ABSTRACT

This paper employs the probabilistic composition of preferences to classify stores by their operational efficiency. Probabilistic composition of preferences is a multicriteria analysis methodology based on the transformation of assessments by multiple attributes into probabilities of choice. The numerical initial measurements provide estimates for location parameters of probability distributions that are compared to measure the preferences. The probabilities of choice according to each attribute separately are aggregated according to probabilistic composition rules. A classification of two sets of stores into five classes is performed.

Keywords: Data Envelopment Analysis, Efficiency, Multiple Criteria, Probabilistic Preferences Composition, Retail Market, Sorting

INTRODUCTION

Composition of probabilistic preferences (CPP) is a multicriteria analysis methodology based on the association of numerical measurements to probability distributions in a way similar to the transformation of crisp numbers into membership intervals in the Theory of Fuzzy Sets (Zadeh, 1965). This association allows for the replacement of the vector of evaluations of a set of comparable options by a vector of probabilities of choice of each of these options. Probabilistic composition rules (Sant’Anna & Sant’Anna, 2001) can then be used to combine these probabilities of choice according to specific attributes to generate overall preferences for the options.

In fact, the evaluation of the preferences according to each attribute in terms of probabilities of choice allows for using the Theorem of Total Probability to combine these preferences by a weighted average as in classical multiattribute theory (Keeney & Raiffa 1976). The probability of choice according to each attribute can be seen as the probability of preference conditional on that attribute being the only one taken into account. If these probabilities of each of the different attributes being chosen can be determined, they can be used as weights for the conditional probabilities.

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Identification of such marginal probabilities is not easy, especially when, as is frequent in a production setup, there is some correlation between the attributes involved. In a production process, disturbances affecting the inputs reflect on the outputs. Larger revenue is associated with a higher number of sales and more sales happen if there are more sellers, who probably sell more if there is more space to display the products in the stores.

The decision-maker may employ, instead of the combination of conditional probabilities, rules based on the computation of probabilities of joint preference that represent other points of view. For instance Data Envelopment Analysis (DEA) algorithms (Charnes, Cooper & Rhodes, 1978) may combine the vectors of probabilities of maximizing desirable attributes and of minimizing undesirable ones.

A comparison of the rankings derived from probabilistic composition with those derived from direct application of DEA highlights the difference between diverse legitimate views. The stores presenting the highest productivities are not necessarily those attaining the largest production. By this reason, for the evaluation of productivity, a comparison to stores of the same size is more informative. A sorting procedure based on the probabilistic composition proposed by Sant’Anna, Costa & Pereira (2014) to divide the stores in groups of stores of similar size is here employed.

On the other hand, the transformation into probabilities of preference that reduces the dependence on scales of measurement is more reliable if the set of alternatives taken in the comparison is larger. To assess the effect of comparison to a larger set in the initial stage, after evaluating a first network, stores of a second network with different features were added to the analysis. The stores were then evaluated by probabilities of preference calculated within the set of stores of the two networks together and within the group of stores of one network alone. It was found high agreement between the scores.

The second network serves a different market, deriving its revenue from the sale of more expensive articles to consumers of a higher social class. This brings, for that network, numbers of sales and employees specialized in the sale function much smaller than those observed in the first. Nevertheless it was perfectly effective the application of the probabilistic approach to the sorting of the stores of the second network in the classes determined by the representative profiles of the first.

**PROBABILISTIC PREFERENCES COMPOSITION**

The preference for an option is essentially the probability of choosing such option. However, often only attributes of the options are known, or only a ranking of the options on the basis of each of such attributes can be objectively established (Sant’Anna, 2007). The technique of probabilistic composition of preferences extracts from the vector of evaluations of the options according to each attribute a vector of probabilities of choice and combines these particular probabilities of choice into a probability of final choice.

To derive from the vector of measurements of attributes, or of ranks according to them, a vector of probabilities of preference, Sant’Anna & Sant’Anna (2001) propose forms to treat these measurements or ranks as centers of intervals of values with different probabilities of occurrence that mirror the membership functions of Fuzzy Sets Theory (Zadeh, 1965).

Even if the attribute is accurately measured, the preference derived from it is inaccurate. The effective value of the attribute depends on the use or the benefit that the decision maker intends to extract from it. Thus, in the process of formation of preferences on the basis of a given attribute, the decision maker effectively establishes a membership interval around each value. The value of each attribute marks only the center of a probability distribution.
How to identify these probability distributions? In a first approximation, we may assume that they have, like disturbances of classical probabilistic models, a symmetric dispersion that depends only on the attribute considered. It may be more natural, however, assign to them different dispersions depending on the distance to each extreme of the distribution. In fact, the decision maker evaluates each option assuming that it may take any possible value with probabilities that decrease more or less steeply depending on how far from each extreme is the observed value.

Four forms of composition can be obtained according to choices of the decision maker between an optimistic and a pessimistic approach and between a conservative and a progressive approach. While the pessimistic approach leads to choose the option most likely to be preferred according to every criterion, the optimist one chooses the option most likely to be preferred by at least one criterion. On the other aspect, the progressive approach seeks maximization of probabilities of reaching the extreme of excellence while the conservative one seeks to maximize the probability of not reaching the extreme of worst performances.

Sant’Anna, Costa & Pereira (2014) employed probabilistic composition to deal with the sorting problem. The classification procedure, named CPP-TRI, is based on comparisons with representative profiles of each class as in ELECTRE-TRI (Yu, 1982) or, more generally, in ELECTRE-TRI-nc (Almeida Dias, Figueira & Roy, 2012), but each option is allocated to that class for which the probabilities of being above and below such class are nearest to each other. Compared to ELECTRE TRI-nC, the main advantage of the probabilistic approach is its dependence on general probabilistic principles to address the imprecision in the preferences evaluation instead of ad hoc determined parameters.

Consider the following terms: \( \{g_1, \ldots, g_m\} \), a set of m criteria; \((a_1, \ldots, a_m)\), a vector of evaluations of a generic option A under the m criteria; \(\{C_1, \ldots, C_r\}\), an ordered set of r classes into which the options will be allocated, so that better options are assigned to classes with a higher index.

For i varying from 1 to r, each class \(C_i\) is identified by n(i) profiles, each of them consisting of the values of m evaluations by the m criteria. The value according to the criterion \(g_k\) of the j-th profile identifying the i-th class is denoted \(c_{ijk}\). To make easier the comparison of the probabilistic distances to different classes, the number of profiles in each class must be the same, i.e., n(i) = n, for all i.

If this is not the case, central profiles with values equal to the average values of the profiles initially reported for the class are joined to those of the classes with a smaller number of initial representative profiles.

For each k from 1 to m, the coordinates \(a_k\), and the coordinates \(c_{ijk}\), for i varying from 1 to r and j from 1 to n, are thought as means of independent normal distributions with the same variance. Let \(v_k\) denote the common variance of the measurements according to \(g_k\), and let \(X_k\) denote the normal random variable with mean \(a_k\) and variance \(v_k\) and \(Y_{ijk}\) the normal random variable with mean \(c_{ijk}\) and variance \(v_k\), for all i, j and k.

Let \(A^{+}_{ik}\) and \(A^{-}_{ik}\) denote, respectively, the probability of option A presenting a value for the evaluation under the k-th criterion above and below those values under such k-th criterion for all the profiles representing the class \(C_i\). By the hypothesis of independence between disturbances affecting different options and different profiles:

\[
A^{+}_{ik} = \prod_j P[X_k \geq Y_{ijk}]
\]

and

\[
A^{-}_{ik} = \prod_j P[X_k \leq Y_{ijk}].
\]
Combining the criteria from a pessimistic point of view, the probabilities of option A being above and below class $C_i$ are given, respectively, by the products $A_i^+=\prod_k A_{ik}^+$ and $A_i^-=\prod_k A_{ik}^-$. Option A will be classified in the class $C_i$ for which the absolute value of the difference $A_i^+-A_i^-$ is minimal.

To gather information on the uncertainty in the final classification, alternative sorting processes based on more and less exacting rules for classifying above and below the classes’ profiles can be applied. These alternative cutting rules are determined by fixed rates of reduction applied to the probabilities of being above or below the representative profiles. A benevolent classification with cutting planes determined by the reduction percent $c$ will place alternative A in the class whose index $i$ minimizes the absolute value of the difference $A_i^+-(1-c)A_i^-$. Analogously, the exacting classification for the same reduction percent will place alternative A in the class whose index $i$ minimizes the absolute value of the difference $(1-c)A_i^+-A_i^-$. More details on the transformation into probabilities of preference and on the composition of the probabilistic preferences are presented in Sant’Anna, Costa and Pereira (2014).

**DATA ENVELOPMENT ANALYSIS**

In addition to those based on joint probabilities, other forms of composing probabilistic preferences without weighting the criteria employ measures of distance to the frontier of most or least preferable options using scores derived by DEA optimization algorithms.

DEA emerged in the late 70’s, with the work of Charnes et al. (1978). Its basic idea consists of comparing Decision Making Units (DMU), by evaluating the ratios between linear combinations of the values of outputs and inputs with coefficients varying freely. DEA was first proposed in deterministic terms, not considering that the measurements of inputs and outputs may be subject to random disturbances. There are two classic models in DEA. The first, which assumes constant returns to scale (CRS), is also called CCR in reference to Charnes et al. (1978). The second was proposed by Banker, Charnes and Cooper (1984) to deal with variable returns to scale (VRS). Successive developments led, after that, to the construction of a long series of DEA models, considering restrictions to the vector of weights and allowing for additive treatment of the variables.

To employ the probabilistic composition of preferences in the DEA context, the more natural form of composition consists of applying the DEA optimization algorithm to the vectors of probability of preferences according to each criterion.

Two other forms of combination of DEA with the probabilistic composition may be considered. The first consists of deriving the scores by combining the probabilities of maximizing each output and minimizing each input, employing any probabilistic rule chosen among those above discussed to perform such combination.

The other form of combination may be applied even if all the attributes are measured in the same direction. If the goal is formulated in terms of maximization, they may be considered as outputs of an invariant input; if it is put in terms of minimization, as inputs for an invariant output. The DEA optimization algorithm for only one input or output with a constant value along all decision-making units may then be applied to them.

**STORES RANKING**

The apparel retail market in Rio de Janeiro has shown strong growth in recent years. It is highly fragmented, including a multitude of small shops and factories that, although unable to compete in price with the large companies, survive by their flexibility in adapting to new trends in the
consumer market. The increased consumption of lower-income social classes is another leverage factor boosting the industry. In such an active scenario, the use of analytical tools can make a big difference to increase profitability. Quantitative methods of identification of stores of superior operational performance that may be studied as benchmarks are then sought.

Stochastic approaches have been added through time to different parts of DEA (Ferrier & Hirschberg, 1999; Simar & Wilson, 2000; Kao & Liu, 2000). Here, besides a classical deterministic DEA analysis, results of the application of this last approach are also examined. The approach of Kao & Liu (2000) consists essentially in simulating the model by replicating the values for input and output variables and deriving stochastic scores from the series of efficiency scores so generated. The DEA stochastic development here considered follows that employed in Kao & Liu (2009) and is presented in more detail by Sant’Anna, Ribeiro & Meza (2014).

Figueiredo & Soares de Melo (2004) obtained more complete results by using to evaluate retail supermarket stores a CCR model together with a model considering also distance to the inverted frontier (Lins & Angulo-Meza, 2000). Therefore, in addition to the methodology proposed by Kao & Liu (2000) and to the direct deterministic DEA approach, results from the combination of direct and inverted frontier scores presented in Sant’Anna, Ribeiro & Meza (2014) are also here examined.

Considering the goals of the company analyzed, were considered in that study, as inputs, store area and number of salespersons and, as outputs, monthly revenue and monthly number of transactions. The variables of revenue, transactions and salespersons were represented by an average of 12 months of observations. In the stochastic DEA model employed, they were treated as stochastic. The area of the store was not subject to variations that could allow for direct estimation of the dispersion and was treated as deterministic.

The first comparison was based on the joint optimization probabilities according to two different points of view applied to the attributes divided into two groups. First was applied an optimistic and progressive viewpoint to the outputs combined with the application of an optimistic and conservative treatment of inputs. This analysis was compared with application of an optimistic and progressive treatment to both groups of attributes.

For the modeling of the dispersion, were considered a triangular distribution, with fixed ends coincident with those observed along the whole set of stores, and a normal distribution with standard deviation equal to the standard deviation of the vector of observations. Regarding these probability distributions, it is noteworthy that the main difference between them is in the weighting of tail values relatively to more central values. The normal distribution is symmetric around the mean with a fast fall after a certain distance from it while the triangular distribution is characterized by a gentle slope toward the points of maximum and minimum, steeper towards that end closer to the modal value. In each case the joint probability was calculated under the two assumptions, of independence and of maximal dependence between the two inputs and between the two outputs.

Besides the computation of joint probabilities, was performed a composition by the distance to the frontier. The DEA-CCR algorithm was applied first with constant input and with outputs given by the probabilities of maximizing revenue and sales and not maximizing area and salespersons. After that, it was applied with two inputs given by the probabilities of maximizing area and number of salespersons and two outputs given by the probabilities of maximizing revenue and sales.

There was considerable agreement between the results of assuming different hypotheses about the distributions. Significant agreement was also found between the results of the different points of view of the probabilistic composition. The rank correlations between the compositions under the optimistic viewpoint, progressive for outputs and conservative for inputs, and under
the conservative treatment of inputs, with and without dependence, maintaining or changing the
distribution, are all high.

A conclusion that can be drawn from the small differences between the results of the ap-
application of the diverse approaches is that the best performances can be clearly identified. On
the other hand, the influence of the choice of different viewpoints can also be traced to identify
structural features of different stores. The preference between some stores reflects a tradeoff
between maximization of net profit and maximization of total production. If, from the point of
view of net profit, revenue maximization and cost minimization should have equal importance,
from a social point of view, maximizing production should receive higher importance than reduc-
tion of the resources employed. Thus, the influence of the values of attributes of the two groups
on the ranks provides a basis for the choice of weights by the decision maker.

APPLICATION TO A SECOND NETWORK

A second network with 46 stores is here added to the analysis to test the possibility of analyzing
together data of stores of different features. This second is a more deeply established network,
recognized by the high quality of the items sold. It serves the national Brazilian market while
the first has a local market in Rio de Janeiro. Offering more expensive articles to a higher social
class of customers, their stores perform relatively few sales along the day and need a smaller
number of sellers.

Rankings for the stores of the second network derived by combining probabilities of present-
ing maximal values for each attribute among the stores of the same network and, in a second
analysis, in the whole set of stores of the two networks are presented in Table 1. Three composition
approaches are displayed in this table. The first is based on a conservative treatment of inputs,
which employs as final scores the joint probabilities of maximizing at least one output and not
maximizing at least one input. The second is based on the fully progressive treatment, which
 corresponds to the benevolent DEA approach. It employs as final scores the joint probabilities
of maximizing at least one output and minimizing at least one input. The first employs the DEA
algorithm to combine the probabilities of maximizing the outputs and of minimizing the inputs.

For the joint probabilities approaches the scores in Table 1 are derived from assumptions
of normality and independence. For the DEA computation, the CCR algorithm is employed.
Spearman correlation coefficients between the vectors of ranks of Table 1 are presented in Table
2. Similar correlations were obtained substituting for these statistical assumptions hypotheses of
dependence and triangular distributions and employing other DEA algorithms.

It can be seen in Table 2 that the correlations between the vectors of ranks obtained by tak-
ing the network isolated or inserted in the set of stores of the two networks are all very high.
The smallest Spearman correlation, corresponding to the DEA algorithm, is 0.919. Fixing the
data set and considering different models, only the correlations between the progressive joint
probability and the DEA model, of 0.932 for the joint data and of 0.949 for the isolated data se-
are higher than 0.919. A similar result of strong concordance between the results obtained with
separated and joint networks applies to the ranks of the stores of the first network.

In the ranks of the different stores for the three models in Table 1 is also noticeable the
concordance between the results derived from the two data sets. The largest distances between
ranks of the same store are between the results of application of different composition models.
It can be seen there how the conservative treatment of inputs makes the analysis less affected
by the smallest values.
Table 1. Stores attributes measurements and final scores

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continued on following page
Especially noticeable is the case of the two stores with very close values for the outputs of, respectively, 110551 and 107205 for revenue and 931 and 1126 for sales. The first, with high values for the inputs, is ranked 28th or 30th by the conservative approaches and last by the other two. The other, with low values for the inputs, is classified as efficient by the DEA approach, is classified as the 8th by the progressive joint probabilities approach and is only the 16th by the conservative approach.

It is also interesting to notice the good ranks of the four stores with the smallest revenue. This may be due to decreasing returns to scale. Possibly, these stores cannot be taken as benchmarks to stores of a larger size. This suggests classifying the stores by size and look for benchmarks within sets of similar stores. To serve this goal, a probabilistic classification is developed in the next section.

### STORES SORTING

The differences in size, and the effect of such differences on productivity detected in the two last sections, make interesting, in the search for benchmarks, trying to compare the efficiency scores only inside classes of similar sizes. This leads to the need of classifying the stores. Probabilistic composition can be applied to that end.

Considering the values of the four variables in the 23 stores in Table 3, the sets of three profiles for five classes were built. Initially, five equally spaced profiles, one for each class were

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**Table 1. Continued**

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**Table 2. Spearman correlation coefficients**

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<th>Inserted</th>
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<th>Isolate</th>
<th>Isolate</th>
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<td>Conserv.</td>
<td>Progres.</td>
<td>DEA</td>
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<td>0.919</td>
<td>0.959</td>
<td>0.888</td>
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<tr>
<td>Inserted DEA</td>
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<td>0.903</td>
<td>0.741</td>
<td>0.949</td>
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<tr>
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<td>0.874</td>
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<td>Isolate Progres.</td>
<td>0.949</td>
<td>0.949</td>
<td>0.741</td>
<td>0.949</td>
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Especially noticeable is the case of the two stores with very close values for the outputs of, respectively, 110551 and 107205 for revenue and 931 and 1126 for sales. The first, with high values for the inputs, is ranked 28th or 30th by the conservative approaches and last by the other two. The other, with low values for the inputs, is classified as efficient by the DEA approach, is classified as the 8th by the progressive joint probabilities approach and is only the 16th by the conservative approach.

It is also interesting to notice the good ranks of the four stores with the smallest revenue. This may be due to decreasing returns to scale. Possibly, these stores cannot be taken as benchmarks to stores of a larger size. This suggests classifying the stores by size and look for benchmarks within sets of similar stores. To serve this goal, a probabilistic classification is developed in the next section.
determined. Then two profiles with larger and smaller cost/benefit ratios were added. Table 4 presents the profiles so obtained.

The classification was then performed assuming normal distributions with dispersion parameters for the measurements of each variable estimated by the observed variances in the vectors of 23 observations of the variable. Lower and upper limits for the classification established according to the exacting and benevolent rules described in Section 2, with a reduction coefficient of 0.5 were also determined. These alternative classifications led to uncertainty about only seven stores. They are in Table 5 together with the rankings derived from the DEA efficiency scores for the probabilities of maximizing the outputs and the probabilities of minimizing the inputs.

In the class of the smallest stores, the best performance is of Store 9 and the worst is of Store 15. In Class 2, the best performance is of Store 18 and the worst is of Store 8. Here it is interesting to notice that benevolent cuts might translate the stores of Class 1 to Class 2. In that case, stores 9 and 15 would outrank stores 8 and 18. So, these alternative extremes might be considered for this class.

In the median class, it is clear the best performance of stores 3 and 1. The worst performance is that of stores 13 and 22. In Class 4, the stores have performances close to each other. In Class

### Table 3. Observed monthly averages

<table>
<thead>
<tr>
<th>Store</th>
<th>Area</th>
<th>Sellers</th>
<th>Revenue</th>
<th>Sales</th>
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</thead>
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<td>278.921</td>
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<tr>
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<td>48.8</td>
<td>13.4</td>
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<tr>
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<td>16.7</td>
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<tr>
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<td>68.0</td>
<td>11.3</td>
<td>129.947</td>
<td>1.723</td>
</tr>
<tr>
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<td>63.0</td>
<td>11.8</td>
<td>109.870</td>
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<tr>
<td>Store6</td>
<td>79.7</td>
<td>11.1</td>
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<td>2.139</td>
</tr>
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<tr>
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<tr>
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</table>
5, is clear the best performance of Store 20, ranked five positions better than Store 14, the other store in this class of the largest stores.

These results also bring information against the hypothesis of a linear relationship between size and efficiency. In fact, two of the three DEA-efficient stores have a median size, while the third is small and the fourth highest efficiency score belongs to one of the two stores classified in the class in the other extreme.

In a second test, to classify the stores of the second network, the same classes determined by the representative profiles of the stores of the first were employed. The classification is presented in Table 6 in a punctual classification and in an intervallic classification with upper and lower limits determined following the rules of Section 2, with the precision reduction coefficient of 0.5.

In Table 6, it can be seen that, though with a larger dispersion of the initial measurements, the stores of the second network are well distributed along the classes, with 6 stores in the class of smaller stores, 7 in the second, 10 in the third, 15 in the fourth and 8 in that of larger stores. In all the 5 classes it is possible to find stores ranked among the 10 most efficient and among the 10 least efficient.

**CONCLUSION**

This paper explained how the transformation of assessments according to multiple criteria into probabilities of choice provides several alternatives for the composition of criteria without assigning weights to them. Formulated in probabilistic terms, the measures of preferences resulting from the application of each different composition viewpoint are easy to interpret directly.

When evaluating the efficiency of stores, it became apparent the possibility of size affecting the performance. The effect of the probabilistic transformation, of increasing distances near the frontier, called attention to that aspect.
Then, probabilistic composition was applied to classify the stores. In this application, benchmarks for the comparison of performances of stores of similar sizes could be found. The classifications also provided information that would be difficult to grasp otherwise.

The analysis performed with two inputs and two outputs can be extended to a larger number of attributes of each type. If this is the case, computing the final probability of preference by a weighted average may make easier to deal with correlation between attributes. In that case, the calculation of joint probabilities may still be used in a preliminary stage to guide the choice of weights.

### Table 5. Classification and DEA ranks for the first network

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Table 6. Classification and DEA ranks for the second network

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REFERENCES


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