A Survey on Image Reconstruction Using Super Resolution

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Abstract:
Imaging plays a key role in many diverse areas of application, such as astronomy, remote sensing, microscopy, and tomography. Owing to imperfections of measuring devices (e.g., optical degradations, limited size of sensors) and instability of the observed scene (e.g., object motion, media turbulence), acquired images can be indistinct, noisy, and may exhibit insufficient spatial and temporal resolution. Super-Resolution (SR) image reconstruction is a promising technique of digital imaging which attempts to reconstruct High Resolution (HR) imagery by fusing the partial information contained within a number of under-sampled low-resolution (LR) images of that scene during the image reconstruction process. Super-resolution image reconstruction involves up-sampling of under-sampled images thereby filtering out distortions such as noise and blur. In comparison to various image enhancement techniques, super-resolution image reconstruction technique not only improves the quality of under-sampled, low-resolution images by increasing their spatial resolution but also attempts to filter out distortions. Not only do such HR images give the viewer a more pleasing picture but also offer additional details that are significant for subsequent analysis in many applications. Super Resolution Reconstruction (SRR) algorithm is considered to be one of the most promising techniques that can help to overcome the limitations due to optics and sensor resolution.

Keywords: Super resolution, digital image reconstruction, higher-resolution images, algorithmic advances.

1. INTRODUCTION
Super-resolution is the task of obtaining a high-resolution image of a scene given low resolution image(s) of the scene. Applications of super-resolution include satellite, forensic, medical imaging, surveillance et al. [1]. Obtaining high-resolution images directly via better hardware (better image sensors, larger chip size) is quite costly. Image interpolation methods are not considered as super-resolution methods since even the ideal sinc interpolation cannot recover the high frequency components that are lost in the low-resolution sampling process. Most of the super-resolution approaches work on the principle of combining multiple slightly-shifted low-resolution images of the scene [2]. This involves image registration, interpolation and deblurring as the basic operations. However, this technique is numerically limited to small increases in resolution. There also exist methods based on techniques like gradient profile priors [3] and sparse representation of images in an over-complete dictionary [4]. A recent approach based on a single image was proposed in [5] that exploits the redundancy of patches within the image and combines this information with example-based techniques such as [6].

There have been several different approaches to super-resolution, with estimation of high-resolution (HR) images from multiple low resolution (LR) observations related by small motions being by far the most common one. Most of these methods are based on accurate registration and solve the super-resolution reconstruction using variants of gradient descent with or without a smoothness prior [7-10]. Super-resolution has also been tried from multiple defocused images [11], varying zoom [12] and photometric cues [13]. Reconstruction based approaches to super-resolution model the low resolution image formation process to establish a relation between the unknown high resolution image and the low resolution observations, and use the relationship to derive algorithms to estimate the high resolution image essentially by an inversion process [11-15]. The inversion process is typically ill-conditioned and it often necessitates the use of smoothness or other priors [14, 16, 17] to obtain reasonable solutions. In [18], Baker and Kanade examine the limits of such processes and derive that for most point spread functions and blur kernels the estimation process is non-invertible or ill-conditioned. Further, the number of possible solutions grow at least quadratically with the desired magnification factor. They also show that this large growth in the number of solutions makes super-resolution difficult even with smoothness priors and the resulting solutions often fail to recover the high frequency details.

2. PRINCIPLE OF SUPER-RESOLUTION IMAGE RECONSTRUCTION
2.1 Definition of Super Resolution
The term super-resolution refers to the construction of an image whose resolution is higher than the resolution provided by the sensor used in the imaging system. Optical resolution is a measure of the ability of a camera system, or a component of a camera system, to depict picture detail. On the other hand, image resolution is defined as the fineness of detail that can be clearly distinguished in an image. Both the definitions apply to digital and analogue camera systems and images. However, in this research, the term resolution will only relate to digital camera systems and digital images. There are two most common classifications of digital image resolution, namely – spatial and bit-depth.

- Spatial resolution refers to the level of detail discernable in an image.
- Bit-depth refers to the number of bits or 0's and 1's that can be used to specify the colour at each pixel of an image.

2.2 Principle
Super-resolution image reconstruction is based on the theory of Analytic Continuation, which means reconstruction of the whole analytic function according to its values in certain
area. Because of diffraction of lights, spectrum distribution of certain image is infinite in space and optical system truncates its frequency to obtain frequency-truncated image that is finite in space. Generally, truncation function cannot be band limited, but a diffraction limited optical system’s truncation is band-like limited, therefore, the reconstruction of whole spectrum function or just spectrum function above certain frequency is possible.

Assume the imaging model:
\[ g(x, y) = h(x, y) * f(x, y) + n(x, y) \]  (1)

Where, \( h(x, y) \) is the point spread function (PSF), ideal image, \( g(x, y) \) is the original image and \( n(x, y) \) is the noise.

Its Fourier transformation is:
\[ G(u, v) = H(u, v) F(u, v) + N(u, v) \]  (2)

Super-resolution reconstruction is to perform analytic continuation to \( F(u, v) \) to extend its support domain by using prior information of objects and posterior processing technologies, and then get a new PSF \( H'(u, v) \). \( H(u, v) \) also has the extended support domain, thus the resolution of image is improved.

3. FREQUENCY DOMAIN METHODS

The frequency domain approach makes explicit use of the aliasing that exists in each LR image to reconstruct an HR image. Tsai and Huang [20] first derived a system equation that describes the relationship between LR images and a desired HR image by using the relative motion between LR images. The frequency domain approach is based on the following three principles: i) the shifting property of the Fourier transform, ii) the aliasing relationship between the continuous Fourier transform (CFT) of an original HR image and the discrete Fourier transform (DFT) of observed LR images, iii) and the assumption that an original HR image is bandlimited. These properties make it possible to formulate the system equation relating the aliased DFT coefficients of the observed LR images to a sample of the CFT of an unknown image. For example, let us assume that there are two 1-D LR signals sampled below the Nyquist sampling rate.

From the above three principles, the aliased LR signals can be decomposed into the unaliased HR signal as shown in Figure below.

1. Aliasing relationship between LR image and HR image.

Let \( x(t_1, t_2) \) denote a continuous HR image and \( X(w_1, w_2) \) be its CFT. The global translations, which are the only motion considered in the frequency domain approach, yield the \( k \)-th shifted image of \( \mathbf{x}_k(t_1, t_2) = \mathbf{x}(t_1 + \delta t_1, t_2 + \delta t_2) \) where \( \delta t_1 \) and \( \delta t_2 \) are arbitrary but known values, and \( k = 1, 2, \ldots, p \). By the shifting property of the CFT, the CFT of the shifted image, \( \mathbf{X}(w_1, w_2) \), can be written as
\[ \mathbf{X}_k(w_1, w_2) = \mathcal{F}\{ \mathbf{x}(t_1, t_2) \} \]  (3)

The shifted image \( \mathbf{x}(t_1, t_2) \) is sampled with the sampling period \( T_1 \) and \( T_2 \) to generate the observed LR image \( \mathbf{Y}(n_1, n_2) \). From the aliasing relationship and the assumption of bandlimitedness of \( \mathbf{X}(w_1, w_2) \) \( (\mathbf{x}(w_1, w_2) = 0 \) for \( |w_1| \geq (L_1 \pi / T_1) \), \( |w_2| \geq (L_2 \pi / T_2) \)), the relationship between the CFT of the HR image and the DFT of the \( k \)-th observed LR image can be written as [24],
\[ \mathbf{Y}_k[w_1, w_2] = \mathcal{F}\{ \mathbf{x}(t_1, t_2) \} = \mathbf{C}_{L_k} \left( j 2 \pi \delta t_1 + \delta w_1, j 2 \pi \delta t_2 + \delta w_2 \right) \mathbf{X}(w_1, w_2) \]  (4)

By using lexicographic ordering for the indices \( n_1, n_2 \) on the right-hand side and \( k \) on the left-hand side, a matrix vector form is obtained as:
\[ \mathbf{Y} = \mathbf{D} \mathbf{X} \]  (5)

where \( \mathbf{Y} \) is a \( p \times 1 \) column vector with the \( k \)-th element of the DFT coefficients of \( y(n_1, n_2) \), \( \mathbf{X} \) is a \( L_1 \times L_2 \) column vector with the samples of the unknown CFT of \( x(t_1, t_2) \), and \( \mathbf{D} \) is a \( p \times L_1 L_2 \) matrix which relates the DFT of the observed LR images to samples of the continuous HR image. Therefore, the reconstruction of a desired HR image requires us to determine \( \mathbf{D} \) and solve this inverse problem. An extension of this approach for a blurred and noisy image was provided by Kim et al. [21], resulting in a Weighted least squares formulation. In their approach, it is assumed that all LR images have the same blur and the same noise characteristics. This method was further refined by Kim and Su [22] to consider different blurs for each LR image. Here, the Tikhonov regularization method is adopted to overcome the ill-posed problem resulting from blur operator. Bose et al. [23] proposed the recursive total least squares method for SR reconstruction to reduce effects of registration errors (errors in \( \mathbf{D} \)).

4. SPATIAL DOMAIN METHODS

Approaching the super-resolution problem in the frequency domain makes a lot of sense because it is relatively simple and computationally efficient. However, there are some problems with a frequency domain formulation. For one, it restricts the inter-frame motion to be translational because the DFT assumes uniformly spaced samples. Another disadvantage is that prior knowledge that might be used to constrain or regularize the super-resolution problem is often difficult to express in the frequency domain. Since the super-resolution problem is fundamentally ill-posed, problem resulting from blur operator. Bose et al. [23] proposed the recursive total least squares method for SR reconstruction to reduce effects of registration errors (errors in \( \mathbf{D} \)). A discrete cosine transform (DCT)-based method was proposed by Rhee and Kang [26]. They reduce memory requirements and computational costs by using DCT instead of DFT. They
also apply multichannel adaptive regularization parameters to overcome ill-posedness such as underdetermined cases or insufficient motion information cases.

**Regularized SR Reconstruction Approach**

Generally, the SR image reconstruction approach is an ill-posed problem because of an insufficient number of LR images and ill-conditioned blur operators. Procedures adopted to stabilize the inversion of ill-posed problem are called regularization. In this section, we present deterministic and stochastic regularization approaches for SR image reconstruction. Typically, constrained least squares (CLS) and maximum a posteriori (MAP) SR image reconstruction methods are introduced [27].

**Deterministic Approach**

With estimates of the registration parameters, the observation. Model can be completely specified. The deterministic regularized SR approach solves the inverse problem by using the prior information about the solution which can be used to make the problem well posed. For example, CLS can be formulated by choosing an \( \mathbf{x} \) to minimize the Lagrangian [28],

\[
\sum_{k \in \mathcal{S}} \| \mathbf{y}_k - \mathbf{W}_k \mathbf{x} \|^2 + \alpha \mathbf{C} \mathbf{x}^T \mathbf{C} \mathbf{x},
\]

where the operator \( \mathbf{C} \) is generally a high-pass filter, and \( \| . \| \) represents a \( L_2 \) norm. In (6), a priori knowledge concerning a desirable solution is represented by a smoothness constraint, suggesting that most images are naturally smooth with limited high-frequency activity, and therefore it is appropriate to minimize the amount of high-pass energy in the restored image. In (6), \( \alpha \) represents the Lagrange multiplier commonly referred to as the regularization parameter that controls the tradeoff between fidelity to the data and smoothness of the solution. The larger values of \( \alpha \) will generally lead to a smoother solution. This is useful when only a small number of LR images are available (the problem is underdetermined) or the fidelity of the observed data is low due to registration error and noise. On the other hand, if a large number of LR images are available and the amount of noise is small, small \( \alpha \) will lead to a good solution. The cost functional in (6) is convex and differentiable with the use of a quadratic regularization term. Therefore, we can find a unique estimate image \( \mathbf{x}^\star \) which minimizes the cost functional in (6). One of the most basic deterministic iterative techniques considers solving

\[
\left( \sum_{k \in \mathcal{S}} \mathbf{W}_k^T \mathbf{W}_k + \alpha \mathbf{C} \mathbf{C}^T \right) \mathbf{x} = \sum_{k \in \mathcal{S}} \mathbf{W}_k^T \mathbf{y}_k,
\]

and this leads to the following iteration for \( \mathbf{x}^\star \)

\[
\hat{\mathbf{x}}^{n+1} = \hat{\mathbf{x}}^n + \beta \left( \sum_{k \in \mathcal{S}} \mathbf{W}_k^T (\mathbf{y}_k - \mathbf{W}_k \hat{\mathbf{x}}^n) - \alpha \mathbf{C} \mathbf{C}^T \hat{\mathbf{x}}^n \right),
\]

where \( \beta \) represents the convergence parameter and contains an upsampling operator and a type of blur and warping operator. Katsaggelos et al. proposed a multichannel regularized SR approach in which regularization functional is used to calculate the regularization parameter without any prior knowledge at each iteration step. Later, Kang formulated the generalized multichannel deconvolution method including the multichannel regularized SR approach. The SR reconstruction method obtained by minimizing a regularized cost functional was proposed by Hardie et al. They define an observation model that incorporates knowledge of the optical system and the detector array (sensor PSF). They used an iterative gradient-based registration algorithm and considered both gradient descent and conjugate-gradient optimization procedures to minimize the cost functional. Bose pointed to the important role of the regularization parameter and a proposed CLS SR reconstruction which generates the optimum value of the regularization parameter, using the L-curve method.

**Stochastic Approach**

Stochastic SR image reconstruction, typically a Bayesian approach, provides a flexible and convenient way to model a priori knowledge concerning the solution. Bayesian estimation methods are used when the a posteriori probability density function (PDF) of the original image can be established. The MAP estimator of \( \mathbf{x} \) maximizes the a posteriori PDF \( P(\mathbf{x}|\mathbf{y}) \) with respect to \( \mathbf{x} \)

\[
\mathbf{x} = \arg \max_{\mathbf{x}} P(\mathbf{x}|\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_p).
\]

Taking the logarithmic function and applying Bayes theorem to the conditional probability, the MAP optimization problem can be expressed as

\[
\mathbf{x} = \arg \max_{\mathbf{x}} \left\{ \ln P(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_p | \mathbf{x}) + \ln P(\mathbf{x}) \right\}.
\]

Here, both the a priori image model \( P(\mathbf{x}) \) and the conditional density \( P(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_p | \mathbf{x}) \) will be defined by a priori knowledge concerning the HR image \( \mathbf{x} \) and the statistical information of noise. Since MAP optimization in (10) includes a priori constraints (prior knowledge represented by \( P(\mathbf{x}) \)) essentially, it provides regularized (stable) SR estimates effectively. Bayesian estimation distinguishes between possible solutions by using a priori image model, and Markov random field (MRF) priors that provide a powerful method for image prior modeling are often adopted. Using the MRF prior, \( P(\mathbf{x}) \) is described by a Gibbs prior whose probability density is defined as

\[
P(\mathbf{X} = \mathbf{x}) = \frac{1}{Z} \exp \{-U(\mathbf{x})\} = \frac{1}{Z} \exp \left\{ -\sum_{c \in \mathcal{S}} \Phi_c(\mathbf{x}) \right\},
\]

where \( Z \) is simply a normalizing constant, \( U(\mathbf{x}) \) is called an energy function, \( \Phi_c(\mathbf{x}) \) is a potential function that depends only on the pixel values located within clique \( c \), and \( \mathcal{S} \) denotes the set of cliques. By defining \( \Phi_c(\mathbf{x}) \) as a function of the derivative of the image, \( U(\mathbf{x}) \) measures the cost caused by the irregularities of the solution. Commonly, an image is assumed to be globally smooth, which is incorporated into the estimation problem through a Gaussian prior.

A major advantage of the Bayesian framework is the use of an edge-preserving image prior model. With the Gaussian prior, the potential function takes the quadratic form

\[
\Phi_c(\mathbf{x}) = (\mathbf{D}^{(m)} \mathbf{x})^T \mathbf{D}^{(m)} \mathbf{x},
\]

where \( \mathbf{D}^{(m)} \) is an \( m \)th order difference. Though the quadratic potential function makes the algorithm linear, it penalizes the high-frequency components severely. As a result, the solution becomes oversmoothed. However, if we model a potential function...
which less penalizes the large difference in \( x \), we can obtain an edge-preserving HR image. If the error between frames is assumed to be independent and noise is assumed to be an independent identically distributed (i.i.d) zero mean Gaussian distribution, the optimization problem can be expressed more compactly as [30],

\[
\hat{x} = \arg \min \left\{ \sum_{k=1}^{F} \| y - W_k \hat{x} \|^2 + \alpha \sum_{i \in \mathcal{S}} \phi_i(x) \right\},
\]

(12)

where \( \alpha \) is the regularization parameter. Finally, it can be shown that the estimate defined in (6) is equal to a MAP estimate if we use the Gaussian prior in (12).

![Figure 2(a)](image1)

![Figure 2(b)](image2)

![Figure 2(c)](image3)

![Figure 2(d)](image4)

2. Regularized SR reconstruction results by (a) nearest neighbor interpolation, (b) CLS with small regularization parameter, (c) CLS with large regularization parameter, and (d) MAP with edge-preserving prior

A maximum likelihood (ML) estimation has also been applied to the SR reconstruction. The ML estimation is a special case of MAP estimation with no prior term. Due to the ill-posed nature of SR inverse problems, however, MAP estimation is usually used in preference to ML. The simulation results of regularized SR methods are shown in Figure 10. In these simulations, the original 256x256 shop image is shifted with one of the subpixel shifts \{(0,0), (0,0.5), (0,0.5), (0.5,0.5)\} and decimated by a factor of two in both the horizontal and vertical directions. Here, only sensor blur is considered and a 20 dB Gaussian noise is added to these LR images. Figure 2(a) is a nearest neighborhood interpolated image from one of the LR images. CLS SR results using a small regularization parameter and a large regularization parameter appear in Figure 2(b) and(c), respectively. In fact, these estimates can be considered as those of MAP reconstruction with Gaussian prior. Figure 2(d) shows the SR result with an edge-preserving Huber-Markov prior [21]. By far, the poorest reconstruction is the nearest neighbor interpolated image.

This poor performance is easily attributed to the independent processing of the LR observations, and it is apparent throughout Figure 2(a). Compared to this method, CLS SR results in Figure 2(b) and(c) show significant improvements by retaining detailed information. We observe that these improvements are further obtained by using the edge-preserving prior as shown in Figure 2(d). Tom and Katsaggelos proposed the ML SR image estimation problem to estimate the subpixel shifts, the noise variances of each image, and the HR image simultaneously. The proposed ML estimation problem is solved by the expectation-maximization (EM) algorithm. The SR reconstruction from an LR video sequence using the MAP technique was proposed by Schultz and Stevenson. They proposed a discontinuity preserving the MAP reconstruction method using the Huber-Markov Gibbs prior model, resulting in a constrained optimization problem with a unique minimum. Here, they used the modified hierarchical block matching algorithm to estimate the subpixel displacement vectors. They also consider independent object motion and inaccurate motion estimates that are modeled by Gaussian noise. A MAP framework for the joint estimation of image registration parameters and the HR image was presented by Hardie et al. in. The registration parameters, horizontal and vertical shifts in this case, are iteratively updated along with the HR image in a cyclic optimization procedure. Cheeseman et al. applied the Bayesian estimation with a Gaussian prior model to the problem of integrating multiple satellite images observed by the Viking orbiter. Robustness and flexibility in modeling noise characteristics and a priori knowledge about the solution are the major advantage of the stochastic SR approach. Assuming that the noise process is white Gaussian, MAP estimation with convex energy functions in the priors ensures the uniqueness of the solution. Therefore, efficient gradient descent methods can be used to estimate the HR image. It is also possible to estimate the motion information and the HR image simultaneously.

**Projection onto Convex Sets Approach**

The POCs method describes an alternative iterative approach to incorporating prior knowledge about the solution into the reconstruction process. With the estimates of registration parameters, this algorithm simultaneously solves the restoration and interpolation problem to estimate the SR image. The POCs formulation of the SR reconstruction was first suggested by Stark and Oskoui [32]. Their method was extended by Tekalp et al. to include observation noise [33]. According to the method of POCS [28], incorporating a priori knowledge into the solution can be interpreted as restricting the solution to be a member of a closed convex set \( C \) that are defined as a set of vectors which satisfy a particular property. If the constraint sets have a nonempty intersection, then a solution that belongs to the intersection set \( C = \cap_{i=1}^{\infty} C_i \), which is also a convex set, can be found by alternating projections onto these convex sets. Indeed, any solution in the intersection set is consistent with the a priori constraints and therefore it is a feasible solution. The method of POCS can be applied to find a vector which belongs in the intersection by the recursion

\[
\gamma_{n+1} = \underbrace{P_{\mathcal{P}_1} \cdots P_{\mathcal{P}_2} \cdots P_{\mathcal{P}_{n-1}}}_{\mathcal{P}} \gamma_n.
\]

(13)
where $x_0$ is an arbitrary starting point, and $P_i$ is the projection operator which projects an arbitrary signal $x$ onto the closed, convex sets, $C_i (i = 1, 2, \ldots, m)$. Although this may not be a trivial task, it is, in general, much easier than finding $P_s$, i.e., the projector that projects onto the solution set $C_s$ in one step. Assuming that the motion information is accurate, a data consistency constraint set based on the observation model in (2) is represented for each pixel within the LR images $y_k(m_1,m_2)$,

$$C_y^{(m_1,m_2)} = \left\{ x | x_{[n_1,n_2]} = \frac{r}{\sum_{n_1,n_2} W[n_{1,2} r_{1,2} ]} + \delta_{[n_1,n_2]} \right\}, \tag{14}$$

where

$$r^{(s)}[n_1,n_2] = y_k[n_{1,2}].$$

And $\delta_k[m_1,m_2]$ is a bound reflecting the statistical confidence, with which the actual image is a member of the set $C_y^{(m_1,m_2)}$. Since the bound $\delta_k[m_1,m_2]$ is determined from the statistics of the noise process, the ideal solution is a member of the set within a certain statistical confidence. Furthermore, the POCS solution will be able to model space- and time-varying white noise processes.

The projection of an arbitrary $x[n_1,n_2]$ onto $C_y^{(m_1,m_2)}$ can be defined as

$$x^{(m_1,m_2)} = x^{(s)}[n_1,n_2] = x[n_{1,2}] = \frac{r^{(s)}[n_1,n_2]}{\sum_{n_1,n_2} W[n_{1,2} r_{1,2} ]} + \delta_{[n_1,n_2]} \tag{16}.$$ 

Additional constraints such as amplitude constraint after (16) can be utilized to improve the results [34]. Reconstruction results by POCS using data constraint and amplitude constraint appear in Figure 3. In this simulation, four LR images are generated by a decimation factor of two in both the horizontal and vertical directions from the 256x256 HR image, and a 20 dB Gaussian noise is added to these LR images. In this simulation, sensor blur is only considered. Figure 3(a) shows a bilinearly interpolated image of one of the LR observations, and parts (b)-(d) are the reconstruction results after 10, 30, and 50 iterations. Comparing the result by bilinear interpolation in Figure 3(a), we observe that the improvement of the results by POCS SR reconstruction is evident. Patti et al. et.al developed a POCS SR technique to consider space varying blur, nonzero aperture time, nonzero physical dimension of each individual sensor element, sensor noise, and arbitrary sampling lattices. Tekalp et al. et.al extended the technique to the case of multiple moving objects in the scene by introducing the concept of a validity map and/or a segmentation map [36].

**ML-POCS Hybrid Reconstruction Approach**

The ML-POCS hybrid reconstruction approach finds SR estimates by minimizing the ML (or MAP) cost functional while constraining the solution within certain sets. Early efforts for this formulation are found in the work by Schultz and Stevenson [31] where MAP optimization is performed while projections-based constraint is also utilized. Here, the constraint set ensures that the down-sampled version of the HR image matched the reference frame of the LR sequence. Elad and Feuer proposed a general hybrid SR image reconstruction algorithm which combines the benefits of the stochastic approaches and the POCS approach. The simplicity of the ML (or MAP) and the nonellipsoid constraints used in POCS are utilized simultaneously by defining a new convex optimization problem as follows:

$$\min_{x} \frac{1}{2} \left\{ y_k - W_k x \right\}^T R_k^{-1} \left\{ y_k - W_k x \right\} + c \{ S_k x \}^T V \{ S_k x \}, \tag{17}$$
subject to
\[ \{ x \in C_k, 1 \leq k \leq M \} \]
where \( R_n \) is the autocorrelation matrix of noise, \( S \) is the Laplacian operator, \( V \) is the weighting matrix to control the smoothing strength at each pixel, and \( Ck \) represents the additional constraint. The advantage of the hybrid approach is that all a priori knowledge is effectively combined, and it ensures a single optimal solution in contrast to the POCS approach.

**Other SR Reconstruction Approaches**

**Iterative Back-Projection Approach**

Irani and Peleg formulated the iterative back-projection (IBP) SR reconstruction approach that is similar to the back projection used in tomography. In this approach, the HR image is estimated by back projecting the error (difference) between simulated LR images via imaging blur and the observed LR images. This process is repeated iteratively to minimize the energy of the error. The IBP scheme to estimate the HR image is expressed by

\[
\begin{align*}
\hat{x}^{n+1}[m_1, m_2] & = \hat{x}^n[m_1, m_2] \\
& + \sum_{m_1, m_2} \left( y_k[m_1, m_2] - \hat{y}_k^n[m_1, m_2] \right) \times h_{BP}^n[m_1, m_2] \\
& \times h_{BP}^n[m_1, m_2]
\end{align*}
\]

where \( \hat{y}_k^n \) is the simulated LR images from the approximation of \( x \) after \( n \) iteration. \( \hat{y}_k^n \) is influenced by \( n_1, n_2 \), where \( n_1, n_2 \in X \) and \( h_{BP}^n[m_1, m_2; m_1, m_2] \) is a back-projection kernel that determines the contribution of the error \( \{ y_k[m_1, m_2] - \hat{y}_k^n[m_1, m_2] \} \) to \( \hat{y}_k^n \) properly. The scheme for \( n \) is illustrated in Figure 12. Unlike imaging blur, \( h_{BP}^n \) can be chosen arbitrarily. In [31], it is pointed out that the choice of \( h_{BP}^n \) affects the characteristics of the solution when there are possible solutions. Therefore, \( h_{BP}^n \) may be utilized as an additional constraint which represents the desired property of the solution. Mann and Picard [39] extended this approach by applying a perspective motion model in the image acquisition process. Later, Irani and Peleg [9], modified the IBP to consider a more general motion model the advantage of IBP is that it is understood intuitively and easily. However, this method has no unique solution due to the ill-posed nature of the inverse problem, and it has some difficulty in choosing the \( h_{BP}^n \). In contrast to the POCS and regularized approach, it is difficult to apply a priori constraints.

**Adaptive Filtering Approach**

Elad and Feuer [2] proposed an SR image reconstruction algorithm based on adaptive filtering theory applied in time axis. They modified notation in the observation model to accommodate for its dependence on time and suggested least squares (LS) estimators based on a pseudo-RLS or R-LMS algorithm. The steepest descent (SD) and normalized SD are applied to estimate the HR image at each time iteratively, and the LMS algorithm is derived from the SD algorithm. As a result, the HR image at each time is calculated without computational complexity of a direct matrix inversion. This approach is shown to be capable of treating any chosen output resolution, linear time and space variant blur, and motion flow [2]. Which makes the progressive estimation of HR image sequence possible. Following this research, they rederive the R-SD and R-LMS algorithm as an approximation of the Kalman filter [40]. Here, convergence analysis and computational complexity issues of these algorithms were also discussed.

**Motionless SR Reconstruction Approach**

The SR reconstruction algorithms presented so far require relative subpixel motions between the observed images. However, it is shown that SR reconstruction is also possible from differently blurred images without relative motion. Elad and Feuer demonstrated that the motionless SR image reconstruction without a regularization term is possible if the following necessary condition is satisfied:

\[
L^2 \leq \min \left\{ \left( 2m + 1 \right)^2 - 2, p \right\},
\]

where \((2m + 1)X(2m + 1)\) is the size of the blurring kernel, and \( L = L1 = L2 \). Hence, although more numbers of blurred observations of a scene do not provide any additional information, it is possible to achieve SR with these blurred samples, provided (20) is satisfied. Note that one can recover the HR image with much fewer LR images if regularization is incorporated to the reconstruction procedure. Rajan and Chaudhuri [39] proposed a similar motionless SR technique for intensity and depth maps using an MRF model of the image field. There have been other motionless attempts to SR imaging. Rajan and Chaudhuri presented the SR method using photometric cues, and the SR technique using zoom as a cue is proposed by Joshi and Chaudhuri [41-42].

5. SUMMARY AND COMPARISON

A general comparison of frequency and spatial domain SR reconstructions methods is presented in Table 1.

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<td>Degradation model</td>
<td>De-aliasing</td>
<td>De-aliasing A- priori info</td>
</tr>
<tr>
<td>SR Mechanism</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Computation req.</td>
<td>Limited</td>
<td>Good</td>
</tr>
<tr>
<td>A-priori info</td>
<td>Limited</td>
<td>Excellent</td>
</tr>
<tr>
<td>Regularization</td>
<td>Poor</td>
<td>Excellent</td>
</tr>
<tr>
<td>App. performance</td>
<td>Limited</td>
<td>Wide</td>
</tr>
<tr>
<td>Applicability</td>
<td>Good</td>
<td>Almost unlimited</td>
</tr>
</tbody>
</table>

**Table 1. Frequency vs. spatial domain SR**

Spatial domain SR reconstruction methods, though computationally more expensive, and more complex than their frequency domain counterparts, offer important advantages in terms of flexibility. Two powerful classes of
spatial domain methods; the Bayesian (MAP) approach and the set theoretic POCS methods are compared in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Bayesian (MAP)</th>
<th>POCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applicable theory</td>
<td>Vast Limited</td>
<td>Limited</td>
</tr>
<tr>
<td>A-priori info</td>
<td>Prior PDF Easy to incorporate, No hard constraints</td>
<td>Convex sets Easy to incorporate Powerful yet simple</td>
</tr>
<tr>
<td>SR solution</td>
<td>MAP estimate Unique</td>
<td>Non-uniqueness of constraint sets</td>
</tr>
<tr>
<td>Computation req.</td>
<td>Iterative</td>
<td>High</td>
</tr>
<tr>
<td>Convergence Good</td>
<td>Good</td>
<td>Possibly slow</td>
</tr>
<tr>
<td>Optimization</td>
<td>High</td>
<td>Iterative</td>
</tr>
<tr>
<td>Complications</td>
<td>Non-convex priors Optimization under</td>
<td>Operators Defn. of projection</td>
</tr>
</tbody>
</table>

Table 2. MAP vs. POCS SR

6. FUTURE RESEARCH DIRECTIONS

Three research areas promise improved SR methods:

Motion Estimation: SR enhancement of arbitrary scenes containing global, multiple independent motion, occlusions, transparency etc. is a focus of SR research. Achieving this is critically dependent on robust, model based, sub-pixel accuracy motion estimation and segmentation techniques presently an open research problem. Motion is typically estimated from the observed undersampled data the reliability of these estimates should be investigated. Simultaneous multi-frame motion estimation should provide performance and reliability improvements over common two frame techniques. For non-parametric motion models, constrained motion estimation methods which ensure consistency in motion maps should be used. Regularized motion estimation methods should be utilized to resolve the ill-posedness of the motion estimation problem. Sparse motion maps should be considered. Sparse maps typically provide accurate motion estimates in areas of high spatial variance exactly where SR techniques deliver greatest enhancement. Reliability measures associated with motion estimates should enable weighted reconstruction. Global and local motion models, combined with iterative motion estimation, identification and segmentation provide a framework for general scene SR enhancement. Independent model based motion predictors and estimators should be utilized for independently moving objects. Simultaneous motion estimation and SR reconstruction approaches should yield improvements in both motion estimates and SR reconstruction.

Degradation Models: Accurate degradation (observation) models promise improved SR reconstructions. Several SR application areas may benefit from improved degradation modeling. Only recently has color SR reconstruction been addressed [43]. Improved motion estimates and reconstructions are possible by utilizing correlated information in color bands. Degradation models for lossy compression schemes (color subsampling and quantization effects) promise improved reconstruction of compressed video. Similarly, considering degradations inherent in magnetic media recording and playback are expected to improve SR reconstructions from low cost camcorder data. The response of typical commercial CCD arrays departs considerably from the simple integrate and sample model prevalent in much of the literature. Modeling of sensor geometry, spatio-temporal integration characteristics, noise and readout effects promise more realistic observation models which are expected to result in SR reconstruction performance improvements.

Restoration Algorithms: MAP and POCS based algorithms are very successful and to a degree, complimentary. Hybrid MAP/POCS restoration techniques will combine the mathematical rigor and uniqueness of solution of MAP estimation with the convenient a-priori constraints of POCS. The hybrid method is MAP based but with constraint projections inserted into the iterative maximization of the a-posteriori density in a generalization of the constrained MAP optimization [44]. Simultaneous motion estimation and restoration yields improved reconstructions since motion estimation and reconstruction are interrelated. Separate motion estimation and restoration, as is commonly done, is sub-optimal as a result of this interdependence. Simultaneous multi-frame SR restoration is expected to achieve higher performance since additional spatio-temporal constraints on the SR image ensemble may be included. This technique has seen limited application in SR reconstruction.

7. CONCLUSION

This paper has surveyed different state-of-art SR methods; we presented only a few methods and insights for specific scenarios of Super-Resolution. Many questions still persist in developing a generic Super-Resolution algorithm capable of producing high-quality results on general image sequences. A thorough study of Super-Resolution performance limits will have a great effect on the practical and theoretical activities of the image reconstruction community. Other issues in the SR techniques to improve their performance are currently focused on the color SR algorithm and the application to compression systems. Adding features such as robustness, memory and computation efficiency, color consideration and automatic selection of parameters in superresolution methods will be the ultimate goal for the Super-Resolution researchers and practitioners in the future. In building a practical Super-Resolution system, many important challenges lay ahead.

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REFERENCES


