

Type of the Paper (Article.)

# Title **A solution of the cosmological constant, using multiverse Penrose CCC cosmology, and Enhanced Quantization compared with Rosen mini-universe constraints as a Future Project outlined**

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## Abstract:

We reduplicate the Book “Dark Energy” by M. Li, X-D. Li, and Y. Wang, zero-point energy calculation with an unexpected “length’ added to the ‘width’ of a graviton wavefunction just prior to the entrance of ‘gravitons’ to a small region of space-time prior to a nonsingular start to the universe. We compare this to a solution worked out using Klauder Enhanced quantization, for the same given problem. The solution of the first Cosmological Constant problem relies upon the geometry of the multiverse generalization of CCC cosmology which is explained in this paper. The second solution, used involves Klauder enhanced quantization. We look at energy given by our methods and compare and contrast it with the negative energy of the Rosen model for a mini sub universe and estimate GW frequencies

**Keywords:** Minimum scale factor, cosmological constant, space-time bubble, bouncing cosmologies

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## 1. Introduction

We bring up this study due to the general failure of field theory methods to obtain a working solution to the Cosmological Constant problem. One of the reasons for this work is that Quintessence studies involving evolution of the Cosmological Constant over time have routinely failed in match ups with Early supernova data. The working hypothesis is that if Dark Energy is commensurate with the Cosmological Constant that the Cosmological Constant, and Dark energy would be the same from the start of inflation. The traditional model of inflation also involves due to the Penrose Singularity theorems [1] a singular point for initial expansion of the Universe. Our study clearly initiates a Cosmological constant which is invariant over time. We digress from using the Penrose Singularity theorem [1] in terms of a Cosmic bounce [2] style nonsingular initial bubble of space-time for reason we state in the manuscript. The importance we have to our work is that we are convinced that the Cosmological Constant [3][4] was set initially for conditions in the interior of a nucleated space-time bubble, with a wavefunction style calculation for collapsing infill into the space-time bubble with that wavefunction of Dark energy [5][6] to be at a minimum 10 to the 30 power times Planck length. We conclude that Polarization states for obtaining early universe high Frequency Gravitational waves detected and analyzed may be more favorable for higher dimensional generalizations of our Dark Energy generating procedures, with Klauder Enhanced Quantization [7],[8] limited to 3+1 Geometry whereas our Vacuum energy [9][10] calculation may be amendable to higher frequency calculation incantations. While considering

[11],[12],[13],[14],[15],[16],[17],[18],[19] issues. In particular [19] gives us future goals to consider. As an example, could multiverse CCC allow incorporating [19] with [13]?

## 2. Methods

We will first start off with the redone calculation as to the Vacuum energy as given in [1] and how we rescale them to be in sync as to the observed experimental value for vacuum energy which is of the present era. This methodology is consistent with the Zero-point energy calculation, we start off with the following as given by [11]

$$\frac{1}{2} \cdot \sum_i \omega_i \equiv V(\text{volume}) \cdot \int_0^{\hat{\lambda}} \sqrt{k^2 + m^2} \frac{k^2 dk}{4\pi^2} \approx \frac{\hat{\lambda}^4}{16\pi^2} \tag{1}$$

$$\xrightarrow{\hat{\lambda}=M_{\text{Planck}}} \rho_{\text{boson}} \approx 2 \times 10^{71} \text{ GeV}^4 \approx 10^{119} \cdot \left( \rho_{DE} = \frac{\Lambda}{8\pi G} \right)$$

In stating this we have to consider that  $\rho_{DE} = \frac{\Lambda}{8\pi G} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}^4}$ , so then that

the equation we have to consider is a wavelength  $\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}$  which is

about  $10^{30}$  times a Plank length radius of a space-time bubble [13] as a nonsingular expansion point for Cosmology, at the start of inflation with the space-time bubble of about a Plank length radius in size. . Having said that , how do we get having the Penrose multiverse condition in this problem, for

$\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}$  just before the near singularity?

### 2.1 Looking now at the Modification of the Penrose CCC (Cosmology)

We now outline the generalization for Penrose CCC(Cosmology) just before inflation which we state we are extending Penrose’s suggestion of cyclic universes, black hole evaporation, and the embedding structure our universe is contained within, This multiverse has BHs and may resolve what appears to be an impossible dichotomy. The following is largely from [13] and has serious relevance to the final part of the conclusion .That there are N universes undergoing Penrose ‘infinite expansion’ (Penrose) [18] contained in a mega universe structure. Furthermore, each of the N universes has black hole evaporation, with Hawking radiation from decaying black holes. If each of the N universes is defined by a partition function, called  $\{\Xi_i\}_{i=1}^N$ , then there exist an information ensemble of mixed

minimum information correlated about  $10^7 - 10^8$  bits of information per partition function in the set  $\left. \{\Xi_i\}_{i=1}^N \right|_{\text{before}}$ , so minimum information is conserved between a set of partition functions per universe [13]

$$\left. \{\Xi_i\}_{i=1}^N \right|_{\text{before}} \equiv \left. \{\Xi_i\}_{i=1}^N \right|_{\text{after}} \tag{2}$$

However, there is non-uniqueness of information put into partition function  $\{\Xi_i\}_{i=1}^N$ . Also

Hawking radiation from black holes is collated via a strange attractor collection in the mega universe structure to form a new inflationary regime for each of the N universes represented

**Our idea** is to use what is known as CCC cosmology[13],[18], which can be thought of as the following. First. Have a big bang ( initial expansion) for the universe which is represented by  $\{\Xi_i\}_{i=1}^N$ . Verification of this mega structure compression and expansion of information with stated non-uniqueness of information placed in each of the N universes favors ergodic mixing of initial values for each of N universes expanding from a singularity beginning. The  $n_f$  stated value, will be using (Ng, 2008)  $S_{entropy} \sim n_f \cdot [20]$ . How to tie in this energy expression, as in Eq.(12) will be to look at the formation of a nontrivial gravitational measure as a new big bang for each of the N universes as by  $n(E_i)$  the density of states at energy  $E_i$  for partition function[13],[14].

$$\{\Xi_i\}_{i=1}^N \propto \left\{ \int_0^\infty dE_i \cdot n(E_i) \cdot e^{-E_i} \right\}_{i=1}^N \quad (9)$$

Each of  $E_i$  identified with Eq.(9) above, are with the iteration for N universes [13],and [18](Penrose, 2006) Then the following holds, by asserting the following claim to the universe, as a mixed state, with black holes playing a major part, i.e.

**CLAIM 1**

See the below[13] representation of mixing for assorted N partition function per CCC cycle

$$\frac{1}{N} \cdot \sum_{j=1}^N \Xi_j \Big|_{j\text{-before-nucleation-regime}} \xrightarrow{\text{vacuum-nucleation-transfer}} \Xi_i \Big|_{i\text{-fixed-after-nucleation-regime}} \quad (10)$$

For N number of universes, with each  $\Xi_j \Big|_{j\text{-before-nucleation-regime}}$  for j = 1 to N being the partition function of each universe just before the blend into the RHS of Eq. (10) above for our present universe. Also, each independent universes as given by  $\Xi_j \Big|_{j\text{-before-nucleation-regime}}$  is constructed by the absorption of one to ten million black holes taking in energy. **I.e. (Penrose)**

[18]. Furthermore, the main point is done in [13] in terms of general ergodic mixing [21][22][23] 96

97

**Claim 2** 98

$$\Xi_j \Big|_{j\text{-before-nucleation-regime}} \approx \sum_{k=1}^{Max} \Xi_k \Big|_{black\text{-holes-}j\text{th-universe}} \tag{5} \quad 99$$

100

What is done in **Claim 1 and 2** [13] is to come up as to how a multi dimensional representation of black hole physics enables continual mixing of spacetime [13],[18],[22],[24] largely as a way to avoid the Anthropic principle[13][25], as to a preferred set of initial conditions 101  
102

**2.2 Looking at the Modification of the Penrose CCC (Cosmology)** 103

**We argue this modification is mandated by having the initial DE wavefunction set as** 104

$$\lambda_{DE} \approx 10^{30} \ell_{Planck} \tag{6} \quad 105$$

After having set this in place we will be comparing this method to the Klauder argument 106

**2.3 Looking at Klauder Enhanced Quantization for Cosmological Constant:** 108

We use the Padmanabhan 1<sup>st</sup> integral [26] of the form , with the third entry of Eq. (1) having a Ricci scalar defined via [27] [28]and usually the curvature  $\mathfrak{R}$  [13] set as extremely small, with the general relativity[13][28] action of 109  
110  
111

$$\begin{aligned} S_1 &= \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda) \\ \&-g &= -\det g_{uv} \\ \&\mathfrak{R} &= 6 \cdot \left( \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 + \frac{\mathfrak{K}}{a^2} \right) \end{aligned} \tag{7} \quad 112$$

**2.3. Next for the idea from Klauder** 113

We are going to go to page 78 by Klauder [8][13 ] of what he calls on page 78 a restricted 114

Quantum action principle which he writes as:  $S_2$  where we write a 1-1 equivalence as in 115

[8][13] , which is 116

$$S_2 = \int_0^T dt \cdot [p(t)\dot{q}(t) - H_N(p(t), q(t))] \approx S_1 = \frac{1}{2\kappa} \int \sqrt{-g} \cdot d^4x \cdot (\mathfrak{R} - 2\Lambda) \tag{8} \quad 117$$

Our assumption is that  $\Lambda$  is a constant[13], hence we assume then use the approximation , 118

from [8], and [13] 119

$$\frac{p_0^2}{2} = \frac{p_0^2(N)}{2} + N; \text{ for } 0 < N \leq \infty, \text{ and } q = q_0 \pm p_0 t$$

$$V_N(x) = 0; \text{ for } 0 < x < 1 \tag{9} \quad 120$$

$$V_N(x) = N; \text{ otherwise}$$

$$H_N(p(t), q(t)) = \frac{p_0^2}{2} + \frac{(\hbar \cdot \pi)^2}{2} + N; \text{ for } 0 < N \leq \infty$$

Our innovation is to then equate  $q = q_0 \pm p_0 t \sim \phi$  and to assume small time step values. 121

Then[13] 122

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2N + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2}\right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} + \left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right) \Bigg|_{t=\tilde{t}} \tag{10} \quad 123$$

If we assume that Eq. (10) was formed within a cosmic bubble of space-time ‘[12] 124

$$\left(\frac{\dot{a}}{a}\right)^2 = H_{\text{bounce-cos}}^2 = \frac{8\pi}{3M_p^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{11} \quad 125$$

Here , we have that  $\rho$  is a space-time density function, whereas  $\sigma$  is the tension of a space-time bubble presumably of the order of a Planck radius. 126  
127

Also within the bubble of space-time  $\left(\frac{\ddot{a}}{a}\right) \approx \varepsilon^+ \approx 0$  and  $a(t) = a_{\text{min}} t^\gamma$  at the surface of the space-time bubble. Hence 128  
129

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2\tilde{N} + \frac{\gamma(3\gamma-1)}{8\pi G t_{\text{Planck}}}\right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_p^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{12} \quad 130$$

$c_1$  is to be determined, whereas the inflaton[13],[26]. 131

$$\phi = \sqrt{\frac{\gamma}{4\pi G}} \ln \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma-1)}} \cdot t \right\} \tag{13} \quad 132$$

And what we will use the ‘‘inflaton potential ‘‘we write as[13],[26] 133

$$V = V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma - 1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{14}$$

Also 135

$$\rho \approx \frac{\dot{\phi}^2}{2} + V(\phi) \equiv \frac{\gamma}{8\pi G} \cdot t^2 + V_0 \cdot \left\{ \sqrt{\frac{8\pi G V_0}{\gamma(3\gamma - 1)}} \cdot t \right\}^{\sqrt{\frac{\gamma}{4\pi G}} - \sqrt{\frac{8\pi G}{\gamma}}} \tag{15}$$

**3. Results: comparing Eq (1) implications with Eq (12)** 138

The significance of the procedure is that with tweaking we may be seeing the actual realization of classical gravity as an Eikonal Approximation to quantum theory[29][30] as what was brought up by Horowitz and Oron in a highly nonstandard way as seen in reference [30 ] which gives a different interpretation to Eq. (15) above,. The term we refer to as Eikonal approximation to a quantum state has the form of the following decomposition. From Powell and Crasemann,[31] we have the decomposition of Geometric style decomposition of the optical wave equation of 139  
140  
141  
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145

$$\nabla^2 \psi - \frac{1}{v_{Velocity}^2} \cdot \frac{d^2 \psi}{dt^2} = 0 \tag{16}$$

Whereas we use the common  $\lambda \cdot \omega = 2\pi v_{Velocity}$  , and if we substitute in an angular frequency dependence of  $e^{-i\omega t}$  in  $\psi$  , we get from (16) that if  $\lambda = h / p$  and  $p = \sqrt{2m \cdot (E - V(potential))}$  , Then we have a transformation of Eq. (17) to Eq.(18) 147  
148  
149

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \cdot \psi = 0 \tag{17}$$

Leads to a Schrodinger equation of the form given by 151

$$\nabla^2 \psi + \frac{8\pi^2}{h^2} \cdot (E - V(potential)) \cdot \psi = 0 \tag{18}$$

Eq. (17) has[32] 153

$$\psi = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (a_{\ell m} j_{\ell}(kr) + b_{\ell m} y_{\ell}(kr)) \cdot Y_{\ell}^m(\theta, \phi) \tag{19}$$

Where we could get away with making a substitution of[31] 155

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h} \cdot (E - V(potential))^{1/2} \tag{20}$$

Where the first and second Bessel Equation solutions[32] are of the form 157

$$j_\ell(kr = x) = (-x)^\ell \left(\frac{1}{x} \frac{d}{dx}\right)^\ell \frac{\sin x}{x} \tag{21} \quad 158$$

$$y_\ell(kr = x) = -(-x)^\ell \left(\frac{1}{x} \frac{d}{dx}\right)^\ell \frac{\cos x}{x} \tag{22} \quad 159$$

Then we have via making use of Kieffer’s result of variance of the energy [33]is 160

$$\Delta E = \frac{\sqrt{2\tilde{n}+1}}{\lambda} \approx \frac{\hbar\sqrt{2\tilde{n}+1} \cdot (E - V(\text{potential}))^{1/2}}{2\pi} \equiv \hbar\sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot t \tag{23} \quad 161$$

We get via [33] on page 239 of this reference,Eq.(23). after we use (23), then 162

$$\Delta E \Delta t \approx \sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot t \cdot \Delta t \cdot \hbar \approx \hbar \tag{24} \quad 163$$

If so, then we have that if so by use of [34] since this is also applicable to black holes 164

$$t \propto t_{\text{Planck}} \propto \Delta t \Rightarrow \Delta E \Delta t \approx \sqrt{2\tilde{n}+1} \cdot \sqrt{\frac{V_{\text{volume}}}{8\pi G}} \cdot (\Delta t)^2 \cdot \hbar \propto \hbar \tag{25} \quad 165$$

$$\Rightarrow (\Delta t)^2 \propto \frac{1}{\sqrt{2\tilde{n}+1}} \cdot \sqrt{\frac{8\pi G}{V_{\text{volume}}}}$$

Using Planck units, what this means is that if  $G = \hbar = \ell_{\text{Planck}} = k_B = 1 = \text{Planck time}$  166

$$(\Delta t)^2 \propto \frac{1}{\sqrt{2\tilde{n}+1}} \cdot \sqrt{\frac{8\pi G}{V_{\text{volume}}}} \equiv \sqrt{\frac{8\pi}{2\tilde{n}+1}} \cdot (t_{\text{Planck}})^2 \tag{26} \quad 167$$

If  $\tilde{n} = \text{quantum - number}$ , then for  $\Delta t$  being proportional to  $t_{\text{Planck}} \propto \Delta t$ , we have definite restrictions on  $\tilde{n} = \text{quantum - number}$ , i.e. of the type given by 168  
169

$$\sqrt{\frac{8\pi}{2\tilde{n}+1}} \leq 1 \tag{27} \quad 170$$

Whereas the Kieffer Wavefunction commensurate as to Eq. (23) for a quantum Dust universe initially is given as[33] on page239 of this reference 171  
172

$$\psi_{\tilde{n},\lambda}(t, r) \equiv \frac{1}{\sqrt{2\pi}} \cdot \frac{\tilde{n}!(2\lambda)^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}} \cdot \left[ \frac{1}{(\lambda + i \cdot t + i \cdot r)^{\tilde{n}+1}} - \frac{1}{(\lambda + i \cdot t - i \cdot r)^{\tilde{n}+1}} \right] \tag{28} \quad 173$$

Here the time t would be proportional to Planck time, and r would be proportional to Planck length, whereas we set 174  
175

$$\lambda \approx \sqrt{\frac{8\pi G}{V_{\text{volume}} \hbar^2 t^2}} \xrightarrow{G=\hbar=\ell_{\text{Planck}}=k_B=1} \sqrt{\frac{8\pi}{t^2}} \equiv \frac{\sqrt{8\pi}}{t} \tag{29} \quad 176$$

Then a preliminary emergent space-time wavefunction would take the form of

$$\begin{aligned} &\psi_{\tilde{n},\lambda}(\Delta t, r) \\ &\equiv \frac{1}{\sqrt{2\pi}} \cdot \frac{\tilde{n}! \cdot (2 \cdot \sqrt{8\pi} \cdot (\Delta t)^{-1})^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}} \cdot \left[ \frac{1}{(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t + i \cdot r)^{\tilde{n}+1}} \right. \\ &\quad \left. - \frac{1}{(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t - i \cdot r)^{\tilde{n}+1}} \right] \end{aligned} \tag{30}$$

Just at the surface of the bubble of space-time, with  $t_{Planck} \propto \Delta t$ , and  $r \propto \ell_{Planck}$

This is from a section, page 239 of the 3<sup>rd</sup> edition of Kieffer’s book, as to a quantum theory of collapsing dust shells, and is a way to use what is referred to as what Isham in 1984 referred to as “group quantization”, as we need to remember that we are assuming  $G = \hbar = \ell_{Planck} = k_B = t_{Planck} = 1$ . Having said that, let us look at Eq.(1) and Eq. (12) again

### 3.1 We compare Eq. (1) and Eq. (12) with the above new information

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2\tilde{N} + \frac{\gamma(3\gamma-1)}{8\pi G t_{Planck}}\right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_P^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{31}$$

$$\Leftrightarrow \rho_{DE} = \frac{\Lambda}{8\pi G} \Leftrightarrow \Lambda \approx \hbar \cdot 8\pi G \cdot \left.\frac{(2\pi)^4}{\lambda_{DE}^4}\right|_{\lambda_{DE} \approx 10^{30} \ell_{Planck}} \equiv 8\pi \cdot \left.\frac{(2\pi)^4}{\lambda_{DE}^4}\right|_{\lambda_{DE} \approx 10^{30}}$$

To compare these two values we can state that within the bubble, that just before the bubble boundary, we have  $\tilde{N} = 0$ , hence we look at, initially

$$\Lambda \approx \frac{-\left[\frac{V_0}{3\gamma-1} + \frac{\gamma(3\gamma-1)}{8\pi G t_{Planck}}\right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_P^2} \cdot \left(\rho - \frac{\rho^2}{2|\sigma|}\right) \tag{32}$$

$$\Leftrightarrow \Lambda \approx \hbar \cdot 8\pi G \cdot \left.\frac{(2\pi)^4}{\lambda_{DE}^4}\right|_{\lambda_{DE} \approx 10^{30} \ell_{Planck}} \equiv 8\pi \cdot \left.\frac{(2\pi)^4}{\lambda_{DE}^4}\right|_{\lambda_{DE} \approx 10^{30}}$$

i.e. we pick  $c_1$  so that the two are equivalent in value,

Note this is just before a very large value set to  $\tilde{N}$  at the boundary of the initial space-time bubble. Hence we also would be looking at



$$\begin{aligned}
 \Lambda &\approx \gamma \cdot t^2 + 8\pi V_0 \cdot \left\{ \sqrt{\frac{8\pi V_0}{\gamma(3\gamma-1)}} \cdot t \right\} \left. \sqrt{\frac{\gamma}{4\pi}} \sqrt{\frac{8\pi}{\gamma}} \right|_{t \approx t_{\text{Planck}}=1} \\
 &\approx \frac{-\left[ \frac{V_0}{3\gamma-1} + \frac{\gamma(3\gamma-1)}{8\pi} \right]}{\frac{1}{\kappa} \int \sqrt{-g} d^3x} + c_1 \frac{16\pi}{M_P^2} \cdot \left( \rho - \frac{\rho^2}{2|\sigma|} \right) \tag{33} \\
 &\approx \hbar \cdot 8\pi G \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Big|_{\lambda_{DE} \approx 10^{30} \ell_{\text{Planck}}} = 8\pi \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Big|_{\lambda_{DE} \approx 10^{30}}
 \end{aligned}$$

In Eq. (31) we will also make the following identification  $\rho \approx \rho_{DE} \approx \hbar \cdot \frac{(2\pi)^4}{\lambda_{DE}^4} \Big|_{\lambda_{DE} \approx 10^{30}}$  194

I.e. just before the bubble, we will factor in a very large bubble tension,  $\sigma$  195

Doing so would be to have an optimal  $c_1$  value, and that this choice of  $c_1$  would allow us to make the identification of the choice of wavefunction for our model which would according to the Eq.(28) model have a quantum flavor, while at the same time in adherence be similar to the Eq.(31) values of Dark Energy density 196-199

### 3.2 What we obtain if we model the cosmological constant this way? 200

*First of all, as noted by Kieffer, the wavefunction in Eq. (28) vanishes if  $r=0$ . We avoid having a vanishing wavefunction by asserting that we will be making an evaluation of it in the neighborhood of a Planck Length. Secondly in making this assumption we are at least proceeding in a direction where we can ditch the Anthropic principle, which I consider a MAJOR blight on cosmology. I also too though this this procedure as outline is experimentally verifiable, once we understand what may constitute embedding of Quantum Mechanics within a deterministic structure. The entire referral of this problem can be contrasted with the solution given in [ 19 ] by Rosen, which is in the mode of Quantum mechanics for a single particle in an isotropic universe,* 201-209

*The primary difference between the Rosen approach, [19] and what is done in this document is that the mass of a graviton, is proportional to the square of the Cosmological constant, an assumption which Rosen does not make. In addition the Rosen document assumes that “ The situation is somewhat analogous to the quantum mechanics of a single particle as compared to quantum field theory” and assumes an isotropic background* 210-215

*We do not confine ourself to Isotropic backgrounds. We also do share with Rosen [19] a procedure as to rendition a linkage between classical to quantum mechanical treatment of our problem, but our work is more in tune with using what Powell and Craseman in their book on quantum mechanics used, [ ] which is similar to a limiting case of geometric optics.* 216-220

*I wish to thank Christian Corda as to alerting me to the Rosen article which is also applicable to a Dust Cosmology. I also wish to thank Fangyu Li , Wen Hao, and others for alerting me as to the crucial role of polarization. As it is, I presented part of this idea in* 221-223

Zeldovich 4, in the ICRANET on line conferences ( a small part of it) and found that the audience and the reviewers did not understand the Klauder treatment of the cosmological constant via Klauder Enhanced quantization. 224  
225  
226

Hence this expanded treatment whereas the Rosen treatment as Corda outlined is most effective for quantum black holes 227  
228

### 3.3 Final reference to high frequency gravitational waves 229

$$(1 + z_{initial-era}) \equiv \frac{a_{today}}{a_{initial-era}} \approx \left( \frac{\omega_{Earth-orbit}}{\omega_{initial-era}} \right)^{-1} \quad (34) \quad 230$$

$$\Rightarrow (1 + z_{initial-era}) \omega_{Earth-orbit} \approx 10^{25} \omega_{Earth-orbit} \approx \omega_{initial-era}$$

Equation (34) can be seen in [35] as well as [36] 231

Whereas We postulate that we specify an initial era frequency via 233  
dimensional analysis which is slightly modified by Maggiore for the speed of a 234  
graviton[[35] whereas 235

$$c(\text{light-speed}) \approx \omega_{initial-era} \cdot (\lambda_{initial-post-bubble} = \ell_{Planck}) \quad (35) \quad 236$$

and that dimensional comparison with initially having a temperature built up so as 237

$$\Delta E \approx \hbar \omega_{initial-era} \quad (36) \quad 238$$

where  $T_{universe} \approx T_{Planck-temperature} = 1.22 \times 10^{19} \text{ GeV}$  . If so then the Planck era 239

temperature would be extremely high leading to a change in temperature from the 240

Pre Planckian conditions to Planck era leading to 242

$$\Delta E = \frac{d(\text{dim})}{2} \cdot k_B \cdot T_{universe} \quad (37) \quad 243$$

In doing so, be assuming 244

$$\omega_{initial-era} \approx \frac{c}{\ell_{planck}} \leq 1.8549 \times 10^{43} \text{ Hz} \quad (38) \quad 245$$

Where we would be assuming  $\omega_{initial-era} \approx \frac{c}{\ell_{planck}} \leq 1.8549 \times 10^{43} \text{ Hz}$  so then we 246

would be looking at frequencies on Earth from gravitons of mass m(graviton) less 247

than of equal to 248  
249

$$\omega_{Earth-orbit} \leq 10^{-25} \omega_{initial-era} \tag{39}$$

And this partly due to the transference of cosmological ‘information’ as given in [13]

for a phantom bounce type of construction .Further point that since we have that gravitons travel at nearly the speed of light [36], that gravitons are formed from the surface of a bubble of space-time up to the electroweak era that mass values of the order of 10<sup>-65</sup> grams (rest mass of relic gravitons) would increase due to extremely

high velocity would lead to enormous  $\Delta E \approx \hbar \omega_{initial-era}$  values per graviton,

which would make the conflation of ultrahigh temperatures with gravitons traveling at

nearly the speed of light as given in Eq.(39) as compared with  $\Delta E \approx \hbar \omega_{initial-era}$

**We can in future work compare this with the Rosen[ 19 ] value of energy for a mini universe of(from a Schrodinger equation) with ground state mass of**

$m = \sqrt{\pi} M_{Planck}$  **and an energy of**

$$E_{\bar{n}} = \frac{-Gm^5}{2\pi^2 \hbar^2 \bar{n}^2} \tag{40}$$

Our preliminary supposition is that Eq. (40) could represent the initial energy of a Pre Planckian Universe and that Eq.(37) be the thermal energy dumped in due to the use of Cyclic Conformal cosmology ( maybe in multiverse form) so that if there is a build up of energy greater than Eq.(40) due to thermal build up of temperature due to infall of matter-energy, we have a release of Gravitons in great number which would commence as a domain wall broke down about in the Planckian era with a temperature of the magnitude of Planck Energy for a volume of radius of the order of Planck Length. This will be investigated in detailed future calculations . All this should be in fidelity, in experimental limits to [37], as well as looking at ideas about Quantum tunneling we may gain from [38] as to understand the transition from Pre Plankian to Planckian physics [39] [40] . Now for expanding Eq. (40), we look at [41], where we have then an information number of N(information) for which we have a total Graviton mass of

$$M_{graviton-total} = n_{graviton} \cdot m_{graviton} \tag{41}$$

Where we will be looking at a value of “information” of initially

$$\mathbb{N}(\text{inf}) = \frac{9\pi}{\ln 2} \cdot \left( \frac{n_{graviton} \cdot m_{graviton}}{m_{planck}} \right)^2 \approx \frac{9\pi \cdot (n_{graviton})^2}{\ln 2} \cdot 10^{-120} \tag{42}$$

Now use the following approximation of the Universe, initially having the entropy of a black hole, i.e, we are using Ng Infinite Quantum statistics, [20],[42]

$$S_{Universe} \propto S_{BH} \cong \frac{A(\text{area})}{4 \cdot \ell_{Planck}^2} \approx \frac{9n_Q}{4} \approx n_{graviton} \tag{43}$$

In taking this step, we are making use of [19] having the following radius used, namely using in our model of a black hole, the quantum “atom” approximation

$$r(n_Q) \approx \frac{3\sqrt{n_Q} \cdot \ell_{Planck}}{2\sqrt{\pi}} \tag{44}$$

In order to have non vanishing information according to [41] we would need to specify having

$$(\text{inf}) = \frac{9\pi}{\ln 2} \cdot \left( \frac{n_{graviton} \cdot m_{graviton}}{m_{Planck}} \right)^2 \approx \frac{729\pi \cdot (n_Q)^2}{16 \cdot \ln 2} \cdot 10^{-120} \tag{45}$$

Given this, if

$$\begin{aligned} \mathbb{N}(\text{inf}) &= \frac{9\pi}{\ln 2} \cdot \left( \frac{n_{graviton} \cdot m_{graviton}}{m_{Planck}} \right)^2 \approx \frac{729\pi \cdot (n_Q)^2}{16 \cdot \ln 2} \cdot 10^{-120} \geq 1 \\ \Rightarrow E_{\tilde{n}=n_Q} &= \frac{-G \cdot (m = \sqrt{\pi n_Q} \cdot m_{Planck})^5}{2\pi^2 \hbar^2 n_Q^2} \approx \frac{-G \cdot (\sqrt{\pi n_Q} \cdot (m_{Planck})^5)}{2\hbar^2} \tag{46} \\ \xrightarrow{\hbar=m_{Planck}=G=1} E_{\tilde{n}=n_Q} &\approx \frac{-(\sqrt{\pi n_Q} \cdot)}{2} \approx l \arg e - \text{negative} - \text{value} \end{aligned}$$

We speculate that in order to have a large negative binding energy that this will mean we have then an enormous initial thermal energy from a multiverse cyclic conformal cosmology input as to undo this so as to initiate a new cycle of creation, i.e breaking the binding energy would require Planck level temperature values. And this is a topic which will be researched furthermore in great detail . And the necessary thermal heat would drive having enormously high initial frequencies

**3.4 And now our concluding words as to a quantization limit to pursue, if the early universe has characteristics of a pre planckian black hole**

In order to do this we adapt an argument used by [43] as using the quantization of an action which we write using [43] first in the case of no cosmological constant, namely if  $\mathfrak{S}$  is an action integral with the form of the Einstein – Hilbert least action of which :L is a radius , and

$$\delta\mathfrak{S} = 0 \tag{47}$$

$$\mathfrak{S} \leq \hbar \tag{48}$$

Equation (48) is an imposed upon quantization limit where we use from [43]

$$\mathfrak{S} = \frac{c^4}{16\pi G} \int \mathfrak{R} \sqrt{-g} d^4x \approx \frac{2\pi^4 \rho L^4 c}{3} \Big|_{t=L/c} \leq \hbar \tag{49}$$

In case of using a black hole limit and constant energy density  $\rho$ , [43] argues for L

$$\text{Quantization near singularity if } L \leq 2\ell_{Planck} \tag{50}$$

In the case of when the cosmological constant is NOT zero we impose

$$\mathfrak{S} = \frac{c^4}{16\pi G} \int (\mathfrak{R} - 2\Lambda) \cdot \sqrt{-g} d^4x \approx \frac{2\pi^4 \rho L^4 c}{3} \Big|_{t=L/c} - \frac{2L^4}{c} \cdot \Lambda \leq \hbar \quad (51)$$

Here,  $\rho$  is an energy density and in the case of no cosmological constant we would use

$$\rho L^2 \leq (3c^2 / 8\pi G) \text{ if } \Lambda = 0 \quad (52)$$

And

$$\mathfrak{S} = \frac{c^4}{16\pi G} \int (\mathfrak{R}) \cdot \sqrt{-g} d^4x \approx \frac{2\pi^4 \rho L^4 c}{3} \Big|_{t=L/c} \leq L^2 c^3 / 4G \quad (53)$$

We argue as does [43] that when there is no cosmological constant that Eq. (53) and

Eq.(49) hold we are obtaining Eq. (50)

In the case where we use Eq. (51) instead of Eq. (49) we would instead see quantization

$$L \leq \ell_{\text{Planck}} \quad (54)$$

This final set of arguments if the early universe had the characteristics of a Black hole

will be pursued in future research

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