# **Effect of Permeability Variation in Vacuum Consolidation**

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**ABSTRACT:** The varying coefficient of permeability plays a major role in increasing the accuracy of the numerical prediction results. This paper aims to evaluate the range of deviation of numerical results if the varying coefficient of permeability is not used during the consolidation time. Four models, one with constant permeability and the others with the varying coefficient of permeability corresponding to different values of  $C_c/C_k$ , were used to simulate an axisymmetric single vacuum wellpoint. Comparing those predicted values together and to the measured values, the results show that: the settlement value and the decrease in pore water pressure value of the model which did not consider the change in permeability coefficient were higher than those of the models which considered the change in permeability coefficient during the consolidation process; the bigger the  $C_c/C_k$  ratio, the smaller the settlement and the smaller the decrease in pore water pressure.

KEYWORDS: Soft ground, Vacuum consolidation, Vertical drain, Vacuum wellpoint systems, Finite element method.

## 1. INTRODUCTION

Since the vacuum preloading was first proposed by Kjellmann (1952), it has been becoming popular for the soil improvement of large areas. In addition to accelerating the consolidation of the soft clays, vacuum preloading also reduces lateral displacements and the surcharge height. Therefore, it has attracted a lot of attention from a lot of scholars in geotechnical engineering field.

Finite element analysis (FEA) is a powerful method for modeling the system of vertical drains combined with vacuum preloading (Chai et al., 2013; Chai et al., 2009; Duong et al., 2012; Ghandeharioon et al., 2011; Indraratna and Redana, 2000; Le et al., 2015; Ong et al., 2012; Rujikiatkamjorn and Indraratna, 2013; Rujikiatkamjorn et al., 2007; Saowapakpiboon et al., 2011; Tran and Mitachi, 2008; Voottipruex et al., 2013; Wu and Hu, 2013). It can be used to model very demanding cases such as complex geometries, loadings and material properties, even for the simulation of a large-scale radial drainage consolidometer (Indraratna et al., 2004), where analytical solutions are hard to obtain.

Many studies have attributed the difference between the numerical predictions and measure data to numerous factors (soil disturbance - smear zones, time-dependent load, well resistance, and partial penetration of drains), but most of them focus more on the material properties such as soil mechanic model and permeability laws. Tarefder et al. (2009) successfully predicted the field behaviour of a full-scale test embankment constructed by using the modified Cam-Clay model and drainage parameters. Indraratna et al. (2005) introduced a Darcian-based analytical model with the effects of a varying coefficient of horizontal permeability and coefficient of compressibility during the consolidation process. Toshifumi et al. (2014) presented a numerical analysis using an elasto-plastic finite element program (FEM) for soil-water coupled problems, incorporating the SYS Cam-clay. Sun et al. (2015) introduced a plain strain FEM program that was coded with the application of the non-linearity constitutive relation Duncan-Chang's model and the non-linear permeability law into the Biot's consolidation theory. Indraratna et al. (2017) developed a numerical solution for largestrain consolidation incorporating non-Darcian (nonlinear) radial flow with varying compressibility and permeability coefficients.

It can be seen that the permeability laws and the varying coefficient of permeability strongly effect on numerical prediction results. This paper aims to determine the deviation range of numerical prediction results if not take the varying coefficient of permeability into consideration. Four models, one with constant permeability and the others with the varying coefficient of permeability corresponding to different values of  $C_c/C_k$ , were used to simulate an axisymmetric single vacuum wellpoint. The effect of variation of permeability coefficients on numerical prediction results

and some relative conclusions would be drawn by comparing the predicted values between numerical models together as well as the values between numerical models and measured values.





Figure 1  $k_h/k_{hi}$  plotted against  $\sigma'/\sigma_i$  for varying  $C_c/C_k$ .

Tavenas et al. (1983) suggested the following two formulas:

$$\overline{e} = e_0 - C_c \log\left(\frac{\sigma'}{\sigma_i}\right) \tag{1}$$

$$\overline{e} = e_0 + C_k \log\left(\frac{k_h}{k_{hi}}\right)$$
(2)

Where *e* and *e*<sub>0</sub> are the void ratio and the in-situ void ratio, respectively;  $k_h$  and  $k_{hi}$  are the permeability and the in-situ permeability, respectively;  $\sigma'$  and  $\sigma_i$  are the effective stress and the initial effective stress, respectively;  $C_k$  and  $C_c$  are permeability change index and the compression index, respectively (for overconsolidated range the recompression index  $C_r$  is used rather than  $C_c$ ).

From Eq. (1) and (2) it follows that:

$$\frac{k_h}{k_{hi}} = \left(\frac{\sigma'}{\sigma'_i}\right)^{-\frac{C_c}{C_k}}$$
(3)

Berry and Wilkinson (1969) found that the typical value of  $C_c/C_k$  for soil is in the range of 0.5÷2.0. Figure 1 shows  $k_h / k_{hi}$  plotted against  $\sigma'/\sigma_i$  for varying  $C_c/C_k$ .

#### 3. LABORATORY TESTING

The test was conducted in a vacuum-surcharge consolidation apparatus at China University of Geosciences (Beijing) (Vu et al., 2016). The schematic of the apparatus is shown in Figure 2. The internal diameter and height of sample tank are 440 and 1000 mm, respectively. The vacuum wellpoint was a 25 mm outside diameter PVC pipe surrounded by very coarse sand.





The soft clay samples were collected at TianJin Province of

China. In the laboratory, the remolded sample was made by mixing the clay samples with the amount of water which was 1.2 times of its fully saturated clay samples in a mechanical mixer. After that, a consolidation pressure 10 kPa was applied to prepare the reconstituted clay. Soil properties of the reconstituted clay sample are shown in Table 1. Test loading scheme is listed in Table 2. At the end of the test, specimens at different locations in the cell were measured to determine the water content, the void ratio, and the permeability coefficient. Soil properties at the end of the test are shown in the Table 3.

Table 1 Soil properties of reconstituted clay sample (Vu et al., 2016)

| Properties                                   | Test results          |
|--|-----------------------|
| Water content, <i>w</i> (%)                  | 35.00                 |
| Unit weight, $\gamma$ (g/cm <sup>3</sup> )   | 1.881                 |
| Void ratio, <i>e</i> <sub>0</sub>            | 0.952                 |
| Particle density, $G_s$ (g/cm <sup>3</sup> ) | 2.71                  |
| Liquid limit, <i>LL</i> (%)                  | 35                    |
| Plastic limit, <i>PL</i> (%)                 | 22                    |
| Horizontal permeability, $k_{hi}$ (cm/s)     | 5.06×10 <sup>-7</sup> |
| Vertical permeability, $k_{vi}$ (cm/s)       | 2.07×10 <sup>-7</sup> |
| Cohesion, $c$ (kPa)                          | 15.6                  |
| Friction angle, $\varphi$ (°)                | 7.0                   |
| Recompression index, Cr                      | 0.024                 |
| Compression index, $C_c$                     | 0.119                 |

Table 2 Summary of the loading for the laboratory test (Vu et al., 2016)

| Time (h)                 | 0-8 | 9-19 | 20-55 |
|--------------------------|-----|------|-------|
| Surcharge pressure (kPa) | 25  | 25   | 25    |
| Vacuum pressure (kPa)    | 50  | 0    | 50    |

| Table 3 | Soil properties at the end of the test (Vu et al., 2016) |  |
|---------|--|--|

| No.        | Sample   | Water   | Unit  | Void  |
|------------|--|---|---|---|
|            | position   | content,  | weight,   | ratio,  |
|            | (Figure 2)   | w (%)   | $\gamma$ (g/cm <sup>3</sup> )   | e   |
| 1          | 5  | 32.77   | 1.906   | 0.888   |
| 2          | 6  | 34.16   | 1.890   | 0.931   |
| 3          | 5  | 31.62   | 1.908   | 0.869   |
| 4          | 6  | 32.44   | 1.910   | 0.879   |
| 5          | 3  | 31.68   | 1.917   | 0.862   |
| 6          | 4  | 32.18   | 1.913   | 0.872   |
| 7          | 3  | 31.60   | 1.919   | 0.858   |
| 8          | 4  | 32.07   | 1.915   | 0.869   |
| 9          | 1  | 31.51   | 1.922   | 0.854   |
| 10         | 2  | 32.44   | 1.910   | 0.879   |
| 11         | 1  | 31.81   | 1.918   | 0.862   |
| 12         | 2  | 31.44   | 1.923   | 0.852   |
|            |  |   |   |   |
| No.        | Sample   | Particle  | Horizontal  | Vertical  |
| No.        | Sample<br>position   | Particle<br>density,  | Horizontal permeability,  | Vertical permeability,  |
| No.        | Sample<br>position<br>(Figure 2)   | Particle<br>density,<br>G <sub>s</sub> (g/cm <sup>3</sup> )   | Horizontal<br>permeability,<br>kh(cm/s)   | Vertical<br>permeability,<br><i>k<sub>v</sub></i> (cm/s)  |
| <b>No.</b> | Sample<br>position<br>(Figure 2)<br>5  | Particle<br>density,<br><i>G<sub>s</sub></i> (g/cm <sup>3</sup> )<br>2.71   | Horizontal<br>permeability,<br><i>k<sub>h</sub></i> (cm/s)  | Vertical<br>permeability,<br>k <sub>r</sub> (cm/s)<br>1.60×10 <sup>-7</sup>   |
| <b>No.</b> | Sample<br>position<br>(Figure 2)<br>5<br>6   | Particle<br>density,<br><i>G<sub>s</sub></i> (g/cm <sup>3</sup> )<br>2.71<br>2.72   | Horizontal<br>permeability,<br><i>k</i> <sub>h</sub> (cm/s)   | Vertical           permeability,           kv(cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup>   |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5  | Particle<br>density,<br><i>G</i> <sub>s</sub> (g/cm <sup>3</sup> )<br>2.71<br>2.72<br>2.71  | Horizontal<br>permeability,<br><i>k</i> <sub>h</sub> (cm/s)<br>—<br>2.29×10 <sup>-7</sup>   | <b>Vertical</b><br>permeability,<br><i>k</i> <sub>r</sub> (cm/s)<br>1.60×10 <sup>-7</sup><br>1.78×10 <sup>-7</sup><br>—   |
| No.        | Sample           position           (Figure 2)           5           6           5           6           5           6           5           6 | Particle           density,           Gs (g/cm³)           2.71           2.72           2.71           2.71  | Horizontal<br>permeability,<br><i>k</i> <sub>h</sub> (cm/s)<br>   | Vertical           permeability,           k_v(cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup>  |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>6<br>3   | Particle           density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71   | Horizontal<br>permeability,<br><i>k</i> <sub>h</sub> (cm/s)<br>   | Vertical           permeability,           k₂(cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup>   |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4   | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71   | Horizontal<br>permeability,<br>$k_h(cm/s)$<br>  | Vertical           permeability,           kv(cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup>   |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4<br>3<br>4<br>3  | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71   | Horizontal<br>permeability,<br>kh(cm/s)<br>   | Vertical<br>permeability,<br>k <sub>ν</sub> (cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup>  |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4<br>3<br>4<br>3<br>4   | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71  | Horizontal<br>permeability,<br>$k_h$ (cm/s)<br>—<br>2.29×10 <sup>-7</sup><br>2.71×10 <sup>-7</sup><br>—<br>1.37×10 <sup>-7</sup><br>1.79×10 <sup>-7</sup>   | Vertical           permeability, $k_{\nu}$ (cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup> —           —           8.03×10 <sup>-8</sup> 1.04×10 <sup>-7</sup> —   |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4<br>3<br>4<br>3<br>4<br>1  | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71                               | Horizontal<br>permeability,<br>$k_h$ (cm/s)<br>   | Vertical<br>permeability,<br>$k_{\nu}$ (cm/s) $1.60 \times 10^{-7}$ $1.78 \times 10^{-7}$ —           8.03 $\times 10^{-8}$ $1.04 \times 10^{-7}$ —           7.50 $\times 10^{-8}$                                       |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4<br>3<br>4<br>3<br>4<br>1<br>2   | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71                | Horizontal<br>permeability,<br>$k_h$ (cm/s)<br>—<br>2.29×10 <sup>-7</sup><br>2.71×10 <sup>-7</sup><br>—<br>1.37×10 <sup>-7</sup><br>1.79×10 <sup>-7</sup><br>—<br>—   | Vertical<br>permeability,<br>$k_{\nu}$ (cm/s) $1.60 \times 10^{-7}$ $1.78 \times 10^{-7}$ 8.03 $\times 10^{-8}$ $1.04 \times 10^{-7}$ 7.50 $\times 10^{-8}$ 9.03 $\times 10^{-8}$   |
| No.        | Sample<br>position<br>(Figure 2)<br>5<br>6<br>5<br>6<br>3<br>4<br>4<br>3<br>4<br>1<br>2<br>1   | Particle<br>density,           Gs (g/cm³)           2.71           2.72           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71           2.71 | Horizontal<br>permeability,<br>$k_h$ (cm/s)           —           —           2.29×10 <sup>-7</sup> 2.71×10 <sup>-7</sup> —           —           1.37×10 <sup>-7</sup> 1.79×10 <sup>-7</sup> —           —           1.29×10 <sup>-7</sup> | Vertical<br>permeability,<br>k₂(cm/s)           1.60×10 <sup>-7</sup> 1.78×10 <sup>-7</sup> —           —           8.03×10 <sup>-8</sup> 1.04×10 <sup>-7</sup> —           9.03×10 <sup>-8</sup> 9.03×10 <sup>-8</sup> — |

After conducting oedometer tests, the recompression index  $C_r$  and the compression index  $C_c$  were 0.024 and 0.119, respectively. Thus, the relationship between void ratio and effective stress was as follows:

$$\overline{e} = e_{0} - 0.119 \log\left(\frac{\sigma}{\sigma_{i}}\right)$$
(4)

Recompressed phase:

$$\bar{e} = e_{0} - 0.024 \log\left(\frac{\sigma'}{\sigma_{i}}\right)$$
(5)

Based on linear regression analysis, permeability change index  $C_k$  was 0.1786 (Figure 3). The relationship between void ratio and horizontal permeability was as follows:

$$\overline{e} = e_{0} + 0.178 \log\left(\frac{k_{h}}{k_{hi}}\right)$$
(6)



Figure 3 The relationship between void ratio and semi – log horizontal permeability (Vu and Yang, 2016)

## 4. THE EFFECT OF VARIATION OF PERMEABILITY COEFFICIENTS ON NUMERICAL PREDICTION RESULTS

In this study, the effect of vacuum pressure was simulated by assigning the negative pore pressure along the drain boundaries, similar to (Indraratna et al., 2004).

A component of GeoStudio 2007 suite of software, SIGMA/W (GEO - SLOPE International Ltd, 2007), was used to simulate the laboratory test. Four models were established with the same twodimensional axisymmetric mesh. After choosing the global element size (0.026 m) and the finite-element mesh pattern (triangles), a finite-element mesh was generated automatically (Figure 4). At the top, the surcharge load of 25 kPa was applied. The horizontal and vertical displacements of the bottom edge and the horizontal displacement of the right and the left sides were restricted. An impervious boundary was assigned to all the boundaries; however the negative pore pressure was applied along the drain boundary. The maximum value of the negative pore water pressure (50 kPa) was at the wellpoint filter and the minimum value of the negative pore water pressure (45 kPa) was at the top of the vacuum well.



Figure 4 Axisymmetric finite-element mesh

Model 1: Model 1 was established to verify subsequent numerical models, which would be used to evaluate the effect of variation of permeability coefficients on numerical prediction results. Soil behaviour was considered to be elastic - plastic. The effect of a varying coefficient of horizontal permeability due to the changing of void ratio, corresponding to  $C_c/C_k = 0.67$ , was taken into account. A reasonable way to improve the accuracy of numerical predictions is to assume that the soil hydraulic conductivity under the vacuum preloading is similar to the permeability of unsaturated soil (Vu and Yang, 2016). Hence, the hydraulic conductivity function was defined as permeability of unsaturated soil: the hydraulic conductivity varied with changes in suction. The assumed suction – unsaturated permeability relationship was shown in Figure 5.



Figure 5 The assumed suction – unsaturated permeability relationship (Vu and Yang, 2016)

Model 2: Conditions were identical to those of Model 1, but did not consider the change of the varying coefficient of horizontal permeability due to the changing of void ratio.

Model 3: Conditions were identical to those of Model 1. However, the varying coefficient of horizontal permeability due to the changing of void ratio was corresponding to  $C_c/C_k = 0.5$ .

Model 4: Conditions were identical to those of Model 1. However, the varying coefficient of horizontal permeability due to the changing of void ratio was corresponding to  $C_c/C_k = 2$ .



Figure 6 The measured settlement and predicted settlement of all the numerical models



Figure 7 The measured and predicted pore water pressure at the piezometer No2

Figure 6 shows the measured settlement as well as predicted settlement through the above numerical models. Figures 7, 8, 9 show the measured and predicted pore water pressure at the piezometers No2, No4, No6 (see Figure 2), respectively. The agreement between the measured values and predicted values from Model 1 in all the figures proved that the parameters used to simulate the laboratory testing were acceptable.



Figure 8 The measured and predicted pore water pressure at the piezometer No4



Figure 9 The measured and predicted pore water pressure at the piezometer No6



Figure 10 The deviation-ratio of the settlement

In order to easily evaluate the effect of variation of permeability coefficients on the numerical prediction results, a formula for calculating a deviation-ratio is proposed as follows:

$$D = \frac{(S_{k=const} - S)}{S_{k=const}} \times 100 \tag{7}$$

Where *D* is deviation-ratio,  $S_k = \text{const}$  is the settlement value without considering the change of permeability coefficient, *S* is the settlement value considering the change of permeability coefficient.

It can be seen from Figure 6 that the settlement value without considering the change of permeability coefficient is higher than that which considering the change of permeability coefficient. The bigger the  $C_c/C_k$  ratio, the smaller the settlement. The smallest settlement is corresponding to  $C_c/C_k = 2$ . This phenomenon is also reflected through the deviation-ratio of the settlement in Figure 10: the maximum deviation ratio curve is in the case of  $C_c/C_k = 2$ , the smallest deviation ratio curve is in the case of  $C_c/C_k = 2$ .

It is indicated from Figure 10 that the deviation ratio value increased rapidly with time in the beginning. However, this value decreased gradually when the vacuum was released and the settlement increased slowly (the state from 8 hours to 20 hours). It could be explained that there was no hydraulic conductivity varying with changes in suction, thus the difference between the settlement values with and without considering the change of permeability coefficient almost unchanged. Meanwhile, the settlement still increased with time. The deviation ratio value increased again when the consolidation rate was higher due to the vacuum reloading. At the end of the test (56 hours) it reached the maximum value of 44.16% and 15.95% in the cases of  $C_c/C_k = 2$  and  $C_c/C_k = 0.5$ , respectively.

The variation in the pore pressure value was similar to the variation in the settlement value and it can be seen in Figure 7-9. The variation in pore water pressure value in the case of constant permeability coefficient is larger than that in the cases of variable permeability coefficients, and the smallest variation is corresponding to the case of  $C_c/C_k = 2$ .

## 5. CONCLUSION

In this study, four models, one with constant permeability and the others with the varying coefficient of permeability corresponding to different values of  $C_c/C_k$ , were used to simulate an axisymmetric single vacuum wellpoint. Comparing those predicted values together and to the measured values, the following conclusions can be drawn.

- (1) The settlement value and the decrease in pore water pressure value given by the model which did not consider the change of permeability coefficient were higher than those given by the model which considered the change of permeability coefficient in the consolidation process. The bigger the  $C_c/C_k$  ratio, the smaller the settlement. The maximum deviation ratio curve was in the case of  $C_c/C_k = 2$ , the smallest deviation ratio curve was in the case of  $C_c/C_k = 0.5$ . And the position of the curve of the model which was used to simulate the laboratory test ( $C_c/C_k = 0.67$ ) between the curves of the models which were corresponding to  $C_c/C_k = 2$  and  $C_c/C_k = 0.5$  also validated that.
- (2) The deviation ratio value of the settlement increased rapidly in the beginning, when the consolidation rate was high. However, while the consolidation rate was low and there was no hydraulic conductivity varying with changes in suction because of the releasing of vacuum pressure, the deviation ratio value gradually decreased. The deviation ratio value increased again when the vacuum reloading. Over time, the deviation ratio value tended to increase slowly and might reach to steady state. At the end of the testing time the deviation ratio value (also the maximum value) were 44.16% and 15.95% for the cases of  $C_c/C_k = 2$  and  $C_c/C_k = 0.5$ , respectively. The model which matched closest to the test 133

values corresponding to  $C_c/C_k = 0.67$  was associated with 20.04% of the deviation of the settlement. Thus, in order to eliminate or minimize the deviation ratio value, it's vital to consider the varying coefficient of horizontal permeability due to the change in void ratio in the consolidation process.

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