Analysis of the Performance Degradation in Uplink MC-CDMA Systems with Doppler Induced Frequency Offsets

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Abstract—In this paper, uplink Time Division Duplex (TDD) Multi-Carrier Code-Division Multiple-Access (MC-CDMA) systems applying pre-equalization to combat channel impairments are considered. Especially, the performance degradation caused by the Doppler induced frequency offsets among the different mobile users is analyzed in detail. Based on this analysis different frequency interleaving schemes are proposed and their potential to reduce the performance degradation is investigated. It turns out, that a frequency interleaver which is allowed to skip a few, well-defined subcarriers is capable of reducing the influence of Doppler induced frequency offsets significantly causing a very little loss in bandwidth efficiency.

I. INTRODUCTION

Multi-Carrier Code-Division Multiple-Access (MC-CDMA) [1], [2] is well recognized as a very promising candidate for the air interface of future mobile wireless downlink systems. However, when considering MC-CDMA for uplink transmission some problems arise. In particular, channel estimation and equalization as well as time and frequency synchronization are more difficult and require more complex algorithms. One of the possibilities to mainly overcome the problems of channel estimation and equalization is to apply channel pre-equalization at the MC-CDMA transmitter as described in [3], [4]. In this paper, the influence of Doppler induced frequency offsets on the performance of uplink time division duplex (TDD) MC-CDMA applying pre-equalization at the transmitter is analyzed in detail. Additionally, countermeasures based on thoroughly designed frequency interleaving schemes are proposed. Sensitivity of uplink MC-CDMA systems to frequency offsets has been considered in several papers [5], [6], but both, an in-depth analysis of the influence of misaligned users on each other and proposals for reducing the resulting performance degradation are not yet available.

Throughout the paper, a slowly varying time-variant channel is assumed. Thus, channel pre-equalization techniques are applicable to the uplink TDD/MC-CDMA transmission which are based on the channel estimation results from the downlink transmission [3], [4]. With TDD mode the effects of the channel on the uplink signal can be pre-equalized at the mobile station based on the channel estimation from the downlink frame.

The desired user is assumed to be synchronized to the receiver frequency, while all other users are misaligned in frequency. Different frequency interleaving schemes are investigated and their performances are analyzed. A novel skip-carrier interleaver is introduced which significantly improves the performance of an uplink MC-CDMA system in the presence of Doppler induced frequency offsets between the different users.

The paper is organized as follows. The TDD/MC-CDMA uplink transmission system considered throughout this paper is described in Section II. In Section III, both the influence of misaligned users on each other is analyzed and frequency interleaving schemes are investigated. The performance of the uplink TDD/MC-CDMA system in the presence of Doppler induced frequency offsets and the performance gains achieved through the proposed skip-carrier interleaver are presented in Section IV. Finally, Section V summarizes the results.

II. TRANSMISSION SYSTEM

A time synchronous uplink MC-CDMA system with $K$ active users is considered. Fig. 1 depicts the block diagram of the considered uplink transmitter of mobile user $k$, $k = 0, \ldots, K - 1$. The uplink transmitter is based on the MC-CDMA system concept with $M$-Modification as proposed in [2]. Note, the $M$-Modification allows each user to transmit $M$ data symbols simultaneously using an additional Orthogonal Frequency-Division Multiplexing (OFDM) component
within the MC-CDMA system. Thus, each MC-CDMA symbol of user $k$ consists of $M$ complex data symbols $d^{(k)} = (d_0^{(k)}, \ldots, d_{M-1}^{(k)})$ obtained from the symbol mapper. An MC-CDMA symbol is formed in the following way. After serial to parallel conversion (S/P) each symbol is spread with the same user specific spreading sequence $c^{(k)} = (c_0^{(k)}, \ldots, c_{M-1}^{(k)})$ of length $L$. It is presumed that Walsh-Hadamard (WH) orthogonal spreading sequences are used for spreading. Note, the duration of a data symbol $d_{m}^{(k)}$, $m = 0, \ldots, M-1$, is $T = LT_c$, while the duration of a spreading chip is $T_c$. After another S/P and frequency interleaving the resulting $N_c = ML$ chips $s^{(k)} = (s_0^{(k)}, \ldots, s_{N_c-1}^{(k)})^T$ are pre- equalized with an $N_c \times N_c$ diagonal channel pre-equalization matrix $G^{(k)}$ to obtain the $N_c$ pre-equalized chips $v^{(k)} = (v_0^{(k)}, \ldots, v_{N_c-1}^{(k)})^T$ according to

$$v^{(k)} = G^{(k)}s^{(k)}. \quad (1)$$

The diagonal elements $G_{ii}^{(k)}$, $i = 0, \ldots, N_c - 1$, of the pre-equalization matrix are calculated from the channel state information derived from downlink channel estimation. The channel fading on each subcarrier is determined using pilot symbols interleaved in time and frequency and applying appropriate interpolation [2]. Finally, the pre- equalized chips $v^{(k)}$ are modulated on the $N_c$ subcarriers using the inverse fast Fourier transform (IFFT). After that, parallel to serial conversion (P/S) each symbol is spread with the same modulo operation. Moreover, due to the block interleaver which is identical to the block interleaver except for the fact that a few well-defined subcarriers are left empty.

Channel pre-equalization can be done in numerous ways [3]. Since investigation of channel pre-equalization is out of the scope of this paper, perfect channel knowledge acquired from the downlink transmission is presumed. The pre-equalization method applied is constrained quasi-MMSE pre-equalization. When applying power constrained pre-equalization, the transmitted power for the system with pre-equalization is kept the same as in the case without pre-equalization. The complex elements $G_{ii}^{(k)}$, $i = 0, \ldots, N_c - 1$, of the diagonal channel pre-equalization matrix $G^{(k)}$ are given by [3]

$$G_{ii}^{(k)} = g_i^{(k)}e^{j\gamma_i^{(k)}} = \frac{H_i^{(k)*}}{(K-1)|H_i^{(k)}|^2 + \sigma^2L} \left( \sum_{k=0}^{N_c-1} |H_i^{(k)}|^2 \right)^{-\frac{1}{2}}, \quad (2)$$

where superscript ‘*’ denotes complex conjugation. The complex channel fading coefficient of the $k$th user on the $i$th subcarrier is $H_i^{(k)}$ and can be represented as $H_i^{(k)} = \rho_i^{(k)}e^{j\phi_i^{(k)}}$. The variance of the additive white Gaussian noise (AWGN) is $\sigma^2$, and $K$ is the number of active users.

The received signal is influenced by frequency selective fading of the time-variant multipath channel, AWGN, and multiple-access interference (MAI) caused mainly by the Doppler induced frequency offsets among the $K$ users. As a result, the received symbol $r_{mn}^{(n)}$, $m = 0, \ldots, M-1$, of a certain user $k = n$, after guard interval removal, fast Fourier transform (FFT), and despreading, can be represented as

$$r_{mn}^{(n)} = \sum_{k=0}^{K-1} \sum_{l=0}^{N_c-1} c_i^{(n)}(k)\rho_i^{(k)}g_i^{(k)}g_i^{(n)}e^{j\phi_i^{(n)}}d_{(i \mod M)}^{(k)}(1)$$

for the case of block interleaving, where

$$S(x) = \frac{\sin(\pi x)}{N_c\sin(\pi x/\pi)}e^{-j\pi \frac{N_c-1}{2} x}. \quad (4)$$

In Equation (3), ‘(div.)’ denotes integer division and ‘(mod.)’ modulo operation. Moreover, $\epsilon^{(k)} (-0.5 < \epsilon^{(k)} < 0.5)$ is the frequency offset normalized to the subcarrier spacing, and $\eta$ represents the AWGN with variance $\sigma^2$. Note, due to the Doppler induced frequency offsets the subcarriers of different users do not coincide anymore, thus, causing MAI.

### III. INFLUENCE OF FREQUENCY OFFSETS

Since the focus of the investigation is on the influence of misaligned users on the desired user, it is assumed that the frequency offset of the desired user $k = n$ is $\epsilon^{(n)} = 0$. The desired signal part $R_{mn}^{(n)}$ in Equation (3) is

$$R_{mn}^{(n)} = \frac{1}{L} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} C_i^{(n)}(k)\rho_i^{(k)}g_i^{(n)}g_i^{(n)}d_{(i \mod M)}^{(k)}d_{mn}^{(n)}, \quad (5)$$

whereas the MAI part $I_{mn}^{(n)}$ is given by

$$I_{mn}^{(n)} = \frac{1}{L} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \sum_{l=0}^{N_c-1} C_i^{(n)}(k)\rho_i^{(k)}e^{j\phi_i^{(n)}}g_i^{(n)}g_i^{(n)}d_{(i \mod M)}^{(k)}d_{mn}^{(n)}(1) \cdot S(1)$$

where $S(1)$ is the channel transfer function.

Using ‘$E\{\cdot\}$’ to denote expectation, the exact analytical expression of the MAI variance $\sigma_M^{2, M, n}$ of the $m$th data symbol can be written as

$$\sigma_M^{2, M, n} = E\{|I_{mn}^{(n)}|^2\}. \quad (7)$$

![Fig. 1. MC-CDMA uplink transmitter of mobile user $k$.](image-url)
and are synchronized and, $\epsilon = 0$, suffer from two-fold symbols profit from the
$\rho_1 = |c_1^{(k)}|$ of the MAI variance is
of the interfering users which are closest to the specific subcarrier of the desired user. For the illustration in Fig. 2 it is assumed, that user ($k$) and despreading code $g_{1-M} + m - 1$, while spreading code $c^{(k)}$ and despreading code $c^{(n)}$ are synchronized and, therefore, do not cause additional MAI. Note, orthogonal WH spreading codes are used for spreading. In Equation (11), the MAI influence is two-fold. Besides the influence from the non-ideal pre-equalization, there is an additional influence in the second part of Equation (11) caused by unsynchronized spreading codes, shifted by one chip. The additional signal degradation due to the unsynchronized spreading codes in Equation (11) is represented by $c_{1-M}^{(n)} r_{1-M}^{(k)} g_{1-M} + m - 1$. The results from Equations (10) and (11) can be used to develop a frequency interleaving scheme which is more resistant to frequency offsets than existing ones, as shown in the following.

An important property of the block interleaver lies in the fact that for $\epsilon^{(k)} > 0$ only for symbol $m = 0$ the MAI influence is two-fold, while the other $M - 1$ symbols show an ‘inner-protection’ property and are affected in a less destructive way by MAI. Analogously, in the case $\epsilon^{(k)} < 0$ the MAI influence is two-fold only when $m = M - 1$. From this considerations, it can be concluded that in a realistic environment, where some users have positive and others have negative frequency offsets, only the symbols $m = 0$ and $m = M - 1$ suffer from two-fold MAI influence, while the other $M - 2$ symbols profit from the ‘inner-protection’ property.

The MAI influence for the case of block interleaving is illustrated in Fig. 3 for parameters $L = 4$, $M = 4$, $K = 2$. It is assumed, that user $k = 0$ is the desired user, the frequency offset of the second user $k = 1$ is $\epsilon^{(1)} > 0$, and that the fading channel is ideally pre-equalized. From Fig. 3 it can be seen, that two different types of MAI influence exist. There is influence from synchronized chips and influence from unsynchronized chips. The ‘inner-protection’ property of the block interleaver is reflected in these two different influences.
It can be noticed, that only each $M$th subcarrier is influenced by unsynchronized spreading code chips and, therefore, suffers from larger degradation due to MAI.

The random interleaver does not possess the ‘inner-protection’ property and, therefore, is much more sensitive to frequency offsets, as will be shown in the next section.

The proposed novel skip-carrier interleaver operates on the premise, that only the two symbols $m = 0$ and $m = M - 1$ are influenced by two-fold MAI, when a block interleaver is used. Moreover, the two-fold influence on these two symbols comes from each other. This leads to the conclusion, that by not transmitting on the subcarriers that correspond to one of these two symbols, the two-fold MAI influence can be mainly eliminated. Thus, for the skip-carrier interleaver block interleaving is performed with the difference that each $M$th chip is left empty. This results in a loss of bandwidth efficiency by a factor of $\frac{1}{32}$. The frequency allocation schemes for the block and the skip-carrier interleaver are illustrated in Fig. 4.

FIG. 4. Illustration of frequency allocation schemes: (a) Block interleaver and (b) Skip-carrier interleaver.

Fig. 3. Desired and interfering user’s signal power spectrum of individual subcarriers for the case of block interleaving: $L = 4$, $M = 4$, $K = 2$, and $\epsilon^{(1)} > 0$.

Fig. 4. Illustration of frequency allocation schemes: (a) Block interleaver and (b) Skip-carrier interleaver.

IV. SIMULATIONS RESULTS

In this section, several simulation results are given that illustrate the performance improvements obtained by the proposed countermeasures for reducing the influence of frequency offsets.

The underlying mobile radio channel for the simulations is based on the assumption of independent Rayleigh fading on each subcarrier. This assumption is justified by the fact, that frequency interleaving is applied leading to a spreading of the chips of one data symbol over the whole transmission bandwidth. QPSK modulation is applied and no channel coding is considered. The spreading is performed with WH codes of length $L = 16$. The number of simultaneously transmitted symbols is $M = 16$ which results in $N_c = 256$ used subcarriers. Channel pre-equalization is performed with quasi-MMSE pre-equalization. The frequency offset of the desired user $k = n$ is set to $\epsilon^{(n)} = 0$, while the frequency offsets $\epsilon^{(k)}$ of the interfering users $k \neq n$ are chosen according to a Jakes distribution.

The distribution of errors over data symbols is illustrated in Fig. 5 for block interleaving. The desired user uses $c^{(14)}$ as spreading code, the signal-to-noise ratio (SNR) is fixed at $E_b/N_0 = 14$ dB, the maximal Doppler shift is 30% of the subcarrier spacing, and a fully-loaded system is considered ($K = 16$). Note, $E_b/N_0$ represents the transmitted energy per bit over the noise spectral density. The unequally distributed MAI influence over data symbols is clearly visible. The data symbols $m = 0$ and $m = 15$ suffer from approximately ten times larger degradation than other symbols. In addition, it can be noticed that also data symbols $m = 1, 2, 13$, and 14 suffer from larger degradation than the remaining symbols. The reason for this lies in the fact, that MAI degradation is not only due to the two closest subcarriers, like approximated in Equation (9), but is also partly caused by the more distant subcarriers.

The different frequency interleaving schemes for TDD/MC-CDMA are compared in Fig. 6. A maximal Doppler shift of 30% of the subcarrier spacing is assumed and the system is fully loaded ($K = 16$). The comparison is performed on
the basis of bit error rate (BER) over SNR curves, where
SNR is given in $E_b/N_0$. As it can be seen, the skip-carrier
interleaver performs best, while block and especially random
interleaver show much larger signal degradation. For the skip-
carrier interleaver the subcarriers which belong to the border-
ing symbol $m = M - 1$ are skipped. Both, the worst-case
performance and the averaged performance are considered for
all interleavers. For the worst-case the spreading code of the desired user is chosen in such a way, that it
causes maximal possible signal degradation. For the averaged
performance, the performance is averaged by assigning all possible spreading codes to the desired user and averaging
the results. The reason for the difference in the worst-case
performance and the averaged performance lies in the fact, that
unsynchronized codes, shifted by one chip, play an important
role in the MAI influence as given in Equation (11). Thus,
when assigning different spreading codes to the desired user,
different influence from the spreading codes of the other active
users will affect the desired user. However, spreading code
selection does not play a vital role for the random interleaver,
since it already averages the effect of spreading code selection
by its random nature. Therefore, the worst-case performance
and averaged performance are almost the same in the case of random interleaver. Although it is expected that the skip-
carrier interleaver almost completely eliminates the influence of frequency offsets, Fig. 6 shows, that residual influence
remains. This is due to the fact, that not only the two closest
subcarriers cause MAI degradation, like already illustrated in
Fig. 5.

Another representation of the simulation results is given in
Fig. 7, where the BER is shown in dependence of the maximal
Doppler shift for a fixed $E_b/N_0 = 14$ dB. Again, the
skip-carrier interleaver outperforms both block and random
interleaver. In addition, it can be noticed from Fig. 7, that the
performance of the block and the skip-carrier interleaver is
comparable for maximal Doppler shifts lower than 10%.

As already noticed, the skip-carrier interleaver does not
eliminate the influence of MAI completely. Fig. 8 illustrates
the distribution of errors over symbols for the skip-carrier
interleaver and a maximal Doppler shift of 30%; desired user uses $c^{(14)}$, $E_b/N_0 = 14$ dB, $L = 16$, $K = 16$, and $M = 15$.
belong to bordering symbols, e.g. to symbol $m = M - 2$ if altogether $2L = 32$ subcarriers are skipped. The additional skipping of subcarriers leads to a further loss of bandwidth efficiency. From Fig. 9, it can be concluded, that a good trade-off between bandwidth efficiency and performance improvements is to skip $L = 16$ or $2L = 32$ subcarriers. This corresponds in the considered case to a loss of $\frac{1}{17} = \frac{1}{16}$ and $\frac{2}{17} = \frac{2}{16}$ in bandwidth efficiency, respectively. Further skipping of subcarriers no longer results in significant performance improvements. Therefore, it is suggested not to skip more than $2L = 32$ subcarriers, since the resulting additional gains no longer justify the additional loss in bandwidth efficiency.

V. CONCLUSIONS

In this paper, the influence of frequency offsets on the performance of uplink TDD/MC-CDMA is considered and countermeasures for the reduction of the effects of frequency offsets are proposed and investigated. A scenario in which the desired user is synchronized to the receiver, while all other interfering users are misaligned in frequency, is considered. It is shown, that frequency interleaving schemes have a great impact on the system performance.

The proposed novel skip-carrier interleaver outperforms both block and random interleaver when the maximal normalized frequency offset is larger than 10%. For a maximal normalized frequency offset lower than 10% the performance of skip-carrier and block interleaver is comparable and the degradation due to frequency offsets is tolerable. The skip-carrier interleaver introduces a loss in bandwidth efficiency by a factor $\frac{1}{17}$. However, this loss is justified due to the large gains obtained with the skip-carrier interleaver as illustrated in this paper.

Further performance gains can be achieved if additional subcarriers are skipped. However, these gains are smaller than for the original skip-carrier interleaver. Moreover, the loss in bandwidth efficiency is increased. Thus, there is a trade-off between the additional performance improvements obtained by additional subcarriers skipping and the increased loss in bandwidth efficiency. Skipping $L$ or $2L$ subcarriers seems to be a good compromise.

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REFERENCES