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# Characterization of Dark Energy and Derivation of Hubble's Constant Using Harmonic Quintessence

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**Abstract**

In this paper, using harmonic quintessence, it is shown that the fundamental parameterization of dark energy can be determined using Planck's constant. This allows the derivation of the mass density, and in turn we obtain the energy density of dark energy. This characterization of dark energy also allows the derivation of an averaged value of Hubble's constant, which is in keeping with results from the Planck telescope. It is also possible to confirm the vacuum electric permittivity, magnetic permeability and the field equations for dark energy and corroborate the continued expansion of the Universe. This establishes the fundamental element of the Universe, derived from Planck's constant, as Harmonic Quintessence.

**Keywords:** dark energy, Quintessence, Planck's constant, Harmonic Quintessence, Hubble's constant, electromagnetism, equation of state

## 1. Introduction

Since the discovery of the presence of dark energy, a number of postulates have been introduced that would explain its presence. Initially, it was thought that the Cosmological constant would be the best candidate, however it is now known that the effects of dark energy on Cosmological expansion is not a constant (Peebles & Ratra, 2003; Adler, Casey, Brendan, *et al.* 1995). Quantum vacuum energy has also been proposed, but conventionally it is some 120 orders of magnitude too large (Adler, Casey, Brendan *et al.* 1995). A third model is quintessence, which is a dynamical field with a large Compton wavelength (Caldwell, Dave, & Steinhardt, 1998). More recently it has been proposed that a photon embodied vacuum could explain dark energy and the phenomenon of inertia and the equivalence principle (Grahn, Annila, & Kolehmainen, 2018). In this paper we advance the concept of a photon embodied vacuum further, and accurately characterize the form of electromagnetic quintessence which would endow the vacuum with the properties of dark energy.

In the quintessence model, dark energy is caused by a scalar field, with a standard equation of state which is given by the ratio [3]:

$$w_q = \frac{p_q}{\rho_q} = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q)}, \tag{1}$$

Where $p_q$ is the radiation pressure, $\rho_q$ is the energy density, $\dot{Q}^2$ is a kinetic term and $V(Q)$ is the potential energy.

Hence, quintessence is dynamic, and has a density and $w_q$ parameter that varies with time (Zlatev, Wang, & Steinhardt, 1999; Bennett, *et al.* 2013). In contrast, the cosmological constant is static, with a fixed energy density of, $w = -1$.

Quintessence initially closely tracks the radiation pressure, which is in itself principally caused by electromagnetism. Conventionally, when matter and radiation are in approximately equal ratios, the equation of state changes to produce the increased effects of dark energy and the accelerated expansion of the Universe.

Given that quintessence tracks the electromagnetic radiation pressure in the early Universe, then electromagnetism itself, in the form of a photon embodied vacuum, would make a good candidate for that tracker field (Steinhardt, Wang & Zlatev, 1999). Here using a photon frequency of a single cycle per unit time, we quantify the essential parameters of dark energy. From here it is also possible to predict dark energy mass density and in turn Hubble's constant, and confirm the vacuum electric permittivity, magnetic permeability and field equations for dark energy, using the single fundamental unit of action, Planck's constant. These derivations corroborate previous publications which have used harmonic quintessence to accurately define the entire spectrum of energy equivalence (Worsley, 2010), particle physics (Worsley, 2011), and the constants of thermodynamics (Worsley, 2013).

The central derivations for the characteristics of dark energy in the 9 results sections are as follows:

(i) Fundamental Considerations, Planck's constant, the unit of quintessential mass, and the energy equivalence of dark energy. (Section 3.1).

(ii) The derivation of the unit of quintessential wavelength (Section 3.2).

(iii) The derivation of the quintessential mass density of free space, (Section 3.3).

(iv) Confirming the energy density for Dark Energy, (Section 3.4)

(v) The derivation of Hubble's constant, (Section 3.5).

(vi) Confirmation of the field equations and constants of free space (Section 3.6).

(vii) Confirmation of the qualitative principles of inertia, the equivalence principle and the broad principles of general relativity (Section 3.7).

(viii) Validation of the principle of the constancy of the speed of light in special relativity (Section 3.8).

(ix) Formalization of the equation of state for dark energy, and understanding the continued acceleration in the expansion of the Universe (Section 3.9).

## 2. Methods

All calculations are based on strict mathematical and algebraic principles. The units of measurement are in S.I units. The values used for *c*, *G* and *h* are taken from the 2018 CODATA values. The speed of light is 2.99792458 x $10^8$ m/sec (exact). Planck's constant $h$ = 6.62607015 x $10^{-34}$ J. sec (exact), and $G$ = 6.67430 (15) x $10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$. The estimated value for the reduced Planck length $l_p$ = 1.616255 (18) x $10^{-35}$ m. The fundamental parameters of quintessence energy, temporal mass, wavelength, and quintessence mass density are given in S.I units in each section.

## 3. Results

### *3.1 Fundamental Considerations, Planck's Constant, the Quintessential Mass and Energy Equivalence*

The precept of quantum physics is that Planck's constant is the minimum unit of action, and that energy is quantised with respect to time. Thus using S.I. units Planck's constant is given in units of J.sec. We find that energy and time in this instance are inextricably linked.

In the standard model the minimum unit of action is Planck's constant $\hbar$, and $h$ itself is a minimum unit of electromagnetic energy over a single wavelength, in keeping with the equation $E = hf$. Here, the quintessential mass is also defined as the minimal unit of mass over a single wavelength. As energy at the quantum level is time dependant, then equally the quintessential mass should also be time dependant. Here, we use the Planck time to define the minimal interval of time, which in turn defines the quintessential mass $(m_q)$. This is given by the standard unreduced Planck mass multiplied by unreduced Planck time.

$$m_q = \sqrt{\frac{hc}{G}} \times \sqrt{\frac{hG}{c^5}} = \frac{h}{c^2}, \quad (2)$$

where $m_q$ is the quintessence mass, $h$ is Planck's constant, $c$ is the speed of light and $G$ is the gravitational constant.

It is possible to use Planck time and length as the minimum interval of time and the minimum value of length respectively. However, because Planck units do not consistently represent minimum values, particularly with respect to mass, then in Planck units $E \neq mc^2$, (Wesson, 1980). Thus in order to follow the laws of physics, it is important to use S.I. units. Thus in S.I. units, harmonic quintessence mass is given in kg.sec, corresponding to the unit of action *h*, given in J.sec.

This means that the unit of mass the quintessence mass ($m_q$) is in keeping with Planck's constant, such that:

$$m_q = \frac{h}{c^2} \ , \tag{3}$$

thus:

$$h = m_q c^2. \tag{4}$$

This results in a quintessential mass $m_q$, where *h* is Planck's constant, and *c* is the speed of light.

Hence, the units of Planck's constant now dovetail in with quintessence mass, such that one unit of action *h* corresponds to one unit of quintessential mass ($m_q$), which both represent a minimal value. The corollary is that 2*h* is equivalent to 2 units of quintessential mass, 3*h* is equivalent to 3 units of quintessential mass, and so on and so forth (Worsley, 2010). So the frequency and mass are directly dependant upon the number of quintessential quanta $(n_q)$ present per unit time, such that:

$$n_q = f \quad , \tag{5}$$

where $n_q$ is the number of quintessence quanta present per unit time, which in turn determines the frequency *f*.

As a consequence the frequency is fundamentally related to the number of number of quintessence quanta present per unit time. From Eq. (5), the total mass (*m*) is also given by the number of quanta multiplied by the quintessential mass ($m_q$).

$$m = \ n_q \text{ x } m_q, \tag{6}$$

and as: $h \ = m_q c^2$ [Eq. (4)].

$$E = hf = \ m_q c^2 . f = \ m_q c^2 . n_q = mc^2, \tag{7}$$

This is not trivial as the minimal unit of temporal mass defines the quintessential mass [Eqs. (2 - 4)]. In addition from the new Equation (7), we can now also see how *E= hf*, correlates with the equation $E = mc^2$, and importantly how the Planck relation now corresponds with mass energy equivalence. Using these concepts we can further determine the fundamental characteristics of harmonic quintessence and in turn dark energy.

### *3.2 Defining the Unit of Wavelength of Quintessence*

In the quintessence model the constituents have a long Compton wavelength. In keeping with previous observations (Grahn, Annila, Kolehmainen, 2018), the quintessence presented here is not a particle but it is electromagnetism and the standard equation *E= hf*, applies. By definition the minimal unit of electromagnetic energy is thus *E = 1h*. Using this formula, in S.I units this unit of time translates to a frequency of 1 Hz. From here it is straight forward to derive the wavelength of a single Planck unit of action. In S.I. units. where the frequency is one per unit of time, then in S.I. units, this means that $\lambda_q$ is *c* in meters, as with standard electromagnetism. Hence the wavelength of a single unit if quintessence is given by:

$$\lambda_q = \frac{v}{f} = \ 2.99792458 \text{ x } 10^8 \text{ m}, \tag{8}$$

This wavelength derives from the speed of light in a vacuum and this represent a single wavelength of quintessence. Longer wavelengths are possible when electromagnetism traverses a liquid or solid. Importantly, however this is the wavelength of electromagnetism with a frequency of 1 unit of time as it traverses a vacuum. Indeed this is corroborated by lowest experimental value for the frequency of electromagnetism, in the near vacuum of the ionosphere of 3 Hz, (Volland, 1995).

### *3.3 Defining the Mass Density of Dark Energy*

Using the quintessential mass, it is possible to accurately estimate the observed energy density of dark energy. Other models cannot predict this, and thus this calculation strongly corroborates the principles embodied here.

From here we can calculate the energy density form first principles using the standard Planck length (see methods), using fundamental geometry. A single wavelength of electromagnetism has two orthogonal oscillation modes one electric and one magnetic. In this model both electromagnetic oscillations are effectively dependant on the Planck length. For clarity let the separate oscillation modes be on the *two* axes, $x$ and $y$ respectively. Starting with the standard Planck length, we can model the number of Planck lengths in each cross section, each with an oscillation of 1 per unit time per 1 meter squared of space-time, in their respective orthogonal axes:

$$nl_P = \frac{1}{l_p} \ / \ \mathrm{m}^2, \tag{9}$$

This gives the number of quintessence quanta crossing a cross sectional area of 1 meter squared in 2 dimensions.

Each single quintessence has a wavelength $\lambda_q$ of *c meters* in direction of motion, in this case in the z axis (section 3,2). Thus the total number of quanta present in one cubic meter is divided by $\lambda_q$. Additionally, due to the amplitude of the oscillations of electromagnetism, there needs to be a separation of the charge density by a factor of $2\pi$ in each of the 3 dimensions.

Hence the number of quintessence ($n_q$) in a cubic meter *per unit time* is given by:

$$n_q = \frac{1}{\lambda_q.(2\pi)^3.l_p} \ / \ (\mathrm{m}^3.\mathrm{sec}), \tag{10}$$

where, $l_P$ is the standard Planck length, $\lambda_q$ is the wavelength of quintessence, and $n_q$ is the number of quintessence quanta present per second in a fixed cubic meter.

This gives the number of quintessence quanta ($n_q$), per meter cubed per unit time.

$$n_q = 8.320124 \ (92) \ x \ 10^{23} \ / \ (\mathrm{m}^3.\mathrm{sec}), \tag{11}$$

which constitutes the number of individual quintessence quanta present per cubic meter, each with a frequency of 1 oscillation per unit time. The error arises solely from the relative uncertainty in the value of the gravitational constant G, and in turn the derived Planck length.

Thus the predicted quintessential mass density of dark energy ($m_{q\rho}$) per cubic meter is given by:

$$m_{q\rho} = n_q \times m_q \,/\, m^3 = \underline{6.134009 \ (69) \ \mathrm{x} \ 10^{-27} \ \mathrm{kg/m}^3}, \tag{12}$$

where $m_q$ is the mass of a single quintessence in S.I units of kg.sec, (as defined in section 2.1) and $n_q$ is the number of quintessence per unit time.

This value predicts the mass density of free space, which is in keeping with mass density and energy density of dark energy (Grahn, Annila, Kolehmainen, 2018). This is also in agreement with the experimental energy density parameter $\Omega_\Lambda = 0.6889 \pm 0.0056$ , (TT, TE, EE +low E +lensing + BAO) (Planck Collaboration, 2020, 2021), using the Hubble constant (see Section 3.5), which gives the predicted experimental value for, $m_{q\rho}$ = 6.134 (65) x $10^{-27}$ kg/m$^3$.

*3.4 Confirming the Energy Density for Dark Energy*

By using the geometric methods above for deriving the mass density [Eq. (10)] of dark energy, taking the Planck length to 6 decimal places, the mass density of free space is 6.134009 (69) x $10^{-27}$ kg/m$^3$. With a relative uncertainty of 1.1137 x $10^{-5}$ (applicable to *G* and in turn to the uncertainty of the Planck length). From here, it is further possible to confirm the energy density of dark energy.

Using the mass density as in Eq (10), the energy density of dark energy is given as:

$$\Omega_{\Lambda E} = 5.512972 \ (62) \ \mathrm{x} \ 10^{-10} \ \mathrm{J/m}^3 \tag{13}$$

which gives an approximate value of 0.55 nJ/m$^3$, in keeping with previous estimates (Grahn, Annila, Kolehmainen, 2018).

Since the relative uncertainty is the same for the number of quintessence and for the energy density, it possible to obtain a ratio of the two terms which is far more accurate. The number of quintessence quanta per mete $n_q$ = 8.3201242238 x $10^{23}$/(m$^3$.sec), and the energy density then $\Omega_{\Lambda E}$ = 5.5129726796 x $10^{-10}$ J/m$^3$. Thus the unit of action of each individual quintessence is:

$$\Omega_{\Lambda E}/n_q = h = 6.62607015 \ \mathrm{x} \ 10^{-34} \ \mathrm{J.sec} \tag{14}$$

Which is exactly equivalent to Planck's constant to eight decimal places and provides proof of principle for the derivation of dark energy from Planck's constant, and suggests the potential for greater accuracy using this methodology.

*3.5 Deriving Hubble's Constant*

From here it is also possible to derive Hubble's constant, by using the derived energy density and mass density of dark energy.

From the mass density of dark energy $m_{q\rho} = 6.134009\ (69) \times 10^{-27}$ kg/m$^3$ [Eqs. (10 -12)], it is possible to derive the critical mass density from Planck. For the mass density of the dark component, $\Omega_\Lambda = 0.6899 \pm 0.0056$ (TT, TE, EE +low E +lensing + BAO) (Planck Collaboration 2020, 2021). Using the mass density derived here, where $\rho_c = \frac{m_{q\rho}}{\Omega\Lambda}$, gives an overall critical mass density of $\rho_c = 8.9041 \pm 0.072. \times 10^{-27}$ kg/m$^3$.

The critical density $\rho_c$ is given by the conventional formula:

$$\rho_c = \frac{3{H_0}^2}{8\pi G} \tag{15}$$

Thus, in turn it is possible to derive Hubble's constant $H_0$ to the 3σ range, giving Hubble's constant as:

$$H_0 = 68.85 \pm 1.2 \text{ (km/sec)/Mpc}$$

This gives a result for the averaged value of Hubble constant of the Universe, calculated from the estimated mass density component of Dark Energy from Planck (Planck Collaboration 2020, 2021), and from the actual mass density determined here (see Section 3.3). This gives a 3σ range of 67.65- 70.05 (km/sec)/Mpc. These ranges encompass the mean results of a number of experimentally derived estimates of Hubble's constant. This includes the final result from Planck (Planck Collaboration, 2020, 2021), the 9 year study from WMAP (Bennett, *et al.* 2013), and a number of other studies (Grieb, *et al.* 2016; Dom ńguez, *et al.* 2019; Freedman, et al. 2019; Sedgwick, *et al.* 2020; Mukherjee, *et al.* 2020). However, this result lies below the range of the measurement of the "local value" for Hubble's constant by 5σ (Riess, *et al.* 2021). This suggests that measurements from the post CMB epoch and the earlier epochs of the Universe are significantly lower than the more recent "local values". These observations support the presence of the more recent acceleration in the expansion of the Universe. This accelerated expansion elevates the recent values for the Hubble constant, and may explain the presence of the Hubble tension (Di Valentino, Mena, Pan. *et al* (2021).

*3.6 Confirming the Field Equations and Constants of Free Space*

Here, the fundamental importance of James Clerk Maxwell's electromagnetic theory (Maxwell, 1873) and Carl Friedrich Gauss electric field law, published posthumously (Gauss, 1867) is brought to the forefront. Gauss's Maxwell's electric (*E*) and magnetic (*B*) field equations for electromagnetism in a vacuum, in S.I units are:

$$\nabla \cdot E = \rho/\varepsilon_0, \tag{16}$$

$$\nabla \cdot B = 0, \tag{17}$$

where $\rho$ is the charge density, and $\varepsilon_0 \approx 8.854 \times 10^{12}$ F/m is the vacuum electric permittivity.

For an individual electromagnetic wave the nett magnetic field is zero and conforms to Equation (18), However, (whilst it is assumed that the vacuum has no potential energy), it is apparent that dark energy does have potential energy. Clearly with a photon embodied vacuum it is the electric field equation [Eq. (17)], for each individual electromagnetic wave that would account for the overall potential energy, as the charge densities repel each other.

In the model presented here, the motion of quintessence would be in all directions through a cubic meter of space time. As the electromagnetism would pass on average through each cross-sectional area in the forward direction and importantly in the contralateral direction in each of the three dimensional axes, this would require the separation of the electromagnetic waves. Allowing for the oscillation of *2π* in each dimension (see Section 3.3), the separation of these charges would govern the energy density and also provide the potential energy on Cosmological scales. However, on the Planck scale the result would be an apparent overall vanishingly small charge density. Nevertheless, on the cosmological scale the charge density would result in the self-repelling nature of quintessence and the potential energy $V(Q)$, of the expanding universe.

Accordingly, when a separate charged particle or current passes through space-time, this would then result in the presence of the vacuum electric permittivity and magnetic permeability constants becoming apparent [19]. Given each single photon has a velocity of $c$, and a wavelength equivalent to c in meters (see Section 3.2), then we arrive at the classical Maxwell unitary equation,

$$c.\,(\varepsilon_0\,\mu_0)^{1/2} \;=\; 1, \tag{18}$$

where $\varepsilon_0$ and $\mu_0$ are the vacuum electric permittivity and magnetic permeability respectively.

*3.7 Qualitative Characteristics of Quintessence Dark Energy*

The qualitative aspects of a vacuum with embodied photons has previously described using a paired photon model (Grahn, Annila, Kolehmainen, 2018). Here a single photon of electromagnetism is used, with a frequency of 1 oscillation per unit time, based on Planck's constant. From here it is possible to determine the fundamental characteristics of quintessence. The quantitative approach has already defined the basic theoretical parameters, of wavelength, mass and energy density of Dark Energy (see Sections 3.1-3.4). Importantly it is also possible to derive a value for Hubbles's constant (see Section 3.5). These characteristics are also in keeping with by Maxwell's equations for electromagnetism and the unitary equation (see section 3.6). Qualitatively, a single photon model described here, as previously outlined (Grahn, Annila, Kolehmainen, 2018), can also explain the presence of instant, and radiative inertia and also rotational inertia based on the vacuum's radiative characteristics (Tuisku, Pernu, & Annila, 2009). Equally, it can explain the equivalence principle with gravitational and inertial mass being equivalent, and at the same time it can explain gravity behaving as an energy density gradient (Sciama, 1953), both principles being in keeping with General Relativity (Annila, 2011; Koskela, Annila, 2011). Using the same principles it is possible to explain such effects as the double slit experiment where particles travelling towards a double slit have forward effects on the state of quintessence space time (Aharonov & Bohm, 1959). Equally the Casimir effect is also readily explained on a priori basis of the energy density of dark energy (Casimir & Polder, 1948), in so far as the maximum force will be obtained when the separation of plates is equivalent to a single Planck length.

*3.8 Quintessence and Special Relativity*

Whilst the phenomenon of special relativity is well established, the presence of dark energy with a defined energy density in the form of quintessence suggests that space-time should also conform to special relativity. The two principal tenets of special relativity are that the speed of light is a constant (Michelson & Morley,1887), and that all observers are equal. In the model presented here only one tenet will suffice in special relativity, in that the speed of light is a constant. The presence of an all-pervasive quintessence, whereby each single individual constituent is travelling at the speed of light and at the same time the constituents are effectively travelling in all directions satisfies the second tenet. Specifically, every observer will be equal in so far as the speed of light will be a constant, whatever the direction of motion and individual speed of the observer. Hence special relativity can now be based on a single tenet, the constancy of the speed of light.

*3.9 Formalization of the Equation of State and Accelerated expansion of the Universe*

In the quintessence model the standard equation for the ratio of the radiation and energy density is given by the equation of state (Caldwell, Dave, Steinhardt, 1998):

$$w_q = \frac{p_q}{\rho_q} = \frac{\frac{1}{2}\dot{Q}^2 - V(Q)}{\frac{1}{2}\dot{Q}^2 + V(Q)}, \tag{1}$$

where $p_q$ is the radiation pressure, $\rho_q$ is the energy density, $\dot{Q}^2$ is a kinetic term and $V(Q)$ is the potential energy.

As with special relativity the speed of light remains a constant as does the wavelength of the quintessence that forms the space time metric. Thus in an expanding Universe then the density of matter itself will decrease with time but the potential energy density of quintessence remains with a trend towards a slow rolling increase. In these circumstances the presence of quintessence cannot be further diluted. Additionally, by the same mechanism, the presence of dark energy will increase over time as more electromagnetism in the radio wave frequency is red shifted to the lowest energy state of quintessence. In the case of the kinetic energy the term

($\frac{1}{2}\dot{Q}^2$) for an electromagnetic wave with the frequency of a single unit of time, this would be vanishingly small.

As regards the charge density of electromagnetism this would create the potential energy $[V(Q)]$. The model described here is with electromagnetism at its lowest frequency of 1 oscillation per unit time, and therefore in its lowest kinetic energy state, where $\frac{1}{2}\dot{Q}^2 \ll V(Q)$. Whilst radiation in the form of electromagnetism should be significantly red shifted during the expansion of the Universe, there is one exception to that expected red shift. That is if the electromagnetism is already in its lowest energy state with the lowest possible frequency of one oscillation per unit time, and therefore cannot be further red shifted. The relative constancy of the quintessence in the post recombination epoch is readily explained as the potential energy density of dark energy is dependent on the electric charge density and remains relatively constant. This is in accordance space-time. This produces the slow rolling of the potential energy $V(Q)$. Conventionally the Universe with the cosmological constant has a value which remains at $w = -1$. However, the data from WMAP suggest that $w$ may vary and the calculated value from the 9 year data suggest a number of possibilities (Bennett, *et al.* 2013).

$$w = -1.073 + 0.090, -0.089 \text{ (flat)}$$

$$\text{WMAP+eCMB+BAO+}H_0.$$

$$w = -1.19 \pm 0.12 \qquad \text{(non-flat)}$$

Adding 472 Type Ia supernovae compiled by Conley *et al.* (2011), improves these limits to

$$w = -1.084 \pm 0.063 \qquad \text{(flat)}$$

$$\text{WMAP+eCMB+BAO+}H_0\text{+SNe},$$

$$w = -1.122 + 0.068, -0.067 \qquad \text{(non-flat)}$$

This view is corroborated by recent experimental evidence at z = 1.5 – 5.1, that dark energy is increasing its effects on the expanding Universe in the $w_q$CDM model at ~ 4σ (Risaliti & Lusso, 2019). In the present epoch when the dark and baryonic matter has stabilised, the increasing predominance of dark energy produces the increased effects of dark energy and the gradually accelerating expansion of the Universe.

## 4. Conclusions and Discussion

In this paper the parameters for dark energy are derived on the presence of harmonic quintessence, based on the standard unit of action, Planck's constant *h*. The parameters for dark energy, in the form of harmonic quintessence, can be further defined by a quintessential mass. This quintessence mass then corresponds with the unit of action *h*. Having defined a quintessential mass, this also means that the Planck relation $E = hf$, corresponds with the energy equivalence formula $E = mc^2$.

In the quintessence model for dark energy presented here, the quintessential mass/energy is on an equal basis to electromagnetism principally with a frequency of 1 oscillation per unit time. This explains the origin of the mass and the energy of dark energy. From here the parameters of quintessence can be accurately determined. As a result, the precise wavelength of quintessence can be determined. Importantly, this also leads to an accurate estimation of the mass and energy density of dark energy. This allows the derivation of the energy density of dark energy of approximately 0.55 nJ/m$^3$, in agreement with current estimates (Grahn, Annila, Kolehmainen, 2018). The accuracy depends on the accuracy of the Planck length in Eq. (10), which in turn derives from the uncertainty of the value for *G*.

In the standard quintessence model the equation of state changes to produce the increased effects of dark energy and the accelerated expansion of the Universe. This observation is corroborated by 4 principal experimental methods. The first arises from the indirect and direct measurements of the Hubble constant (see Section 3.5). The measurements from the post CMB epoch and the earlier epochs of the Universe are significantly lower than the more recent "local values". These observations support the presence of the more recent acceleration in the expansion of the Universe. This accelerated expansion results in an increase in the recent values for the Hubble constant, and may explain the presence of the Hubble tension (Domínguez, *et al.* 2019). Moreover, the equation which determines the ratio between Hubble's constant and the critical density [Eq. (15)] incorrectly assumes a cosmological constant and requires modification to encompass the accelerating Universe. Secondly there is a disparity between the post CMB epoch measurement of Hubble constant using the Planck telescope, the Hubble time and the age of the Universe. Even within the results of the Planck telescope measurements, the Hubble constant is in disagreement with the Hubble time in so far as the Hubble time appears far greater than the age of the Universe as measured by Planck itself (Planck Collaboration, 2020, 2021). Thirdly the WMAP data for dark energy suggests $w$ is not a constant and differs from $w$ = -1 (see Section 3.9) (Bennett, *et al.* 2013). In addition, recent evidence comparing higher $z$ values strongly suggest that the expansion of the Universe is accelerating (Risaliti G., Lusso E. 2019). Indeed, a wholesale modification of Cosmology is required to fully understand dark matter and dark energy, the latter which is addressed in this paper.

Importantly, these observations on dark energy now dovetail in with previous works on the energy equivalence equations not only Planck's relation and the mass energy equivalence equation, but also the relativistic energy momentum equation and Shrodinger's wave equation using harmonic quintessence (Worsley, 2010). Harmonic quintessence also leads to the derivation of the characteristics of the electron, proton, quarks and other fundamental particles (Worsley 2011). The progenitor of the particles is the spherical configuration of the electron containing 1.23 x $10^{20}$ harmonic quintessence, which determine the frequency and wavelength of the electron in the ground state. Equally, the equations for thermodynamics (Worsley 2013) can be determined from harmonic quintessence. This indicates that harmonic quintessence underpins dark energy, particle physics and thermodynamics, and the forces of nature, in so far as everything is composed of harmonic quintessence, depending on the configuration and number of quintessence.

As a result, on a qualitative level, harmonic quintessence, can explain the presence of instant, and radiative inertia based on electromagnetic characteristics formalized by Maxwell's equations. Here a single unit of electromagnetism, can equally explain the presence of inertia. These equations also conform to Maxwell's equations for electromagnetism, the electric permittivity and magnetic permeability for free space, and explain the relation with the constancy of the speed of light. It is also in keeping with special relativity where all observers measure the speed of light as a constant. Equally, it can explain the equivalence principle and at the same time it can explain gravity behaving as an energy density gradient, both principles being in keeping with General Relativity. Finally, it is possible to formalize the equation of state for quintessence, and account for the acceleration in the expansion of the Universe.

The concepts embodied in this work transform the accuracy of quantum theory in relation to dark energy, with quantum theory being 120 magnitudes too large. In this quintessence model it is possible to accurately predict the characteristics of dark energy to 4 decimal places. This puts the presence of dark energy well within the realm of being calculated using Planck's constant, and quantum electrodynamics at the smallest scale using harmonic quintessence.

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