# Advanced Tutorial on Coevolution

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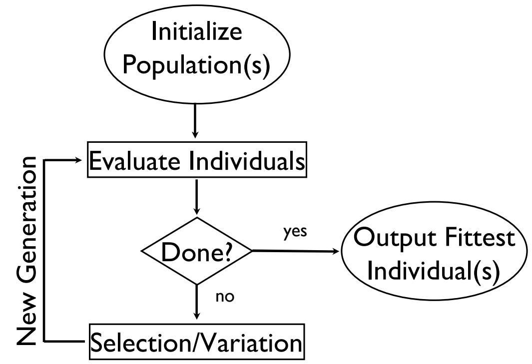
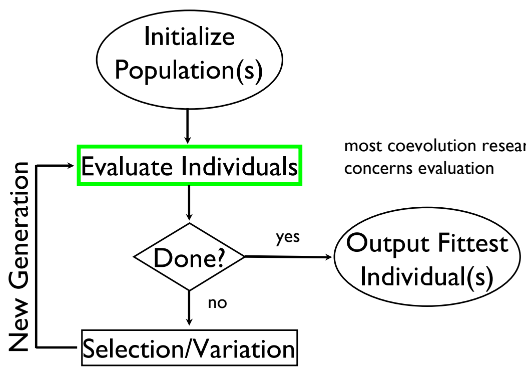
## Conventional Coevolution

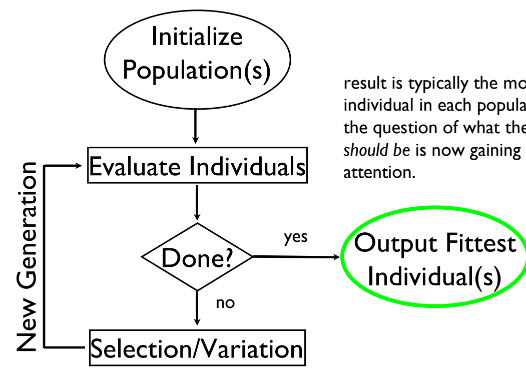



## Conventional Coevolution




most coevolution research concerns evaluation

## Conventional Coevolution




result is typically the most-fit individual in each population; the question of what the result *should be* is now gaining more attention.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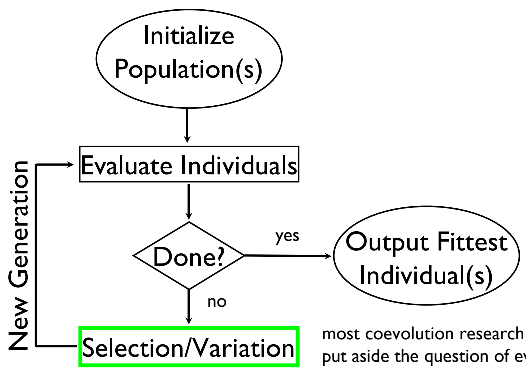
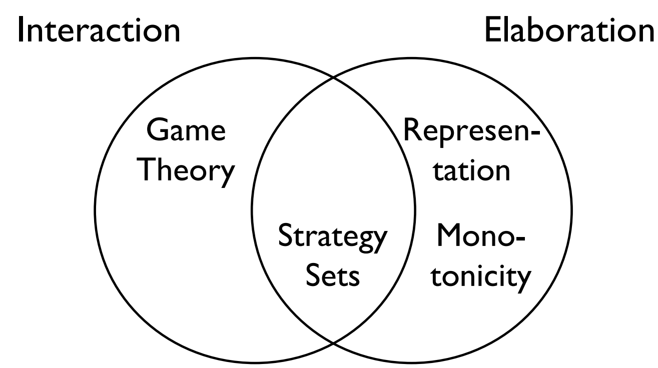
## Conventional Coevolution




most coevolution research has put aside the question of evolutionary representation; this is an area that is gaining more attention now.

## Main Themes




## Interaction

- To evaluate an individual in coevolution, we must have it interact with others
- The outcome of evaluation is contingent upon whom the individual interacts with
- The individual may appear good in one context and poor in another context
- This context sensitivity is game theoretic in nature
- Solutions may be sets of individuals

## Elaboration

- We want the evolving individuals to improve over evolutionary time
- Coevolutionary “arms race” is an example
- Improvement can be viewed as an accumulation of competences, or elaboration
- We will discuss different forms of elaboration

## Main Topics

- Game theory
  - game, strategies, payoffs
  - solution concepts: implementation
- Strategy sets
  - Mixtures, Pareto front, archives, ...
- Representation
- Monotonic improvement over time

## Motivation: Coevolutionary Pathologies

- Cycling: algorithm revisits a portion of state-space periodically—no progress
- Disengagement: loss of fitness gradient
- Overspecialization: lack of elaboration
- Forgetting: loss of potentially useful traits
- Relative overgeneralization: favoring of versatile components over those of optimal solution

## Game Theory

- Mathematics of strategic reasoning [Fundenberg & Tirole 1998]
- If we have a number of interacting agents...
  - How will they behave; what will be outcome?
  - If we interact, how should we behave?
- Provides descriptive predictions of how players will behave
- Provides prescriptive (normative) instructions on how to behave

## Game Theory

- Provides predictions and instructions about behavior
- Assumes all agents are rational, selfish
- Nash equilibrium [Nash 1951]
  - A configuration of strategic choices such that no player has incentive to deviate unilaterally from its current strategy
  - All finite games have at least one Nash equilibrium

## Game Theory: Components

- Game specifies for each player…
  - strategies that are available
  - outcomes that result for each strategy when interacting with other players' strategies
- Solution concept
  - formal specification of what configuration of players' behaviors (strategies) constitutes a solution to the game

## Rock Paper Scissors

Player 2

| Player 1 | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0 | -1 | 1 |
| Paper | 1 | 0 | -1 |
| Scissors | -1 | 1 | 0 |

## Rock Paper Scissors

Player 2

| Player 1 | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0 | -1 | 1 |
| Paper | 1 | 0 | -1 |
| Scissors | -1 | 1 | 0 |

- Pure strategies: rock, paper, scissors
- Mixed strategy: any probability distribution over pure strategies

## Rock Paper Scissors

Player 2

| Player 1 | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0 | -1 | 1 |
| Paper | 1 | 0 | -1 |
| Scissors | -1 | 1 | 0 |

- Payoffs (outcomes) for all possible pure-strategy interactions
- For mixed strategies, we calculated expected payoffs based on probability distributions used

## Rock Paper Scissors

Player 2

Player 1

| | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0 | -1 | 1 |
| Paper | 1 | 0 | -1 |
| Scissors | -1 | 1 | 0 |

- Rock > Scissors > Paper > Rock
- No pure strategy is universally best
- Solving this game requires a <u>set</u> of strategies

## Rock Paper Scissors

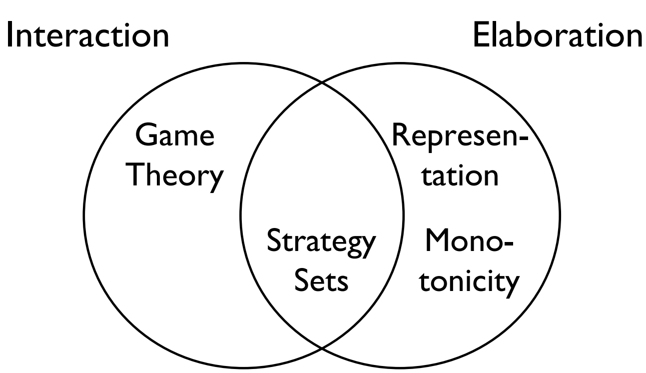
Player 2

Player 1

| | Rock | Paper | Scissors |
|---|---|---|---|
| Rock | 0 | -1 | 1 |
| Paper | 1 | 0 | -1 |
| Scissors | -1 | 1 | 0 |

- Nash equilibrium strategy is mixed
  - R, P, S each played with probability = 1/3
  - expected payoff of Nash player is zero, regardless of what other player does
  - expected payoff of other player is also zero, regardless of what it does

## Main Themes




## Interaction and Elaboration

- From the outcomes of pure-strategy interaction...
  - we find that no single pure strategy provides all needed competences
- The Nash mixed-strategy...
  - is a set of pure strategies...
  - and represents an elaboration of pure-strategies

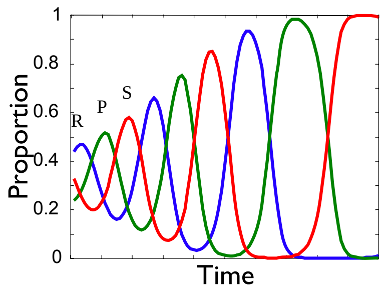
## Solution Concept

- Specifies <u>properties</u> of a solution
  - (not the solution itself)
- But must be implemented in search algorithm
- Incorrect implementation of solution concept will cause search algorithm to diverge from desired solution properties

## Solution Concept

- Examples where algorithm fails to implement Nash equilibrium in a game
  - Proportional selection and Rock-Paper-Scissors: mixed Nash equilibrium? [Hofbauer & Sigmund 1998]
  - Alternative selection methods and Hawk-Dove game [Ficici et al. 2000, 2005]
  - Diversity maintenance methods and Hawk-Dove game [Ficici 2001]

## Rock-Paper-Scissors




- Under fitness-proportional selection...
  - Nash equilibrium represented as polymorphic population of pure-strategists is unstable
  - Nash equilibrium also unstable for mixed strategists
- Nash concept not properly implemented here

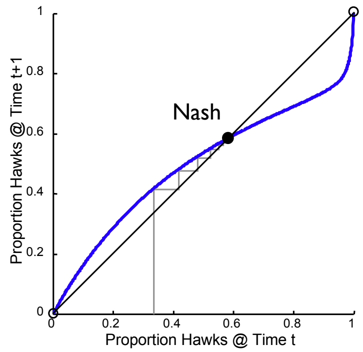
## Hawk-Dove Game

[Maynard Smith 1982]

| | Hawk | Dove |
|---|---|---|
| Hawk | -25 | 50 |
| Dove | 0 | 15 |

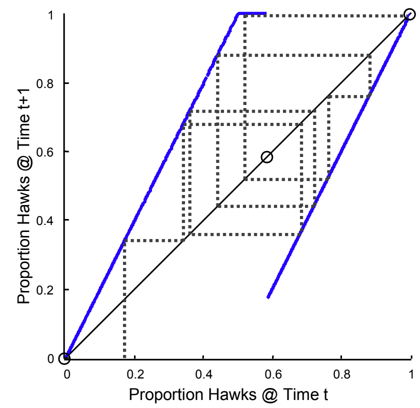
- Nash equilibrium strategy for these payoffs:
  - 7/12 Hawk, 5/12 Dove
  - probability distribution for a mixed strategy...
  - OR proportions for polymorphic population of pure-strategists

## Proportional Selection



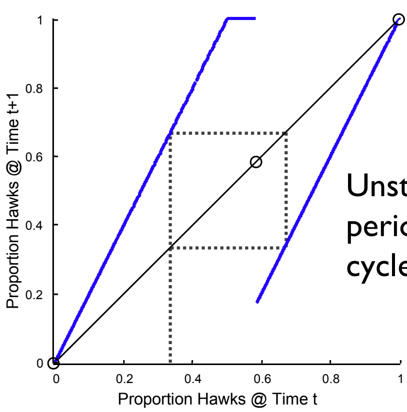

Nash equilibrium is dynamical attractor

## Truncation Selection



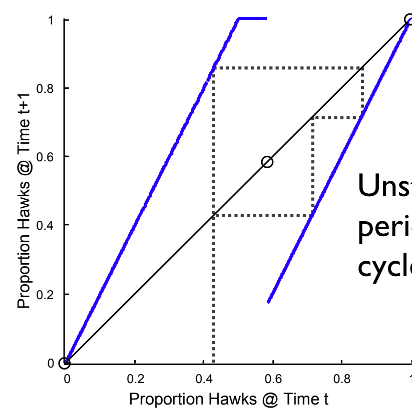

All-Hawks is now an attractor

## Truncation Selection

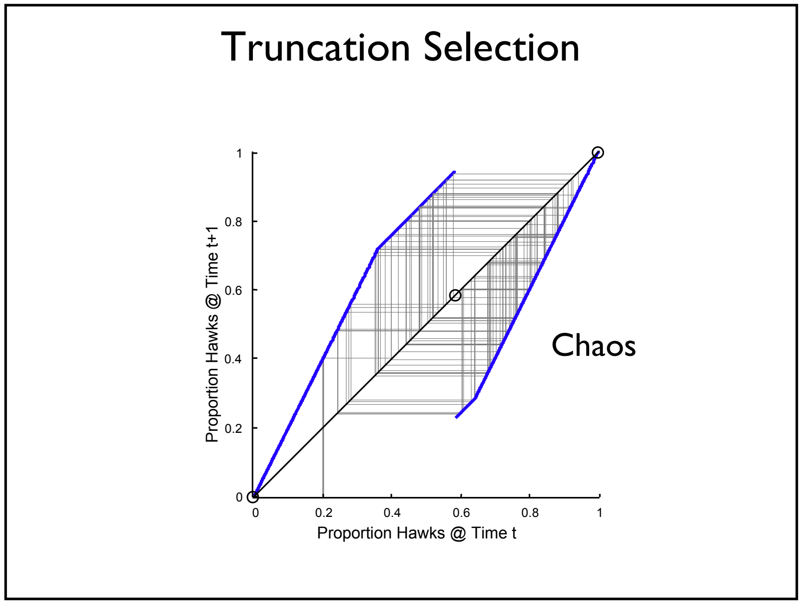


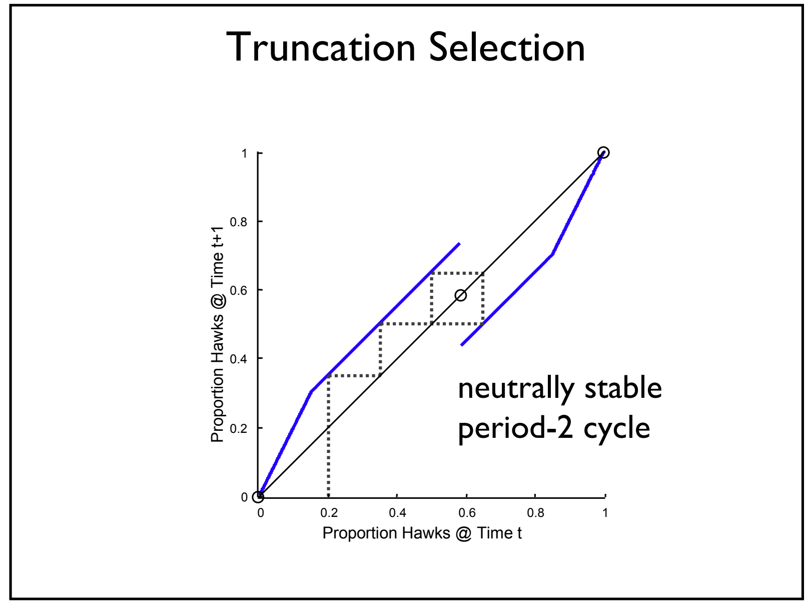

Unstable period-2 cycle

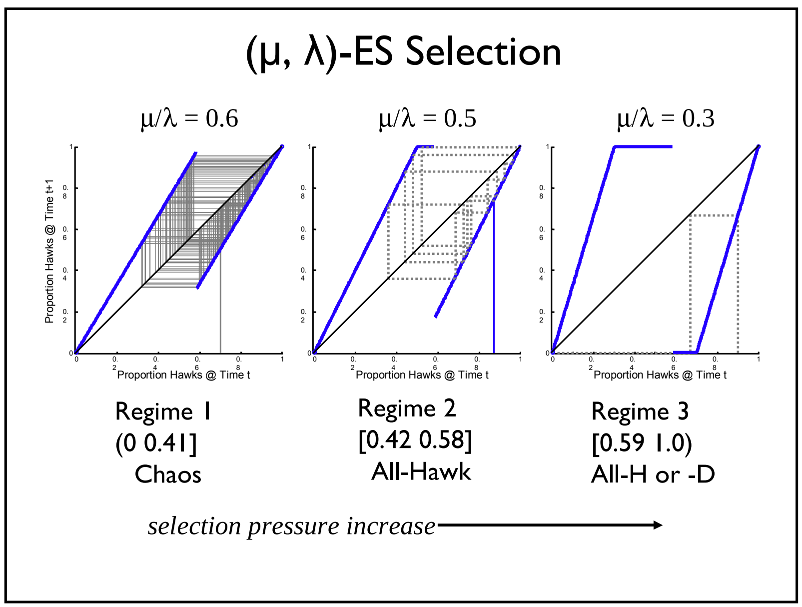
## Truncation Selection

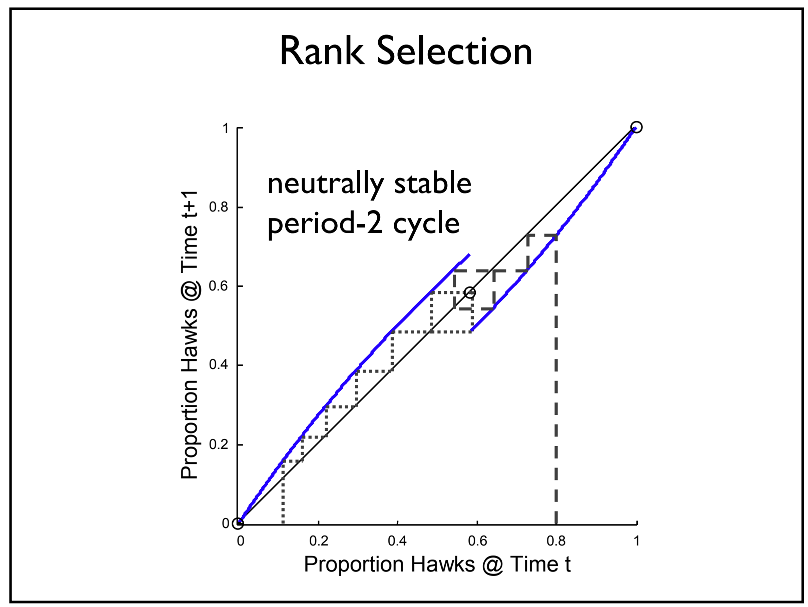



Unstable period-3 cycle


Truncation Selection
Chaos
Proportion Hawks @ Time t+1
Proportion Hawks @ Time t


Truncation Selection
neutrally stable
period-2 cycle
Proportion Hawks @ Time t+1
Proportion Hawks @ Time t

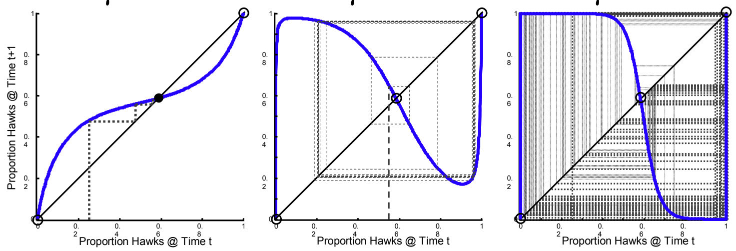

(μ, λ)-ES Selection
μ/λ = 0.6
μ/λ = 0.5
μ/λ = 0.3
Regime 1
(0 0.41]
Chaos
Regime 2
[0.42 0.58]
All-Hawk
Regime 3
[0.59 1.0)
All-H or -D
selection pressure increase


Rank Selection
neutrally stable
period-2 cycle
Proportion Hawks @ Time t+1
Proportion Hawks @ Time t

## Boltzman Selection

$$f_{\text{Boltz}} = e^{\gamma f}$$

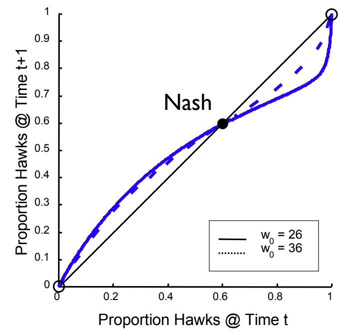
$\gamma = 0.05$ $\gamma = 0.2$ $\gamma = 0.5$




Nash Intact | Attr. Limit Cycle | Chaos

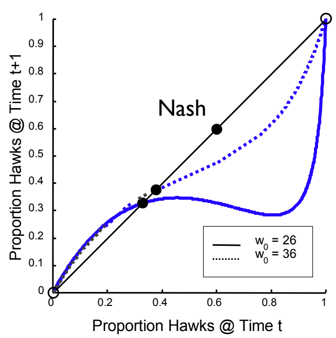
*selection pressure increase* →

## Proportional Selection



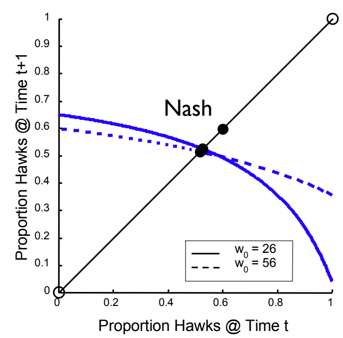

## Competitive Fitness Sharing

[Rosin 1997]




## Similarity-Based Fitness Sharing

[Goldberg 1989]



## Discussion

- We use different selection methods and diversity-maintenance methods to improve search for a particular domain
- Evolving population expected to both:
  - contain sufficient genetic diversity for search
  - represent solution to search task (may be a polymorphism)
- These tasks not necessarily orthogonal
- Above illustrates pitfalls

## Discussion

- Why not separate tasks?
- Let population perform search
- Let another mechanism (not population) represent best solution found so far
- Leads us to archive methods

## Archive Methods

- Archives provide a way to
  - collect (according to some organizing priniple) "good" individuals over evolutionary time
  - encapsulate wider phenotypic range (than a population contains at any one moment in time)
  - broaden evaluation (and selection pressure) via augmented phenotypic diversity
  - ameliorate evolutionary forgetting
  - represent the result of the evolutionary process

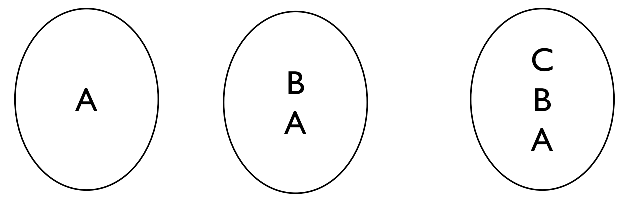
## Archive Methods

- Hall-of-Fame [Rosin & Belew 1997]
  - accumulate fittest of each generation
  - sample *k* members for testing current generation
  - shown to help, but weak organizing principle
- Dominance Tournament [Stanley & Miikkulainen 2002]
- Nash memory [Ficici & Pollack 2003]
- Pareto archives [de Jong 2004]

## Dominance Tournament

- For zero-sum games, symmetric or not
- Organizing principle is Pareto dominance
- Add strategy X to DT archive if and only if X outperforms each member of archive
- Each new member inserted into archive has a broader demonstrated range of competence
- Avoids intransitive cycles
- Most recently added member is "solution"

## Dominance Tournament




If B beats A

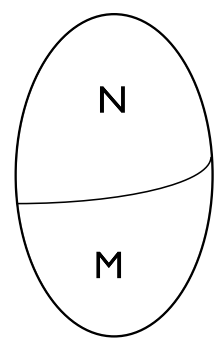
If C beats B and A

- If D beats C, but not B, then D excluded

## Nash Memory

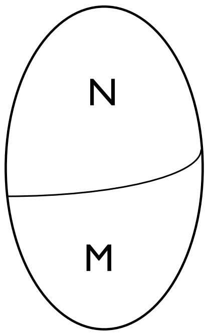
- For zero-sum games, symmetric or not
- Organizing principle is Nash equilibrium
- Begin with arbitrary approximation to Nash equilibrium N of game, and empty "memory" M
- If strategy S beats N, then update N and M to obtain a new Nash approximation that doesn't lose to any strategy in S ∪ N ∪ M
- Final approximation N is "solution"

## Nash Memory



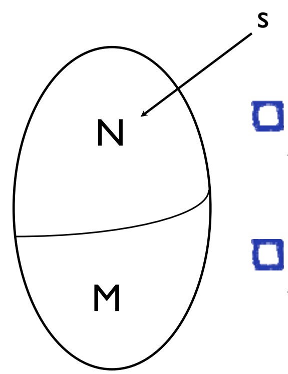

- N and M are mutually exclusive sets of pure strategies
- N is initially an arbitrary pure strategy
- M is initially the empty set

## Nash Memory



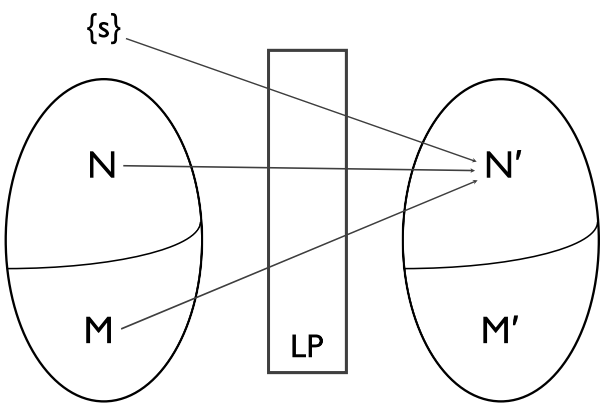

- N is Nash strategy with respect to pure strategies in N ∪ M
- M is set of strategies that used to be in N earlier and may be again in the future
- M is of bounded size

## Nash Memory

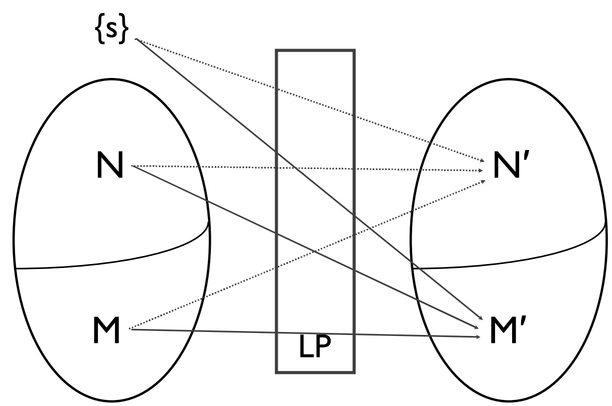



- If s does not beat N, then discard s; keep searching
- Otherwise, s indicates a weakness in N; update N

## Nash Memory




## Nash Memory



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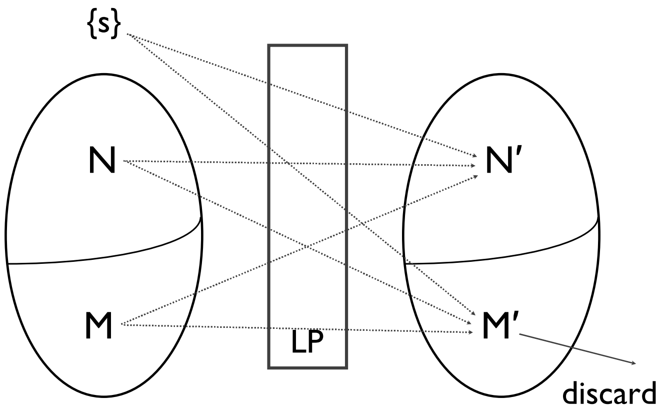
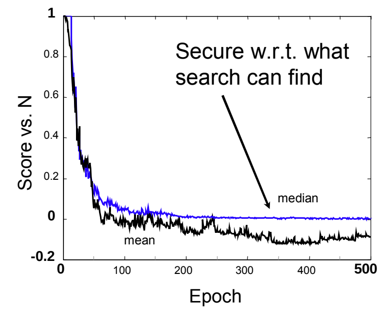



## Nash Memory

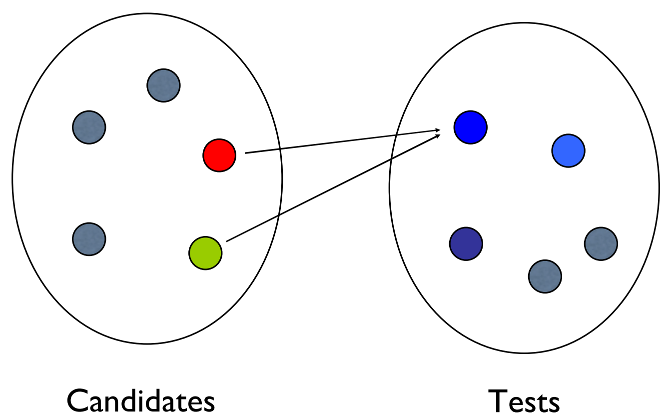


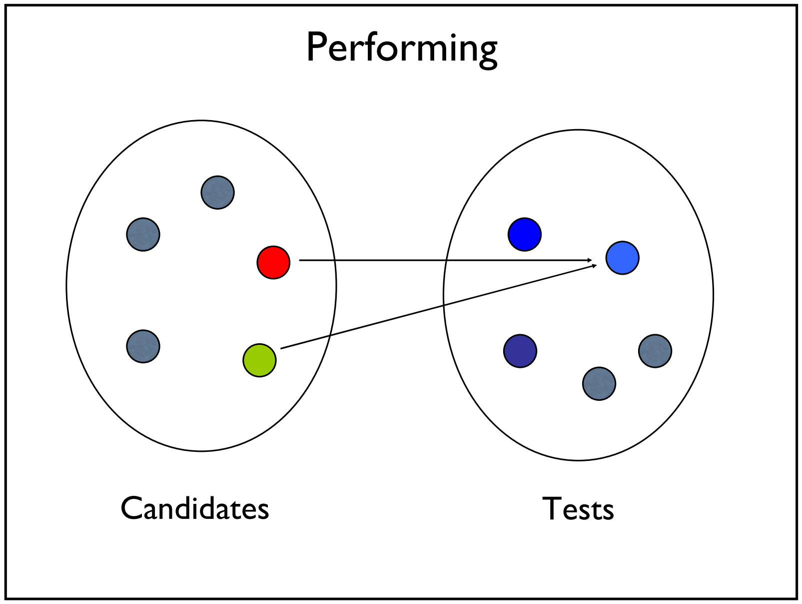

- Even though M is finite, and strategies discarded, monotonic improvement of N is approximated

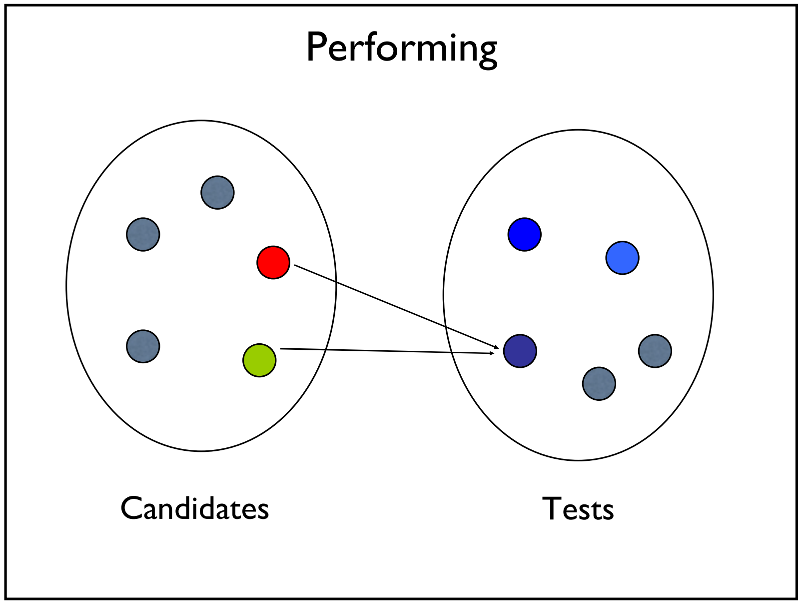
## Pareto Archives

- *Pareto Coevolution* [Ficici & Pollack 2001; Noble & Watson 2001] treats entities with two roles: *candidates* and *tests* (sometimes *learners* and *teachers*)
- **Candidates** are incented to *perform*
- **Tests** are incented to *inform* about candidates
- Key insight: performing ≠ informing

## Performing




Performing
Candidates
Tests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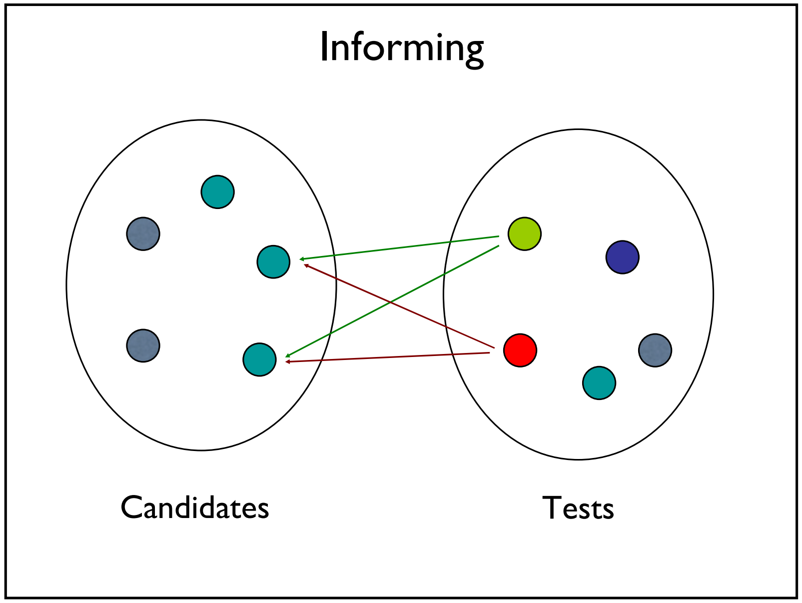
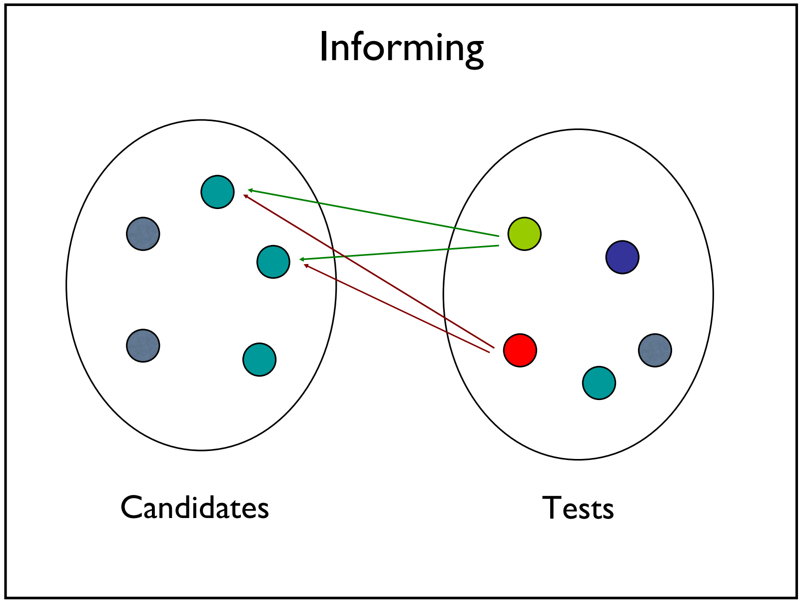
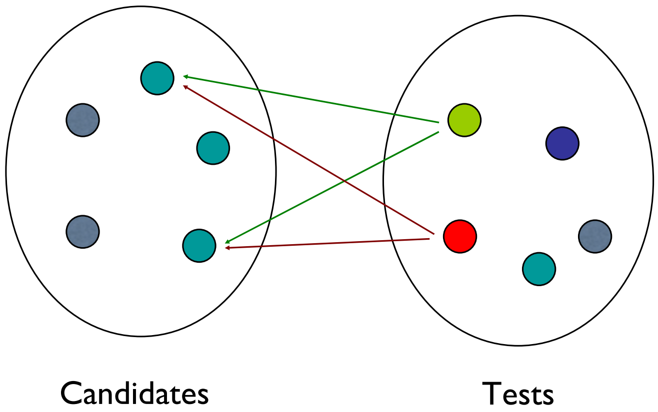

Informing
Candidates
Tests


Informing
Candidates
Tests

## Informing




## Pareto Archives: IPCA

- Incremental Pareto Coevolution Archive [de Jong 2004]
- Theoretically ensures monotonic progress for Pareto Coevolution
- Allows the candidate population to explore
- Test archive maintains candidate distinctions and can grow without bound

## Pareto Archives: LAPCA

- LAyered Pareto Coevolution Archive [de Jong 2004]
- Keeps a tunable number of Pareto layers
- Approximates IPCA, but bounds the archive – loses monotonicity guarantee
- Combined with NEAT and applied to coevolve Pong players [Monroy et al. 2006]

## Test-Based Problems

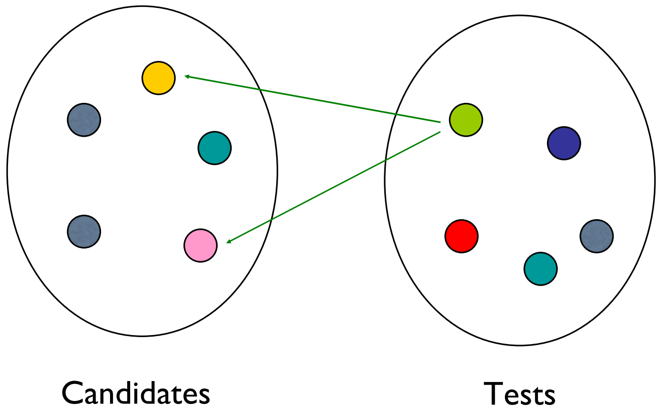
- Candidate solutions are *tested* by interacting with other entities, as in:

| ***Domain*** | ***Candidate*** | ***Test*** |
|---|---|---|
| Design | Sorting network | Unsorted list |
| Classification | Classifier | Data point |
| Function/model regression | Function or model | Input |
| Strategy learning | First player | Second player |

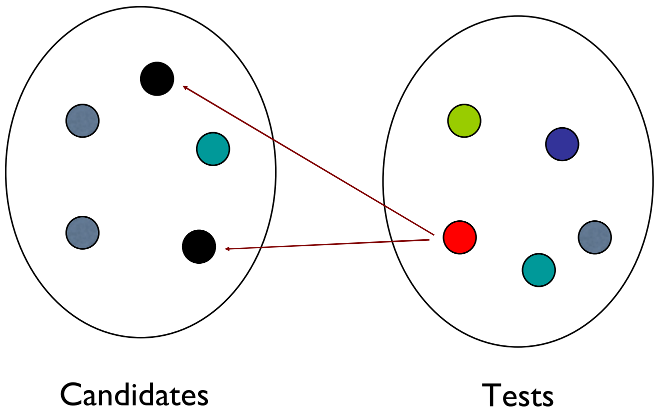
## Pareto Coevolution

- Maintains two populations, candidate solutions and tests
- **Candidates** are compared using Pareto dominance: *A* dominates *B* if it does at least as well as *B* against all tests and better on at least one
- **Tests** are compared using *distinctions* [Ficici & Pollack 2001] or *informativeness* [Bucci & Pollack 2003]
- Solution set is non-dominated front of candidates and an informative set of tests

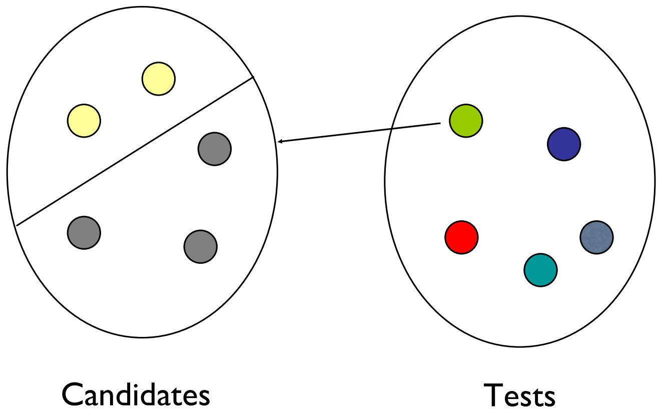
## Shows a Distinction



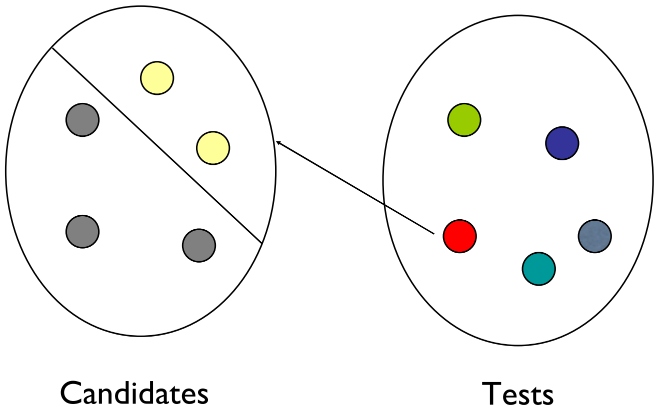

## Shows No Distinction



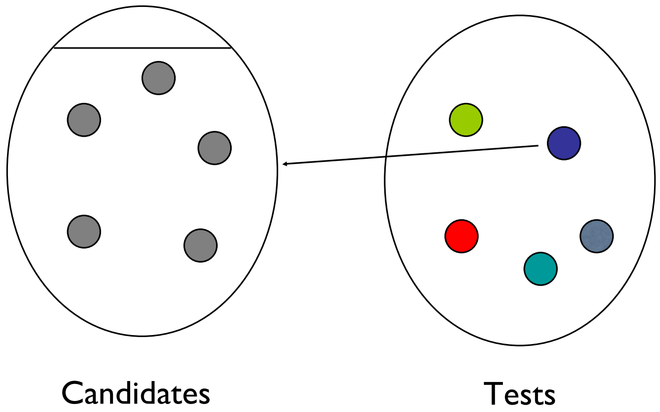

## Informative



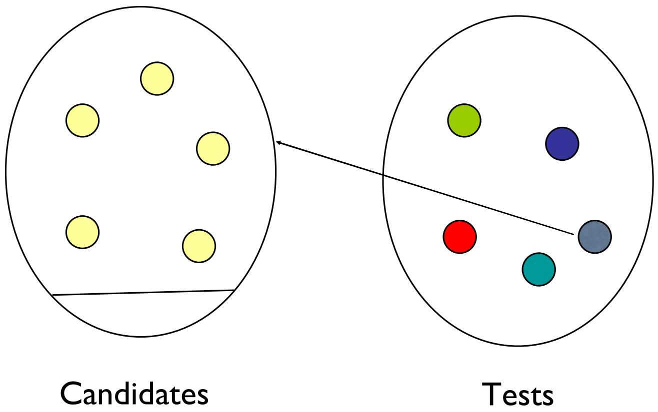
## Differently Informative




## Uninformative




## Differently Uninformative



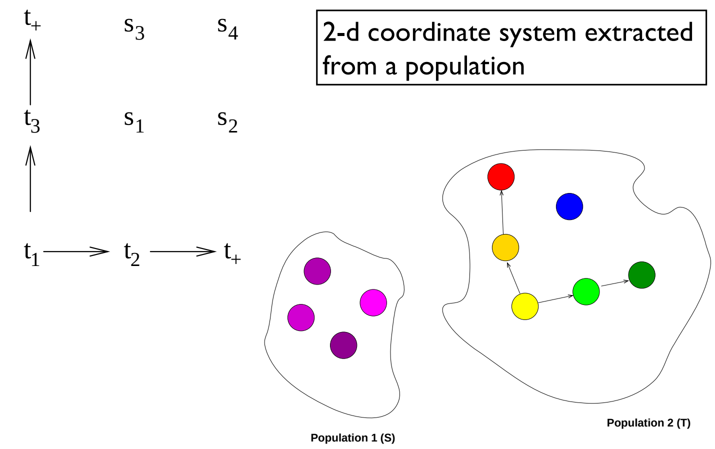

## Reducing the Number of Tests

- Ideal Test Set [Bucci & Pollack 2003]
- Complete Evaluation Set [de Jong & Pollack 2004]
- [de Jong & Pollack 2004] posited the existence of multiple *underlying objectives* akin to fitness functions
- [Bucci et al. 2004] grounded the idea as *coordinate systems* (*informative dimensions*)

## Dimension Extraction

- Coordinate systems collect several tests into a composite *axis*
- Set of axes forms a coordinate system analogous to a basis for a vector space
- [Bucci et al. 2004] proved coordinate systems exist and gives a polynomial-time algorithm to extract one
- [de Jong & Bucci 2006] gave a CEA, DECA, which extracts coordinate systems from populations to inform selection

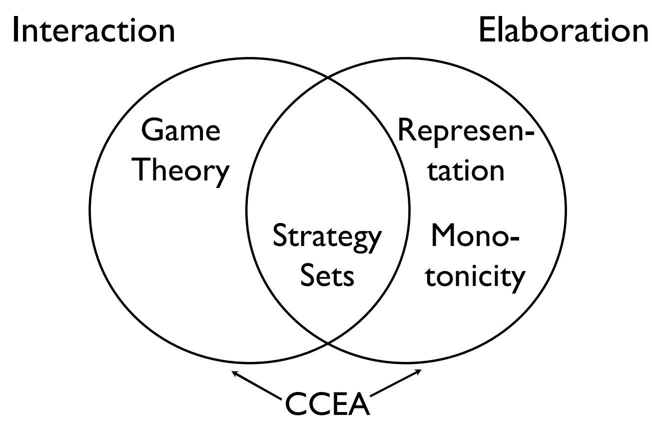
## Dimension Extraction




## Reducing the Amount of Testing: EEA

- Estimation-Exploration Algorithm [Lipson et al. 2005]
- **Candidates** are models of a system
- **Tests** are probes of the real system (assumed to be expensive)
- Aim is to evolve a model of the real system using as few probes as possible

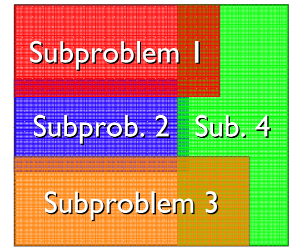
## Main Themes



## Cooperative Coevolution
[Potter & De Jong 1994]

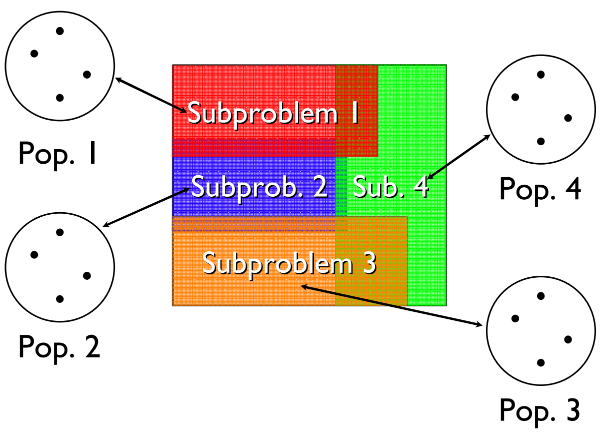

- Monolithic problem may be too difficult

## Cooperative Coevolution
[Potter & De Jong 1994]




- Decompose into mosaic of semi-independent sub-problems

## Cooperative Coevolution
[Potter & De Jong 1994]




## Cooperative Coevolutionary Algorithms (CCEAs)

- [Potter & De Jong 1994] argued that CCEAs optimize functions
- [Wiegand 2003] argued they do not optimize functions, but rather optimize for robustness.
- Used EGT to argue certain Nash equilibria are preferred
- Same work suggested biasing CCEA such that they do optimize

## Biasing CCEA Towards Optimization

- [Panait et al.  2004] aimed to bias the CCEA by mixing evaluation with another term biasing towards its optimal evaluation
- [Bucci & Pollack 2005] used Pareto dominance comparison with no bias term; collaborators were tests
- [Panait et al. 2006] proposed an archive of good collaboration choices, iCCEA

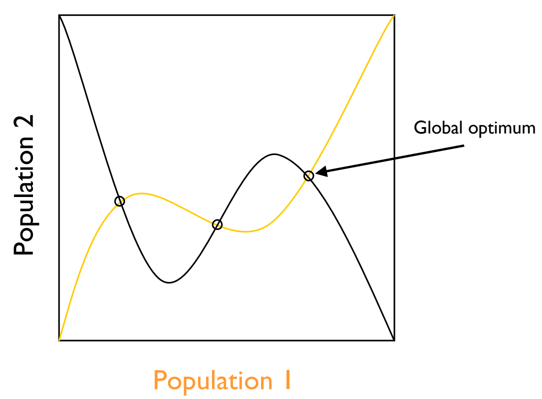
## Analyzing Collaboration Schemes

[Popovici & De Jong 2005]

- Best response curves are a property of a problem
- In CCEA, intersection points of best response curves are Nash equilibria
- Trajectories of individuals is a propety of an algorithm; e.g., the collaboration scheme
- Trajectories which land at best response curve intersection points get stuck even if they are suboptimal

## Analyzing Collaboration Schemes

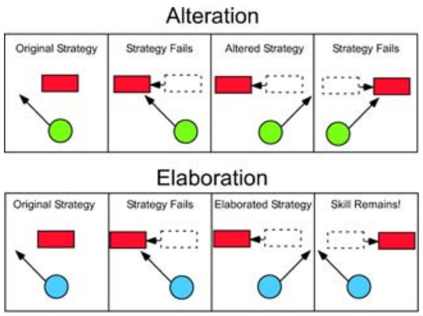
[Popovici & De Jong 2005]




## NeuroEvolution of Augmenting Topologies (NEAT)

- Evolves increasingly complex neural network topologies [Stanley & Miikkulainen 2004]
- Mutations occasionally add new structure
- Speciation protects innovative structures
- In combination, these mechanisms support elaboration

## Alteration vs. Elaboration




Alteration alone may damage capabilities
Elaboration accumulates capabilities
Can we abstract this idea?

## Progress in Coevolution

- A core theme in coevolution research: How to ensure progress—is it possible?
- Evaluation: individual interacts with others
- Measured quality of an individual is function of which other individuals interact with it
- Constantly shifting landscape!
- Open-ended search spaces problematic

## Progress in Coevolution

- Monitoring progress
  - Miller & Cliff 1994
  - Floreano & Nolfi 1997
  - Rosin 1997
  - Stanley & Miikkulainen 2002
  - Bader-Natal & Pollack 2004, 2005

## Approach

- Examine the issue of progress from viewpoint of solution concepts
- Some solution concepts intrinsically “support” monotonic progress
- Not a value judgment—use whatever solution concept is appropriate
- But something to be aware of!

## Desirable Property

- As your knowledge of a search-space increases…
- … your estimations of a solution should improve

The longer the algorithm runs,
the better the output should be!

(Experience tells us this is not the case in coev.)

## Desirable Property

$\mathcal{P}$

- As your knowledge of a search-space increases…
- … your estimations of a solution should improve

## Desirable Property

$\mathcal{P}$

- As your knowledge of a search-space increases… $\mathcal{W} \subset \mathcal{P}$
- … your estimations of a solution should improve

## Desirable Property

$\mathcal{P}$

- As your knowledge of a search-space increases… $\mathcal{W} \subset \mathcal{P}$
- … your estimations of a solution should improve ~~$\mathcal{C} \in \mathcal{P}$~~
  $\mathcal{C} \in 2^{\mathcal{P}}$

## Desirable Property

- As your knowledge of a search-space $\mathcal{P}$ increases... $\mathcal{W} \subset \mathcal{P}$
- ... your estimations of a solution should improve ~~$\mathcal{C} \in \mathcal{P}$~~ $\mathcal{C} \in 2^{\mathcal{P}}$

## Monotonic Improvement

Knowledge — Complete Knowledge

$$\mathcal{W}_t \subset \mathcal{W}_{t+1} \subset \mathcal{W}_{t+2} \subset \cdots \subset \mathcal{W}^*$$

$$\mathcal{C}_t \preceq \mathcal{C}_{t+1} \preceq \mathcal{C}_{t+2} \preceq \cdots \preceq \mathcal{C}^*$$

Estimations — Global Solution

## Monotonic Improvement

Complete Knowledge

$$\mathcal{W}_t \subset \mathcal{W}_{t+1} \subset \mathcal{W}_{t+2} \subset \cdots \subset \mathcal{W}^*$$

$$\mathcal{C}_t \preceq \mathcal{C}_{t+1} \preceq \mathcal{C}_{t+2} \preceq \cdots \preceq \mathcal{C}^*$$

Global Solution

Possible in coevolution when used to solve game of strategy?

## Reasoning

$$\mathcal{W}_t \subset \mathcal{W}_{t+1} \subset \mathcal{W}_{t+2} \subset \cdots \subset \mathcal{W}^*$$

$$\mathcal{C}_t \preceq \mathcal{C}_{t+1} \preceq \mathcal{C}_{t+2} \preceq \cdots \preceq \mathcal{C}^*$$

- Should get monotonic improvement because...
  - Knowledge of strategy space strictly increasing
  - Evaluation increasingly comprehensive

## Reasoning

$$\mathcal{W}_t \subset \mathcal{W}_{t+1} \subset \mathcal{W}_{t+2} \subset \cdots \subset \mathcal{W}^*$$

$$\mathcal{C}_t \preceq \mathcal{C}_{t+1} \preceq \mathcal{C}_{t+2} \preceq \cdots \preceq \mathcal{C}^*$$

- Might not get monotonic improvement because…
  - Evaluation never fully complete
  - New strategy may radically shift evaluations

## Shift of Perspective

1. Begin with Rock
2. Discover Paper; Rock loses to Paper
3. Then discover Scissors; Paper loses, Rock wins

- Paper appears to lose quality; Rock gains it
- Directional improvement ill-defined concept

## Depends on Solution Concept

$$\mathcal{W}_t \subset \mathcal{W}_{t+1} \subset \mathcal{W}_{t+2} \subset \cdots \subset \mathcal{W}^*$$

$$\mathcal{C}_t \preceq \mathcal{C}_{t+1} \preceq \mathcal{C}_{t+2} \preceq \cdots \preceq \mathcal{C}^*$$

- If solution concept is "monotonic"…
  - then monotonic increase in knowledge $\Rightarrow$
  - monotonic improvement of estimation

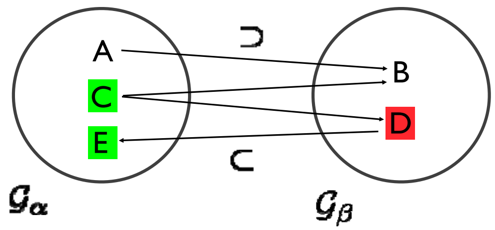
## Monotonicity

With a monotonic solution concept…

If $\mathcal{C}_\alpha$ is solution to X and Z, where $X \supset Z$

Then $\mathcal{C}_\alpha$ will be a solution to <u>any</u> Y where $X \supset Y \supset Z$

## Monotonicity




$\mathcal{C}_\alpha$ solution to games C and E, where $C \supset E$

$\mathcal{C}_\alpha$ not solution to some game D, where $C \supset D \supset E$

Then solution concept is non-monotonic

## Monotonicity

- A monotonic solution concept means:
  - once you discard an estimation in favor of another...
  - you will never return the to earlier estimation
  - ... regardless of whatever new strategies you discover in the future
- Non-monotonic solution concept means:
  - you may return to an estimation from some earlier point in time as you discover new strategies

## Monotonic Solution Concepts

- Solution concepts that are monotonic
  - Nash equilibrium
  - Pareto optimality, but only if you exclude newly discovered strategies that appear identical to ones previously discovered
- Non-monotonic
  - Maximal expected payoff; best response
- This notion of monotonicity subsumes that of [de Jong 2005]

# Advanced Tutorial on Coevolution—References[1]

## 1 Background

### 1.1 Game Theory

[Fudenberg and Tirole, 1998], [Nash, 1951]

### 1.2 Dynamical Systems

[Strogatz, 1994]

## 2 Solution Concepts

[Fudenberg and Tirole, 1998], [Ficici, 2004], [de Jong, 2005], [Bucci and Pollack, 2007], [Wiegand, 2003]

### 2.1 Solution Concept and Evolutionary Dynamics

[Maynard-Smith and Price, 1973], [Maynard-Smith, 1982], [Fogel and Fogel, 1995], [Fogel et al., 1997], [Fogel et al., 1998], [Hofbauer and Sigmund, 1998], [Liekens et al., 2004], [Ficici et al., 2005], [Ficici, 2006], [Ficici and Pollack, 2007]

## 3 Representation

[Moriarty and Miikkulainen, 1997], [Stanley and Miikkulainen, 2002b], [Stanley and Miikkulainen, 2004], [Ashlock et al., 2006]

## 4 Evaluation

[Bull, 2001], [Panait et al., 2004], [Popovici and De Jong, 2005a], [Popovici and De Jong, 2005b], [Popovici and De Jong, 2006c], [Popovici and De Jong, 2006b], [Popovici and De Jong, 2006a]

### 4.1 Test-Based Evaluation

[Juillé and Pollack, 1996b], [Juillé and Pollack, 1996a], [Juillé and Pollack, 1998], [Juillé, 1999], [Juillé and Pollack, 2000], [Watson and Pollack, 2000], [Ficici and Pollack, 2001], [Ashlock et al., 2004], [Bucci and Pollack, 2002], [Bucci and Pollack, 2003], [de Jong and Pollack, 2003], [Bucci et al., 2004], [de Jong, 2004a], [de Jong, 2004b], [Bongard and Lipson, 2005], [de Jong and Bucci, 2006]

# 5 Pareto Coevolution

[Watson and Pollack, 2000], [Ficici and Pollack, 2001], [Noble and Watson, 2001], [Bucci and Pollack, 2002], [Bucci and Pollack, 2003], [de Jong and Pollack, 2003], [Bucci et al., 2004], [de Jong, 2004a], [de Jong, 2004b], [Bongard and Lipson, 2005], [de Jong and Bucci, 2006], [Watson, 2006]

# 6 Archive Methods, design and use

[Rosin and Belew, 1997], [Stanley and Miikkulainen, 2002a], [Ficici and Pollack, 2003], [de Jong, 2004a], [de Jong, 2004b], [Monroy et al., 2006]

# 7 Progress in Coevolution

[Miller and Cliff, 1994], [Floreano and Nolfi, 1997], [Bader-Natal and Pollack, 2004], [de Jong, 2005], [Bader-Natal and Pollack, 2005], [Ficici, 2005]

# 8 Cooperative Coevolution

[Potter and Jong, 1994], [Potter and Jong, 2000], [Wiegand et al., 2001], [Wiegand et al., 2002b], [Wiegand et al., 2002a], [Wiegand et al., 2003], [Wiegand, 2003], [Jansen and Wiegand, 2004], [Panait et al., 2004], [Bucci and Pollack, 2005], [Popovici and De Jong, 2005a], [Popovici and De Jong, 2005b], [Popovici and De Jong, 2006c], [Popovici and De Jong, 2006b], [Popovici and De Jong, 2006a]

# 9 Markov Analyses

[Bull, 2001], [Schmitt, 2003a], [Schmitt, 2003b]

# 10 No Free Lunch

[Wolpert and Macready, 1997], [Wolpert and Macready, 2005]

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