benefits in another, many of these costs are channel specific. Therefore, just because a manufacturer finds information sharing optimal in one channel, does not automatically mean it is optimal in a second. This cost structure implies that it is possible to see a manufacturer exchange demand information in selected retail channels, but not in all.  

Though the initial analysis focuses on one-sided information sharing—from the manufacturer to a retailer—supply chain collaboration in practice usually involves sharing information in both directions (Min et al. 2005). The lessons from the one-sided analysis serve as a benchmark for the subsequent analysis of two-sided sharing in which information is transmitted not only from a manufacturer to a retailer, but also from a retailer to a manufacturer. As indicated above, alleviation of the inference effect is most beneficial when a retailer has a relatively imprecise signal of demand. With two-sided sharing, the manufacturer would receive poorer information in return than if it had shared with the better-informed retailer. We provide conditions indicating when and with which retailer(s) the manufacturer gains the most benefit from a two-sided information-sharing arrangement. Then, we use the results of this analysis to derive testable hypotheses to determine whether alleviating the adverse consequences of the inference effect may provide an incentive for the manufacturers to share their information.

The subject of information sharing has attracted the interest of many academics in different fields. In marketing, Villas-Boas (1994) examines whether competing firms should share the same advertising agency, which can make a firm’s private information available to competitors. In accounting, Radhakrishnan and Srinidhi (2005) study the implication of channel members’ bargaining power on the implementation of information exchange (IE) arrangements. In economics, there is a literature on information sharing in oligopoly. This research (e.g., Novshek and Sonnenschein 1982, Vives 1984, Gal-Or 1985, Li 1985) investigates information-sharing incentives of firms engaged in horizontal competition. We distinguish our work from these by the fact that we investigate the incentive for information sharing in a distribution channel that involves vertical relations. Another strand of literature in economics that is directly related to our analysis is the literature on signaling (e.g., Spence 1973, Milgrom and Roberts 1982, Gal-Or 1987). Similar to the inference effect that arises in our analysis, in this literature, as well, the absence of direct communication among asymmetrically informed parties leads to distortions in their behavior.

As mentioned previously, there have been a number of studies on information sharing in the supply chain management literature, which identify benefits relating to improving logistics, alleviating order batching, and inventory shortages (e.g., Gavirneni et al. 1999, Cachon and Fisher 2000, Lee et al. 2000, Raghunathan 2001). However, these papers primarily use channel structures, in which there is no horizontal competition between the retailers, and information transmission is only from the retailer to the manufacturer. In addition, we focus on the role demand uncertainty plays in the pricing decisions of the manufacturer.

A relevant paper, Li (2002), incorporates horizontal competition to a vertical information-sharing model to examine the leakage effect of information sharing. This effect is caused by the reaction of competitors that are not involved in information sharing. However, in Li’s (2002) research as well, the downstream retailer transmits its information to the upstream supplier. He et al. (2007) and Niraj and Narasimhan (2002), on the other hand, investigate information transmission not only from the retailer to the manufacturer, but also from the manufacturer to the retailer. However, He et al. (2007) and Niraj and Narasimhan...
(2002) do not allow the retailers to infer the manufacturer’s private information from the wholesale price, and are thus unable to detect the channel distortions caused by the inference effect.

Our work is also similar in spirit to the growing literature on asymmetric channel structures. For example, Dukes et al. (2006) consider retailers that are asymmetric in terms of their retailing costs. In Geylani et al. (2007), the asymmetry is in retailers’ differential ability to dictate their terms to the upstream supplier, and in Raju and Zhang (2005), it is in price leadership. In the channel structure analyzed here, on the other hand, retailers differ in the quality of their demand signals.

As with much of the marketing literature on channels, our research relates to the manufacturer’s use of strategies to coordinate channel efforts. Calderaro and Coughlan (2007) and Jeuland and Shugan (1983), for instance, examine the use of sales force incentives and wholesale pricing contracts, respectively, as a means to reduce channel inefficiencies, while Luo et al. (2007) proposes an approach to new product development that aligns channel interests. The manufacturer, in our setting, by contrast, is concerned with the impact of demand information on channel decisions.

Subsequent sections are organized as follows. Section 2 provides the basic modeling framework. Section 3 characterizes the equilibrium in a channel where a manufacturer considers transmitting his demand information to two asymmetrically informed retailers. Section 4 extends this model to incorporate bilateral information exchange. Section 5 proposes two hypotheses to test whether alleviation of the inference effect is a benefit from vertical information sharing, and §6 includes concluding remarks and extensions. Finally, most of the technical details are provided in the appendix to this paper. The remaining details are contained in a Technical Appendix available on the Marketing Science website at http://mktsci.pubs.informs.org.

2. Modeling Framework

Consider a channel with two retailers who compete in the sale of a single product that is supplied by a manufacturer. Without loss of generality, assume that marginal costs for both the manufacturer and the retailers are zero. The retailers are symmetric in every aspect except the accuracy of their private information about the state of the demand. We make this assumption to focus solely on the implication of asymmetry in the accuracy of information of the retailers on the manufacturer’s incentives to share information. Similar to the retailers, the manufacturer also has access to private information about the state of the demand. This information may be shared within the channel and this decision is strategically made by the manufacturer.

We further assume that the manufacturer offers a uniform wholesale price $p^W$ to both retailers. Following the manufacturer’s wholesale pricing decision, the retailers $i = 1, 2$ compete on retail prices $p^i$. Each retailer faces the following linear stochastic demand:

$$q^j = \frac{a}{b + d} - \frac{bp^j}{b^2 - d^2} + \frac{dp^j}{b^2 - d^2} + u,$$

(1)

where $u$ is normally distributed with mean 0 and variance $\sigma$. All the channel members observe noisy signals of $u$. The signal observed by the manufacturer is $x_0$ and by the retailers are $x_1$ and $x_2$. Specifically, for $i = 0, 1, 2$, we assume $x_i = u + e_i$, $(u, e_0, e_1, e_2) \sim N(0, \Sigma)$, where $\Sigma = \text{diag}(\sigma, s_0, s_1, s_2)$. Notice that in this formulation, the noise in a signal is captured by $s_i$. The better-informed $i$ is the smaller $s_i$ is. We assume $s_i, i = 0, 1, 2$ and $\sigma$ to be known by all the channel members. The model’s information structure is illustrated in Figure 1.3

$^{3}$This is an equivalent form of $q^j = \alpha - \beta p^j + \gamma p^i + u$, with $\alpha > 0$ and $\beta > \gamma > 0$. We use the demand function given in (1) because of its nice boundary cases of no differentiation and local monopolies when $d = b$ and $d = 0$, respectively. The direct demand functions $q^j = a/(b + d) - bp^j/(b^2 - d^2) + dp^j/(b^2 - d^2)$ are equivalent to the system of inverse demand functions $p^j = a - bq^j - dq^j$, $i, j = 1, 2; i \neq j$. Note that when $d = 0$, the inverse demand reduces to $p^j = a - bq^j$, which is the schedule facing a monopolist. Hence, $d = 0$ can be interpreted as maximum differentiation between retailers. In contrast, when $d = b$, the inverse demand reduces to $p^j = a - b(q^j + p^i)$, implying that the price of $i$ declines at the same rate irrespective of whether retailer $i$ or $j$ expands its physical sales. Hence, $d = b$ corresponds to no differentiation between the retailers.

$^{9}$Notice that we assume perfectly correlated disturbances for the retailers $(u_1 = u_2 = u)$. It is also possible to assume market-specific disturbances $u_i$ for retailers $i = 1, 2$, and an overall market demand signal that is observed by the manufacturer in the form of the average of these signals: $(u_1 + u_2)/2$. For simplicity, our analysis is based on the perfectly correlated disturbances assumption, but the reader may bear in mind that incentives for information sharing revealed
The sequence of events and decisions are as follows. First, the manufacturer decides whether and with which retailer to share information. This decision involves investment in the infrastructure necessary to support the sharing of information with the selected retailers. This infrastructure may include ITs and personnel dedicated to the interaction with the retailers. Subsequent to the information-sharing decision, nature determines the demand signals $x_i$, $x_j$, and $x_k$ and channel members observe a subset of these realizations. Second, the manufacturer and retailers observe their private signals and any signals which they are privy to as determined by the information-sharing arrangement made in the first stage. Third, after signal realizations have been observed, the manufacturer sets the wholesale price. Last, the retailers choose their prices. The timing of this game is graphically depicted as a four-stage process in Figure 2.

We distinguish between two types of information-sharing arrangements: one sided and two sided. With one-sided sharing, information is transmitted only from the manufacturer to the retailer. Specifically, one-sided information sharing implies that a retailer is informed of the manufacturer’s private signal $x_i$. In §3, we develop the model in the case of one-sided information. Later, in §4, we extend the analysis to two-sided information sharing where information is transmitted not only from the manufacturer to the retailer, but also from the retailer to the manufacturer. Thus, in the two-sided case, a decision to share information with retailer $i$ implies that the manufacturer and retailer $i$ both observe $x_i$ and $x_j$ in stage 2.

The timing depicted in Figure 2 reflects the fact that the installation of communication infrastructure is a long-term decision, relative to pricing. The manufacturer must, therefore, commit to its information-sharing policy before observing its own signal.10 In addition, the assumed timing implies the manufacturer can adjust its wholesale price $p^w$ contingent on the demand signal $x_i$ that it observes. It follows that a retailer can draw inferences about the realization of the demand signal based on the observation of $p^w$,

by our model will be present as long as there is positive correlation between the individual market disturbances ($0 < \rho(u_1, u_2) < 1$). See the Technical Appendix at http://mktsci.pubs.informs.org.

10 An alternative timing, in which the manufacturer’s sharing decision is made after it observes his signal, yields the same basic results. See the Technical Appendix at http://mktsci.pubs.informs.org.

11 Previous literature has demonstrated that this environment leads to a unique Bayesian-Nash equilibrium with all parties following linear decision rules when decisions are simultaneous (see Radner 1962, Theorem 5; Basar and Ho 1974; Vives 1999, Ch. 8). Our channel setting, however, requires us to extend this result to a sequential setting.
the No-Sharing (NS) game, the manufacturer does not reveal its private signal to either retailer. If the manufacturer decides to share this signal with retailer \( i = 1 \) or 2, exclusively, then the parties play a Partial Sharing (PSi) game. Finally, it may share the signal with both retailers in which case we are in the Full Sharing (FS) game.

As will be shown, an uninformed retailer’s use of wholesale price to infer the manufacturer’s private signal on demand plays an important role in the choice of wholesale price, and, consequently, for the incentives to share information. The role this inference plays can be illustrated by a careful analysis of the NS scenario. Consider the subgame following the manufacturer’s decision in stage 1 not to reveal his private signal, as inferred by his price \( p^w \). Th

The manufacturer’s signal as inferred by his price \( p^w \).

That is, retailer \( i \) can, itself, evaluate the manufacturer’s decision rule, which has the assumed form

\[
p^w = f_0(x_0) = \alpha_0 + \alpha x_0,
\]

by solving for \( x_0 \) if \( \alpha \neq 0 \).\(^{12}\) Equation (6) implies that the retailers can perfectly infer \( x_0 \) from the wholesale price \( p^w \), since it is only the manufacturer’s demand signal that affects its pric

Equation (8) accounts for the fact that despite no information sharing, retailers can perfectly infer the manufacturer’s private signal by inverting the manuf

Retailers can be solved for the six unknown parameters \( \bar{D}_k \), \( k = 0, 1, 2 \) for \( i = 1, 2 \), where the \( \hat{\cdot} \) notation corresponds to the retailer’s optimal decision rule.

\({12}\) The condition \( \alpha \neq 0 \) ensures that the manufacturer’s decision rule is an invertible function of its private signal. In fact, it is intuitive that \( \alpha > 0 \), since the manufacturer will respond to a higher demand parameter with a higher price. As we show in Lemma 1, this is indeed the case in equilibrium.

\({13}\) Wholesale prices may depend on many other factors, including different production costs, competitive positioning, and profit targets. If all of these factors are deterministic, drawing perfect inferences about the stochastic demand signal \( x_0 \) is still possible. The additional factors are simply reflected in this case in the values of the coefficients \( \alpha_0 \) and \( \alpha \). However, if there is some stochastic element in determining the value of additional factors, retailers will be able to draw only imperfect inferences from \( p^w \). In particular, a higher value of \( p^w \) would imply that demand was likely to be higher only in a probabilistic sense (provided that \( \alpha > 0 \)).
The optimal $\hat{D}$’s are functions of exogenous demand parameters $a, b, d$, the distribution parameters $s_0, s_1, s_2$, and $\sigma$, and the coefficients $a_0$ and $\alpha$ of the manufacturer’s pricing decision rule. The algebraic details of this solution offer little insight and are thus relegated to the appendix (in proof of Lemma A.1).

At the second stage, the manufacturer chooses its pricing decision rule using the decision rules of the retailers derived above to maximize expected profits conditional on its own signal

$$E[\Pi_m | x_0] = E[(q' + q)p^w | x_0],$$

which can be rewritten as follows:

$$E[\Pi_m | x_0] = \left( \frac{2a - E[p | x_0] - E[p | x_0]}{b + d} + 2E[u | x_0] \right)p^w$$

$$= \left( (2a - (D_0^b + D_1^bE[x_i | x_0] + D_2^bE[p^w]) \right) \cdot (b + d)^{-1} + 2E[u | x_0])p^w.$$ (9)

The manufacturer’s expectations of retailers’ decision rules for prices have been substituted into the respective demand equations (1). The expected profit function, expressed above, reveals an adverse incentive for manufacturer’s wholesale pricing decision. To see this, first note that because the manufacturer cannot use nonlinear pricing rules, it is faced with the familiar channel distortions of double marginalization. Second, each retailer uses wholesale price to statistically assess the state of demand. As previously discussed, a higher $p^w$ indicates a higher state of demand (through retailers’ inference on $u$). Therefore, because of an inference effect, the marginal effect of wholesale price on output (retail sales) is exaggerated. In this sense, asymmetric information exacerbates channel inefficiencies associated with double marginalization.

To see this mathematically, observe in (9) each retailer’s reaction to $p^w$. The coefficients $D_1^2, D_2^2$ reflect the sensitivity of output with respect to wholesale price. In a model of no uncertainty, these coefficients are easily computed to be $b/(2b - d)$. However, with uncertainty and private information, $\hat{D}_1^2, \hat{D}_2^2 > b/(2b - d)$, which reflects greater retailer sensitivity to wholesale price. Therefore the manufacturer’s optimal wholesale price is lower than with full information. To put it differently, the manufacturer must distort $p^w$ downward to convey a lower state of demand and induce retailers to set lower retail prices. Note that in spite of the attempt of the manufacturer to convey information about a low state of demand, the retailers are never deceived. When the schedule of $p^w$ is strictly increasing, a separating equilibrium prevails, and there is a one-to-one mapping from $x_0$ to $p^w$. Hence the retailers can perfectly infer the value of $x_0$ from the observation of $p^w$. This suggests that the manufacturer may benefit if it could credibly commit to truthfully revealing its private signal by means other than through the unit price, $p^w$.

To determine the equilibrium wholesale price, we evaluate the optimization problem of the manufacturer. The FOC for the maximization of (9) with respect to $p^w$ leads to

$$p^w = \frac{1}{2(D_2^2 + D_2^2)} \cdot (2a - (\hat{D}_0^b + \hat{D}_1^bE[x_i | x_0] - \hat{D}^2 + \hat{D}_1^bE[x_i | x_0])$$

$$+ 2(b + d)E[u | x_0]),$$ (10)

where the manufacturer’s expectations of retailers’ signals can be expressed as follows (DeGroot 1970, pp. 51–55):

$$E[x_i | x_0] = E[x_i | x_0] = E[u | x_0] = \frac{\sigma x_0}{\sigma + s_0}. \quad \text{(11)}$$

Intuitively, (11) expresses the manufacturer’s posterior expectation of the retailers’ signals. As the precision of its own signal improves (a decrease in $s_0$), its expectation of $x_i$ gets closer to its own signal.

Using (10) to match the coefficients in (6) leads to two equations, which can be solved for the optimal $\hat{w}$ and $\hat{a}_0$ in terms of exogenous demand and distribution parameters, details of which are omitted from the main text, but found in the appendix. The solution ($\hat{w}, \hat{a}_0$), presented in Lemma 1, characterizes the manufacturer’s optimal wholesale price conditional on the realization of $x_0$ in the NS game. Lemma 1 also presents the manufacturer’s optimal wholesale price in the other information-sharing regimes.

In the partial information-sharing game, the manufacturer informs retailer $i$ of its demand signal $x_{0i}$ but does not inform retailer $j$. In the third stage, the manufacturer chooses its decision rule $p^w$ using decision rules of the retailers and given his own signal to maximize $E[\Pi_m | x_0]$. The main differences between the solution of this game and that of the NS game are in the conditional expectations and the linear decision rule retailer $i$ uses. Instead of the conditional

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14 If there are additional cost uncertainties affecting $p^w$, a higher $p^w$ implies a higher probability that demand is higher instead of the definite deterministic inferences arising in our model.

15 To make this commitment credible, the manufacturer may invest up front in costly infrastructure or personnel training that supports the collaboration with the retailers. However, this up front commitment is not necessarily monetary. It can be any action that binds the manufacturer to truthfully transmit its information. If, however, such credibility is not established, there is an incentive for the manufacturer to report untruthfully a signal lower than the true realization to further mitigate inefficiencies from double marginalization. (See Technical Appendix at http://mktsci.pubs.informs.org.)
expectations given in (8), retailer $i$ estimates $u$ directly from the manufacturer’s signal using (7) rather than inferring it from the wholesale price $p^w$. This softens the inference effect since retailer $i$’s expectation $E[x_i | x_r, x_o]$ does not depend on wholesale price $p^w$ as in (9). Retailer $j$, on the other hand, has not been informed of $x_o$ and infers it from $p^w$ by inverting the manufacturer’s decision rule as in (9). Hence the inference effect is present in the $PSi$ game, but not to the extent as in the $NS$ game.

Finally, in the $FS$ game, the manufacturer shares $x_o$ with both retailers, so that retailers’ inference about $u$ are independent of $p^w$. Therefore the inference effect is not present in the $FS$ game. The following lemma summarizes the derivation of the manufacturer’s decision rules in each $NS$, $PSi$, and $FS$ games and illustrates the impact of information sharing on wholesale pricing.

**Lemma 1.** The wholesale price selected by the manufacturer in the first stage is given as

$$p^w = y^{k}\left(a + \frac{(b + d)\sigma}{\sigma + s_j} x_o\right), \quad \text{for } k = NS, PSi, FS,$$

where

$$y^{NS} = \frac{4bd(\sigma + s_0)^2 s_i + \sigma s_0(\sigma + s_i)(s_j + s_i)}{2(4b^2 + b^2 + d^2) + \sigma^2 s_0^2(4b^2 - d^2)},$$

$$y^{PSi} = \frac{\left[(4b^2 - d^2)\sigma s_0^2 s_i + (\sigma + s_0)^2 s_i s_j (2b^2 + bd + d^2)\right]}{2(4b^2 + b^2 + d^2 + 2bd^2)s_i} + \frac{1}{2}\sigma s_0(\sigma + s_i)(8b^3 - 2b^2 d + bd^2) s_i \sigma^2 s_0^2,$$

$$y^{FS} = \frac{1}{2}, \quad \Delta_i = (\sigma + s_0)s_i + \sigma s_i.$$

It is directly shown that $y^{FS} > y^{PSi} > y^{NS}$, implying that as the manufacturer increases the extent of information sharing with the retailers, it raises also the overall level of the wholesale prices that it charges. Hence the inference effect has adverse consequences for the manufacturer’s ability to set a high price. In fact, the expected wholesale price is highest, $E[p^w] = a/2$, when the manufacturer shares information with both retailers. Otherwise, whenever both or one of the retailers does not have access to $x_o$, expected wholesale price is lower: $E[p^w] < a/2$ (since $y^{PSi}$, $y^{NS} < 1/2$ when $d < b$). Therefore the asymmetry of information that arises in our model forces the manufacturer to cut its wholesale price, on average.\(^{16}\) Whenever the manufacturer does not reveal directly its private demand signal, retailers draw indirect inferences about this signal from the wholesale price charged by the manufacturers. Specifically, a higher price is interpreted as a signal of an improved state of demand. Such inferences have the potential to exacerbate the problem of double marginalization that arises in the channel. To alleviate this problem, the manufacturer ends up cutting its wholesale price below the full information optimum. The extent of price cut depends, however, on the extent of competition between the retailers. When the competition between the retailers intensifies, which, in our model implies that the retailers become less differentiated, and the difference $(b - d)$ declines, the manufacturer does not have to cut the price to a very large extent. With a more competitive retailing market, the problem of double marginalization in the channel diminishes, thus reducing the adverse consequences of the inference effect. In particular, when $(b - d)$ is 0, which implies that the retailers are perfectly homogenous, the adverse consequences disappear completely, and $E[p^w] = a/2$ for all informational regimes, including $NS$ and $PSi$. In contrast, the problem of double marginalization is most severe when there is no competition between the retailers at all; namely, when $d = 0$. In this case, the adverse consequences of the inference effect are maximized, thus leading to a significant cut in the wholesale price below the full information level ($E[p^w] \ll a/2$, in this case, for the $NS$ and $PSi$ regimes).

We summarize the above insights in Corollary 1.

**Corollary 1.** The adverse consequences of the inference effect:

(i) are mitigated as the manufacturer raises the extent of information sharing with the retailers; namely,

$$p^w_{FS} > p^w_{PSi} > p^w_{NS},$$

(ii) are more severe the more highly differentiated the retailers are (the larger the difference $b - d$).

The adverse consequences of the inference effect\(^ {17}\) on the manufacturer’s pricing decision are similar to those derived in the signaling literature in economics. In this literature, similar distortions arise whenever asymmetrically informed parties interact without being able to credibly and directly share their private information. In Spence (1973), the distortions are characterized in the context of the interaction

\(^{16}\) It is interesting to note as well that in all regimes $E[p^w] \to a/2$ as $(s_i, s_j) \to 0$. This is driven by the reduced value of the wholesale price as a signal and suggests the manufacturer has an incentive to help the retailer acquire more precise information.

\(^ {17}\) It is interesting to note that the inference effect would have similar adverse consequences on the manufacturer if the uncertainty were about the slope of the demand rather than its intercept. Hence the manufacturer would be inclined to directly communicate with the retailer information about price sensitivity of the consumers instead of allowing the retailer to infer this sensitivity via $p^w$. (See the Technical Appendix at http://mktsci.pubs.informs.org.)
between an employer and potential job applicants. In Milgrom and Roberts (1980), the distortions arise in the context of the interaction between an incumbent monopolist and a potential entrant, and in Gal-Or (1987), the distortions are derived in the context of sequential play of firms in the marketplace.

Note that we have referred to the adverse consequence of the inference effect on wholesale price only. However, to evaluate the manufacturer’s expected benefit from the information sharing, we put the equilibrium pricing decision rules back into the objective function of the manufacturer for each of the informational regimes the manufacturer may choose in the first stage of the game. In Lemma 2, we state the expressions for those expected profits.

**Lemma 2.** The maximized first-stage expected profits of the manufacturer are

\[
E(\Pi_{m}^{NS}) = V \left\{ 2b(y^{NS})^2 + \frac{2b(b-d)(\sigma+s_i)y^{NS}}{4b^2 \Delta_j - d^2 \sigma^2 s_i^2} \right\},
\]

\[
E(\Pi_{m}^{PS}) = V \left\{ 2b(y^{PS})^2 + \frac{(2b+d)(b-d)(\sigma+s_i)y^{PS}}{4b^2 \Delta_j - d^2 \sigma^2 s_i^2} \right\},
\]

\[
E(\Pi_{m}^{FS}) = V \left\{ 2b(y^{FS})^2 \right\},
\]

where \( V \equiv \left[ a^2 + \frac{(b+d)^2 \sigma^2}{\sigma + s_i} \right] \frac{1}{(2b-d)(b+d)} \).

Using the expressions of Lemma 2, we can now establish our first proposition.

**Proposition 1.**

(i) \( E(\Pi_{m}^{FS}) > E(\Pi_{m}^{PS}) > E(\Pi_{m}^{NS}) \).

(ii) Let \( E(\Pi_{m}^{PS}) \) and \( E(\Pi_{m}^{PS2}) \) denote the expected manufacturer profits when sharing with retailers 1 and 2, respectively. Then, \( E(\Pi_{m}^{PS}) > E(\Pi_{m}^{PS2}) \) if \( s_i > s_2 \).

Proposition 1 states that if the manufacturer chooses to transmit information to exactly one of the retailers, it will choose the less-informed one. To understand why, recall that it is the reduction of the inference effect that benefits the manufacturer when sharing information. When the manufacturer is in the no-information-sharing regime and considers revealing \( x_i \) to exactly one retailer, it evaluates in which channel the inference effect is most severe. Equation (8) indicates that inferences on the demand component \( u \) are more sensitive to wholesale price \( p^w \) for the retailer with lower signal precision (i.e., larger \( s_i \)). Thus, losses because of the inference effect on wholesale price are greatest with retailer 1. If we interpret \( s_i > s_2 \) as saying retailer 1 is less informed than retailer 2, then the manufacturer has greater benefit from informing the less-informed channel member.

However, according to part (i) of the proposition, even if the manufacturer shares information with the less-informed retailer, it can further benefit from revealing its demand signal to the better-informed retailer as well. The question that remains, however, is whether this benefit is greater or smaller now that the competing retailer has been informed. This question is relevant in view of our earlier assertion that sustaining information sharing with each retailer may be costly, given the investment in physical and human capital it requires. If the benefit from information sharing with the second retailer increases after the first retailer is informed, it would imply that the manufacturer always chooses to inform both retailers if it is optimal to inform one. We show, however, in Proposition 2 that this may not be the case.

**Proposition 2.**

(i) \( [E(\Pi_{m}^{PS}) - E(\Pi_{m}^{PS2})] > 0 \) \( i = 1, 2 \).

(ii) \( [E(\Pi_{m}^{PS}) - E(\Pi_{m}^{NS})] - [E(\Pi_{m}^{PS2}) - E(\Pi_{m}^{PS})] > 0 \) \( i = 1, 2 \), if \( [(2b+d)\sigma_s / b][s_i (b-d) - b_s] - 2d_s s_i (\sigma + s_i) > 0 \).

Hence, if \( s_i > s_2 \) and \( d \) is sufficiently small, there are diminishing returns to information sharing.

Under the condition of Proposition 2(ii), the manufacturer may transmit its information exclusively to the less-informed retailer if the cost associated with sharing information is sufficiently high. Let \( c \) denote the cost of sharing information with any given retailer. The manufacturer will share its information with exactly one retailer if

\[
[E(\Pi_{m}^{PS}) - E(\Pi_{m}^{NS})] > c > [E(\Pi_{m}^{PS}) - E(\Pi_{m}^{PS2})].
\]

Thus, if there are diminishing returns to revealing information to the second retailer, given that one retailer has already been informed, the manufacturer may choose to share its information with only one retailer rather than with both retailers.\(^\text{18}\)

Proposition 2 states that when the retailers are sufficiently differentiated and the quality of their signals is markedly different, there are diminishing returns to revealing information for the manufacturer. Higher differentiation, and therefore relaxed competition between the retailers makes the double-marginalization problem severe, and the manufacturer is

\(^\text{18}\) If \( [E(\Pi_{m}^{PS}) - E(\Pi_{m}^{PS2})] < c \), the manufacturer does not share its information with any of the retailers, and if \( [E(\Pi_{m}^{PS}) - E(\Pi_{m}^{PS2})], E(\Pi_{m}^{PS}) - E(\Pi_{m}^{PS2}) > c \), it shares with both. If there are no diminishing returns to sharing information, the asymmetry (i.e., sharing with only one retailer) cannot arise.
tempted to reduce its wholesale price. However, when the well-informed retailer’s signal is of sufficiently higher quality, there is little gain for the manufacturer from sharing information with this retailer. The well-informed retailer relies only a little on its inferences from the manufacturer’s wholesale price when forming its expectations about the demand, and the manufacturer is not tempted to make significant price reductions. Therefore the manufacturer does not benefit much from sharing information with the well-informed retailer given that it already has shared with the less-informed one. This result, together with Proposition 1, implies that if the cost associated with information sharing is sufficiently high, the manufacturer may choose to share information with the less-informed retailer rather than both retailers.19

4. Two-Sided Information Sharing
The model of §3 involved one-sided information transmission. In this section, we extend this model to include bilateral IE. When the manufacturer decides to share information with a retailer, information is transmitted not only from the manufacturer to the retailer, but also from the retailer to the manufacturer. In this section, we show that the inference effect may still sway the manufacturer in the direction of sharing information with the less-informed retailer in spite of the fact that it receives less precise information in return.

To simplify the derivation, we restrict attention to the case that one of the retailers is perfectly informed, and the other does not have access to any valuable information about the state of the demand. Let retailer 1 be the perfectly informed retailer and retailer 2 be the perfectly uninformed retailer, then using our earlier notation, \( s_1 = 0 \) and \( s_2 \to \infty \). We now investigate the relative benefit to the manufacturer of sharing information with retailer 1 versus retailer 2. To evaluate this relative benefit, in Lemma 3, we derive the expected profits of the manufacturer from partial information sharing with only one retailer (retailer 1 or retailer 2).

**Lemma 3.** When \( s_1 = 0 \) and \( s_2 \to \infty \), the expected profit of the manufacturer from partial information sharing with only one retailer is given as follows:

\[
E(\Pi_{m}^{PS1}) = \frac{1}{(2b - d)(b + d)128b^6} \left[ a^2 + \frac{(b + d)^2 \sigma^2}{\sigma + s_0} \right] \cdot \left[ 8b^3 + d(2b + d)(b - d) \right] \frac{8b^3 - d(2b + d)(b - d)}{b^3 - d}. \]

From Lemma 3, we can calculate the net benefit, denoted by \( DD \), to the manufacturer from sharing information with the uninformed instead of the informed retailer, as follows:

\[
DD = E(\Pi_{m}^{PS2}) - E(\Pi_{m}^{PS1}) = \frac{[a^2 + (b + d)^2 \sigma][b - d][16b^4(b + d) + 2b^2d(b + d) - d^3(2b + d)]}{(2b - d)(b + d)128b^6} - \frac{[6b^2 - bd - d^2][2b^2 + bd + d^2](b + d)\sigma s_0}{(2b - d)32b^3(\sigma + s_0)}. \tag{12}
\]

Communicating with the uninformed retailer, retailer 2, instead of the informed retailer, introduces the following trade-off for the manufacturer. On the one hand, the benefits of alleviating the inference effect is higher with the uninformed retailer. On the other hand, the manufacturer gets worse information in return with this retailer. The expression for \( DD \) in (12) captures these counteracting incentives. The first term, which is positive, precisely measures the benefit accruing to the manufacturer when retailer 2 does not use wholesale price \( p^a \) to make inferences about the signal \( x_0 \). The second term, which is negative, reflects the opportunity cost of the manufacturer from not communicating with retailer 1 and learning the precise signal, \( x_1 \).

It is directly seen that the first term of \( DD \) dominates the second for relatively small values of the parameters of \( d \) and \( s_0 \). Hence, as the degree of differentiation between the retailers increases, and as the precision of the private signal of the manufacturer improves, the manufacturer is more likely to communicate with the uninformed retailer, if it chooses to communicate with only one retailer. As was pointed out earlier, the adverse consequences of the inference effect are especially pronounced when the retailers are highly differentiated because the problem of double marginalization is severe in this case. As well, when the signal of the manufacturer is relatively precise, the added benefit of obtaining more precise information from the perfectly informed retailer is relatively small.

When the values of the parameters \( d \) and \( s_0 \) are not sufficiently small, the manufacturer prefers communication with the informed retailer, retailer 1. In Proposition 3, we summarize the relative incentives of the manufacturer to communicate with the uninformed versus informed retailer.

**Proposition 3.** When there is two-way communication between the manufacturer and a single retailer and \( s_1 = 0 \) and \( s_2 \to \infty \), the manufacturer will communicate with the uninformed retailer (retailer 2) if the parameters...
Recall from §3 that the inference effect for a given channel is intimately related to the level of double marginalization. Our modeling framework provides a mechanism for controlling the extent of this double marginalization through the differentiation parameter $d$. By considering $d$ at 0 and at $b$, we see in Corollary 2 how double marginalization affects the relative trade-off of partial communication described in (12) by $DD$.

**Corollary 2.** With partial communication,

(i) when $d = 0$, the manufacturer will share information with the uninformed retailer if

$\frac{\sigma_s}{\sigma + s_1} < (a^2 + b^2\sigma)/3b^2$.

The latter inequality holds, in particular, when $s_1 \leq \sigma/2$.

(ii) when $d = b$, the manufacturer will always share information with the perfectly informed retailer.

Corollary 2, part (i) states a formal condition for when two-sided information exchange with the uninformed retailer is optimal for the manufacturer, as mentioned in the above discussion. Part (ii) of the corollary confirms the following intuition. When retailers are perfectly competitive ($d = b$), the double-marginalization problem does not exist. Therefore the manufacturer has nothing to gain from sharing information with the uninformed retailer because there is no inference effect. In terms of (12), the first term is zero, which implies that $DD$ is negative.

## 5. Detecting the Incentives for Information Sharing

As mentioned in the introduction, smoothing production and delivery schedules is a common benefit attributed to information sharing. Our model isolates a second benefit: the reduction of the inference effect. The comparative statics we conduct in our analysis allows us to derive two hypotheses to help us identify the possible existence of this second incentive.

The first hypothesis relates to the degree of retail competition in a given setting. Recall that on the one hand, the inference effect is less pronounced in highly competitive retail environments as competition reduces double marginalization. On the other hand, highly competitive retailers have incentives to cut operating costs to price more aggressively. One way to cut costs is to improve supply chain efficiency through information sharing. Thus we arrive at the following:

**Hypothesis 1.** In more competitive retail environments, there are fewer information-sharing alliances (two-sided information-sharing arrangements) between manufacturers and retailers.

If empirical research reveals an inverse relationship between the extent of retail competition and the incidence of information-sharing alliances, then it is likely that the inference effect is indeed present in addition to efficiency considerations. If, on the other hand, a direct relationship between those two variables is empirically observed, then improving supply chain efficiencies may be the dominating force behind the incentives to share information between manufacturers and retailers.21 (According to Proposition 3, although the inference effect may still be a factor, it is impossible, in this case, to distinguish it from supply chain efficiency considerations.)

A second hypothesis that is implied by our model offers another test to identify the possible existence of the inference effect. This hypothesis relates to the precision of the information that is available to the retailers. According to our analysis, the inference effect is more pronounced in channels with poorly informed retailers. Hence, when retailers make relatively small investments in IT, manufacturers may be more inclined to form information-sharing alliances with them to eliminate the distortions that arise because of the inference effect. However, when manufacturers are motivated by supply chain efficiencies, they gain more by forming alliances with well-informed retailers that make significant investments in information infrastructure. Hence we obtain the following hypothesis:

**Hypothesis 2.** In channels where retailers make large investments in ITs, there are fewer information-sharing alliances between manufacturers and retailers.22

If empirical research shows an inverse relationship between the extent of investments undertaken by the retailers and the incidence of information-sharing alliances, then it is likely that the inference effect plays a role in such initiatives.23 If, on the other hand, the

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21 The example of Delta Faucet that was mentioned in the introduction is consistent with Hypothesis 1. The fact that Delta Faucet chose to establish an information-sharing alliance with a niche retailer like True Value Hardware, and not with a large-scale big-box retailer like Home Depot, implies that Delta Faucet might have been motivated by the elimination of the price distortions caused by the inference effect. Niche retailers face moderate pressures to cut prices in comparison to the big-box retailers, thus giving rise to significant distortions because of the inference effect.

22 Note that both Hypotheses 1 and 2 assume that manufacturers use linear pricing. These hypotheses cannot be used in environments where nonlinear wholesale pricing is employed (see the related discussion in §6.2).

23 Note that retailers that can afford to make large-scale investments in IT may also carry greater channel power. The greater extent to which this additional channel power permits the retailer greater say in decisions made with the manufacturer, the further we depart from our assumption of the manufacturer as the Stackelberg leader.
relationship between those two variables is shown to be direct, then it is more likely that improving supply chain efficiencies is the dominating force in determining the incentives to form information-sharing alliances in retail channels.24 (Once again, Proposition 3 supports the possible existence of the inference effect in this case as well. With a direct relationship, it is impossible to distinguish, however, between this effect and the benefit of improving supply chain efficiencies.)

6. Concluding Remarks and Extensions

6.1 Summary of the Results

This paper investigates the nature of information-sharing arrangements in a distribution channel when retailers are asymmetrically informed. Specifically, we asked: Should a manufacturer, possessing key information about the state of demand for its product, share this information with the retailer better or worse informed than a rival? Indeed, casual intuition might suggest a retailer already poised with superior information will offer a better return on the manufacturer’s investment on an information-sharing arrangement. We showed, however, that this reasoning misses the role of retailers’ inferences on the state of demand communicated through the wholesale price set by the manufacturer. Accounting for what we called the inference effect, this research suggests that there may be larger benefits from sharing with the more poorly informed retailer. When the manufacturer does not share its private information about the state of demand, the inference effect forces the manufacturer to distort wholesale price downward to alleviate double-marginalization problems. This effect is present for either retailer. However, it is more pronounced the less precise a retailer’s own information is about demand conditions. Of course, there can be benefits from sharing information uniformly across all retail channels. But, if setting up and installing such systems is costly, then trade-offs arise.

We also investigated the benefits accruing to a manufacturer when it can acquire retailer-specific information from a sharing arrangement. This complicates the above trade-off because exclusive information exchange with the less-informed retailer, offers the manufacturer, in return, a less precise signal of the demand than if it had shared with the other, better-informed retailer.

6.2 Extensions

To investigate the issues mentioned here, we constructed an analytical model that employed a number of simplifying assumptions. It is instructive, however, to anticipate the implication of relaxing some of them.

We assumed, for instance, that the manufacturer makes its wholesale pricing decision based solely on the demand signal observed. This implies that the retailers can perfectly infer the manufacturer’s private signal when observing its wholesale price. We made this simplifying assumption to highlight the inference effect. However, in reality, there may be additional supply-side uncertainties that face the manufacturer and affect its pricing decision, and therefore introduce noise in the wholesale price. When there is noise in the wholesale price, the degree of inference is lessened, thus reducing the manufacturer’s losses through the inference effect distortions. Note that information sharing, in this case, introduces additional benefits to the retailers as well, because they can directly observe the private demand information of the manufacturer. In the absence of sharing, the retailers can only infer a noisy signal of the manufacturer’s private information.

We also assumed that the manufacturer is restricted to uniform wholesale pricing. If the manufacturer is able to price discriminate, retailers may still infer the manufacturer’s signal through the channel-specific wholesale price. Recall that a retailer’s inference is possible because it can solve the manufacturer’s optimal decision rule to learn the signal. Thus the manufacturer benefits through the reduction of the inference effect when sharing information with a retailer. However, with the ability to set retailer-specific prices, the economics of price discrimination suggests that the manufacturer appropriates more of the channel profit, which makes information sharing more likely.

The inference effect in our model is driven by the double-marginalization problem in the channel. As is well known, nonlinear wholesale pricing can perfectly alleviate this problem. To the extent that the manufacturer can use nonlinear pricing, the benefits of information sharing revealed here may be reduced. However, given that linear (or unit) pricing is commonly observed in practice (Iyer and Villas-Boas 2003), we expect the inference effect to be present in many actual retail settings.

24 Hypothesis 2 is consistent with another example we have discussed in the introduction. The snack producer, Nabisco, initiated an information-sharing arrangement with the relatively small (67 stores in 2004) grocery chain, Wegmans (Boyle 2005, VICS 1999). Such a choice would seem to be inconsistent with a recent observation made by the Grocery Manufacturers of America (GMA). Based on surveys conducted by GMA, mass merchandisers were the retailers most likely to adopt CPFR, and the main reason cited for such information-sharing arrangements was to “reduce out-of-stocks” (GMA: 2002 CPFR Baseline Study). It seems, therefore, that reducing the adverse consequences of the inference effect may have guided Nabisco in choosing a smaller chain that is likely to be far less informed about demand conditions than the mass merchandisers.
Our model also assumed that the manufacturer posts a nonnegotiable wholesale price. It is reasonable, however, to imagine that a retail buyer can haggle or negotiate wholesale terms. To consider this possibility in the context of our model, suppose a retailer and the manufacturer enter a bargaining process in stage 3 in which both parties make alternating offers of price (Rubinstein 1982).\(^{25}\) In principle, offers made by each party reflect its own private information. Specifically, the manufacturer’s offers can be used by the retailer to infer \(x_0\), leading to the familiar distortions of the inference effect.\(^{26}\)

Finally, in contrast to our model, a retailer may have supply alternatives rather than a single manufacturer. It is therefore interesting to consider the consequence of upstream competition. The presence of a competing supplier puts downward pressure on wholesale prices. Nevertheless, the retailer’s ability to infer the manufacturer’s private signal is, in principle, undeterred. Therefore we expect the inference effect to remain present even in the case of multiple suppliers.

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Appendix
In the Technical Appendix at http://mktsci.pubs.informs.org, we establish that it is sufficient to restrict attention to linear decision rules when random variables are normally distributed and demand is linear. Based on this result, we assume that the manufacturer follows a linear pricing rule and deduce, in Lemma A.1, the retailers’ decision rules in all information regimes of the one-way sharing game.

**Lemma A.1.** Assuming that the manufacturer follows a linear decision rule in choosing wholesale price (i.e., \(p^w = a_0 + \alpha x_0\)), then retailer \(i\) also follows a linear decision rule of the form \(p^i = B_i^0 + B_i^1 x_j + B_i^2 p^w\) when it has access to \(x_0\), and of the form \(p^i = D_i^0 + D_i^1 x_j + D_i^2 p^w\) when it does not.

Proof of Lemma A.1. When retailer \(i\) has access to information about the signals \(x_j\) and \(x_0\), it can update beliefs concerning the state of the demand, \(u\), and the private signal observed by retailer \(j\), \(x_j\). Using DeGroot (1970, pp. 51–56), we obtain the posterior expectations as given in (7). If there is direct communication with retailer \(i\), it can observe the value of \(x_0\). Otherwise, this retailer uses the observed value of \(p^w\) to draw inferences about the value of \(x_0\) by inverting \(f_0^i\) in (6). We now consider, in sequence, the three informational regimes that can arise.

**No-Information Sharing (NS):** Note that the NS game was partially analyzed in §3. The system of FOC’s leads to the following system of six equations in six unknowns:

\[
a(b - d) - 2bD_0^i + dD_1^i = \frac{dD_1^i}{\alpha_i} + \frac{(b^2 - d^2)x_0}{\Delta_i} = 0
\]

\[
-2bD_2^i + \frac{dD_1^i}{\alpha_i} + \frac{x_0(b^2 - d^2)}{\Delta_i} = 0
\]

\[
-2bD_2^i + \frac{dD_1^i}{\alpha_i} + \frac{x_0(b^2 - d^2)}{\Delta_i} + b = 0,
\]

where \(\Delta_i \equiv (\sigma + s_i) + \sigma s_{0i}, i = 1, 2, \) and \(i \neq j\).

Solving system (13) for the coefficients \(D_i^1\) in terms of \(\alpha_i\) and \(\alpha\) yields

\[
D_0^i = \frac{a(b - d)}{2b - d} - \frac{2b\sigma(b^2 - d^2)x_0(2b(2b s_j + d s_j) + d s_{0j}(2b s_j + d s_j))}{\alpha(4b^2\Delta_j - d^2 s_j^2)}
\]

\[
D_1^i = \frac{(b^2 - d^2)x_0(2b \Delta_j + d s_{0j})}{(4b^2\Delta_j - d^2 s_j^2)}
\]

\[
D_2^i = \frac{b}{2b - d} + \frac{2b\sigma(b^2 - d^2)(2b(2b s_j + d s_j) + d s_{0j}(2b s_j + d s_j))}{\alpha(4b^2\Delta_j - d^2 s_j^2)} + \frac{2b\sigma(b^2 - d^2)(2b(2b s_j + d s_j) + d s_{0j}(2b s_j + d s_j))}{\alpha(4b^2\Delta_j - d^2 s_j^2)}
\]

where \(\Delta_j \equiv (\sigma + s_j) + \sigma s_{0j}, \Delta_j \equiv (\sigma + s_j) + \sigma s_{0j}, i = 1, 2, \) and \(i \neq j\).

**Partial Information Sharing (PSI):** Let retailer \(i\) be the one with whom the manufacturer shares information, and let retailer \(j\) be the competitor, then the retailers choose their prices to maximize the following objectives:

\[
\max_{p^i} E\left[\left(\frac{a}{b + d} - \frac{b p^i}{b^2 - d^2} + \frac{d p^i}{b^2 - d^2} + u\right)(p^i - p^w)\right| x_j, x_0\]

\[
\max_{p^i} E\left[\left(\frac{a}{b + d} - \frac{b p^i}{b^2 - d^2} + \frac{d p^i}{b^2 - d^2} + u\right)(p^i - p^w)\right| x_j, p^w\]

where the asserted linear decision rules followed by the retailers are given as

\[
p^i = B_j^0 + B_j^1 x_j + B_j^2 p^w + B_j^3 x_0
\]

\[
p^i = D_j^0 + D_j^1 x_j + D_j^2 p^w
\]

In (15) therefore for \(i\)’s objective: \(E(p^i | x_j, x_0) = D_j^0 + D_j^1 (1/\Delta_j)(\sigma s_j x_j + \sigma s_{0j} x_0) + D_j^2 p^w\), and for \(j\)’s objective: \(E(p^j | x_j, p^w) = B_j^0 + B_j^1 (1/\Delta_j)(\sigma s_j x_j + \sigma s_{0j} (p^w - \alpha_0/\alpha)) + B_j^2 p^w + B_j^3 (p^w - \alpha_0/\alpha)\).

Optimizing (15) with respect to retail prices yields the FOC of the retailers as follows:

\[
\frac{a}{b + d} = \frac{2bp^i}{b^2 - d^2} + \frac{dE(p^i | x_j, x_0)}{b^2 - d^2} + E(u | x_j, x_0) + \frac{bp^w}{b^2 - d^2} = 0
\]

\[
\frac{a}{b + d} = \frac{2bp^j}{b^2 - d^2} + \frac{dE(p^j | x_j, p^w)}{b^2 - d^2} + E(u | x_j, p^w) + \frac{bp^w}{b^2 - d^2} = 0
\]
Requiring that the FOC (17) is valid for every possible realization of the arguments of the pricing rules \(x_i, p^w,\) and \(x_0\) for retailer \(i,\) and \(x_i, p^w\) for retailer \(j\) yields a system of seven equations in seven unknowns \((B_i, k = 0, 1, 2, 3,\) and \(D_j, l = 0, 1, 2, 3,\)) Solving for the unknowns yields the solution

\[
B_i^0 = \frac{a(b - d)}{2b - d} - \frac{d(b^2 - d^2)\sigma \alpha \lambda [2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i)]}{(2b - d)\alpha (4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
B_i^1 = \frac{(b^2 - d^2)\sigma \alpha}{(4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
B_i^2 = \frac{b}{2b - d} + \frac{d(b^2 - d^2)\sigma [2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i)]}{(2b - d)\alpha (4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
D_i^0 = \frac{a(b - d)}{2b - d} - \frac{2b(b^2 - d^2)\sigma \alpha [2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i)]}{(2b - d)\alpha (4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
D_i^1 = \frac{(b^2 - d^2)\sigma \alpha (2b\Delta_i + d\sigma s_i)}{(4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
D_i^2 = \frac{b}{2b - d} + \frac{2b(b^2 - d^2)\sigma [2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i)]}{(2b - d)\alpha (4b^2\Delta_i - d^2\sigma_2 s_i^2)}.
\]

Full Information Sharing (FS): With full information sharing, the objective functions of the retailers become

\[
\max_{p^i} \left[ \frac{a}{b - d} + \frac{bp^i}{b^2 - d^2} + \frac{DE(p_i \mid x_i, x_0)}{b^2 - d^2} + E(u \mid x_i, x_0) \right] [p^i - p^w],
\]

where \(i = 1, 2,\) and \(i \neq j,\) and the asserted linear pricing rule takes the form

\[
p^i = B_i^0 + B_i^1 x_i + B_i^2 p^w + B_i^3 x_0 \quad i = 1, 2.
\]

Using derivations similar to those used earlier, yields a system of eight equations in the above coefficients as unknowns. The solution of the system is given as follows:

\[
B_i^0 = \frac{a(b - d)}{2b - d}, \\
B_i^1 = \frac{(b^2 - d^2)\sigma \alpha (2b\Delta_i + d\sigma s_i)}{(4b^2\Delta_i - d^2\sigma_2 s_i^2)}, \\
B_i^2 = \frac{b}{2b - d}, \\
B_i^3 = \frac{2b(b^2 - d^2)(2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i))}{(2b - d)(4b^2\Delta_i - d^2\sigma_2 s_i^2)}. \quad \Box
\]

Proof of Lemma 1. The manufacturer chooses \(p^w\) in the second stage to maximize expected profits expressed as follows:

\[
\max_{p^w} E(q^i + q^j \mid x_0),
\]

where \(q^i = a(b + d) - bp_i / (b^2 - d^2) + dp_i / (b^2 - d^2) + u, p^j,\) and \(p^j\) are linear decision rules with coefficients expressed by (14), (18), (20) for the NS, PSI, and FS environments, and \(E(x_i \mid x_0) = E(x_j \mid x_0) = E(u \mid x_0) = \sigma(\sigma + s_i)x_0,\)

No-Information Sharing (NS): From (14),

\[
E(q^i + q^j \mid x_0) = \frac{2ab - 2bp^w}{(b + d)(2b - d)} + \frac{2b\sigma x_0[4b\Delta_i b + (b - d)\sigma s_i(b + d) - d\sigma^2 s_i^2]}{(\sigma + s_i)(4b^2\Delta_i - b^2\sigma^2 s_i^2)} + \frac{2b\sigma(b - d)(p^w - a_0)[4b(\sigma + s_i)s_i + (2b + d)\sigma s_i(s_i + s_j)]}{\alpha(2b - d)(4b^2\Delta_i - d^2\sigma^2 s_i^2)}.
\]

Optimizing (21) with respect to \(p^w\) yields

\[
-\frac{(D_j^2 + D_j^1)p^w}{b + d} + E(q^i + q^j \mid x_0) = 0.
\]

Substituting into (24), the expressions obtained in (22) and (23) yields the FOC for the manufacturer. This condition should hold for every possible realization of the signal \(x_0.\) Recall that the coefficients \(D_i^k\) were derived under the assumption that \(p^w\) is a linear decision rule of the form \(p^w = \alpha_0 + \alpha x_0.\) We substitute this assumption back into (24), and require this condition to hold irrespective of the value of \(x_0.\) This requirement yields a system of two equations in the coefficients \(\alpha_0\) and \(\alpha\) as unknowns. Solving the system yields that

\[
p^w = y^0\left(a + \frac{(b + d)\alpha}{\sigma + s_i}x_0\right),
\]

as claimed in the lemma.

Since there exist coefficients \(\alpha_0\) and \(\alpha\) that satisfy the equilibrium condition (24), the derivation implies that a linear decision rule for \(p^w\) is indeed a Bayesian Nash equilibrium.

Partial Information Sharing (PSI): From (18),

\[
E(q^i + q^j \mid x_0) = \frac{2ab - 2bp^w}{(b + d)(2b - d)} + \frac{\sigma x_0[2b\Delta_i \Delta_j (3b + d) - (b - d)\sigma s_i \Delta_j (2b + d) - d\sigma^2 s_i^2(b + d)]}{(\sigma + s_i)(4b^2\Delta_i - b^2\sigma^2 s_i^2)} + \frac{(b - d)(2b + d)(p^w - a_0)[2b(\sigma + s_i)s_i + (2b + d)\sigma s_i(s_i + s_j)]}{\alpha(2b - d)(4b^2\Delta_i - d^2\sigma^2 s_i^2)},
\]

and

\[
B_j^0 = \frac{2b}{2b - d} - \frac{(2b + d)(b^2 - d^2)[2b(\sigma + s_i)s_i + \sigma s_i(2bs_i + ds_i)]}{(2b - d)(4b^2\Delta_i - d^2\sigma^2 s_i^2)}.
\]
Optimizing (21) with respect to \( p^w \) yields

\[
-\frac{(B^j_0 + D^j_0)p^w}{b + d} + E(q' + q^1 | x_0) = 0. \tag{27}
\]

Substituting into (27), the expressions obtained in (25) and (26) and requiring that the FOC (27) holds for all possible realizations of \( x_0 \) yields a system of two equations in \( \alpha_0 \) and \( \alpha \) as unknowns. The solution of the system yields

\[
p^w = y^{FSI} \left[ a + \frac{(b + d)\alpha}{\sigma + s_0} x_0 \right],
\]

as claimed in the lemma.

*Full Information Sharing (FS):* From (20),

\[
E(q' + q^1 | x_0) = \frac{2ab - 2bp^w}{(b + d)(2b - d)} + \frac{2b\sigma x_0}{(\sigma + s_0)(2b - d)},
\]

and

\[
B^j_0 + B^j_0 = \frac{2b}{2b - d}.
\]

Optimizing (21) with respect to \( p^w \) yields

\[
-\frac{(B^j_0 + D^j_0)p^w}{b + d} + E(q' + q^1 | x_0) = 0. \tag{29}
\]

Substituting (28) into (29) and requiring that (29) holds for all values of \( x_0 \) yields that

\[
p^w = y^{FS} \left[ a + \frac{(b + d)\alpha}{\sigma + s_0} x_0 \right], \quad \text{where } y^{FS} = 1/2. \quad \square
\]

**Proof of Corollary 1.** From Lemma 1,

(i) \( y^{PSI} - y^{NS} = \frac{1}{2(4b^2\Delta_j - d^2\sigma^2 s^2)}(2b - d)(\sigma + s_0)(b - d)\)

\[\cdot \left[ (\sigma + s_0)(s_j) + \sigma s_0 s_j (2b + d) (2b)^{-1} \right] > 0 \]

(ii) \( y^{FS} - y^{PSI} = \frac{1}{2(4b^2\Delta_j - d^2\sigma^2 s^2)}(2b + d)(\sigma + s_0)(b - d)\)

\[\cdot \left[ (\sigma + s_0)(s_j) + \sigma s_0 (2d s_j + d s_j) (2b)^{-1} \right] > 0. \tag{30}\]

Hence \( p^{FS} > p^{PSI} > p^{NS} \).

(ii) The gaps expressed in (30) are increasing with the difference \( b - d \). Hence, greater differentiation increases the gaps. \( \square \)

**Proof of Lemma 2.** From (24), (27), and (29) in the proof of Lemma 1, it follows that the equilibrium first-stage expected profits of the manufacturer are given as follows:

\[
E(\Pi^{NS}) = \frac{(D^j_0 + D^j_0)^{NS}E(p^w_{S_0})^2}{b + d},
\]

where \( (D^j_0 + D^j_0)^{NS} \) is given by (23),

\[
E(\Pi^{PSI}) = \frac{(B^j_0 + D^j_0)^{PSI}E(p^w_{S_0})^2}{b + d}, \tag{31}
\]

where \( (B^j_0 + D^j_0)^{PSI} \) is given by (26),

\[
E(\Pi^{FS}) = \frac{(B^j_0 + D^j_0)^{FS}E(p^w_{S_0})^2}{b + d},
\]

where \( (B^j_0 + D^j_0)^{FS} \) is given by (28).

From Lemma 1, \( E(p^w_{S_0})^2 = (y^f)^2 [a^2 + ((b + d)^2\alpha^2 / \sigma + s_0)] \)

since \( E(x_0)^2 = \sigma + s_0 \).

Substituting \( E(p^w_{S_0})^2 \) into (31) yields the given profit expressions. \( \square \)

**Proof of Proposition 1.**

(i) From the expressions obtained in Lemma 2, it follows that

\[
E(\Pi^{NS}) - E(\Pi^{PSI}) = V(2b(y^{PSI} - y^{NS}))^2 + [(2b + d)(b - d)(\sigma + s_0)(y^{PS} - y^{NS})] \]

\[\cdot \left( 4b^2 \Delta_j - d^2\sigma^2 s^2 \right)^{-1}, \tag{32}\]

and

\[
E(\Pi^{FS}) - E(\Pi^{PSI}) = V2b(y^{FS} - y^{PSI})^2. \]

Since \( y^{FS} > y^{PSI} > y^{NS} \), it follows that \( E(\Pi^{NS}) > E(\Pi^{PSI}) > E(\Pi^{FS}) \).

(ii) From the expressions obtained for \( E(\Pi^{PSI}) \) in Lemma 2, it follows that

\[
E(\Pi^{PSI}) - E(\Pi^{PSI}) = V(2b(y^{PSI} - y^{PSI}))^2 + [(2b + d)(b - d)(\sigma + s_0)(y^{PSI} - y^{PSI})] \]

\[\cdot \left( 4b^2 \Delta_j - d^2\sigma^2 s^2 \right)^{-1}, \tag{32}\]

where

\[y^{PSI} - y^{PSI} = \frac{(\sigma + s_0)(\sigma s_0)(b - s_0)(2b - d^2)}{4b(4b^2 \Delta_j - d^2\sigma^2 s^2)}. \]

Hence, if \( s_1 > s_2 \), \( y^{PSI} > y^{PSI} \) and \( E(\Pi^{NS}) > E(\Pi^{PSI}) \). \( \square \)

**Proof of Proposition 2.** From (32), a sufficient condition that \( E(\Pi^{PSI}) - E(\Pi^{PSI}) > E(\Pi^{PSI}) \) is that \( y^{PSI} - y^{PSI} > y^{PSI} - y^{PSI} \). From (30), the latter inequality holds if the condition stated in the proposition is valid. \( \square \)

**Proof of Lemma 3.** When \( s_j \rightarrow \infty \) and \( s_j = 0 \) (sharing with retailer 2), retailer 2 is perfectly uninformed. Hence, this case is equivalent to one-way communication given that the information from retailer 1 (2 in this case) is worthless.

Substituting, therefore \( s_j \rightarrow \infty \) and \( s_j = 0 \) into the expression obtained for \( E(\Pi^{PSI}) \) in Lemma 2 yields \( E(\Pi^{PSI}) \).

When \( s_j = 0 \) and \( s_j \rightarrow \infty \) (sharing with retailer 1), retailer 1 is perfectly informed, and the manufacturer uses this retailer’s private signal in updating its belief about the uncertainty. Therefore we need to explicitly solve for the equilibrium of the partial information-sharing game for the two-sided communication case. Using similar techniques as in the proof of Lemma A.1, we can get the coefficients of the retailers’ decision rules, \( p' = B^j_0 + B^j_0 x_0 + B^j_0 + B^j_0 x_0 \) and...
where

\( p' = D'_i + D'_j x + D'_j p' \), given the manufacturer's decision rule, \( p'^* = \alpha_0 + \alpha x + \beta x \), as follows:

\[
R'_0 = \frac{a(b - d)}{2b - d} - \frac{b d}{2b - d} - \left[ \frac{b d}{2b - d} + \frac{d(b^2 - d^2)(a + \beta)\alpha_s + d \alpha_s d_s}{(2b - d)[4b^2 D \Delta_j - d^2 \sigma^2(\alpha_s - \beta_s)^2]} \right],
\]

\[
R'_1 = \frac{(b^2 - d^2)\alpha_s}{4b^2 D \Delta_j - d^2 \sigma^2(\alpha_s - \beta_s)^2},
\]

\[
R'_2 = \frac{b}{2b - d} + \frac{d(b^2 - d^2)(a + \beta)\alpha_s + d \alpha_s d_s}{(2b - d)[4b^2 D \Delta_j - d^2 \sigma^2(\alpha_s - \beta_s)^2]},
\]

\[
D'_0 = \frac{a(b - d)}{2b - d} - \frac{b d}{2b - d} - \left[ \frac{b d}{2b - d} + \frac{d(b^2 - d^2)(a + \beta)\alpha_s + d \alpha_s d_s}{(2b - d)[4b^2 D \Delta_j - d^2 \sigma^2(\alpha_s - \beta_s)^2]} \right],
\]

\[
D'_1 = \frac{(b^2 - d^2)\sigma^2(\alpha_s + \beta_s)^2 + d \alpha_s d_s}{4b^2 D \Delta_j - d^2 \sigma^2(\alpha_s - \beta_s)^2},
\]

\[
D'_2 = \frac{b}{2b - d} + \frac{b^2 - d^2}{(2b - d)\beta_s}.
\]

Using (34) and similar techniques as in the proof of Lemma 1, we get the manufacturer's pricing rule

\[
p'^* = \frac{2b^2 + b d + d^2}{8b^2} [a + (b + d) x].
\]

Upon substitution of \( p'^* \) into the objective of the manufacturer (similar to the derivation of (27) in Lemma 1), it follows that the equilibrium first-stage expected profits of the manufacturer can be expressed as follows:

\[
E(\Pi'^*_{m}) = \frac{(B'_i + D'_j)E(p'^*)^2}{b + d}.
\]

Substituting (35) and the expressions for \( B'_i \) and \( D'_j \) from (34) into (36), yields \( E(\Pi'^*_{m}) \). □

Proof of Proposition 3. It follows since the first term of DD is relatively big when \( d \) is small (the gap \( b - d \) is big), and the second term is relatively small when \( d \) is relatively small, (the expression \( s \sigma_s/\sigma_s + s \) \( s \) is an increasing function of \( s \)). □

Proof of Corollary 2.

(i) Follows by substituting \( d = 0 \) into the expression obtained for DD.

(ii) Follows since the first term of DD vanishes when \( d = b \), implying that \( E(\Pi'^*_{m}) - E(\Pi'^*_{m}) < 0 \) when \( d = b \). □

References


