Reducing the Dispersion Effects in Multiwavelength Optical CDMA Systems by Using MCM Signaling

Ngoc T. Dang and Anh T. Pham

Abstract—We propose a novel multiwavelength optical code-division multiple-access (OCDMA) system using multicore modulation (MCM) signaling. For the multicore de-modulator, we employ a maximum-likelihood detection and optical hard limiters. The work in this paper theoretically analyzes the performance of the proposed system over linear dispersive channels, while taking into account the impacts of multiple-access interference (MAI), optical-beating interference (OBI), and receiver noise. It is seen that the use of MCM can significantly relax the effects of chromatic dispersion. In addition, MAI and OBI can be effectively mitigated in the proposed system. As a result, in comparison with the conventional systems using on-off keying (OOK) modulation and pulse-position modulation (PPM), the proposed system can support a higher user bit rate and a larger number of users. Also, a lower transmitted power is required and a longer transmission length can be achieved.

Index Terms—Multiwavelength optical code-division multiple access (MW-OCDMA); Chromatic dispersion; Multicode modulation (MCM); Optical hard limiter (OHL).

I. INTRODUCTION

Optical code-division multiple access (OCDMA) has been considered as a promising candidate for the next-generation optical access networks [1]. The OCDMA systems that use multiple wavelengths (MW-OCDMA) to encode the transmitted signal have recently received increasing attention thanks to the use of two-dimensional (2-D) codes that make better use of the vast optical fiber bandwidth [2].

In the next generation of optical access networks, it is expected that the multiple-access technique is able to support a large number of users with an ultrahigh user bit rate (Gbits/s or higher). In MW-OCDMA systems, as each bit is further spread into chip sequences, the chip rate is relatively high, and the transmitted pulse is therefore very narrow. Chromatic dispersion in MW-OCDMA systems consequently becomes one of the most critical performance degradation factors even in the short-distance access environment [3–6].

In MW-OCDMA systems, the effects of chromatic dispersion include time skewing, pulse broadening, and peak power reduction [6]. Time skewing is the phenomenon in which relative delays occur among chips at different wavelengths. In MW-OCDMA systems, time skewing results in incorrect decoding and then errors in bit detection [3]. This effect, however, can be effectively compensated by using the preskewing/postskewing technique at the encoder/decoder [4]. On the other hand, to minimize the last two effects, a wide pulse width is required since the power loss caused by chromatic dispersion is especially strong for short pulses [7]. This requirement introduces another issue in the design of MW-OCDMA systems because the long code (in the time domain) and/or modulations with multiple positions (such as pulse-position modulation—PPM) are usually used to reduce the impact of the multiple-access interference (MAI) and the optical-beating interference (OBI) [8,9].

In this paper, we therefore propose a novel method of M-ary multicode modulation (MCM). In this modulation method, we also use M-ary code words as in PPM, which would help relax the impact of MAI and OBI. Nevertheless, instead of using multiple positions for M-ary symbols, multiple MW-OCDMA codes are used as shown in Fig. 1, a unique MW-OCDMA code for each symbol. Since the symbol duration (Ts) in MCM is log2 M times wider than the bit duration (Ts) and M times wider than the symbol duration in PPM, the chip duration (Tc) in MCM is increased by the same factors. This allows the use of a wide pulse width that consequently reduces the chromatic dispersion effects. To further diminish the impact of MAI and OBI, we use maximum-likelihood (ML) detection and optical hard limiters (OHLs) at the multicode demodulator. As a result, the proposed systems can support a large number of users with a high bit rate as required for next-generation optical access networks.

The rest of the paper is organized as follows. Section II presents the multicode configuration for MCM. The system descriptions and performance analysis are presented in Section III and IV, respectively. Section V shows the numerical results and discussion. Finally, Section VI concludes the paper.
The algorithm determines the place of a pulse within a block as follows:

$$a_{xy} = [x,y], \quad x,y = 0, 1, \ldots, p_x - 1,$$

(1)

where \( p_x \) is a prime number, and \([\cdot]\) denotes modulo \( p_x \) operation [2]. The algorithm determines the place of a pulse within a block of length \( p_x \). A code pattern consists of \( p_x \) such blocks. Similarly, a WH pattern is generated from a prime number \( p_h \) (\( p_h \leq p_b \)). The process of generating the TS and the WH patterns is illustrated in Table I.

The WH pattern \( H_0 \) comprises pulses at one wavelength only, as evident from Table I, and is therefore discarded. Hence the number of WH patterns is \( p_h - 1 \), the number of TS patterns being \( p_x \). Thus a 2-D code set, including \( p_x \times (p_h - 1) \) distinctive 2-D prime codes of length \( p_h x p_x \), can be generated. The code weight is \( p_x \) and the maximum cross-correlation between two 2-D codes is one [2].

### B. Multicode Configuration for MCM

In MCM, each user requires \( M \) codes for the encoding of its \( M \) symbols. \( p_x \times (p_h - 1) \) 2-D prime codes of a code set are hence equally divided to all users, i.e., \( M \) distinctive 2-D codes to each user. The maximum number of users is therefore equal to \( [p_x(p_h - 1)]/M \), where \([\cdot]\) defines the floor function of \( x \). For example, a 2-D prime code constructed from \( p_x = 11 \) and \( p_h = 31 \) can theoretically support a maximum of 82 users when 4-MCM is used.

It is worth noting that, in conventional systems, which assign one code to each user, there are many unused codes. This is mainly because conventional systems can support only a few users under the impact of physical impairments such as noise, interference, and dispersion [6,10,11]. In our proposed system, we make use of these unused codes for improving the system performance.

### III. System Descriptions

A schematic diagram of a 2-D OCDMA system using MCM and OHLs is shown in Fig. 2. Signals from \( K \) transmitters are combined by a combiner and broadcast to all receivers by a splitter. For illustrative purposes, one transmitter and one receiver are depicted in detail in the figure.

#### A. MCM Transmitter

At the transmitter, each block of \( \log_2 M \) bits is first converted into \( M \) symbols denoted as \( s_u \) \((u=0, \ldots, M-1)\) by a symbol converter. Each symbol is then encoded by one of \( M \) 2-D codes at a multicode implanter, which can be implemented by using the well-studied fiber Bragg grating (FBG) arrays as shown in Fig. 3(a). The positions and wavelengths of the FBG arrays in one FBG array are tuned for the code of the corresponding MCM symbol. M FBG arrays are connected to an encoding controller that uses the information symbol to select the corresponding FBG array. The encoded signal, which is illustrated in Fig. 2, is a sequence of chips including chips “1” and “0.” A chip “1” corresponds to an optical pulse, whereas a chip “0” means that no pulse is transmitted. In a chip sequence, the positions of chips “1” and “0” are determined by the TS pattern, whereas the wavelengths of chips “1” are determined by the WH pattern. Finally, the encoded signal is combined with signals from other transmitters and sent to the receiver side over an optical fiber.

#### B. Pulse Propagation Model

During propagation over the fiber, optical pulses are affected by fiber attenuation and chromatic dispersion. To analyze their effects, each optical pulse is modeled as a Gaussian pulse and optical fiber is considered as a linear dispersive channel. The amplitude of a transmitted Gaussian pulse is written as

$$G_0(t) = \sqrt{P_0} \exp\left(-\frac{t^2}{2T_0^2}\right),$$

(2)

where \( P_0 \) is the transmitted peak power of the optical pulse, which is assumed to be the same for all users. \( T_0 \) is the half-width of the pulse (at the 1/e intensity point), which is derived from the chip duration so that this pulse is located completely inside the chip duration.

For a fair comparison with other systems, the analysis is considered under a constraint on the average power per bit denoted as \( P_b \). The relation between \( P_0 \) and \( P_b \), which is presented in detail in the Appendix, is given by...
\[ P_0 = \frac{T_c \log_2 M}{T_0 P_b \sqrt{\pi}}. \] (3)

After propagating over the optical fiber of length \( L \) km, the amplitude of the received Gaussian pulse can be expressed as [7]

\[ G_L(t) = \sqrt{P_0} \exp(-aL) \frac{T_0}{(T_0^2 - j\beta_2 L)^{1/2}} \exp\left(-\frac{t^2}{2(T_0^2 - j\beta_2 L)}\right), \] (4)

where \( a \) and \( \beta_2 \) are the attenuation and dispersion coefficients of the fiber, respectively. The average received power per chip considering the splitter loss \( (1/M) \) caused by the demodulator can be written as

\[ P_s = \frac{1}{MT_c} \int_{-T_c/2}^{T_c/2} |G_L(t)|^2 dt. \] (5)

C. MCM Receiver

At the receiver, the received signal including MAI is input into a multicode demodulator, which consists of \( M \) decoding branches as shown in Fig. 3(b). At each branch, the received signal is decoded by a FBG-based decoder (D). Assuming that the targeted transmitter sends symbol \( s_d \), the decoding process needs to be considered in two cases (Fig. 4): at (1) the desired decoder \( (D_d) \) and (2) the undesired decoder \( (D_u, u \neq d) \).

At the desired decoder, all \( p_s \) desired pulses of the chip sequence representing symbol \( s_d \) will be collected since their wavelengths are matched to this decoder. Moreover, FBGs of the desired decoder are tuned in inverse order of those in the encoder of the targeted transmitter. The relative delays among desired pulses are therefore compensated so that they appear at the same time at the output of the decoder and generate an autocorrelation. MAI pulses from interfering users also contribute to the autocorrelation. However, by using OHL to limit the number of optical pulses at each wavelength to the maximum number, all MAI pulses are blocked and the autocorrelation peak has a height of \( p_s \).

Considering an undesired decoder, each user has the maximum MAI pulse that can pass through it since the cross-correlation between any two 2-D codes is at most one. Considering the system with \( K \) simultaneous users, the maximum number of MAI pulses that are visible at the output of an undesired decoder will be \( K \). These pulses are distributed over \( p_s \) wavelengths. Let \( k_i, \ i = 1 \) to \( p_s \), denote the number of MAI pulses at the \( i \)th wavelength; then \( k_i \) is a random variable whose value is in the range of 0 and \( K \) \( (K = \sum_{i=1}^{p_s} k_i) \). At the output of an OHL, there is a pulse at the \( i \)th wavelength only when \( k_i \geq 1 \) [12]. Considering all \( p_s \) wavelengths, the total number of pulses at the output of an OHL, denoted as \( \kappa \), will be a random variable, where \( 0 \leq \kappa \leq p_s \).

After the decoding process, optical signals at all decoding branches are converted into photocurrents by photodetectors (PDs). As the maximum number of visible pulses at a wavelength is one thanks to the OHL, OBIs are relaxed because there is no beating interference between optical

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pulses at the same wavelength. Next, at the symbol detector, the photocurrents at its $M$ inputs are compared in parallel to determine the detected symbol corresponding to the largest value. Finally, the detected symbol is converted into binary data.

### IV. Performance Analysis

In this section, we first present the method to calculate the symbol error rate $P_e$. The bit error rate of the proposed system then can be derived as $\text{BER}=0.5P_e$ [9].

We assume that the transmitted data is large enough that the probability of sending any symbols are the same. Without loss of generality, we also assume that symbol $s_0$ is transmitted, i.e., $d=0$. Employing a union bound, $P_e$ can be given as

$$P_e = 1 - \Pr[I_0 > I_u | s \in \{1, \ldots, M-1\}, s = s_0] = \sum_{u=1}^{M-1} \Pr[I_u]$$

$$= \sum_{k=0}^{ps} \Pr[I_k]$$

$$= \sum_{k=0}^{ps} \Pr[I_0 = 0] = (M-1) \sum_{k=0}^{ps} \Pr[k]$$

where $s$ represents the transmitted symbol, $k$ represents the total number of MAI pulses at the output of the OHL, and hence $\Pr[k]$ is the probability that there are totally $k$ MAI pulses. $Q(\cdot)$ is the $Q$ function. $I_0$ is the photocurrent at the desired input of the ML detector and is calculated by $I_0 = g_{ps} P_s$. $I_u = g_{ps} P_{ps}$ is the photocurrent at the undesired branch of the ML detector. $\sigma^2_{01}$ is the receiver noise, which is the total of the shot noise ($\sigma^2_{01} = 2e I_0/B_e$) and thermal noise ($\sigma^2_{01} = 8nk_b T_e B^2_e C$) at the zeroth/first input. Here, $e$ is the electron charge and $B_e$ is the electrical bandwidth. $k_B$ is Boltzmann’s constant, $T_e$ is the receiver noise temperature, and $C$ is the receiver capacitance.

In Eq. (6), $\Pr[k=k]$ is equal to the probability that the number of MAI pulses at the input of the OHL is different from zero (i.e., $k_i \neq 1$) at $k$ (out of $p_s$) wavelengths. This probability is equivalent to the probability that the number of MAI pulses at the input of the OHL is empty at $k = p_s - k$ wavelengths. Employing the model of the occupancy problem [13], $\Pr[k=k]$ can be calculated as

$$\Pr[k=k] = \sum_{l=0}^{p_s} (-1)^{k-l} \binom{l}{k} (1-lp_s)^k,$$

where $K$ is the number of simultaneous users. $p_s$ is the probability that a pulse from one user becomes a MAI pulse and is visible at one of $p_s$ wavelengths.

For 2-D prime code, $p_s$ can be calculated as $p_s = \langle \mu_s \rangle / \langle \mu_o \rangle$, where $\langle \mu_s \rangle$, the average number of wavelengths common to any pair of codes, can be estimated as [10]

$$\langle \mu_s \rangle = \frac{1}{p_s} \left( \frac{p_h-1}{p_h} \right) \left( \frac{p_s-1}{p_s} - 2 \right) + \frac{p_h-2}{p_h-1} \frac{p_s-1}{p_s} \frac{p_s-2}{p_s-1}.$$

### V. Numerical Results

In this section, we first analyze the impact of dispersion on the performance of the conventional systems using on-off keying (OOK) modulation and PPM. We then compare the performance of the proposed system using MCM signaling to that of the conventional systems. We use the 2-D prime code set with $p_s=11$ and $p_h=31$. $p_s$ wavelengths are in the window of 1550 nm with a spacing of 0.4 nm. As the proposed system is considered for access networks, the transmission length $L=20$ km is chosen. Other system parameters are shown in Table II.

First, Fig. 5 shows BER versus $P_b$ when $K=30$ users and $R_b=1$ Gbit/s. We consider the two cases with and without the impact of dispersion for OOK and PPM systems with $M=2, 4,$ and 8. It is seen that, due to MAI and OBI, the BER floor in the OOK system is very high ($10^{-4}$) and cannot satisfy the requirement of $\text{BER} \leq 10^{-9}$. MAI and OBI can be relaxed by using PPM so that the BER floor can be reduced. In PPM systems, however, due to the effects of dispersion, high transmitted power is required. For example, the system us-

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver capacitance</td>
<td>$C$</td>
<td>$0.02 \times 10^{-12}$ F</td>
</tr>
<tr>
<td>Noise temperature</td>
<td>$T_e$</td>
<td>300 K</td>
</tr>
<tr>
<td>PD responsivity</td>
<td>$\eta$</td>
<td>1 A/W</td>
</tr>
<tr>
<td>Attenuation coefficient</td>
<td>$\alpha$</td>
<td>0.2 dB/km</td>
</tr>
<tr>
<td>Dispersion coefficient</td>
<td>$\beta_2$</td>
<td>20 ps²/km</td>
</tr>
<tr>
<td>Bit rate per user</td>
<td>$R_b$</td>
<td>bits/s</td>
</tr>
<tr>
<td>Electrical bandwidth</td>
<td>$B_e = R_b / \log_{10} M$</td>
<td>Hz</td>
</tr>
<tr>
<td>Chip duration</td>
<td>$T_c = \log_{10} M / (R_b p_s^2)$</td>
<td>seconds (s)</td>
</tr>
<tr>
<td>Half-width of pulse</td>
<td>$T_0$</td>
<td>$T_c / (8 \sqrt{2})$</td>
</tr>
</tbody>
</table>
ing 4-PPM suffers a power penalty of 25 dB due to the impact of dispersion. It is also seen that, in the presence of dispersion, $M=4$ is the best choice for PPM systems since the pulse width becomes narrower when $M/\log_2(M)$ increases as $M$ increases, a MCM system with higher $M$ will have better resistance against the dispersion. This results in lower required transmitted power in MCM systems. For example, a gain of 9 dB is achieved when $M$ increases from 2 to 4 in the proposed system. The better performance is seen when $M$ increases to 8; however, it is necessary to note that the maximum number of users in that case will be reduced as more codes are required for each user.

The advantage of MCM in reducing the effects of dispersion is further highlighted in Fig. 7. In this figure, we fix $P_b=-5$ dBm and investigate the BER versus the transmission length ($L$) for PPM and MCM systems (an OOK system is not considered here because its performance is worse than that of PPM systems). The number of simultaneous users and the user bit rate are kept the same as in Fig. 6. Thanks to the reduction of dispersion effects, the maximum transmission length (at BER=$10^{-9}$) in the proposed system can be extended as $M$ increases. For example, the maximum transmission length in the 4-MCM system is 30 km longer than that of the 4-PPM system.

In Fig. 8, we compare the 4-PPM system with the M-MCM systems in terms of the maximum user bit rate at BER=$10^{-9}$. The number of simultaneous users is 30 and $L=20$ km. In the transmitted power range of −14 and 0 dBm, the proposed M-MCM system can support a significantly higher user bit rate than the 4-PPM system can (i.e., the best PPM system). For example, the 4-MCM system can support a user bit rate of 2.5 Gbits/s at $P_b=-3$ dBm, whereas the 4-PPM system can support only 1.2 Gbits/s.

The advantage of the proposed systems using MCM in the number of simultaneous users is shown in Fig. 9 when $P_b=-5$ dBm. It is seen that, using a longer code length (i.e.,
ps = 17) cannot help the OOK system to improve its performance, which even becomes worse due to dispersion effects. On the other hand, MCM systems are able to support 40 (or 60) users when 2-MCM (or 4-MCM) is used, which is 2 (or 3) times more than that of the 4-PPM system.

Figure 9 also shows that when the number of simultaneous users increases, the BER of the 4-PPM system is not increased as much as that of the M-MCM systems. This is because the impact of MAI and OBI, which becomes dominant with a large number of users, is better relaxed in 4-PPM systems since the cross-correlation between two PPM symbols is zero while it is at most one in MCM systems.

The required transmitted power per bit is investigated versus the number of simultaneous users (K) in Fig. 10 with BER = 10^{-9}. As shown in this figure, the proposed systems require less power in comparison with the 4-PPM system. It is, however, seen that when K increases, the required power in the proposed systems increases faster than that of 4-PPM. This is because the impact of MAI and OBI gradually becomes stronger than that of dispersion when K increases.

Finally, we consider the relation between the number of supportable users and the maximum user bit rate in the proposed systems in Fig. 11 with different M. Also, $P_b = -5$ dBm and $L = 20$ km are used. Using this figure, we can find the maximum user bit rate for a specific number of users and vice versa. It is obvious that, in order to keep the required BER of $10^{-9}$, the maximum user bit rate should be decreased when the number of simultaneous users increases. However, it is seen that the decrease of the maximum user bit rate in the system using 4-MCM is faster than that of the 2-MCM system. This is because the higher 4-MCM system is more affected by MAI and OBI, which become dominant compared with dispersion when K increases.

VI. CONCLUSION

We have proposed a novel MW-OCDMA system using MCM signaling. The performance of the proposed system was analyzed and compared with that of the conventional systems using OOK and PPM. The major advantages of the proposed system are the ability to simultaneously mitigate chromatic dispersion, MAI, and OBI. The numerical results reveal that the proposed system outperforms the conventional systems in terms of the number of supportable users, user bit rate, and required transmitted power.
APPENDIX: DERIVATION OF EQUATION (3)

The average transmitted power per chip \( P_c \) is defined as

\[
P_c = \frac{1}{T_c} \int_{-T_c/2}^{T_c/2} |G_d(t)|^2 dt.
\]

(9)

We assume that the amplitude of the transmitted Gaussian pulse is decreased so that the borders of the chip duration, \(-T_c/2 \) and \( T_c/2 \), can be replaced by \(-\infty \) and \( \infty \) as integration limits. Equation (9) can be written as

\[
P_c = \frac{P_0}{T_c} \int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{T_0^2}\right) dt = P_0 \frac{T_0}{T_c} \int_{-\infty}^{\infty} \exp(-x^2) dx = P_0 \frac{T_0}{T_c} \sqrt{\pi},
\]

(10)

where \( x = t/T_0 \) and the Gaussian integral \( \int_{-\infty}^{\infty} \exp(-x^2) dx \) is equal to \( \sqrt{\pi} \).

Moreover, there are \( p_s \) Gaussian pulses (chips “1”) in one MCM symbol duration \( (T_s) \), where \( T_s = T_b \log_2 M \). The relation between \( P_c \) and \( P_b \) is given by

\[
P_c P_s = P_b \log_2 M.
\]

(11)

From Eqs. (10) and (11), Eq. (3) can be derived.

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Anh T. Pham received the B.E. and M.E. degrees, both in electronics engineering, from the Hanoi University of Technology, Vietnam, in 1997 and 2000, respectively, and the Ph.D. degree in information and mathematical sciences from Saitama University, Japan, in 2005. From 1998 to 2002, he worked as a Telecommunication Network Engineer for NTT Vietnam Corp. Since April 2005, he has been an Assistant Professor at the School of Computer Science and Engineering, University of Aizu. His present research interests are in the area of the spread spectrum technique and optical communications. Dr. Pham received a Japanese government scholarship (MonbuKagaku-sho) for his Ph.D. study. He also received a Vietnamese government scholarship for his undergraduate study.