The assessment of seismic hazards in mines

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Synopsis

Methods for the analysis of seismic hazards in mines are reviewed as illustrations of the techniques that are being used in some South African mines. The classical parameters that describe a hazard are the seismic activity rate, λ, the Gutenberg-Richter value, b, *and, occasionally, the maximum regional magnitude,* m_{max}. *The activity rate is calculated readily owing to the inherent simplicity of its definition. The evaluation method presented for the Gutenberg-Richter* b *value takes into account the presence of* m_{max}. *Three statistical procedures are described for the assessment of* m_{max}, *in which the maximum observed magnitude,* X_{max}, *plays a crucial role.*

Often, the configuration of mine seismic networks is changed, or the networks are improved, which inevitably gives rise to changes in the completeness of the seismic catalogues. The routine analysis of seismic hazards must account for these time-dependent changes.

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Introduction

The classical description of seismic activity is based on the seismic activity rate, λ, which is equal to the number of events with magnitudes greater than, or equal to, a defined magnitude level, m_{min}, during a specified time period, T; the parameter b in the Gutenberg-Richter relation; and, sometimes, the maximum regional magnitude, m_{max}.

The Gutenberg-Richter relation takes the form

$$\log n(m) = a - bm, \qquad [1]$$

where $n(m)$ is the number of events not less than magnitude m, and a and b are parameters. A high b value indicates that a small fraction of the total number of seismic events in the region of interest has high magnitudes, whereas a low b value implies a large fraction of high magnitude events. Establishing the b value for a region can be important in the assessment of seismic hazard, since high levels of ground motion are often associated with damage or rockmass weakening that occurs as a result of seismic events of high magnitude.

Despite the value of the above approach, the quantitative comparison of seismic hazards between different mining areas and periods is difficult if one looks only at the parameters λ, b, and m_{max}, or at probability tables. A concept that allows for these difficulties is the integrated volume of ground motion (VGM)[1,2]. The VGM value is a single number that integrates the probabilities of seismic events occurring at different magnitudes described by the Gutenberg-Richter relation [1], and a relation between magnitude, distance, and peak ground velocity, over the magnitude range of interest. The resulting VGM is equal to the total volume of the rockmass affected by peak ground velocities not smaller than a certain critical value.

The following sections describe both the classical approach and the VGM parameter for the assessment of seismic hazards in mines, and applications are given for the mining region in South Africa known as the Far West Rand (FWR).

Procedure for the Assessment of Seismic Hazards

Assessment of m_{max}

At present, there is no generally accepted method for the estimation of the maximum magnitude, m_{max}. It can be estimated from empirical relationships between the magnitude and various tectonic and fault parameters[3-5], or from the extrapolation of frequency-magnitude curves and the use of strain rate or rate of seismic moment release[6,7]. McGarr[8] used such an approach to evaluate the maximum possible magnitude of seismic events in mines. The value of m_{max} can also be evaluated from statistical procedures[9-12]. We here describe three statistical procedures for the elevation of m_{max}, in which the maximum observed magnitude, X_{max}, plays a crucial.

Method I

In the case of limited and/or doubtful seismic data, or when one wants to obtain quick results without going into sophisticated analysis, the following rule[13] would apply to the evaluation of m_{max} from a data set of $X_1 \leq X_2 \leq \ldots \leq X_{n-1} \leq X_{max}$:

$$\hat{m}_{max} = X_{max} + (X_{max} - X_{n-1}), \qquad [2]$$

where $\hat{m}_{max}$ is an estimation of m_{max}, and X_{max} and X_{n-1} are the largest and the second-largest magnitudes observed in the region (fore-shocks and after-shocks excluded).

Method II

In this approach, the condition for the evaluation of m_{max} is based on some properties of the end-point estimator of a uniform distribution. It is known that, if a random variable, ξ, follows a uniform distribution in the range <0,a>, where a is an unknown, the unbiased estimation of a is equal to[14]

$$\hat{a} = \frac{n+1}{n}\xi_{max}, \qquad [3]$$

A method for the evaluation of hazards is reviewed that accommodates varying levels of quality in the catalogues.

Finally, the concept of the volume of ground motion (VGM) is presented. This is a technique that allows for most aspects of seismic hazards in mines. The advantage of the VGM is that a hazard can be described by one physically significant value: the volume of rock (cumulated over a time period) in which the ground motion exceeds some critical particle velocity.

Data from the gold-mining region in South Africa known as the Far West Rand is used in a demonstration of the discussed methods of hazard analysis.

where ξ_{max} is the maximum observed value of ξ, $\xi_{max} = \max(\xi_1, \ldots \xi_n)$, and n is the number of observations. Since the value of any cumulative distribution function, $F(m)$, follows the uniform distribution within the interval <0,1> then, by the replacement of ξ by $F(m)$, where $F(m)$ is the cumulative probability function of magnitude m, the following relation is obtained:

$$1 = \frac{n+1}{n} F(X_{max}), \qquad [4]$$

where X_{max} is the maximum observed magnitude in the catalogue. Let us assume that the magnitude of the seismic event, m, follows the double-truncated Gutenberg–Richter magnitude distribution[15,16]:

$$F(m) = \frac{1 - \exp[-\beta(m - m_{min})]}{1 - \exp[-\beta(m_{max} - m_{min})]}, \qquad [5]$$

$$\text{for } m_{min} \le m \le m_{max},$$

where m_{min} is the known threshold of completeness of the catalogue. When the number of events, n, is substituted by λT, in [4], where λ is the activity rate and T is the span of the catalogue, $\hat{m}_{max}$, we get, after simple rearrangement,

$$\hat{m}_{max} = -\frac{1}{\beta} \ln \left\{ \exp(-\beta m_{min}) - \left[\exp(-\beta m_{min}) - \exp(-\beta X_{max})\right] \frac{\lambda T + 1}{\lambda T} \right\}. \qquad [6]$$

Method III

In this approach, the evaluation of m_{max} is obtained from the condition that, during the span of the catalogue, T, the largest *observed* magnitude, X_{max}, is equal to the largest *expected* magnitude:

$$X_{max} = \mathbf{EXPECTED}(X_{max}|T). \qquad [7]$$

The formula for the largest *expected* magnitude, **EXPECTED**$(X_{max}|T)$, is given by [11]

$$\mathbf{EXPECTED}(X_{max}|T) = m_{max} - \frac{E_1(Tz_2) - E_1(Tz_2)}{\beta \exp(-Tz_2)} - m_{max} \exp(-\lambda T), \qquad [8]$$

where $z_i = -\lambda A_i/(A_2 - A_1)$, $A_1 = \exp(-\beta m_{min})$, $A_2 = \exp(-\beta m_{max})$, and $E_1(\cdot)$ denotes an exponential integral function[17]:

$$E_1(x) = \int_x^{\infty} \exp(-\zeta)/\zeta \, d\zeta. \qquad [9]$$

Since, for most real seismic data, $Tz_1 > 1$ and $Tz_2 > 1$, $E_1(x)$ can be approximated as

$$E_1(x) = \frac{1}{x} \exp(-x) \frac{x^2 + a_1 x + a_2}{x^2 + b_1 x + b_2}, \qquad [10]$$

where a_1 = 2,334 733, a_2 = 0,250 621, b_1 = 3,330 657, and b_2 = 1,681 534. Formula [10] is an approximation of the exponential integral function with a maximum error of $5 \cdot 10^{-5}$ for $1 \le x \le \infty$.

Relations [6] and [8] are in full agreement with our intuitive expectations: the larger the period of the observations, the less the estimated maximum possible magnitude, $\hat{m}_{max}$, deviates from the maximum observed magnitude, X_{max}.

Assessment of λ

The simplest formula for the assessment of the seismic activity rate, λ, follows from its definition:

$$\lambda = n/T, \qquad [11]$$

where n is the total number of events with magnitudes greater than, or equal to, the threshold of completeness, m_{min}, and T is the span of the event catalogue. On the assumption that the occurrence of seismic events in time follows a Poisson distribution, the approximate standard deviation[14] of λ is equal to $\lambda^{1/2}$. It can be shown that the activity rate corresponding to any magnitude between m_{min} and m_{max} is equal to

$$\lambda(m) = \lambda[1 - F(m)], \qquad [12]$$

where $F(m)$ is the Gutenberg–Richter double-truncated cumulative distribution of magnitude [5].

In addition to relation [12], several other relations can be obtained from this model. One immediate consequence of the model is that $Rp(m)$, the *mean return period* (mean time interval) between seismic events having a magnitude equal to, or greater than, m is given by

$$Rp(m) = 1/\lambda(m). \qquad [13]$$

Such a simple model of seismicity and evaluation of mean return period are applicable only if the catalogue of seismic events is complete and the magnitude uncertainties are small. If the catalogue is incomplete, or if some of the information was recovered from inaccurate records, or if the catalogue contains different types of magnitude estimations, then more advanced procedures must be applied, as described later.

Assessment of b

Numerous observations indicate that seismic events induced by mining in many respects follow the same rules as those obeyed by natural earthquakes. A prominent concept in the analysis of both types of seismicity is the frequency–magnitude relation [1] introduced by Gutenberg and Richter[18].

Several efficient statistical procedures are available for the evaluation of parameter *b*. To demonstrate the maximum likelihood method, we describe the classic procedure for the evaluation of the *b* parameter introduced by Aki[19] and Utsu[20,21], which is performed under the assumption that magnitudes are continuous and unlimited from the top. Then, the formula obtained for the *b* evaluation will be corrected for the presence of m_{max}.

If the magnitudes of seismic events are assumed to be independent, identically distributed random variables, the frequency–magnitude Gutenberg–Richter relation [1] can be written in the following form:

$$f(m) = \beta \exp\left[-\beta\left(m - m_{\min}\right)\right] \quad [14]$$

or

$$F(m) = 1 - \exp\left[-\beta\left(m - m_{\min}\right)\right], \quad [15]$$

where $f(m)$ and $F(m)$ are the probability density and the cumulative distribution functions of magnitude m respectively, m is regarded as a continuous variable that may assume any value above the threshold value m_{min}, and $\beta = b \ln(10)$. If the magnitudes are considered to be independent, then the joint probability density for the set of N magnitudes m_i ($i = 1, \ldots, N$) is equal to the product of the individual probability densities $f(m_i)$. The maximum likelihood estimate of the parameter β is the value of $\hat{\beta}$ for which the likelihood function $L(\cdot)$, proportional to the joint probability density, is maximum. The maximum likelihood condition can be written as

$$L\left(\beta|m_1,\ldots,m_N\right) = \text{const} \prod_{i=1}^{N} f\left(m_i|\beta\right) = \max, \quad [16]$$

or equivalently

$$\sum_{i=1}^{N} \frac{\partial}{\partial\beta} \ln f\left(m_i|\beta\right) = 0. \quad [17]$$

The classic Aki–Utsu estimate of the maximum likelihood of β is then obtained[19,20]:

$$\hat{\beta} = \frac{1}{<m> - m_{\min}}, \quad [18]$$

where the sample mean magnitude $<m> = \sum m_i/N$. From the central-limit theorem, it follows that, for sufficiently large N, $\hat{\beta}$ is approximately normally distributed about its mean value equal to [18], with the standard deviation equal to

$$\hat{\sigma}_\beta = -\left(\frac{\partial^2 \ln L}{\partial\beta^2}\right)^{-1/2} = \hat{\beta}\ \sqrt{N}. \quad [19]$$

The standard deviation of $\hat{b}$ is obtained by dividing $\hat{\sigma}_\beta$ by ln (10).

For an estimate of time-varying seismic hazard, and for the prediction of impending seismic events, the approximation of σ_β obtained for a time-dependent β is more useful. Shi and Bolt[22] showed that, for large samples, N, and slow temporal changes of β, the standard deviation of $\hat{\beta}$ is

$$\hat{\sigma}_\beta = \hat{\beta}^2 \left\{\sum_{i=1}^{N}\left(<m> - m_i\right)^2 \Big/ \left[N(N-1)\right]\right\}^{1/2}. \quad [20]$$

Since its first derivation in 1965, the Aki–Utsu formula [18] has been used successfully in a great number of studies that had entirely different patterns of seismicity. This approach has, nevertheless, a significant shortcoming, i.e. the assumption that magnitudes are unbounded from the top.

The maximum likelihood estimate of β for continuous magnitudes between m_{min} and its upper limit, m_{max}, was derived for the first time by Page[15]. It is easy to show that, if the upper limit of magnitude is taken into account, the cumulative probability-distribution function [15] takes the form [5], and the density distribution [14] is

$$f(m) = \frac{\beta \exp\left[-\beta\left(m - m_{\min}\right)\right]}{1 - \exp\left[-\beta\left(m - m_{\min}\right)\right]}. \quad [21]$$

Following the described procedure, the maximum likelihood equation for β is[15]

$$1/\hat{\beta} = <m> - m_{\min} + \frac{\left(m_{\max} - m_{\min}\right)\exp\left[-\beta\left(m_{\max} - m_{\min}\right)\right]}{1 - \exp\left[-\beta\left(m_{\max} - m_{\min}\right)\right]}. \quad [22]$$

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The exact evaluation of $\hat{\beta}$ from equation [22] requires a knowledge of m_{min} and m_{max}, and can be obtained only by recursive solutions. Nevertheless, a simple approximation of $\hat{\beta}$ is possible. From relation [22] it is clear that the maximum likelihood estimate of β by the classic Aki–Utsu formula [18] is greater when the presence of m_{max} is ignored. Thus, a proper estimation of β can be obtained by use of simple formula [18] and then by correction of the result for the expected bias. Consequently, following relations [18], with accuracy to the second term of the Taylor expansion of [22], the β value becomes[23]

$$\hat{\beta} = \hat{\beta}_0\left(1 - \kappa_{max}\right), \qquad [23]$$

where

$$\kappa_{max} = \hat{\beta}_0 \frac{\left(m_{max} - m_{min}\right)\exp\left[-\hat{\beta}_0\left(m_{max} - m_{min}\right)\right]}{1 - \exp\left[-\hat{\beta}_0\left(m_{max} - m_{min}\right)\right]}, \qquad [24]$$

and $\hat{\beta}_0$ is the Aki-Utsu estimator [18].

Evaluation from Incomplete and Uncertain Data

It often happens that, as a result of changes in the detection capability of a seismic network arising from changes in the instrumentation and station coverage, the catalogues of seismic events can differ in different intervals of time. This time-dependent incompleteness of catalogues makes them unsuitable for use in the evaluation of seismic hazards based on existing procedures. In this section, a procedure is described that facilitates the use of catalogues with different thresholds of completeness, and incorporates the uncertainty in the determination of earthquake magnitudes. The proposed procedure is a maximum-likelihood method for the estimation of hazard parameters, namely the maximum regional magnitude, m_{max}, the seismic activity rate, λ, and the b parameter of the Gutenberg–Richter relation.

Following Tinti and Mulargia[24], let us assume that the observed (apparent) magnitude is distorted by an observational stochastically independent error, ε. If the error is assumed to be normally distributed with the standard deviation σ, the density and cumulative probability-distribution functions of the apparent magnitude become[12]

$$\tilde{f}\left(x \mid m_{min}\right) = \frac{f\left(m \mid m_{min}, \sigma\right)}{1 - F\left(m_{min} \mid m_{min}, \sigma\right)}, \qquad [25]$$

$$\tilde{F}\left(m \mid m_{min}, \sigma\right) = \frac{F\left(m \mid m_{min}, \sigma\right) - F\left(m_{min} \mid m_{min}, \sigma\right)}{1 - F\left(m_{min} \mid m_{min}, \sigma\right)},$$

where

$$f\left(m \mid m_{min}, \sigma\right) = \beta \cdot A(m) / \left(A_1 - A_2\right) \cdot C_\sigma\left(m \mid m_{min}, \sigma\right),$$

$$F\left(m \mid m_{min}, \sigma\right) = \left[A_1 - A(m)\right] / \left(A_1 - A_2\right) \cdot D_\sigma\left(m \mid m_{min}, \sigma\right),$$

$$C_\sigma\left(m \mid m_{min}, \sigma\right) = \frac{\exp\left(\gamma^2\right)}{2}\left[\operatorname{erf}\left(\frac{m_{max} - m}{\sigma\sqrt{2}} + \gamma\right) + \operatorname{erf}\left(\frac{m - m_{min}}{\sigma\sqrt{2}} - \gamma\right)\right],$$

$$D_\sigma\left(m \middle| m_{min}, \sigma\right) = \left\{A_1\left[\operatorname{erf}\left(\frac{m - m_{min}}{\sigma\sqrt{2}}\right) + 1\right] + A_2\left[\operatorname{erf}\left(\frac{m_{max} - m}{\sigma\sqrt{2}}\right) - 1\right] - 2C_\sigma\left(m \middle| m_{min}, \sigma\right) \cdot A(m)\right\} \quad 2\left[A_1 - A(m)\right], \qquad [26]$$

where $A(m) = \exp(-\beta m)$, erf(·) is the error function, $\gamma = \beta\sigma/\sqrt{2}$, and $m \geq m_{min}$.

Finally, if the model in which the density function vanishes below the cut-off magnitude, m_{min}, is unrealistic (in practice the transition occurs gradually), the relation between the apparent activity rate, $\tilde{\lambda}(m)$, and the 'true' rate takes the form

$$\tilde{\lambda}(m) = \lambda(m)\frac{\exp\left(\gamma^2\right)}{2}\left[1 + \operatorname{erf}\left(\frac{m_{max} - m}{\sigma\sqrt{2}} + \gamma\right)\right] \qquad [27]$$

Let us assume that our catalogue of seismic events can be divided into s subcatalogues (Figure 1). Each of them has its span T_i and is complete, starting from the known magnitude, m^i_{min}. For each subcatalogue i, let $\mathbf{x}_i = \lfloor x_{ij}, \sigma_{ij} \rfloor$, the apparent magnitude and its standard deviation, with $j = 1,...,n_i$, where n_i denotes the number of events in each complete subcatalogue and $i = 1,...,s$. If the size of seismic events is independent of their number, the likelihood function $L_i(\lambda,\beta|\mathbf{x}_i)$ of the unknown parameters (β, λ) is the product of two functions: the likelihood function of parameter β, $L_\beta(\beta|\mathbf{x}_i)$, and the likelihood function of parameter λ, $L_\lambda(\lambda|\mathbf{x}_i)$. If the occurrence of seismic events in time forms a Poisson process, and if the magnitudes obey the Gutenberg–Richter relation, the maximum likelihood functions of parameters λ and β are respectively equal to

$$L_\lambda\left(\lambda \middle| \mathrm{x}_i\right) = const\ \exp\left[-\tilde{\lambda}\left(m^i_{min}\right) \cdot T_i\right]\left[\lambda\left(m^i_{min}\right) \cdot T_i\right]^{n_i}, \qquad [28]$$

and

$$L_\beta\left(\beta \middle| x_i\right) = const\prod_{j=1}^{n} \tilde{f}\left(x_{ij} \middle| m_i, \sigma_{ij}\right). \qquad [29]$$

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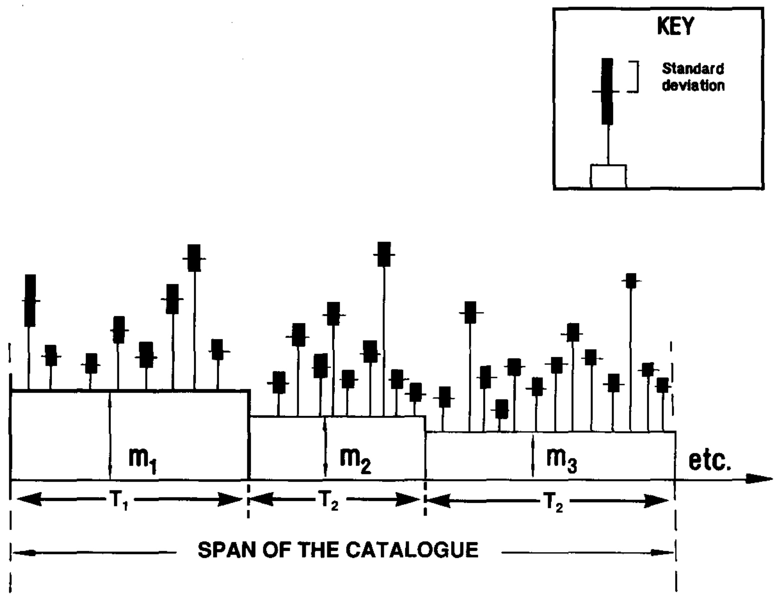

Figure 1—Illustration of the data that can be used to obtain seismic-hazard parameters by the procedure described in the text. The approach permits the combination of the parts of the catalogue with variable thresholds of completeness (m_1, m_2, m_3,...) and uncertain magnitudes. It is assumed that the observed magnitude is the true magnitude distorted by a random error that is normally distributed with the mean equal to zero and the standard deviation, σ (marked by thick vertical bars)

In the relations [28] and [29], *const* is a normalizing factor, the apparent activity rate is defined by relation [27], $\lambda(m^{i}_{min}) = \lambda[1\text{-}F(m^{i}_{min}|m_{min})]$, and m_{min} is the 'total' threshold value of magnitude. The only condition in the choice of this value is that m_{min} cannot exceed the threshold magnitude value of any part of the catalogue. Relations [28] and [29] define the likelihood function of the sought parameters for each complete subcatalogue, *i*, where $i = 1, \ldots s$. The joint-likelihood function based on the whole catalogue is given by

$$L(\lambda, \beta|\mathrm{x}) = \prod_{i=1}^{s} L_i(\lambda, \beta|\mathrm{x}_i). \quad [30]$$

The maximum likelihood estimate of $\hat{\lambda}$ and $\hat{\beta}$ are the values of λ and β that maximize the likelihood function [30]. The estimation of m_{max} can be carried out by the introduction of condition [6] or [7], or of some additional information.

Assessment of Potential VGM

The VGM integrates the probability of occurrence; the rate of events; and the relation between magnitude or logarithm of seismic-event energy, distance, and peak ground velocity. It does this over the magnitude range of interest[1,2]. The VGM defines the total volume of the rockmass affected by the peak velocities not smaller than a critical velocity, *v*, over a period of time, *T*:

$$VGM = \lambda_0 T \int_{m_0}^{X_{max}} f(m)V(m,v)\,\mathrm{d}m, \quad [31]$$

where m_0 is the minimum magnitude of a seismic event considered to be important from a damage point of view, X_{max} is the maximum observed magnitude during the time interval *T*, and λ_0 is the activity rate of seismic events with a magnitude not smaller than m_0:

$$\lambda_0 = \lambda \frac{\exp(-\beta m_0) - \exp(-\beta m_{max})}{\exp(-\beta m_{min}) - \exp(-\beta m_{max})}. \quad [32]$$

$f(m)$ is the probability density-distribution function [21], where m_{min} is replaced by m_0, and $V(m, v)$ is the volume around a seismic event of magnitude *m* in which the peak ground velocity was higher than some critical value, *v*. $V(m,v)$ can be derived from the following equation:

$$\ln v = a_1 m + a_2 \ln R + a_3 \quad [33]$$

which, after simplification, takes the form

$$V(m,v) = Av^{c}\exp(-3a_1 m/a_2), \quad [34]$$

where $A = \frac{4\pi}{3}\exp(-3a_3/a_2)$, and $C = 3/a_2$. Including equations [32], [33], and [34] into [31] and integrating give the analytical formula for the calculation[1,2] of VGM as a function of m_0 and *v*:

$$VGM(m_0, v) = Av^{c}\lambda T\beta / B \frac{\exp(-Bm_0) - \exp(BX_{max})}{\exp(-Bm_{min}) - \exp(\beta m_{max})}, \quad [35]$$

where $B = \beta + 3a_1/a_2$. In this case, the peak ground velocity, *v*, is expressed in millimetres per second (mm/s), and the value of the ground motion in cubic kilometres (m^3).

Examples of Application

Evaluation of m_{max}

The described procedures for the estimation of m_{max} are applied now to seismic data from the FWR mining area in South Africa. The catalogue was compiled from bulletins issued by the Geological Survey of South Africa. The data cover the period between January 1972 and December 1991, and are complete from a threshold magnitude of M_L=2,8. The maximum observed magnitude (X_{max}) for the catalogue is M_L=4,8.

Method I for the evaluation of m_{max} is the most rudimentary procedure, and gives an $\hat{m}_{max}$ of 5,0 for the FWR. The more sophisticated methods, II and III, both give the same $\hat{m}_{max}$ of 4,83±0,32.

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Evaluation of Seismic Hazards from Incomplete Data

For the purpose of illustration, the catalogue from the FWR was modified to give varying levels of completeness and uncertainty. From 1972 to 1985, the data were assumed to have a standard deviation of 0,3 magnitude units and an m_{min} of 3,3. Between 1985 and 1990, m_{min} was set to M_L=3,0, with a standard deviation in the data of 0,2 magnitude units. Finally, between 1990 and 1991, the magnitude uncertainty was established at 0,1, with an m_{min} of 2,8.

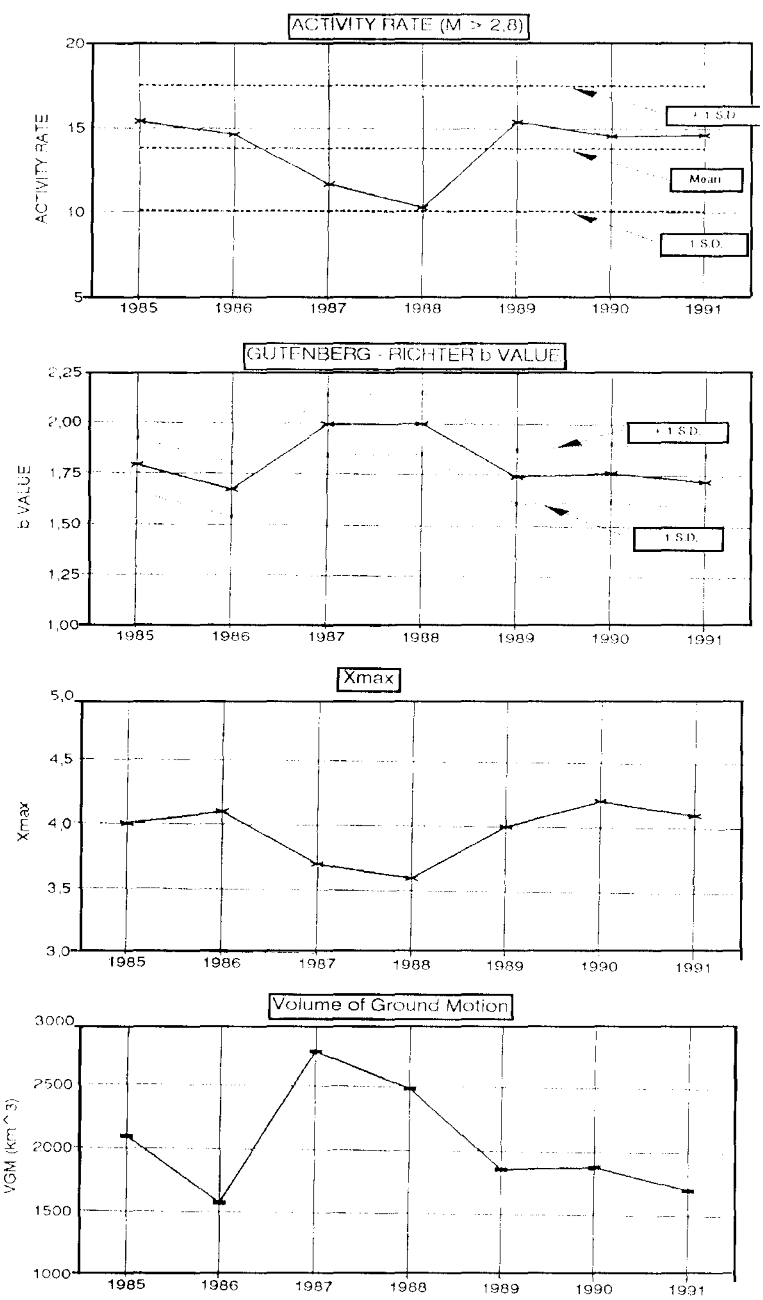

Figure 2—Illustration of the VGM and the seismic-hazard parameters that are used in the calculation of the VGM. These data apply to the FWR region

From the complete catalogue with m_{min} = 2,8, $\hat{\lambda}(m = 2,8)$ is known to be 8,48 ± 0,18 events per month, and $\hat{\beta}$ is 2,50 ± 0,06 ($\hat{b}$ = 1,06 ± 0,02). By use of the methodology given earlier and of the incomplete catalogue, $\hat{\lambda}(m = 2,8)$ was found to be 11,35 ± 0,34 events per month and $\hat{\beta}$ was 3,82 ± 0,10 ($\hat{b}$ = 1,62 ± 0,04). The differences are significant between the two catalogues, even though both sets of data are from the same area (FWR). This shows that variations in the quality of the data within a seismic catalogue can have a significant effect on the evaluation of seismic hazards.

Assessment of Potential VGM

Figure 2 illustrates the results from an assessment of the VGM variations in the FWR between January 1985 and December 1991. The catalogue from 3,1 was used for the evaluation, and a window of 1 year was used in the analysis. From a damage point of view, the minimum values of local magnitude and peak particle velocity considered to be important are M_L=1,0 and 100 mm/s, respectively. Figure 2 shows the activity rate, λ, in events per month for m_{min}=2,8, the estimated Gutenberg–Richter $\hat{b}$ value, X_{max}, and the variations in VGM. This figure serves as an illustration of the information that is integrated to give the potential VGM. It should be noted in this example that the large variation in the b value between 1986 and 1989 has a significant influence on the VGM, despite the drop in activity rate and X_{max}.

The advantage in the use of the VGM concept is that a hazard can be described by one physically significant value. However, one must realize that the VGM filters out small variations in seismic hazard. It is inevitable that some information contained in a hazard evaluation will be lost by the smoothing together of X_{max}, b, and λ. Nevertheless, for the routine evaluation of hazards in mines, the VGM is essential. The analysis is relatively simple to complete, and the results are easy to interpret. Furthermore, the VGM is a useful tool in the presentation of seismic-hazard analyses to mine management in that the concept can be understood by people who have no background in seismology.

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Summary

Standard methods for the evaluation of seismic hazards in mines are reviewed in this paper. A typical hazard study in a mining area would involve the estimation of the Gutenberg–Richter value, *b*, and the activity rate, λ, and the determination of the maximum magnitude, X_{max}, that occurred within some period. A more detailed analysis would also include an estimate of the maximum possible magnitude, m_{max}, that could occur in the mining region.

From time to time, the configurations of mine seismic networks change and improve, which reduces both the level of completeness and the uncertainty in seismic catalogues. The methods described here can be used in evaluations of seismic hazards over periods in which the statistical nature of the catalogues have changed.

Finally, the paper gives an additional parameter for the evaluation of hazards; the potential volume of ground motion (VGM). VGM is a one-dimensional description of a multi-dimensional seismic hazard. Information is inevitably lost with the implementation of the VGM, but the parameter is essential for the routine evaluation of hazards in mines, and also serves as a useful concept in the presentation of hazard evaluations to mine management. ◆